# A game theoretic framework for distributed self-coexistence among IEEE 802.22 networks

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***Abstract*—The cognitive radio based IEEE 802.22 wireless regional area network (WRAN) is designed to operate in the under–utilized TV bands by detecting and avoiding primary TV transmission bands in a timely manner. Such networks, deployed by competing wireless service providers, would have to self-coexist by accessing different parts of the available spectrum in a distributed manner. Obviously, the goal of every network is to acquire a clear spectrum chunk free of interference from other IEEE 802.22 networks so as to satisfy the QoS of the services delivered to the end–users. In this paper, we study the distributed WRAN self–coexistence problem from a minority game theoretic perspective. We model the spectrum band switching game where the networks try to minimize their cost in finding a clear band. We propose a mixed strategy that the competing networks must adhere to in order to achieve the Nash equilibrium. Simulation experiments have also been conducted and results corroborate with the theoretical analysis.**

## I. Introduction

IEEE 802.22 based on cognitive radios (CRs) is a wireless regional area networks (WRAN) standard that can operate in the sub–900 MHz licensed bands on an non-interfering basis [1], [2]. Cognitive radio is the key enabling technology in this standard that can periodically perform spectrum sensing and can operate at any unused frequency in the licensed bands [9]. The most important regulatory aspect is that cognitive radios must not interfere with the operation in licensed bands and must identify and avoid such bands in timely manner [3], [4]. If any of the spectrum bands used by WRAN is accessed by the licensed incumbents, the IEEE 802.22 devices (e.g., base stations (BS) and consumer premise equipments (CPE)) are required to vacate the channels within the channel move time and switch to some other channel [6].

One of the major challenges in the newly proposed IEEE 802.22 standard is ensuring quality of service (QoS) among IEEE 802.22 networks themselves, i.e., in other words, maintaining self–coexistence. Though most of the work on IEEE 802.22 has been on the enhancement of reliable spectrum sensing, there is hardly any investigation on issues related to self–coexistence. In areas with significant high primary incumbents (licensed services), open channels will be a commodity of demand. Therefore, dynamic channel access among IEEE 802.22 networks will be of utmost importance so that the interference among IEEE 802.22 networks can be minimized; else the throughput and quality of service (QoS) will be compromised. Different from other IEEE 802 standards where self-coexistence issues are only considered after the specification essentially is finalized, it is required for IEEE 802.22 to take the proactive approach and mandate to include self-coexistence protocols and algorithms for enhanced MAC as revision of the initial standard conception and definition [8].

In this paper, we focus on the self–coexistence of IEEE 802.22 networks from a game theoretic perspective. We use the tools from *minority game theory* [5] and model the competitive environment as a distributed *Modified Minority Game* (MMG). We consider the system of multiple overlapping IEEE 802.22 networks operated by multiple wireless service providers that compete for the resources and try to seek a spectrum band void of interference from other coexisting IEEE 802.22 networks. If interfered by other IEEE 802.22 networks at any stage of the game, the networks face a binary choice of whether to stick to the band (assuming the interferers might move away) or move to another band itself. As networks do not have information about which bands other IEEE 802.22 networks will choose, the game is played under incomplete information. So, the questions that need to be answered are: how can each network decide to co-operate or not co-operate in this minority game? Is there an equilibrium solution to this problem? How will the solution change if some common information is available to all the networks? With the help of proposed MMG model, we intend to help all the networks make better decisions even without direct knowledge of other networks' strategies. There are several advantages for taking the MMG approach. First, the MMG model works in a distributed manner where a central authority or a centralized allocating mechanism is not needed thus making the system scalable. Second, direct communications or negotiation messages are not needed among the networks thus reducing overhead in communication. Third, being rational entities in the game, IEEE 802.22 networks individually try to maximize their own payoffs or minimize the cost of channel switching subject to constraints on resource usage. We investigate both pure and mixed strategy mechanisms from networks' perspectives. The important investigation in such a game is the existence of any equilibrium point– with a set of strategies played by each of the networks such that no network can benefit any more by changing its own strategy unilaterally while the other networks keep their strategies unchanged. This equilibrium is known as the Nash equilibrium [10]. We conduct simulation experiments with multiple competing IEEE 802.22 networks the results of which successfully corroborates with theoretical analysis. To the best of our knowledge, this research is the first attempt to apply the minority game framework in IEEE 802.22 networks and to solve the self–coexistence problem in a distributed manner.

The rest of the paper is organized as follows. In section II, we design the self–coexistence problem as dynamic channel switching game from a minority game model perspective. We analyze the game in section III with pure strategy and obtain sub-optimal solution. We extend the analysis to the mixed strategy space in section IV to achieve Nash equilibrium. Simulation experiments and results are discussed in section V. Conclusions are drawn in last section.

## II. Game Formulation

In this section, we formulate the self–coexistence problem as a dynamic channel[1] switching game. We assume that $N$ IEEE 802.22 networks (players) operated by $N$ separate wireless service providers in a region are competing for one of $M$ separate orthogonal spectrum bands not used by primary incumbents. The IEEE 802.22 networks can be partially or completely overlapped geographically (i.e., coverage area) with each other. If one network is in the interference range of another, they can not use the same spectrum band; otherwise QoS of both the networks will suffer. In this scenario, we model the dynamic channel switching as MMG where the aim of each network is to capture a spectrum band free of interference. We assume the only control information needed for participating successfully in the MMG is the number of overlapping competitors in the region which can be known from the broadcasting beacons by each of the IEEE 802.22 networks in Foreign Beacon Period (FBP) [2]. Before, we formulate the minority game, let us briefly discuss this theory.

### *A. Minority Game Theory*

Minority game theory, originally proposed by Challet and Zhang [5], is a branch of game theory for studying competition and self–imposed cooperation in a non–cooperative game with limited resources. Players in this game usually play with binary strategy set and do not interact or negotiate with each other directly regarding the strategy set. Classical Minority game or the El Farol bar problem was first proposed in [5]. In the bar problem, a group of $n$ persons have to decide independently and at the same time if they want to go to the El Farol bar on Friday night. At each step, a player has *binary strategy set*: to go or not to go to the bar. Going to the bar is enjoyable only if the bar is not too crowded. Now, if all $n$ players decide not to go to the bar thinking that the bar will be crowded then the bar will be empty. However, if they all decide that the bar will be empty and decide to go, then the bar will be overcrowded.

[1]Throughout this paper, we use the words "channel", "band" and "chunk" interchangeably unless explicitly mentioned otherwise.

### *B. Decision Problem*

We consider the most generic abstraction of "always greedy and profit seeking" model of $N$ competing IEEE 802.22 networks operated by wireless service providers. Without loss of generality, we focus our attention on a particular network $i \in N$. Due to homogeneity of the networks, the same reasoning applies to all other networks.

At the beginning of the game, each network dynamically chooses one of the $M$ allowable spectrum bands for its operations. If two or more overlapped networks operate using the same spectrum band, then interference will occur and their transmissions will fail. Thus the networks will have to make new decisions for channel switching in the next stage of the game. Each of these stages of the game is formulated using modified minority game theoretic framework. The game ends when all the networks are successful in capturing a clear spectrum band, and is re-initiated if the primary TV transmission starts using IEEE 802.22 occupied band(s), and thus the spectrum usage report changes for one or more networks. In such a case, the IEEE 802.22 network(s) involved will again try to access new band(s). The optimization problem is to find the mechanism of achieving minimum number of failed transmission stages from the networks.

As far as the decision strategy in this MMG model is concerned, if interfered at any stage of the game, network $i$ has the binary strategy set of switching to another band (expecting to find a free spectrum band) or staying on the current band (assuming the interferers will move away). Using game theoretic notation, the binary strategy set for network $i$ can be represented as

$$S_i \;=\; \{\text{switch}, \text{stay}\} \qquad (1)$$

To generalize, we assume the existence of strategy sets $S_1, S_2, \cdots, S_N$ for the networks $1, 2, \cdots, N$. In this game, at every stage, if network 1 chooses strategy $s_1 \in S_1$, network 2 chooses strategy $s_2 \in S_2$ and so on, we can describe such a set of strategies chosen by all $N$ networks as one ordered $N$-tuple, $\mathbf{s} = \{s_1, s_2, \cdots, s_N\}$. This vector of individual strategies is called a strategy profile (or sometimes a strategy combination). For every different combination of individual choices of strategies, we would get a different strategy profile $\mathbf{s}$. The set of all such strategy profiles is called the space of strategy profiles $\mathbf{S}'$. It is simply the cartesian product of the vectors $S_i$ for each network which can be written as $\mathbf{S}' = S_1 \times S_2 \times \cdots \times S_N$.

### *C. Channel Switching Cost Function*

We use multi–stage modified minority game where each stage of the game can be represented in a *strategic form* [7]. Each stage of the game is played by having all the networks (players) simultaneously pick their individual strategies[2]. This set of choices results in some strategy profile $\mathbf{s} \in \mathbf{S}'$, which

[2]We assume that networks are synchronized with decision making at every stage of the game. However, a closer look at the game indicates that even asynchronization at any stage does not impact any change to the results.

we call the outcome of the game. Each network faces a cost of preference over these outcomes $\mathbf{s} \in \mathbf{S}'$.

At the beginning of a stage, when an interfered network $i$ chooses either *"switch"* or *"stay"*, it faces one of two possible costs in terms of time units. Note that, throughout this paper, we assume the cost as time units consumed. Then if the network $i$ chooses to switch, it faces a cost of finding a clear spectrum band in the game. Note that, in a game of $N$ networks competing over $M$ spectrum bands, the network $i$ might find the clear channel just after 1 switching, or it might take more than 1 switching as multiple networks might choose the same band chosen by network $i$ resulting in a subgame. Moreover, note that, with varying $N$ and $M$, the average cost of finding a clear band will also vary. However, how this cost will vary is not known. In this regard, we propose a multiplicative form for the cost for finding a clear band in the MMG. We define the expected cost of finding a clear channel, if the network chooses the strategy of switching, as

$$E[C_i(s_i, \mathbf{s_{-i}})] = c^{f(N,M)} \quad (2)$$

over all possible resulting subgames where, $s_i$ and $\mathbf{s_{-i}}$ denote the strategies chosen by network $i$ and rest of the networks respectively. We assume that $c$ is the cost of single switching and $f(\cdot)$ is a function that depicts the varying behavior of the cost with $N$ and $M$; we discuss about the nature of $f(\cdot)$ later.

At the beginning of the stage, if the network $i$ chooses the strategy of "stay", it might fall in one of three different scenarios. (i) All the other networks which were attempting to operate using the same band as network $i$, might move away thus creating a clear band for network $i$. (ii) All the other networks which were attempting to operate using the same band as network $i$, might also "stay", thus wasting the stage under consideration and repeating the original game $G$, which started at the beginning of the stage. (iii) Some of the networks move ("switch") while some networks end up being in the same band ("stay"), thus wasting the stage under consideration and creating a subgame $G'$ of the original game $G$. More detailed explanations for subgame $G'$ will be presented later. We define the cost functions as

$$C_i(s_i, \mathbf{s_{-i}}) = \begin{cases} 0 & \text{Scenario (i)} \\ 1 + C_i(G) & \text{Scenario (ii)} \\ 1 + C_i(G') & \text{Scenario (iii)} \end{cases} \quad (3)$$

## III. Game Analysis

With the strategy set and cost functions defined, the optimization problem in this game is to find a mechanism of switching or staying such that cost incurred can be minimized and an equilibrium can be achieved. We typically assume all the players are rational and pick their strategy keeping only individual cost minimization policy in mind at every stage of the game. We intend to find if there is a set of strategies with the property that no network can benefit by changing its strategy unilaterally while the other networks keep their strategies unchanged (Nash equilibrium).

### A. Modified Minority Game in Strategic form

We analyze the game $G = (P : S : C)$ in strategic form by taking the iterated dominance approach. $P$ denotes the set of players (competing networks), $S$ denotes the strategy set and $C$ denotes the cost. At this point, we start with the *pure strategy space* played by all the networks. This means that network $i$ will choose a strategy, say "switch", with probability "1" or "0". To simplify investigation of Nash equilibrium with pure strategy space, we consider the reduced strategic–form minority game with two players (network $i$ and $j$) coexisting on one band. The game is represented in strategic form in table I. Each cell of the table corresponds to a possible combination of the strategies of the players and contains a pair representing the costs of players $i$ and $j$, respectively.

| $i \backslash j$ | **Switch** | **Stay** |
|---|---|---|
| **Switch** | (c,c) | (c,0) |
| **Stay** | (0,c) | Back to Original Game |

TABLE I
STRATEGIC–FORM MINORITY GAME WITH NETWORK $i$ AND $j$

Once the game is expressed in strategic form, it is usually interesting to find if Nash equilibrium exists and if the equilibrium is helpful for the system in minimizing the cost incurred. In this regard, we present the following lemma.

***Lemma 1:*** *Pure strategy dominant response results in sub–optimal solution of the channel switching minority game.*

*Proof:* We proceed with the iterated strict dominance to solve the game as presented in table I. If we consider the strategy space from the point of view of network $i$, then it appears that the "switch" strategy is strictly dominated by the "stay" strategy. This means that we can eliminate the first row of the matrix, since a rational network $i$ will never choose this strategy. Similar reasoning is applicable for network $j$, leading to the elimination of the first column of the matrix. As a result, the dominant response from both the networks in this minority game is {stay,stay} resulting in increased cost as they are back to the original game and the stage under consideration is wasted. Thus pure strategy dominant response leads to a sub–optimal solution for the channel switching minority game and Nash equilibrium can not be achieved. The special case of two–player game can be easily applied to generalized $N$–player game and the result would still be the same. ∎

With the pure strategy space proving to be ineffective in this minority game, we lean towards mixed strategy space for the networks for finding the Nash equilibrium with the optimal solution. In next section, we discuss the modified minority game with mixed strategy.

## IV. Modified Minority Game with Mixed Strategy

We analyze the game in this section from two perspectives: (i) a special case where all the networks are coexisting on a single band at the start of the game; and (ii) the generalized case where we start at any random stage of the game where network $i$ is coexisting on a spectrum band with a few other networks (say, $n-1$, where $n-1 < N-1$).

### A. Special Case MMG Model

With the mixed strategy space for the networks, we deviate from the pure strategy space game by assigning probabilities to each of the strategies in the binary strategy space. We define the mixed strategy space of network $i$ as

$S_i^{mixed} = \{(\text{switch} = p), (\text{stay} = (1-p))\}$ where, network $i$ chooses the strategy "switch" with probability $p$ and chooses the strategy "stay" with probability $(1-p)$. Since all networks are assumed to behave identically, we assume similar mixed strategy space for all the networks. The question now is what values of $(p, 1-p)$ tuple will help us achieve the optimal solution, i.e., in other words, if there exists any finite non-zero probability of "switch" and "stay"?

In the special case, we start the game with all $(N-1)$ other networks coexisting with network $i$ on one band and choose a strategy from mixed strategy space. Then regardless of the strategy chosen by network $i$, the resulting subgame will obtain one of the following possible outcomes: all $N-1$ networks choose "switch", or $N-2$ networks choose "switch", or $\cdots$, or $0$ networks choose "switch". To find the Nash equilibrium, we then determine the expected cost if network $i$ under consideration chooses to "switch" or "stay". Following the switching cost for finding a non-occupied band as indicated previously in equation (2), the expected cost over all possible resulting subgames for network $i$, if it chooses to switch, is

$$E[C_i^{switch}] = \sum_{j=0}^{N-1} Q_j \times c^{f(N,M)} \quad (4)$$

where, $j$ denotes the number of other networks choosing to "switch" and $Q_j$ denotes the probability of $j$ networks switching out of other $N-1$ networks and is given by

$$Q_j = \binom{N-1}{j} p^j (1-p)^{(N-1-j)} \quad (5)$$

With the help of equation (5), equation (4) can be reduced to

$$E[C_i^{switch}] = c^{f(N,M)} \quad (6)$$

On the other hand, the expected cost for network $i$, if it chooses "stay" can then be given as

$$E[C_i^{stay}] = \sum_{j=0}^{N-2} Q_j (1 + E[C_i(G'_{(N-j)})]) + Q_{(N-1)} \times 0 \quad (7)$$

where, $E[C_i(G'_{(N-j)})]$ denotes the expected cost incurred in subgame $G'_{(N-j)}$. Note that, if $E[C_i^{switch}] < E[C_i^{stay}]$, being the rational player, network $i$ will always choose the strategy "switch" thus going back to the pure strategy and as a result can not achieve the Nash equilibrium (refer Table I in lemma 1). Again, if $E[C_i^{switch}] > E[C_i^{stay}]$, similar reasoning can be applied for the strategy "stay" and Nash equilibrium can not be achieved. Thus for the existence of mixed strategy Nash equilibrium, $E[C_i^{switch}] = E[C_i^{stay}]$, i.e., network $i$ is indifferent between "switch" or "stay" regardless of strategies taken by other networks. In other words, the probability tuple $(p, 1-p)$ helps in choosing the strategy such that network $i$ is never dominated by response from any other networks and thus will not deviate from the mixed strategy space $(p, 1-p)$ unilaterally to obtain lower cost. To find the optimal values for mixed strategy space, we equate equations (6) and (7) as

$$E[C_i^{nash}] = c^{f(N,M)} = \sum_{j=0}^{N-2} Q_j (1 + E[C_i(G'_{(N-j)})] \quad (8)$$

Note that, the expected cost of the game at Nash equilibrium is actually not dependent on $j$ as evident from first part of the equation (8), i.e., how many networks are actually switching; rather, the cost varies with $N$, the number of networks and $M$, the number of bands. Thus the expected cost for network $i$ in the subgame $G'_{(N-j)}$ can be deduced to be same as that in the original game. Then, we can rewrite equation (8) as

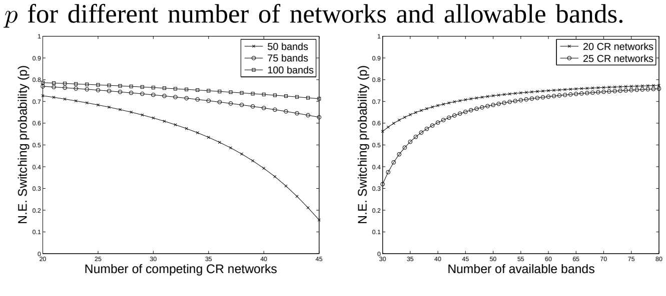
$$\sum_{j=0}^{N-2} Q_j = \frac{c^{f(N,M)}}{1 + c^{f(N,M)}} \quad (9)$$

Using binomial expansion, equation (9) can be reduced to $Q_{N-1} = \frac{1}{1+c^{f(N,M)}}$. Expanding $Q_{N-1}$, we obtain the closed form for $p$ as

$$p = \left(\frac{1}{1 + c^{f(N,M)}}\right)^{\frac{1}{N-1}} \quad (10)$$

For any values of $N$ and $M$, we find that $p$ has a non-zero finite value thus proving the existence of mixed strategy Nash equilibrium point. In other words, the mixed strategy tuple, $(p, 1-p)$ presented in equation (10) constitutes the dominant best response strategy in this channel switching game.

To have a better insight into the analysis[3], we assume a simple closed form of $f(N, M) = \frac{NM}{M-N}$. The intuitive reason behind proposing such function is that expected cost to find a clear band increases with increasing $N$ but fixed $M$; while the cost decreases with increasing $M$ but fixed $N$; however, with both $N$ and $M$ increasing the cost varies simultaneously with the ratio of $M : N$ and the difference between them. Note that, we could choose any other form for $f(N, M)$ as long as the above conditions are satisfied. With the above discussion, we calculate the mixed strategy Nash equilibrium probabilities $p$ for different number of networks and allowable bands.




(a) (b)

Fig. 1. Mixed strategy Nash equilibrium probabilities a) with number of competing networks; b) with number of bands

In figure 1(a), we present the mixed strategy Nash equilibrium "switch" probability ($p$) with varying number of competing networks. The probability calculation shows non-zero finite values of $p$ thus proving the existence of mixed strategy space for achieving Nash equilibrium. It is also clear that with less number of networks competing for the available resources, the networks are better off with a higher probability of switching. Figure 1(b) strengthens the above claim where the mixed strategy Nash equilibrium is plotted against varying

[3]Later, in simulation results section we conduct comparative analysis between mathematical analysis and simulation experiments and we find accurate corroboration between them.

number of bands with number of networks fixed. With less bands available, networks show a less inclination of switching, while with higher number of bands available the optimal probability of switching is high. Next, we extend the above result to a more generalized game framework.

### B. Generalized Case MMG Model

Here, we focus our attention on a generalized stage of the game where network $i$ is coexisting on a single band with $(n-1)$ other networks ($n-1 < N-1$). Then $N-1$ networks (all networks other than network $i$) can be separated into two separate sets: set of networks $\mathcal{N}_{\mathbf{i}}$ residing on the same band as of network $i$ and set of networks $\mathcal{N}_{-\mathbf{i}}$ not residing on the same band as of network $i$. To find Nash equilibrium, we proceed with similar approach as before and try to find the expected cost when network $i$ chooses "stay". In table II, we present the costs for network $i$ with all possible subgame scenarios.

| Scenario | cost |
|---|---|
| $(n-1) \in \mathcal{N}_{\mathbf{i}}$ chooses "switch"; $0 \in \mathcal{N}_{-\mathbf{i}}$ chooses "switch" | $0$ |
| $(n-2) \in \mathcal{N}_{\mathbf{i}}$ chooses "switch"; $0 \in \mathcal{N}_{-\mathbf{i}}$ chooses "switch" | $1 + E[C_i(G'_{2,0})]$ |
| $\cdots$ | $\cdots$ |
| $(n-1) \in \mathcal{N}_{\mathbf{i}}$ chooses "switch"; $(N-n) \in \mathcal{N}_{-\mathbf{i}}$ chooses "switch" | $1 + E[C_i(G'_{0,N-n})]$ |
| $\cdots$ | $\cdots$ |
| $0 \in \mathcal{N}_{\mathbf{i}}$ chooses "switch"; $(N-n) \in \mathcal{N}_{-\mathbf{i}}$ chooses "switch" | $1 + E[C_i(G'_{n,N-n})]$ |

TABLE II
COSTS ASSOCIATED WITH ALL POSSIBLE SUBGAME SCENARIOS

Then the expected cost with network $i$ choosing "stay" is

$$\begin{aligned} E \quad [C_i^{stay}] &= \sum_{j=0}^{n-2} Q_{j\in\mathcal{N}_{\mathbf{i}},0\in\mathcal{N}_{-\mathbf{i}}}(1 + E[C_i(G'_{(n-j),0})]) \\ + \quad & \sum_{j=0}^{n-1} Q_{j\in\mathcal{N}_{\mathbf{i}},1\in\mathcal{N}_{-\mathbf{i}}}(1 + E[C_i(G'_{(n-j),1})]) + \cdots + \\ & \sum_{j=0}^{n-1} Q_{j\in\mathcal{N}_{\mathbf{i}},(N-n)\in\mathcal{N}_{-\mathbf{i}}}(1 + E[C_i(G'_{(n-j),(N-n)})]) \quad (11) \end{aligned}$$

where, $Q_{j\in\mathcal{N}_{\mathbf{i}},k\in\mathcal{N}_{-\mathbf{i}}}$ is the probability of $j \in \mathcal{N}_{\mathbf{i}}$ and $k \in \mathcal{N}_{-\mathbf{i}}$ simultaneously choosing the strategy "switch" and so on. The probability expression for $Q_{j\in\mathcal{N}_{\mathbf{i}},k\in\mathcal{N}_{-\mathbf{i}}}$ can be given as

$$\binom{n-1}{j} p^j (1-p)^{n-1-j} \times \binom{N-n}{k} p^k (1-p)^{N-n-k} \quad (12)$$

Again, if network $i$ chooses "switch", the expected cost (over all possible subgames) of finding a non-occupied band is

$$E[C_i^{switch}] \;=\; c^{f(N,M)} \quad (13)$$

To achieve the Nash equilibrium and to find the optimal values for mixed strategy space, we equate equations (11) and (13). Applying the same deduction that the expected cost for network $i$ starting from any subgame $G'$ is same as that from the original game $G$, we derive $Q_{(n-1)\in\mathcal{N}_{\mathbf{i}},0\in\mathcal{N}_{-\mathbf{i}}} = \frac{1}{1+c^{f(N,M)}}$. Expanding $Q_{(n-1)\in\mathcal{N}_{\mathbf{i}},0\in\mathcal{N}_{-\mathbf{i}}}$, the expression for $p$ becomes

$$p^{n-1} \times (1-p)^{N-n} = \frac{1}{1+c^{f(N,M)}} \quad (14)$$

Solving the equation (14) with numerical analysis and averaging over all possible values of $n$, the mixed strategy Nash equilibrium probability can be found. For any values of $N$ and $M$, we find that $p$ has a non-zero finite value thus proving the existence of mixed strategy Nash equilibrium point even in the generalized MMG.

Note that, in the study so far, we have assumed that $N$ networks operated by multiple wireless service compete among each other, i.e., any network is in the interference range of all other $(N-1)$ networks. However, if all $N$ networks are not in competition with each other in the region, i.e., the networks do not form a completely connected interference constrained graph, the solution though simple and can be derived from the complete competition scenario described before, is worth mentioning. In this case where, network $i$ has $l$ other networks ($l \leq N-1$) in its interference range, the game becomes an *assymetrical modified minority game* (AMMG). The function $f(N,M)$ is modified as $\frac{NM\alpha(l)}{M-N\alpha(l)}$ where, $\alpha(l)$ is a function depending on the number of interfering networks. We assume $0 < \alpha(\cdot) \leq 1$ such that, when $l = N-1$, $\alpha(l) = 1$. The only additional information network $i$ needs in the AMMG to successfully calculate its mixed strategy Nash equilibrium probability $p_i$, is the number of interfering networks of each of the $l$ networks which again can easily be obtained using the foreign beacon broadcasting in IEEE 802.22. Rest of the analysis for AMMG can be carried out in the exact same manner as MMG and mixed strategy Nash equilibrium probability can be found.

## V. Simulation Experiments and Results

We conducted simulation experiments to evaluate the improvements achieved by the proposed mixed strategy. Source code for the experiment has been written in C under Linux environment. We assumed $N$ IEEE 802.22 networks, operated by $N$ separate wireless service providers, compete for one of $M$ available spectrum bands. Each of the networks is associated with a mixed strategy space of "switch" and "stay". The system converges when all the networks capture a spectrum band free of interference from other IEEE 802.22 networks. $N$ and $M$ are given as inputs to the experiment.

In figure 2(a), we present the average system convergence cost with $25$ competing cognitive radio (CR) networks. Switching probability is varied for this simulation experiment and different scenarios of available bands are considered. The inference is that with increase in number of available bands, the convergence cost decreases as claimed earlier through the game analysis. However, the interesting observation is the convex nature of the curves in figure 2(a), proving that a point of minima exists for each of the curve. This minima corresponds to the probability $(p)$ for the mixed strategy Nash equilibrium. Similarly, in figure 2(b), we kept the number of available bands fixed but varied the number of competing networks from $10$ to $25$. Similar convex plots are obtained proving the existence of mixed strategy Nash equilibrium point. Moreover, as proposed in the cost function, the convergence cost increases exponentially with the decrease in difference between $M$ and $N$.

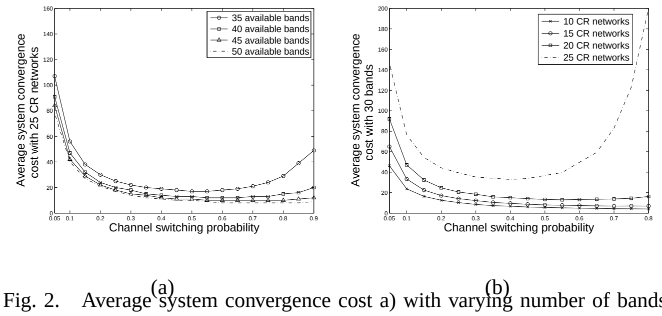



Fig. 2. Average system convergence cost a) with varying number of bands; b) with varying number of competing networks

We show the switching probability (p) for achieving minimized cost Nash equilibrium from the simulation experiments and compare them with that found through game analysis in table III. In figure 3, we plot the comparison results. It is found that the Nash equilibrium probabilities calculated through theoretical analysis corroborates with simulation experiments thus justifying the proposed cost function and MMG model.

| 25 **competing networks** | | |
|---|---|---|
| **Number of available bands** | **Theoretical analysis** | **Simulation experiment** |
| 35 | 0.524421 | 0.53 |
| 40 | 0.602632 | 0.60 |
| 45 | 0.652256 | 0.65 |
| 50 | 0.683970 | 0.69 |

TABLE III
MIXED STRATEGY NASH EQUILIBRIUM PROBABILITY COMPARISON

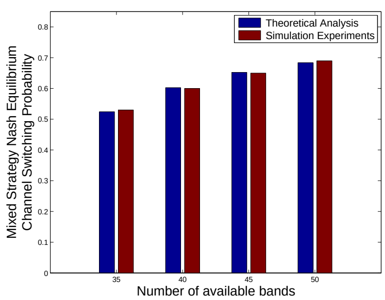



Fig. 3. Mixed strategy Nash equilibrium probabilities with number of bands

Next, in figure 4, the comparison between pure and mixed strategy space mechanisms are conducted. For the experiment, we varied both the number of available bands (from 30 to 70) and number of competing networks; however the network:band ratio were always kept fixed at $70\%$. We find that with increase in number of bands and networks, the mixed strategy space always performs better than the pure strategy space.

Last but not the least, we present different system convergence costs following mixed strategy space for varying network:band ratio ($50\% - 70\%$) in figure 5. We find that with increase in ratio of network:band, the system convergence cost increases. However, the important insight into this convergence cost lies in that fact that the cost does not increase in a simple additive manner; rather in a more complex multiplicative behavior with increase in ratio of number of networks $N$ and number of available bands $M$, justifying the proposed cost function nature in the mathematical analysis.

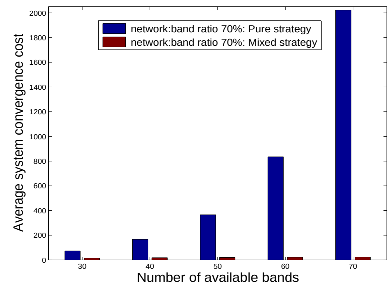



Fig. 4. Average system convergence cost with Pure and Mixed strategy space

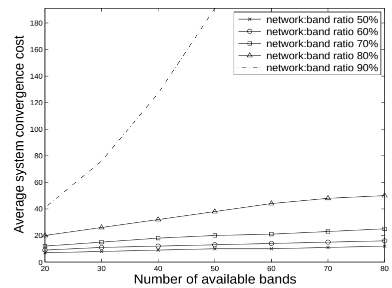



Fig. 5. Average system convergence cost with varying network:band ratio

## VI. CONCLUSIONS

In this research, we investigate the cognitive radio based IEEE 802.22 networks that are being standardized for operation in the under-utilized TV bands. We studied the problem of self–coexistence, i.e., how multiple overlapped IEEE 802.22 networks controlled by different service providers can operate on the available spectrum and coexist. We use modified minority game (MMG) to model the problem. We found that the mixed strategy space for decision making perform better than the pure strategy space in achieving optimal solution. We also proved that the cost (time duration) of finding an unoccupied band follows a complex multiplicative behavior with increase in number of networks $N$ and number of available bands $M$. Simulation results demonstrated that the IEEE 802.22 networks would incur minimum cost by adhering to the calculated mixed strategy switching probability and would achieve the Nash equilibrium.

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