A Galileon Design of Slow Expansion

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We show a model of the slow expansion, in which the scale invariant spectrum of curvature perturbation is adiabatically induced by its increasing mode, by applying a generalized Galileon field. In this model, initially $\epsilon \ll -1$, which then is rapidly increasing, during this period the universe is slowly expanding. There is not the ghost instability, the perturbation theory is healthy. When $\epsilon \sim -1$, the slow expansion phase ends, and the available energy of field can be released and the universe reheats. This scenario might be a viable design of the early universe.

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I. INTRODUCTION

The observations imply that the primordial curvature perturbation is scale invariant. Thus how generating it has been still a significant issue, especially for single field. The curvature perturbation on large scale consists of a constant mode and a mode dependent of time [1]. When one of which is dominated and scale invariant, the spectrum of curvature perturbation will be scale invariant. When the scale factor is rapidly changed while ϵ is nearly constant, the constant mode is responsible for that of inflation [2],[3],[4],[5], while the increasing mode is for the contraction with matter [6],[7],[8], both are dual [6].

In principle, the increasing mode of metric perturbation, which is scale invariant for $\epsilon \gg 1$ [9] or $\epsilon \ll -1$ [10], might dominate the curvature perturbation. The constant mode of metric perturbation is same with the constant mode of curvature perturbation. The duality of scale invariant spectrum of metric perturbation has been discussed in [11],[12],[13]. The evolution with $\epsilon \gg 1$ is the slowly contracting, which is that of ekpyrotic universe [14]. While $\epsilon \ll -1$ gives the slow expansion [10], which has been applied for island universe [15]. In certain sense, in Ref.[10] it was for the first time observed that the slow expansion might adiabatically generate the scale invariant spectrum of curvature perturbation, see [16] for that induced by the entropy perturbation.

When the available energy of field is released, the slow expansion phase ends and the universe reheats. Thus the slow expansion might be a viable scenario of the early universe. In principle, when ϵ is constant, whether the increasing mode of the metric perturbation can be inherited by the curvature perturbation depends of the physics around the exiting [17]. However, when ϵ is rapidly changed, the thing is altered, see [18] for that of the slow contraction. During the slow expansion, the scale invariant curvature perturbation can be naturally induced by its increasing mode [19], or its constant mode [20],[21].

The perturbation mode can leave the Hubble horizon during the slow expansion requires $\epsilon < 0$ [10],[19], or a period after it is required to extend the perturbation mode out of the Hubble horizon [20]. Thus in [10],[19], the phantom was applied for a phenomenological studying. However, there is a ghost instability. Thus it was argued that the evolution of $\epsilon < 0$ emerges only for a period, the phantom field might be only a simulation of a full theory without the ghost below certain physical cutoff [22].

Recently, the cosmological application of Galileon, [23],[24], or its nontrivial generalization [25],[26],[27], has acquired increasing attentions [28],[29],[30],[31],[32]. It has been found for generalized Galileon that $\epsilon < 0$ can be implemented stably, there is not the ghost instability. We, in this paper, will show a model of the slow expansion given in [19], by applying a generalized Galileon field. In this model, the perturbation theory is healthy, the scale invariant curvature perturbation is given by itself increasing mode, which can be consistent with the observations. As will be argued, this in certain sense validates the argument and calculations in [10],[19]

The models of early universe, builded by applying generalized Galileon, have been studied. In Ref.[26], the inflation model is implemented by using generalized Galileon field. However, here what we discuss is an alternative to inflation. There is a slightly similar scenario in [33]. However, in [33], the adiabatic perturbation is not scale invariant, the scale invariant curvature perturbation is obtained by the conversion of the perturbations of other light scalar fields. Here, we will see how the adiabatic perturbation is naturally scale invariant.

II. AS A GENERAL RESULT

We begin with a brief review on the slowly evolving model in [19]. The quadratic action of the curvature perturbation $\mathcal{R}$ is

$$S_2 \sim \int d\eta d^3x \frac{a^2 Q}{c_s^2}\left(\mathcal{R}'^2 - c_s^2(\partial\mathcal{R})^2\right), \qquad (1)$$

which is actually general for single field, like $P(X,\varphi)$ [34], generalized Galileon [25],[26],[35], and the modified gravity [36],[37]. Q and c_s^2 are generally different for different models. However, $Q > 0$ and $c_s^2 > 0$ should be satisfied to avoid the ghost and gradient instabilities.

The equation of $\mathcal{R}$ is [38],[39]

$$u_k'' + \left(c_s^2 k^2 - \frac{z''}{z}\right) u_k = 0, \qquad (2)$$

after defining $u_k \equiv z\mathcal{R}_k$, where $'$ is the derivative for η, $z \equiv a\sqrt{2M_P^2 Q}/c_s$. We here only care the case with constant c_s^2. When $k^2 \ll z''/z$, the solution of $\mathcal{R}$ given by Eq.(2) is

$$\mathcal{R} \sim C \quad is \text{ constant mode} \tag{3}$$

$$or \quad D\int \frac{d\eta}{z^2} \quad is \text{ changed mode,} \tag{4}$$

where D mode is increasing or decaying dependent of different evolutions.

The scale invariance of $\mathcal{R}$ requires $\frac{z''}{z} \sim \frac{2}{(\eta_*-\eta)^2}$, which implies

$$z \sim \frac{a\sqrt{Q}}{c_s} \sim \frac{1}{\eta_*-\eta} \quad for \text{ constant mode} \tag{5}$$

$$or \quad (\eta_*-\eta)^2 \quad for \text{ increasing mode} \tag{6}$$

has to be satisfied, where initially $\eta \ll -1$. In certain sense, both evolutions are dual [6]. The results will be different if c_s^2 is changed, however, which we will not involve here. In principle, both a and Q can be changed, and together contribute the change of z. However, only one among them is changed while another is hardly changed might be interesting, e.g. the inflation, given by (5), or the contraction dominated by the matter, given by (6), in which a is rapidly changed while Q is hardly changed.

However, the case can also be inverse. When Q is rapidly changed while a is hardly changed, the scale invariant spectrum of curvature perturbation can also be induced by either its constant mode [18],[20],[21], given by (5), or its increasing mode [19], given by (6). Though both cases give the scale invariant spectrum, both pictures are distinct. In general, for the picture in [19], initially $|\epsilon| \gg 1$, which then is rapidly decreasing, the slow evolution of the scale factor ends when $|\epsilon| \sim 1$. While for that in [18],[20],[21], initially $|\epsilon| \lesssim 1$, which then is rapidly increasing. In addition, for [18],[20],[21], during the slow evolution, the perturbation mode is actually still inside the Hubble horizon. Thus a period after it is required to extend the perturbation mode out of the Hubble horizon, while in [19], the perturbation mode can naturally leave the Hubble horizon during the slow evolution. There is also not the problem pointed in [40].

Here, we will discuss that in [19]. We have generally $Q = \epsilon$ for single field action $P(X,\varphi)$ [34]. While the case is slightly complex for generalized Galileon [25],[26]. However, as will be showed in following section, we actually have $Q \sim |\epsilon|$.

Thus $Q = |\epsilon|$ will be set for general discussions in the following. In principle, $|\epsilon|$ is dependent of a. However, it can be observed that a is nearly constant for $|\epsilon| \gg 1$. Thus for (6), we have

$$Q = |\epsilon| \sim \Lambda_*^4 (t_*-t)^4, \tag{7}$$

since $\eta \sim t$, where Λ_* is $1/t_*$ dimension. The Hubble parameter is given by

$$H \sim \frac{1}{\Lambda_*^4 (t_*-t)^5}. \tag{8}$$

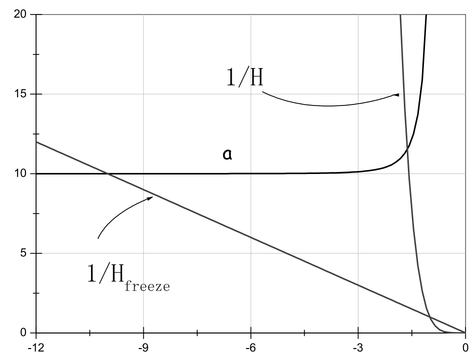

FIG. 1: The evolutions of a, the Hubble horizon and the $\mathcal{R}$ horizon during the slow expansion given by Eq.(7). $a_* = 10$ is set. During this phase, due to the rapidly change of H and H_{freeze}, the perturbation mode initially inside both horizons, i.e. $\lambda \sim a \ll 1/H_{freeze} \ll 1/H$ will naturally leave the $\mathcal{R}$ horizon, i.e.$\lambda \sim a > 1/H_{freeze}$, and then the Hubble horizon, i.e.$\lambda \sim a > 1/H$.

Thus a is given by

$$|\ln\left(\frac{a}{a_*}\right)| \sim \frac{1}{\Lambda_*^4 (t_*-t)^4} \sim \frac{1}{|\epsilon|}. \tag{9}$$

When initially $\Lambda_*(t_*-t) \gg 1$, i.e.$|\epsilon| \gg 1$, the evolution corresponds to the slow expansion for $\epsilon \ll -1$, or the slowly contraction for $\epsilon \gg 1$, since $a/a_* \simeq 1$. The slow evolution ends when $\Lambda_*(t_*-t) \simeq 1$, at which $|\epsilon| \sim 1$.

When $k^2 \simeq z''/z$, the perturbation mode is leaving the horizon, and hereafter it freezes out. This horizon might be called as the $\mathcal{R}$ horizon

$$1/\mathcal{H}_{freeze} = \sqrt{\left|\frac{z}{z''}\right|} \simeq \eta_* - \eta. \tag{10}$$

Thus the physical $\mathcal{R}$ horizon is $a/\mathcal{H}_{freeze} \simeq t_* - t$, While the Hubble horizon is $1/H$ given by Eq.(8). Here, the evolutions of the $\mathcal{R}$ horizon and the Hubble horizon are different. While when a is rapidly changed and $|\epsilon|$ is unchanged, e.g.inflation, both evolutions are almost same. The reason is that for inflation, $z''/z \sim a''/a$, thus

$$1/\mathcal{H}_{freeze} \simeq \sqrt{\left|\frac{z}{z''}\right|} \simeq \sqrt{\left|\frac{a}{a''}\right|} \sim 1/\mathcal{H}, \tag{11}$$

while here a is constant and $|\epsilon|$ is rapidly changed, we have not $z''/z \sim a''/a$.

When $k^2 \gg z''/z$, i.e. the perturbation is deep inside the $\mathcal{R}$ horizon, u_k oscillates with a constant amplitude. The quantization of u_k is well defined for $Q \sim |\epsilon| > 0$, which gives its initial value. The evolutions of a, $1/H$ and $a/\mathcal{H}_{freeze}$ are plotted in Fig.1 for the slow expansion. It can be found that the perturbation mode firstly leaves the $\mathcal{R}$ horizon, after which it is freezed out, but it is still inside the Hubble horizon. However, since the Hubble horizon is decreasing, after a while the perturbation mode will be inevitably extended outside it, and become the primordial perturbation on super Hubble scale.

When $k^2 \ll z''/z$, the amplitude of perturbation spectrum is $\mathcal{P}_{\mathcal{R}}^{1/2} \simeq \sqrt{k^3}\left|\frac{u_k}{z}\right|$. Thus

$$\mathcal{P}_{\mathcal{R}} \simeq \frac{|\epsilon|}{c_s M_P^2} H^2, \tag{12}$$

where $Q \sim |\epsilon|$ is applied. The perturbation is given by the increasing mode (4), because a is hardly changed and $|\epsilon|$ is decreasing. When $|\epsilon| \sim 1$, the change of a begins to become not negligible. Though $|\epsilon|$ is still decreasing, a is increased exponentially. Thus this mode will become the decaying mode at certain time $t_f \sim \mathcal{O}(t_*)$ shortly after $|\epsilon| \sim 1$. In principle, the spectrum of $\mathcal{R}$ should be calculated around t_f. Thus

$$\mathcal{P}_{\mathcal{R}}^{1/2} \sim \sqrt{\frac{1}{c_s M_P^2}} H_f. \tag{13}$$

The universe reheats around or after t_f, and hereafter the perturbation is dominated by its constant mode, until it enters into the Hubble horizon during the radiation or matter domination. $|\epsilon_f| \sim 1$ brings $\Lambda_*^4 (t_* - t_f)^4 \sim 1$. Thus Eq.(13) becomes

$$\mathcal{P}_{\mathcal{R}}^{1/2} \sim \frac{\Lambda_*}{M_P \sqrt{c_s}}, \tag{14}$$

which is general result of the slow evolution in [19], i.e. the evolution of $|\epsilon|$ follows Eq.(7) and c_s^2 is constant.

III. A GALILEON DESIGN OF SLOW EXPANSION

Here, we will detailed show a model of the slow expansion given in [10],[19]. While the scenario of the slow contraction given in [19] is slightly alike with that in [18], which might be studied in detail elsewhere.

A. The background

We consider a generalized Galileon as

$$\mathcal{L} \sim -e^{4\varphi/\mathcal{M}} X + \frac{1}{\mathcal{M}^8} X^3 - \frac{1}{\mathcal{M}^7} X^2 \Box \varphi, \tag{15}$$

where $\mathcal{M}$ is the energy scale. Here, the sign before $e^{4\varphi/\mathcal{M}} X$ is negative. However, as will be showed that this model has not the ghost and gradient instabilities, since $Q > 0$ and $c_s^2 > 0$. The evolution of background is determined by

$$\begin{aligned} &\left(-e^{4\varphi/\mathcal{M}} + \frac{15}{\mathcal{M}^8} X^2 + \frac{24}{\mathcal{M}^7} H \dot{\varphi} X\right) \ddot{\varphi} \\ &+ 3\left(-e^{4\varphi/\mathcal{M}} + \frac{3}{\mathcal{M}^8} X^2\right) H \dot{\varphi} \\ &+ \left(-\frac{4}{\mathcal{M}} e^{4\varphi/\mathcal{M}} + \frac{6\dot{H}\dot{\varphi}^2}{\mathcal{M}^7} + \frac{18 H^2 \dot{\varphi}^2}{\mathcal{M}^7}\right) X = 0, \end{aligned} \tag{16}$$

$$3H^2 M_P^2 = -e^{4\varphi/\mathcal{M}} X + \frac{5}{\mathcal{M}^8} X^3 + \frac{6}{\mathcal{M}^7} X \dot{\varphi}^3 H. \tag{17}$$

We require that initially $\epsilon \ll -1$, and behaviors as Eq.(7). This can be found by requiring $e^{4\varphi/\mathcal{M}} X \simeq \frac{5X^3}{\mathcal{M}^8}$ in Eq.(17). This gives

$$e^{\varphi/\mathcal{M}} = \left(\frac{5}{4}\right)^{1/4} \frac{1}{\mathcal{M}(t_* - t)}. \tag{18}$$

Thus

$$\dot{\varphi} = \frac{\mathcal{M}}{(t_* - t)}. \tag{19}$$

Thus

$$H \simeq \frac{\dot{\varphi}^5}{\mathcal{M}^7} \simeq \frac{1}{\mathcal{M}^2 M_P^2 (t_* - t)^5} \tag{20}$$

is induced. Thus for $\mathcal{M} M_P \sim \Lambda_*^2$, Eq.(8) is obtained. This gives Eq.(7), which is just required evolution.

Eqs.(16) and (17) are numerically solved in Fig.2 and Fig.3. We can see that Eqs.(19) and (20) can be highly consistent with accurate solutions for a long range of time. The significant deviation only occurs around $t_f \sim \mathcal{O}(t_*)$. We might think that the slow expanding phase ends when the significant deviation appears, and the reheating begins. However, it might be possible that the reheating of universe begins some time after the significant deviation occurs, since the perturbation generated during this period only are the perturbation on small scale, which has not to be scale invariant.

Eqs.(19) and (20) implies $H\dot{\varphi}\mathcal{M} \ll X$, $H\dot{\varphi}/\mathcal{M}^3 \ll e^{2\varphi/\mathcal{M}}$, and $H\dot{\varphi} \ll \ddot{\varphi}$, since

$$H \sim \frac{1}{(t_* - t)^5} \ll \frac{1}{(t_* - t)} \tag{21}$$

for $|\epsilon| \gg 1$, i.e.$\sqrt{\mathcal{M} M_P}(t_* - t) \gg 1$. Thus Eq.(16) is approximately

$$\left(-e^{4\varphi/\mathcal{M}} + \frac{15}{\mathcal{M}^8} X^2\right) \ddot{\varphi} - \frac{4}{\mathcal{M}} e^{4\varphi/\mathcal{M}} X \simeq 0 \tag{22}$$

for $\sqrt{\mathcal{M} M_P}(t_* - t) \gg 1$. It can be found that Eq.(22) is consistent with Eqs.(18) and (19). Thus the equation of the perturbation $\delta\varphi$ of φ is

$$\begin{aligned} &\left(-e^{4\varphi/\mathcal{M}} + \frac{15}{4\mathcal{M}^8} \dot{\varphi}^4\right) \delta\ddot{\varphi} - \frac{4}{\mathcal{M}} e^{4\varphi/\mathcal{M}} \dot{\varphi} \delta\dot{\varphi} \\ &+ \frac{15}{\mathcal{M}^8} \dot{\varphi}^3 \ddot{\varphi} \delta\dot{\varphi} - \left(\frac{4}{\mathcal{M}} \ddot{\varphi} + \frac{8}{\mathcal{M}^2} \dot{\varphi}^2\right) e^{4\varphi/\mathcal{M}} \delta\varphi \simeq 0. \end{aligned}$$

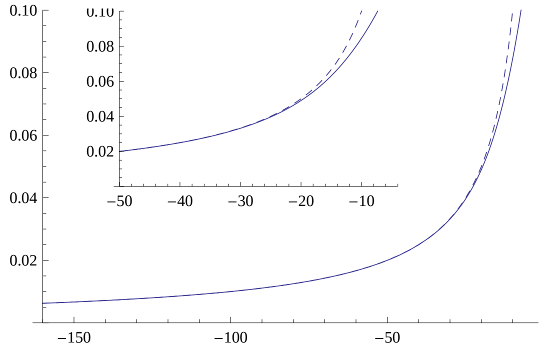

FIG. 2: The evolution of $\dot{\varphi}$ with respect to the time. The initial values of φ and $\dot{\varphi}$ are required to satisfy Eqs.(18) and (19), respectively. The parameter $\mathcal{M} = 0.01 M_P$. The dashed line is that of Eq.(19). The inset is that around $t_f \sim \mathcal{O}(t_*)$.

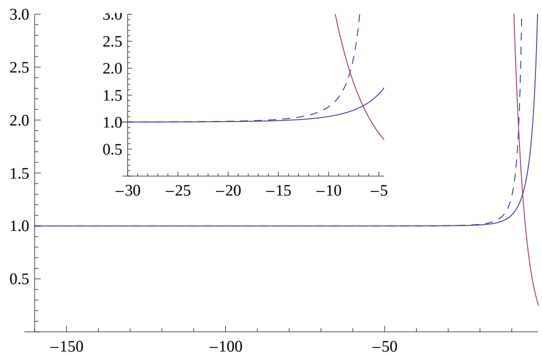

FIG. 3: The evolutions of a and H with respect to the time. The red line is that of H. The black line is that of a, while the black dashed line is that of Eq.(9). The inset is that around $t_f \sim \mathcal{O}(t_*)$.

When Eqs.(18) and (19) are considered, the solution is

$$\delta\varphi \sim (t_* - t)^6, \ \ is \text{ decaying mode} \tag{23}$$

$$or \ \ 1/(t_* - t), \ \ is \text{ increasing mode}. \tag{24}$$

The decaying mode is negligible. The increasing mode is dominated. Thus $\delta\varphi \sim \frac{\dot{\varphi}}{\mathcal{M}}$. Thus for $\mathcal{M}\Delta t \gg 1$, $\delta\varphi \ll \Delta\varphi$. Thus if initially $\delta\varphi \ll \varphi$ is satisfied, it will be valid all along. When the time arrives around t_f, Eq.(21) will be not right. Thus Eq.(22) can not be found. This explains why there will be significant deviation for Eq.(19) around t_f.

There might be other fluids, However, their energies generally do not increase, since the expansion is slow. Thus for $|\epsilon| \gg 1$, i.e.$\sqrt{\mathcal{M}M_P}(t_* - t) \gg 1$, the evolution of background, given by Eqs.(19) and (20), is stable.

B. The curvature perturbation

$\mathcal{R}$ satisfies Eq.(2). We follow the definitions and calculations of Refs.[26],[35] Here, the generalized Galileon

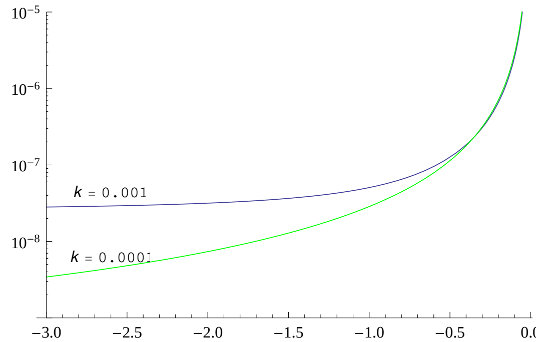

FIG. 4: The evolutions of the amplitude of curvature perturbation for different k with respect to the time. The green and black lines are that with different k. Here, the time axis is rescale as $\mathcal{M}t$ for the convenience of numerical calculation, t is that in Fig.2 and Fig.3, $\mathcal{M} = 0.01$.

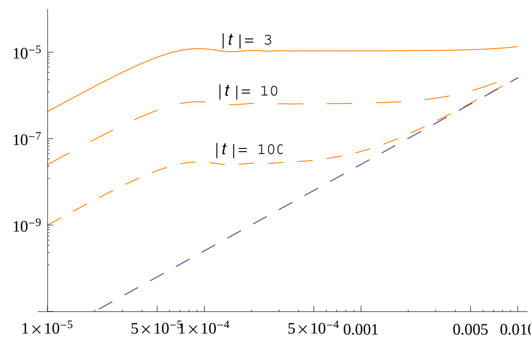

FIG. 5: The spectrum of curvature perturbation at different times with respect to k. The black dashed line is initial spectrum. The short dashed, long dashed and solid orange lines are the spectra at different times, respectively. There is a cutoff $k_{cutoff} \sim 5 \times 10^{-5}$, below which the spectrum is not scale invariant, which is explained in the text.

action is (15). Thus it is found that

$$\begin{aligned}
\mathcal{F} &= -e^{4\varphi/\mathcal{M}} + \frac{3X^2}{\mathcal{M}^8} + \frac{8X}{\mathcal{M}^7}(\ddot{\varphi} + H\dot{\varphi}) - \frac{8X^4}{\mathcal{M}^{14}M_P^2} \\
&\simeq \frac{7}{2\mathcal{M}^4(t_* - t)^4} \qquad (25) \\
\mathcal{G} &= -e^{4\varphi/\mathcal{M}} + \frac{15X^2}{\mathcal{M}^8} + \frac{12H\dot{\varphi}^3}{\mathcal{M}^7} + \frac{12X^4}{\mathcal{M}^{14}M_P^2} \\
&\simeq \frac{5}{2\mathcal{M}^4(t_* - t)^4} \qquad (26)
\end{aligned}$$

for $\mathcal{M}(t_* - t) \gg 1$. In [26], the results are applied to that of inflation, however, which are actually general for arbitrary evolution. Thus Q is given by

$$Q = \frac{\mathcal{F}X}{M_P^2(H - \frac{2\dot{\varphi}X^2}{\mathcal{M}^7 M_P^2})^2} \sim \frac{M^{14}M_P^2\mathcal{F}}{\dot{\varphi}^8} \simeq \mathcal{M}^2 M_P^2 (t_* - t)^4, \tag{27}$$

where Eqs.(19) and (20) are applied. Thus $Q \sim |\epsilon| > 0$, which is just required here, satisfies Eq.(7). There is not the ghost instability. Here, the importance of $X^2\Box\varphi$ is obvious, because if it disappears in (15), $\mathcal{F}$ is given by

$$\mathcal{F} = -e^{4\varphi/\mathcal{M}} + \frac{3X^2}{\mathcal{M}^8} \simeq -\frac{1}{2\mathcal{M}^4(t_*-t)^4} < 0, \quad (28)$$

$Q > 0$ will hardly be obtained, which is consistent with $Q = \epsilon < 0$ for this case. This indicate that it is $X^2\Box\varphi$ that alters the sign of Q, and leads $Q \sim |\epsilon| > 0$. The c_s^2 is given by

$$c_s^2 = \frac{\mathcal{F}}{\mathcal{G}} \sim 1.4. \quad (29)$$

Thus $c_s^2 > 0$ is constant, which is also just required. The sign of c_s^2 is determined by the signs of $\mathcal{F}$ and $\mathcal{G}$, both are positive. Here, obviously $\mathcal{F} > 0$ is also required to assure $c_s^2 > 0$. Thus there are not the ghost and gradient instabilities, the effective theory is healthy.

We plot the evolution of the amplitude of the curvature perturbation in Fig.4, and the spectrum of perturbation in Fig.5. We can see that the perturbation is initially not increasing, since it is inside the $\mathcal{R}$ horizon. The increase begins until the perturbation mode leaves the $\mathcal{R}$ horizon. The longer the wavelength of perturbation is, the earlier the perturbation leaves the $\mathcal{R}$ horizon, the earlier it begins to increase. However, since the shorter the wavelength of perturbation is, the larger its initial amplitude is, all perturbation modes will eventually have same amplitude.

There is a cutoff k_{cutoff} in Fig.5, which is given by

$$k_{cutoff} \sim \mathcal{H}_{inifr}, \quad (30)$$

where $\mathcal{H}_{inifr}$ is $\mathcal{H}_{freeze}$ at initial time, and can be changed with the difference of the initial parameters in the numerical calculation. The spectrum is scale invariant for $k > k_{cutoff}$. However, for $k < k_{cutoff}$, since the corresponding perturbation modes are outside the $\mathcal{R}$ horizon all along, only are their amplitudes increasing but not the shape of the spectrum is not altered [41],[42].

The spectrum of $\mathcal{R}$ is scale invariant. The amplitude of spectrum is given by Eq.(14)

$$\mathcal{P}_{\mathcal{R}}^{1/2} \sim \sqrt{\frac{\mathcal{M}}{c_s M_P}}, \quad (31)$$

where $\Lambda_* \sim \sqrt{\mathcal{M}M_P}$ is applied. Thus $\mathcal{P}_{\mathcal{R}}^{1/2} \sim 10^{-5}$ requires $\mathcal{M} \sim 10^{-10}c_s M_P$. Thus $\mathcal{M} \sim 10^9$Gev for $c_s \simeq 1$. The only adjusted parameter in this model is fixed by the observation. There is not other finetuning.

C. The reheating

When the slowly expanding phase ends, the energy of Galileon field is required to be released into the radiation, and the universe reheats. Hereafter, the evolution of hot "big bang" cosmology begins. We can notice that before this, the perturbation mode has leaved the Hubble horizon.

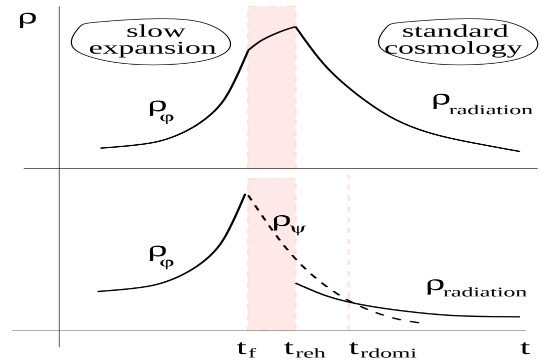

FIG. 6: The sketch of the evolution of the energy density ρ for different reheating courses discussed here.

Here, in certain sense, the reheating is alike with that for inflation. The preheating theory after inflation has been developed in [43],[44]. In general, during the preheating phase after inflation the energy of inflaton will be rapidly released by the parametric resonance effects, due to the coupling of inflaton with other fields. Then this issue has been extensively studied, see [45],[46],[47] for reviews.

We will apply the instant preheating mechanism [48] for given case here. We consider the straight coupling of φ with χ particle as

$$\mathcal{L} \sim g^2(\varphi - \varphi_{reh})^2\chi^2, \quad (32)$$

where g is the coupling constant. The effective mass of χ particle is $M_{\chi eff}^2 \sim g^2(\varphi - \varphi_{reh})^2$. When the φ field arrives at the region around φ_{reh}, $M_{\chi eff}^2 \lesssim \dot{M}_{\chi eff}$, the adiabatic condition is broke, and the productions of χ particles will be inevitable. This generally occurs in a region around φ_{reh}, $\Delta\varphi \lesssim \dot{\varphi}_{reh}/g$, in which $\dot{\varphi}_{reh}$ is the velocity of φ through φ_{reh}. Thus the productions of χ particles is instantaneous, $\Delta t_{reh} \sim 1/\sqrt{g\dot{\varphi}_{reh}}$.

The number density n_χ of χ particle is

$$n_\chi = \frac{1}{2\pi^2}\int n_k k^2 dk \simeq \frac{g^{3/2}\dot{\varphi}_{reh}^{3/2}}{8\pi^3}, \quad (33)$$

where n_k is the occupation number of χ particle. Thus $\rho_\chi = n_\chi M_\chi \sim g^2\dot{\varphi}_{reh}^2$, since $M_{\chi eff} \sim g(\varphi - \varphi_{reh}) \sim g\dot{\varphi}_{reh}\Delta t_{reh}$. Thus the energy drained by the production of χ particle is

$$\frac{\rho_\chi}{\rho_{\varphi reh}} \sim \frac{g^2}{8\pi^3}\mathcal{M}^6 M_P^2(t_* - t_{reh})^8, \quad (34)$$

where Eqs.(19) and (20) are applied, and $\rho_{\varphi reh}$ is the energy density of φ around t_{reh}. We assume $t_f \sim t_{reh}$ for simplicity, i.e. the reheating occurs at the time when the slow expansion ends. Thus $\mathcal{M}^2 M_P^2(t_* - t_{reh})^4 \sim 1$.

This implies

$$\frac{\rho_\chi}{\rho_{\varphi reh}} \sim \frac{g^2\mathcal{M}^2}{8\pi^3 M_P^2}. \tag{35}$$

We generally require $\mathcal{M} \ll 1$ and $g < 1$. Thus $\rho_\chi/\rho_{\varphi reh} \ll 1$, which indicates that for such a single preheating, the energy of φ can hardly be released completely, the universe is still dominated by ρ_φ, which will continue all along, since the energy density of φ is increasing with the expansion of universe while that of χ particle is decreasing.

However, there might be $\mathcal{N}$ couplings, one of which is alike with (32). We can find, after doing similar calculations, that when

$$\mathcal{N} > \frac{M_P^2}{g^2\mathcal{M}^2}, \tag{36}$$

the release of the energy of φ will be complete. The sketch of this reheating course is plotted in upper panel in Fig.6. We assume that the χ particle produced is rapidly transferred into the radiation. In this case, the reheating temperature T_r is approximately determined by $\rho_{\varphi reh} \sim T_r^4$. Thus we have

$$T_r \sim \left(\frac{\dot{\varphi}^{10}}{\mathcal{M}^{14}M_P^2}\right)^{1/4} \sim \mathcal{M}^{1/4}M_P^{3/4}, \tag{37}$$

where $\mathcal{M}^2 M_P^2(t_* - t_{reh})^4 \sim 1$ is applied again. Thus if $\mathcal{M} \sim 10^{-10}M_P$, we have $T_r \sim 10^{15}$Gev.

Here, $\mathcal{N} \gg 1$ is feasible, however, might be uncomfortable. $\mathcal{N} \gg 1$ is required is because the energy of φ has to be released completely for one time, or since the energy density of φ is increasing, the universe will dominated by φ all along. However, we also could consider another channel of the reheating, likes that in phantom inflation. The energy of φ is firstly shifted to the kinetic energy of a normal field, e.g.ψ, and then the energy of ψ is released by the instant preheating. The sketch of this reheating course is plotted in lower panel in Fig.6. Here, the energy of ψ is not required to be released completely, since $\rho_\psi \sim 1/a^6$ is decreasing faster than that of the radiation, the universe will be dominated by that of the radiation early or late.

We can implement it by considering a potential of φ, illustrated in Fig.7. We require that it is only significant around or after $|\epsilon| \sim 1$, and is negligible $|\epsilon| \gg 1$. Then we introduce a waterfall field ψ, coupled to φ. The effective mass of ψ is initially positive and becomes negative around $|\epsilon| \sim 1$. Thus ψ will roll down along its potential. Thus almost all energy of φ will be shifted to $\rho_\psi \sim \dot{\psi}^2$. This energy will be expected to be released by the instant reheating. Thus there could be a suitable reheating after the slow expansion ends, after which the evolution of hot "big bang" cosmology begins.

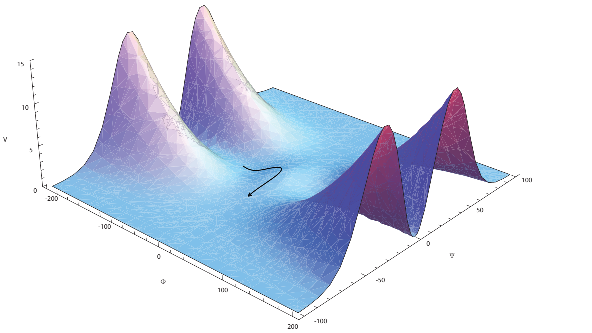

FIG. 7: The figure of the effective potential for the exiting from the slow expansion. The black solid line is the motive trajectory of field in (φ, ψ) space.

IV. DISCUSSION

When initially $\epsilon \ll -1$ and is rapidly increasing, the universe is slowly expanding. The spectrum of curvature perturbation generated during such a phase of slow expansion can be scale invariant. This provides a mechanism by which an alternative scenario of early universe can be imagined. Here, we show a model of such a scenario by applying an effective action of generalized Galileon.

In principle, $\epsilon < 0$ implies the ghost instability. However, in this model, because of the introduction of Galileon field, there is not the ghost instability, the perturbation theory is healthy. In Refs.[10],[19], the phantom was applied for an implementing of slow expansion. In the calculations of perturbation, for consistence, $|\epsilon|$ is used, though the initial value of perturbation is still pathologically defined. However, in the model given here, it can be found that actually $Q \simeq |\epsilon|$. This in certain sense validates the argument and calculations used in [10],[19], i.e. the phantom field might be a simple simulation of a full theory without the ghost below certain physical cutoff, which can give same results with that of a full theory, when the replacement of ϵ with $|\epsilon|$ is done.

When $\epsilon \sim -1$, the slow expansion ends. The exiting to a hot universe is only a simple reheating, since the universe expands all along. Thus there is not the problem how the bouncing is implemented in bouncing cosmologies [14],[50],[51]. We have discussed possible implements of reheating, and found that the available energy of Galileon field can completely released, the universe can reheat to a suitable temperature. Thus the model of the slow expansion given here might be a viable design of the early universe.

The material compares of model with the observations is certainly interesting, which will place rigid constrains for the model. The results obtained will be expected to either improves or rules out this model. We will investigate it elsewhere. However, it should be pointed that we only bring one of all possible implements of the slow expansion. In principle, there might be other effective actions of generalized Galileon, or modified gravity, which could give the same evolution of background. Thus for the slow expansion, it might be also significant to find alternative implements to the model given here, which will help to uplift the flexility of the slow expansion to the observations.

Here, the scale factor is asymptotic to a constant value in infinite past, there is not singularity point. Thus in certain sense, the slow expansion scenario brings a solution to the cosmological singularity problem. However, it also can be imagined that after the available energy of the field is released, it might be placed again in the bottom of its effective potential, and after the universe undergoes the radiation and matter periods, the field might dominate again and roll again with increasing energy. This models an eternally expanding cyclic universe [52],[53],[54], i.e. H oscillates periodically while a expands all along. The implement of this cyclic universe might be interesting for refining with the model given here.

Here, c_s is constant is set. However, its change will obviously enlarge the space of solutions of the scale invariance of curvature perturbation [55],[56],[57],[58],[59]. In certain sense all possibilities of the changes of a, Q and c_s^2 might be interesting for further exploring.

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