A Game Theory-based Approach for Modelling Mandatory Lane-Changing Behaviour in a Connected Environment

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Abstract

The connected environment provides real-time information about surrounding traffic; such information can be helpful in complex driving manoeuvres, such as lane-changing, that require information about surrounding vehicles. Lane-changing modelling in the connected environment has so far received little attention. This is due to the novelty of connected environment, and the consequent scarcity of data. A behaviourally sound lane-changing model is not even available for the traditional environment; that is, an environment without driving aids. To address this need, this study develops a game theory-based mandatory lane-changing model (AZHW model) for the traditional environment and extends it for the connected environment. The CARRS-Q advanced driving simulator is used to collect high-quality vehicle trajectory data for the connected environment. The developed models (for traditional environment and connected environment) are calibrated using NGSIM and simulator data in a bi-level calibration framework. The performance of the models has been rigorously evaluated using various performance indicators. These include the true positive, false positive, detection rate, false alarm rate, time prediction error, and location prediction error. Results consistently show that the developed game theory-based models can effectively capture mandatory lane-changing decisions with a high degree of accuracy. Furthermore, the performance of the developed AZHW models is compared with representative game theory-based lane-changing models in the literature. The comparative analysis reveals that the AZHW models developed in this study outperform existing models.

Keywords: Lane-changing; Decision-making; Game theory; Connected vehicles; Driving simulator.

1. Introduction

The connected environment is expected to contribute to solving many transport issues related to safety, efficiency, mobility, and environmental impact. The connected environment provides drivers with aids that help them to navigate current and upcoming driving situations, especially the provision of information on possible but unseen hazards. As a result, drivers in a connected environment can make more informed and safer lane-changing decisions. Unfortunately, however, lane-changing decision (LCD) modelling in a connected environment has received little attention. This is due to the novelty of the connected environment, which is not yet in large-scale operation, and to the consequent scarcity of relevant data.

Lane-changing is often performed either to change the driving situation (discretionary lane-changing) or to reach the planned lane position (mandatory lane-changing). Unfortunately, the benefits of lane-changing often come at the cost of neighbouring road users by slowing down the following vehicles in the target lane; this, in turn, causes negative impacts on traffic flow such as breakdowns, capacity drops (Cassidy and Rudjanakanoknad, 2005), stop-and-go oscillations (Ahn and Cassidy, 2007, Zheng et al., 2011), and safety hazards. Due

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to these adverse lane-changing impacts on traffic safety and traffic flow, modelling lane-changing behaviour remains an area of interest. Compared to discretionary lane-changing, mandatory lane-changing's negative impacts on traffic flow and road safety are generally more pronounced because lane-changes (more specifically, forced lane-changes) frequently occur during merging scenario, which have significant impact on traffic stream as mentioned in previous research. Due to the greater disruption caused by its mandatory and forceful nature (Ali et al., 2018), this study focusses on modelling mandatory lane-changing behaviour.

Zheng (2014) points out several issues in LCD modelling literature in general and modelling methodology in particular that need to be addressed. The former includes calibration and validation issues in LCD models. Except for Kang and Rakha (2017), for example, existing studies ignore the impact of a waiting period before mandatory lane-changing (i.e., non-merging events) on the model's performance. Additionally, non-merging events in the data for calibrating the mandatory LCD model are predominant because merging events are relatively rare. This unbalanced representation of merging and non-merging events can affect both model calibration and model validation. Moreover, the performance of a mandatory LCD model is often evaluated using numerical errors such as Mean Absolute Error (MAE) and Root Mean Square Error, and these are inadequate for testing the prediction capability of a model for discrete events such as mandatory lane-changing. Rather, the performance of a mandatory LCD model should be rigorously evaluated using a frequency-based matrix that is capable of reliably testing its accuracy in predicting discrete events.

Apart from general LCD modelling problems, the adopted LCD modelling methodology is another important issue that plays a vital role in capturing mandatory LCD making behaviour of drivers. Driving behaviour varies across different lane-changings (particularly mandatory), which may be classified as free, cooperative, and forced (Hidas, 2002). In the cooperative and forced lane-changings, at least two decision-makers are involved in the decision-making process. For example, in a merging scenario, a merging vehicle either waits or accelerates to attain an acceptable gap, while an immediate follower in the mainline traffic often responds to the situation either by showing courtesy (i.e., by decelerating or changing lanes) or by discouraging the mandatory lane-changing (i.e., accelerating or maintaining their speed as they have the right-of-way). This shows the strong interaction of the merging and the immediate following vehicles during the mandatory LCD making process, as each player's decision depends on the (expected) response of the other. If the decision of either decision-maker is ignored, a collision could result. Unfortunately, most mandatory lane-changing decision models in the literature consider mandatory lane-changing as a one-way decision process by focusing on the lane-changer only. In other words, there is a great need to expand both the behavioural scope and the consistency of the existing approaches to modelling mandatory lane-changing decisions, as concluded in Zheng (2014).

To address this need, this study employs the game theory approach, as this approach has the ability to simultaneously incorporate the decision of two players. Although the game theory approach has been used for modelling mandatory lane-changing decisions in the literature for both traditional environment (Kita, 1999, Liu et al., 2007, Kang and Rakha, 2017) and connected environment (Talebpour et al., 2015, Weng et al., 2016), several important issues are yet to be addressed, as discussed in detail in Section 2.3.

The objective of this paper, therefore, is threefold: (a) to develop a game theory-based mandatory lane-changing model for traditional environment and for connected environment by addressing the aforementioned issues in the previous game theory models; (b) to rigorously test the developed models using more reliable performance indicators; and (c) to compare the performance of the developed models with the existing game theory-based mandatory lane-changing models.

The main contribution of this study is a game theory-based mandatory lane-changing model in a connected environment. The performance of model is assessed using the real data from a connected environment where drivers make decisions with the help of driving aids. The developed model is rigorously tested using two minimisation algorithms and different waiting periods. Result shows behavioural soundness and consistency of the model. Furthermore, the following vehicle's actions are validated for the first time in a game theoretical framework, which provides further insights into prediction capability and efficacy of the proposed game theory approach for modelling mandatory lane-changing behaviour.

The rest of the paper is organised as follows: Section 2 reviews major modelling approaches and game theory for modelling mandatory lane-changing behaviour; Section 3 describes the methodology, including model formulation and payoff matrices; Section 4 explains the data sources, processing, and empirical evidence of strategies; Section 5 presents the model calibration and validation results; Section 6 compares the performances of the developed models with those of the existing game theory-based mandatory lane-changing decision models; Section 7 discusses issues and main findings.

2. Literature review

For the most part, the literature reviewed comprises two main themes: (a) previous mandatory lane-changing decision modelling approaches; and (b) mandatory lane-changing decision modelling using the game theory approach and their main issues. Providing a comprehensive and exhaustive review of lane-changing decision models is beyond the scope of this paper; however, interested readers can refer to Zheng (2014).

2.1. Lane-changing decision modelling approaches

Major lane-changing decision modelling approaches in the literature include rule-based, utility-based, and game theory-based approaches. Gipps (1986) was among the first to develop a rule-based deterministic lane-changing decision model that evaluates the possibility, necessity, and desirability of a lane-changing. This model also considers factors such as the existence of a safety gap, the locations of permanent obstructions, the intent of turning movement, the presence of heavy vehicles, and speed advantage. These factors are evaluated based on a set of sequential deterministic rules according to their importance. When more than one lane is available for the lane-changing, this model selects a lane that is deterministically based on a set of priority rules that depend on factors such as the location of any obstruction, the presence of a heavy vehicle, and speed gain. Gipps' lane-changing model can be viewed as a decision tree that generates a binary outcome (i.e., change lane/not change lane), which is based on various fixed conditions. Due to its deterministic nature, this model fails to incorporate driver heterogeneity, especially under varying traffic conditions and different interactions between the subject vehicle and the surrounding traffic stream. To overcome some of the limitations of Gipps' model, several rule-based models were developed (Yang and Koutsopoulos, 1996, Hidas, 2002, Kesting et al., 2007).

Ahmed et al. (1996) developed a utility-based mandatory lane-changing model that can incorporate driver heterogeneity and state dependence. The lane-changing process in a utility-based approach consists of two steps: target lane selection, and gap acceptance (Toledo et al., 2003). A driver compares the utilities of the available lanes and selects the lane that will best improve his/her driving condition. The gap acceptance in the target lane is evaluated as a binary problem in which a driver decides to accept or reject the available gap by comparing it with the critical (that is, the minimum acceptable) gap. The critical gaps are modelled as random variables to capture the uncertainty associated with decision-making. Many extensions of, and improvements to utility-based models can be found in the literature (Toledo et al., 2005, Choudhury et al., 2006, Toledo and Katz, 2009).

Game theory-based approaches incorporate the decisions of the lane-changer and the immediate follower in the target lane in a competing situation, where the outcome of one decision-maker depends on the actions of the other. The game theory approach captures the complexity of human behaviour, and determines the optimal outcome from a set of choices by analysing the cost and benefit to each player as they compete. Exploiting the inherent capability of game theory, Arbis et al. (2016) study drivers' interactions at a signalised intersection. The ensuing sub-section further explains the game theory approach in the context of mandatory lane-changing decision modelling. The ensuing sub-section further explains the game theory approach in the context of mandatory lane-changing decision modelling.

Besides these widely used approaches, researchers have adopted several other approaches to modelling lane-changing decision behaviour; e.g., artificial intelligence technique (Moridpour et al., 2009), cellular automata (Maerivoet and De Moor, 2005), markov process (Toledo and Katz, 2009), and hazard-based models (Hamdar, 2009).

2.2. The game theory approach to mandatory lane-changing decision modelling

As one of the first studies that considers merging as a two-player non-zero-sum non-cooperative game (that is, where both players do not cooperate, and their aggregate gain or loss is not equal to zero), Kita (1999) developed a merging model using the game theory approach. Each player has two strategies: the strategies for the subject vehicle (SV) are merging and waiting, whilst the strategies for the following vehicle (FV) are giving way or not. This model was based on the safety criterion of time-to-collision, and was calibrated using the maximum likelihood method. Although the developed model shows promise, it does not consider the remaining distance in the acceleration lane, a critical factor in the merging process. Additionally, speed is kept constant during the merging process in this model, and this is unrealistic (Liu et al., 2007). Moreover, Kita considers time-to-collision as the payoff for both players. This results in unrealistic Nash equilibria, as both players are similarly affected by time-to-collision.

Liu et al. (2007) present an enhanced game theoretic model for the merging situation where a merging vehicle's motive is to minimise the time spent in the acceleration lane without causing a collision, whilst a through vehicle's objective is to minimise interruption to its speed. Strategies in the model of Liu et al. (2007) are similar to those in the model of Kita (1999). However, Liu et al. (2007) propose robust payoff matrices for defining the strategies. To solve the game, a bi-level calibration framework was used, with the upper level as an ordinary least square problem, and the lower level as a linear complementarity problem for finding the Nash equilibrium. The units of payoff for both players are different, resulting in a trivial equilibrium solution. In addition, the acceleration of the merging vehicle in a merging strategy has not been explicitly considered, and the speed variation of the following vehicle during the yield strategy is ignored.

Wang et al. (2015) presented a game theory-based lane-changing control approach for connected and automated vehicles in which a lane-changing game can be formulated as non-cooperative as well as cooperative. This study does not predefine a set of finite strategies, but rather evaluates different combination of lane-change time and acceleration in a prediction time window to optimise some performance function (own cost and collective cost). Results indicate that the proposed control approach reasonably generates future lane-changing decisions whilst maintaining drivers' safety and comfort.

Talebpour et al. (2015) pioneered the game theory approach to modelling lane-changing in a connected environment by applying the Harsanyi transformation (Harsanyi, 1967) to transform the game from incomplete information to imperfect information. They developed a generic model for both mandatory and discretionary lane-changing. However, their study does not explicitly define the formulation of payoff matrices, and does not consider same payoff units for both players. All of these factors might have resulted in a high prediction error in their model in replicating observed mandatory lane-changing behaviour. In contrast, the game theory-based mandatory lane-changing model in a connected environment is calibrated and validated using NGSIM data, which do not have merging events in a connected environment.

Kang and Rakha (2017) model merging behaviour as a two-player game in which the merging vehicle has three strategies: merging, waiting, and overtaking, and the strategies for the following vehicle are the same as in previous studies (Kita, 1999, Liu et al., 2007). Payoffs with different units are formulated on the basis of safety, expected travel time and efficiency, and acceleration. They adopt the calibration approach proposed in Liu et al. (2007), and report their model's effective performance.

Arbis and Dixit (2019) recently modelled mandatory lane-changing behaviour in a traditional environment using game theory approach by incorporating conflict risks into utilities of the players, and reported that longer acceleration lanes and reduced speed limits tend to reduce the likelihood of a conflict.

2.3. Issues in the previous studies using the game theory approach

A thorough literature review of previous game theory-based mandatory lane-changing models revealed several important issues in the previous studies that are yet to be addressed, as discussed below.

First, following vehicle strategies that they consider are either incomplete or improperly defined. For example, some studies only consider the yield/give way strategy (Kita, 1999, Liu et al., 2007, Kang and Rakha, 2017) and ignore other strategies such as doing nothing (Talebpour et al., 2015). It has been frequently observed in the field that drivers of following vehicles remain unaffected (or maintain their speed as they have the right-of-way) by the lane-changing action of merging vehicles, considering such action as safe. Thus, it is necessary to capture this behaviour of following vehicles to realistically mimic the mandatory lane-changing decision-making process. Furthermore, some studies consider changing lane as a new strategy (Talebpour et al., 2015) whereas it is more appropriate to treat changing lane by the immediate follower as a new game, as the following vehicle would need to play a game with players in the adjacent lane for changing lanes, which requires a separate formulation and thus, should not be considered as a strategy in the merging game. In addition, the field data show that changing lane strategy is rarely selected, which implies that due to insufficient data, reliably estimating parameters for this strategy would be difficult. This brings another shortcoming of many existing studies, i.e., the lack of empirical evidence for the selected strategies in the field data. Mandatory lane-changing strategies in the previous studies are not extracted from, or verified by field observations. This is important because: (a) different strategies should be separated based on their availability in the data; and (b) each strategy must have a reasonable sample size for calibration purpose. Ignoring this aspect in game theory modelling process would lead to unrealistic and biased parameter estimates of the model.

Second, with the exception of Liu et al. (2007) and Kang and Rakha (2017), the formulation of payoff matrices is not explicitly defined. However, the payoff units for two players are different in these two studies, and this can result in trivial equilibrium solutions, which may not be realistic in representing drivers' selection from a set of choices.

Third, the previous studies did not fully utilise the advantage of using game theory approach as they only focused on actions of the merging vehicle during the model validation process whilst ignoring the actions of the following vehicle. As the game theory approach simultaneously evaluates the decisions of two players, it is important to consider the actions of both merging and following vehicles when assessing the model's performance.

Finally, and more importantly, the developed model for connected environment in previous studies is either tested by using NGSIM data (which do not contain any data for connected environment) or by numerical simulation, and this can be unrealistic or (even) misleading. NGSIM data do not contain decisions under the connected environment whilst numerical simulations ignore human factor involved in mandatory lane-changing decision-making.

In summary, all of the aforementioned approaches (except game theory) consider mandatory lane-changing as one-way decision-making process focusing on decision of lane-changer whilst ignoring the corresponding action of following vehicle in response to merging action of the subject vehicle. In a typical mandatory lane-changing scenario, at least two drivers are engaged in decision-making process affecting each other's decision. Ignoring decision of any decision maker could result in a crash. In addition, by reviewing the literature, many issues in existing game-theory based mandatory lane-changing models are identified. Addressing these issues is critical for developing more realistic and behaviourally sound mandatory lane-changing models.

3. Methodology

3.1. Game and its components

This study first develops a mandatory lane-changing decision model for traditional environment (LCD_TE hereon), and this is then used as the foundation for developing the mandatory lane-changing model in a connected environment (LCD_CE hereon). (Note that the mandatory lane-changing model developed in this study is referred as AZHW model and LCD_TE, LCD_CE, two-strategy, and three-strategy models are mainly the variation of the AZHW model.) In the traditional environment drivers perform driving tasks without driving aids. The connected environment, in contrast, provides driving aids for assisting drivers to perform merging manoeuvres. A typical merging scenario, as shown in Figure 1, includes a merging vehicle (subject vehicle [SV] in this study) on the acceleration lane; an immediately following/lag vehicle (FV) on the target lane; and (possibly) a lead vehicle (LV) on the target lane. In the merging scenario, both SV and FV are assumed to act in a rational way. This situation can be modelled as a game with various components such as a number of players, player strategies, payoff matrix, and the cooperative or non-cooperative nature of the game.

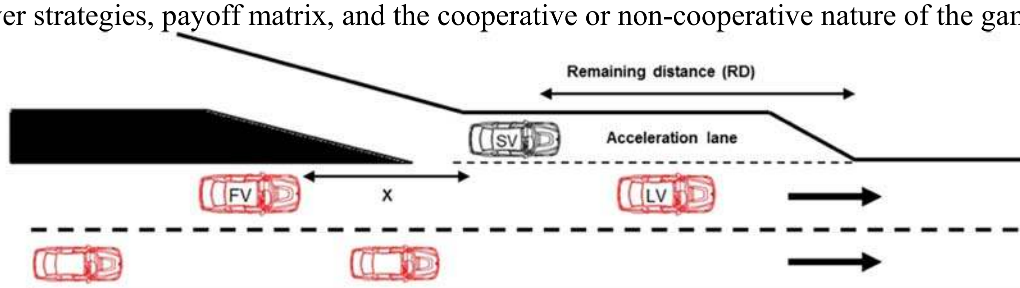


Fig. 1. A typical merging scenario

This study considers a two-player non-zero-sum non-cooperative game under incomplete information for the LCD_TE model, where the two players in a game are SV and FV. The interaction between these vehicles is dominant, and the effect of other vehicles (for example, the effect of LV) can be implicitly taken into FV's payoff (more discussion on this in ensuing sub-sections). A non-zero-sum game refers to a game in which all players receive a payoff corresponding to their actions and the sum of their payoffs is not zero. In non-cooperative games under incomplete information, a player has inaccurate information about another player's strategy (i.e., a game in a traditional environment where players make assumptions/predictions of each other's actions).

One of the shortcomings of earlier studies is the incomplete formulation of strategies. This study, in contrast, formulates a comprehensive list of strategies for both players, as shown in Table 1. The strategies for SV are merging and waiting, while FV has four strategies: accelerating, decelerating, doing nothing, and changing lane. Note that this study does not consider overtaking behaviour of SV, which is considered by Kang and Rakha (2017), because overtaking in a high density traffic is rare; in addition, overtaking is prohibited during merging manoeuvres in Queensland, Australia. Table 1 represents a merging game that can be either played in a traditional environment or in a connected environment.

Table 1. Merging game for traditional and connected environments in the normal form

FV	SV				Probability for FV
	Traditional environment		Connected environment		
	Merging (S_1)	Waiting (S_2)	Merging (S_1)	Waiting (S_2)	
Accelerating/Forced merging (F_1)	P_{11}, Q_{11}	P_{12}, Q_{12}	S_{11}, R_{11}	S_{12}, R_{12}	y_1
Decelerating/Courtesy yielding (F_2)	P_{21}, Q_{21}	P_{22}, Q_{22}	S_{21}, R_{21}	S_{22}, R_{22}	y_2
Doing nothing (F_3)	P_{31}, Q_{31}	P_{32}, Q_{32}	S_{31}, R_{31}	S_{32}, R_{32}	y_3
Changing lane (F_4)	P_{41}, Q_{41}	P_{42}, Q_{42}	S_{41}, R_{41}	S_{42}, R_{42}	y_4
Probability for SV	z_1	z_2	z_1	z_2	

P and Q respectively denote the payoffs for SV, FV in the traditional environment and the corresponding payoffs in the connected environments are respectively S and R; y and z are probabilities of FV's and SV's action, respectively.

The study adopts the Harsanyi transformation (Harsanyi, 1967), which transforms a game of incomplete information (that is, a game in a traditional environment) to a game of imperfect[2] information (that is, a game in a connected environment). Furthermore, the Harsanyi transformation introduces "nature" as a player who chooses the type of each player. Nature's role can be perceived as another player in the game with no payoffs. Nature's choice can be represented by a game tree as shown in Figure 2. Edges coming from a nature's choice node are labelled with the probability of the event that occurs. Without loss of generality, assume that nature selects the driver of SV who is playing a game in the traditional environment. Following the approach presented by Talebpour et al. (2015), the game can be transformed into an extensive form, as shown in Figure 2, which indicates that nature first selects the driver of SV in traditional environment with probability (p), and the driver of SV in the connected environment with probability ($1-p$). (Note that nature can select driver of FV as well, however, for simplicity and explanation purpose, the case of SV is reported herein.) It should be noted that these probabilities are from FV's perspective, and both players have similar information about these probabilities. However, SV perceives nature's move and has information about the selection of strategy, whilst FV is unaware of nature's move. After applying this transformation, and combining the transformed game into a normal form, SV will have four strategy, whilst FV will have 16 action sets, as it can be seen in the figure below (see Table 2 for more details).

[2] In an imperfect information game, players are simply unaware of actions chosen by each player; however, each player knows who the other player is in the game, his/her possible strategies, etc (Harsanyi, 1967). Such information is (directly or indirectly) provided by the connected environment.

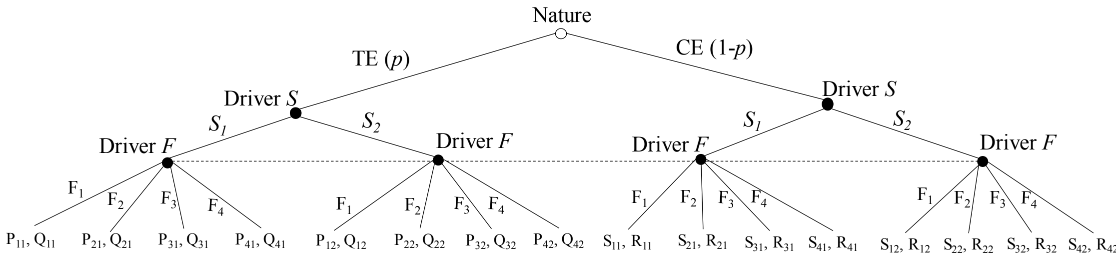


Fig. 2. Transformed merging game in the extensive form; TE: Traditional environment; CE: Connected environment

Table 2. Transformed merging game in the normal form

Strategy FV	SV $\boldsymbol{S_1^{TE}S_1^{CE}}$	$\boldsymbol{S_1^{TE}S_2^{CE}}$	$\boldsymbol{S_2^{TE}S_1^{CE}}$	$\boldsymbol{S_2^{TE}S_2^{CE}}$
F_1 (Accelerate/ Forced yielding)	$(pP_{11} + (1-p)S_{11}, pQ_{11} + (1-p)R_{11})$	$(pP_{11} + (1-p)S_{12}, pQ_{11} + (1-p)R_{12})$	$(pP_{12} + (1-p)S_{11}, pQ_{12} + (1-p)R_{11})$	$(P_{12} + (1-p)S_{12}, pQ_{12} + (1-p)R_{12})$
F_2 (Decelerate)	$(pP_{21} + (1-p)S_{21}, pQ_{21} + (1-p)R_{21})$	$(pP_{21} + (1-p)S_{22}, pQ_{21} + (1-p)R_{22})$	$(pP_{22} + (1-p)S_{21}, pQ_{22} + (1-p)R_{21})$	$(P_{22} + (1-p)S_{22}, pQ_{22} + (1-p)R_{22})$
F_3 (Doing nothing)	$(pP_{31} + (1-p)S_{31}, pQ_{31} + (1-p)R_{31})$	$(pP_{31} + (1-p)S_{32}, pQ_{31} + (1-p)R_{32})$	$(pP_{32} + (1-p)S_{31}, pQ_{32} + (1-p)R_{31})$	$(P_{32} + (1-p)S_{32}, pQ_{32} + (1-p)R_{32})$
F_4 (Changing lane)	$(pP_{41} + (1-p)S_{41}, pQ_{41} + (1-p)R_{41})$	$(pP_{41} + (1-p)S_{42}, pQ_{41} + (1-p)R_{42})$	$(pP_{42} + (1-p)S_{41}, pQ_{42} + (1-p)R_{41})$	$(P_{42} + (1-p)S_{42}, pQ_{42} + (1-p)R_{42})$

$S_i^{TE}S_j^{CE}$ (i, j =1, 2) shows that SV performs mandatory lane-changing in the traditional or connected environments based on the nature's move.

The Harsanyi transformation can be applied to any problem where one phenomenon may have two or more options. Talebpour et al. (2015), for instance, applied Harsanyi transformation to lane-changing types, i.e., mandatory and discretionary lane-changing. The nature was introduced into the game and selected the mandatory lane-changing with probability (*p*) and discretionary lane-changing with probability (1-*p*).

The Harsanyi transformation also states that an incomplete information game (i.e., a game in traditional environment) is Bayes equivalent to a game of imperfect information (i.e., a game in connected environment) if strategies space, and payoffs are the same; however, the attribute vectors are different, and the model needs to be reinterpreted. In the context of this study, connected environment data (obtained from the advanced driving simulator) contain additional attributes (i.e., additional advisory information); and merging decisions are obviously influenced by the connected environment, and will be used to reinterpret the LCD_CE model.

In the connected environment, drivers receive an uninterrupted supply of information leading to more informed and safer merging decisions. The drivers in the connected environment also have the information about the nature of actions of the following vehicles (traditional environment game versus connected environment game). As such, the connected environment game in a normal form can be formulated as non-zero-sum non-cooperative game, and the structure of the game can be seen in Table 3. (Note that although connected environment provides information about the surrounding traffic, the game still remains non-cooperative[3] because the connected environment does not force the drivers to act in a cooperative way and decision is still at the discretion of drivers either to accept or ignore the information.)

Table 3. A merging game in the connected environment in the normal form

SV	FV			
	Accelerating/Forced yielding (F_1)	Decelerating/Courtesy yielding (F_2)	Doing nothing (F_3)	Changing lane (F_4)
Merging (S_1)	S_{11}, Q_{11} or R_{11}	S_{21}, Q_{21} or R_{21}	S_{31}, Q_{31} or R_{31}	S_{41}, Q_{41} or R_{41}
Waiting (S_2)	S_{12}, Q_{12} or R_{12}	S_{22}, Q_{22} or R_{22}	S_{32}, Q_{32} or R_{32}	S_{42}, Q_{42} or R_{42}

Each player selects one of the strategies to achieve the goal of a game (Kang and Rakha, 2017). However, finding the entire set of optimal/best strategies remains an area of research in the field of economics (Talebpour et al., 2015). To determine the entire set of best responses, the concept of Nash equilibrium is utilised. This is a solution point where no player can unilaterally gain more than his/her expected payoff by changing his individual strategy to another. In a two-player game, consider that player *a* (i.e., SV) has two strategies – $S = (S_1, S_2)$ – and player *b* (i.e., FV) has four strategies: $F = (F_1, F_2, F_3, \text{and } F_4)$. This suggests that this game has eight possible sets of strategies, and the Nash equilibrium can be defined as:

$$\begin{cases} E_1(S^*, F^*) \geq E_1(S, F^*) \\ E_2(S^*, F^*) \geq E_2(S^*, F) \end{cases} \tag{1}$$

where, E_1 and E_2 represent the expected payoff at equilibrium, and $\boldsymbol{S}^*$ and $\boldsymbol{F}^*$ are the equilibrium set of strategies for SV and FV, respectively. The solution approach and solution of the game are presented in Section 5.

3.2. Payoff formulations

Earlier studies consider various motives for payoff formulations of players in a game. This leads to different units in the payoffs of different players (Liu et al., 2007, Talebpour et al., 2015, Kang and Rakha, 2017), and results in trivial and unrealistic equilibrium solutions. Therefore, in this study, the payoffs of both players are formulated by using the same motive for acceleration. More specifically, the payoff for SV is defined as the acceleration required for merging or for waiting for the next available gap, while the payoff for FV is defined as the acceleration required to avoid a collision (i.e., forced yielding); showing courtesy (i.e., deceleration and changing lanes); or doing nothing in response to SV's action. Moreover, in a typical merging scenario, the following assumptions are made: (a) prior to the merging event, FV and lead vehicle are in car-following mode; (b) both players (SV and FV) construct their respective payoffs as soon as SV appears on the acceleration lane; and (c) the distance between SV and FV is less than 60 m. Vehicles beyond this range are normally unaffected by each other's decision (Toledo et al., 2003, Liu et al., 2007). The time when both players construct their respective payoff matrices is termed *decision time* (that is, the time when SV appears in the acceleration lane).

[3] A key difference between cooperative and non-cooperative game is that in cooperative games, players can make binding agreements before playing the game, e.g., how to share payoffs. On the other hand, agreements are not binding in non-cooperative games. The individual players are the cornerstone in non-cooperative games whilst cooperative games consider coalition of players (d'Aspremont and Jacquemin, 1988, Dockner and Van Long, 1993).

3.2.1. Payoffs for FV

At decision time, FV needs to decide their action in response to SV's action. For FV, who has the right-of-way over SV, the available strategies are: accelerating to avoid merging; decelerating or changing lane to show courtesy; and remaining unaffected by SV's action (i.e., doing nothing). Similar to Talebpour et al. (2015) study, this study assumes that maintaining safety and minimising speed variations are FV's two main motives. For the forced merging case, the driver of FV prioritises safety and adopts acceleration to avoid a collision with SV. To show courtesy (by decelerating and changing lanes), FV calculates the required deceleration or change in speed.

Table 4 shows the payoff matrix for FV. In this table, *Acc* stands for acceleration; subscripts *M* and *W* represent merging and waiting, respectively; subscripts *A, D, DN,* and *CL* respectively indicate acceleration, deceleration, doing nothing, and changing lane; $Acc_{FV}^{LV\,TL}$ is the acceleration required for FV, considering the lead vehicle in the target lane as a new leader; $Acc_{FV}^{FV\,TL}$ is the acceleration required for FV in the target lane, considering FV (i.e., lane-changer) as the new lead vehicle in the target lane; ΔV is speed change; G is the available gap in the adjacent lane; ε & δ represent the error terms that capture the unobserved variation, and are assumed to follow a standard normal distribution, N ~ (0,1); and α & β are parameters to be estimated.

For the purpose of illustration, consider a case where SV decides to merge straight away and FV is accelerating; FV has to brake hard to avoid a collision with SV. The projected required acceleration is shown in Equation (2). The initial states and projected states are calculated using Newtonian equations, which are similar to those in Liu et al. (2007), and are explained in Appendix A. (Note that the formulae for calculating each variable [in the payoffs] are also explained in Appendix A.)

Table 4. Payoff matrices for FV and SV

Players			SV	
		Strategies	Merging (S_1)	Waiting (S_2)
Payoff for FV	FV	Accelerating (F_1)	$Q_{11} = \alpha_{11}^0 + \alpha_{11}^1 Acc'_{M-A} + \varepsilon_{11}$	$Q_{12} = \alpha_{12}^0 + \alpha_{12}^1 Acc_{W-A} + \varepsilon_{12}$
		Decelerating (F_2)	$Q_{21} = \alpha_{21}^0 + \alpha_{21}^1 Acc_{M-D} + \varepsilon_{21}$	$Q_{22} = \alpha_{22}^0 + \alpha_{22}^1 Acc_{W-D} + \varepsilon_{22}$
		Doing nothing (F_3)	$Q_{31} = \alpha_{31}^0 + \alpha_{31}^1 Acc_{M-DN} + \varepsilon_{31}$	$Q_{32} = \alpha_{32}^0 + \alpha_{32}^1 Acc_{W-DN} + \varepsilon_{32}$
		Changing lane (F_4)	$Q_{41} or\ Q_{42} = \alpha_{41}^0 + \alpha_{41}^1 Acc_{FV}^{LV\,TL} + \alpha_{41}^2 Acc_{FV}^{FV\,TL} + \alpha_{41}^3 \Delta V + \alpha_{41}^4 G + \varepsilon_{41}$	
Payoff for SV	FV	Accelerating (F_1)	$P_{11} = \beta_{11}^0 + \beta_{11}^1 Acc_{M-A} + \delta_{11}$	$P_{12} = \beta_{12}^0 + \beta_{12}^1 Acc_{W-A} + \delta_{12}$
		Decelerating (F_2)	$P_{21} = \beta_{21}^0 + \beta_{21}^1 Acc_{M-D} + \delta_{21}$	$P_{22} = \beta_{22}^0 + \beta_{22}^1 Acc_{W-D} + \delta_{22}$
		Doing nothing (F_3)	$P_{31} = \beta_{31}^0 + \beta_{31}^1 Acc_{M-DN} + \delta_{31}$	$P_{32} = \beta_{32}^0 + +\beta_{32}^1 Acc_{W-DN} + \delta_{32}$
		Changing lane (F_4)	$P_{41} = \beta_{41}^0 + +\beta_{41}^1 Acc_{M-LC} + \delta_{41}$	$P_{42} = \beta_{42}^0 + \beta_{42}^1 Acc_{W-LC} + \delta_{42}$

Note that $\alpha_{12}^1 \neq \alpha_{22}^1 \neq \alpha_{32}^1$ *as it is expected that FV places different weights to different scenarios. Similarly,* $\beta_{21}^1 \neq \beta_{41}^1$, $\beta_{22}^1 \neq \beta_{42}^1$.

$$Acc_{M-A} = \frac{2(X' - v'_{FV} t_b)}{t_b^2} \quad (2)$$

$$Acc'_{M-A} = minimum \begin{cases} Acc_{M-A},\ if\ v_{SV} \sim v_{FV} \\ -4.5\ m/s^2 \quad ,if\ v_{SV} \ll v_{FV} \end{cases} \quad (3)$$

$$Acc_{M-D} = minimum\ (Acc_{M-D}, -3\ m/s^2) \quad (4)$$

$$Acc_{M-DN} = \begin{cases} Acc_{M-A}\ , \ speed\ of\ SV < speed\ of\ FV \\ Acc_{FV}\ OR\ v_{FV}\ , \qquad\qquad Otherwise \end{cases} \tag{5}$$

Where v'_{FV} is the projected state (refer to appendix A) of FV; t_b is the time taken by FV to react (i.e., 2 s (AUSTROADS, 1993)); X' is a gap between SV and FV when the former joins the through traffic.

In the merging case, FV's motion is governed by the leader in the current state of car-following. Also, there are two existing conditions (Equation 3) based on the speed of SV: (a) if the speed of SV is equal or close to the speed of FV, FV will adopt an acceleration (Acc_{M-A}), using the projected states; and (b) if the speed of SV is slower than FV but SV wants to merge anyway, FV will need to brake hard to avoid a collision.

Another option for FV is to show courtesy early by decelerating or changing lanes. For the deceleration case, the payoff of FV is Q_{21}, as shown in Table 4. In this case, FV signals that SV can merge by adopting a comfortable deceleration (Acc_{M-D}), obtained from Equation (4). However, even if a lower deceleration rate is required to avoid a collision, FV would adopt a higher deceleration rate.

Meanwhile, FV can also decide to remain unaffected by SV (i.e., do nothing), given that FV has the right-of-way. This leads to two situations (refer to Equation 5): (a) if the speed of SV is greater than the speed of FV, SV will merge without causing any disruption to FV; and (b) if the speed of SV is lower than the speed of FV, SV will cause FV to decelerate. In such a scenario, FV's payoff will be Q_{31}, as shown in Table 4.

FV can also choose to change lanes in response to SV's merging attempt. Then FV will need to calculate speed change (ΔV) and the available gap (G) in the target lane. The payoff for FV will be Q_{41} (Table 4).

For the cases where SV decides to wait for the next available gap, possible scenarios and their corresponding payoff matrices can be obtained in a similar way. (Refer to Table 4 for details.)

3.2.2. Payoffs for SV

At the decision time, SV decides either to merge into the through traffic or to wait for the next available gap. Table 4 shows the payoffs for SV that are calculated according to the initial and the projected states of both vehicles. (For a detail description of payoffs, refer to Appendix B.)

For the purpose of illustration, consider a case where SV decides to merge right away, and FV accelerates to avoid the merging. In this case, SV has to increase acceleration to reach the merging point prior to FV to avoid a collision (Acc_{M-A}, refer to SV's payoff in Table 4). On the other hand, if FV shows early courtesy by decelerating, and SV still decides to merge, SV will merge with a comfortable amount of acceleration (Acc_{M-D}). As shown in Table 4, SV's payoff will be P_{21}.

When SV has the right-of-way and FV is neither accelerating nor decelerating but continuing its current state as dictated by the leader (i.e., doing nothing), SV has a similar payoff to that achieved by acceleration. SV has to calculate the acceleration (Acc_{M-DN}) needed to avoid a collision with FV, and its payoff will be P_{31}. Another option for FV is to show courtesy by changing lanes. In this case, SV has to calculate the acceleration (Acc_{M-LC}) required for the merge, and its corresponding payoff will be P_{41}.

For the case where SV decides to wait for the next available gap, SV calculates their acceleration with respect to the remaining distance in the acceleration lane. The possible scenarios, and their corresponding payoff matrices, can be obtained in a similar manner. (For more details, refer to Table 4.)

4. Data sources and processing

4.1 Data

1) NGSIM Data

The celebrated NGSIM data are used (FHWA, 2007) to calibrate and validate the LCD_TE model. This data contains vehicle speeds and positions for every 0.1 s. Montanino and Punzo (2015), however, report the inaccuracy of NGSIM data for microscopic models, and propose a methodology for reconstructing the data. They also applied the proposed methodology to reconstruct 15 minutes I-80 data-1 (from 4.00 pm to 4.15 pm). Thus, this study first uses the I-80 reconstructed data (I-80-R hereon) to assess the behavioural soundness and consistency of the LCD_TE model. Later, the full I-80 (i.e., 45 mins) data (represented as I-80-F from here onwards), denoised by Zheng et al. (2013), is used for the LCD_TE model calibration and validation. Figure 3 shows the study site that features an on-ramp and an off-ramp, where many mandatory lane-changings are expected. For the specific purpose of this study, however, only lane-changings from the on-ramp merge to the freeway are considered. From NGSIM database, it can be determined when SV is not performing merging. Such instances are termed as “waiting” or “non-merging events” in this study. (Note that in such cases no merging point is observed, and model’s predictive capability is assessed against non-merging events.)

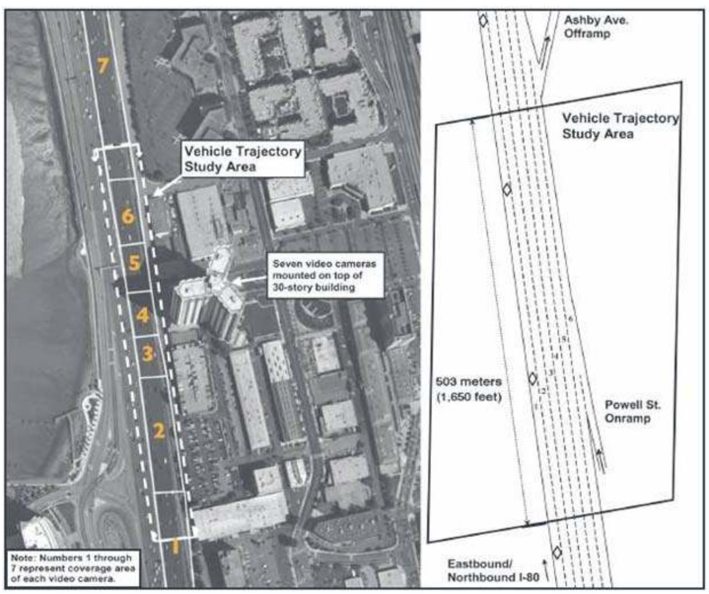


Fig. 3. I-80 study site

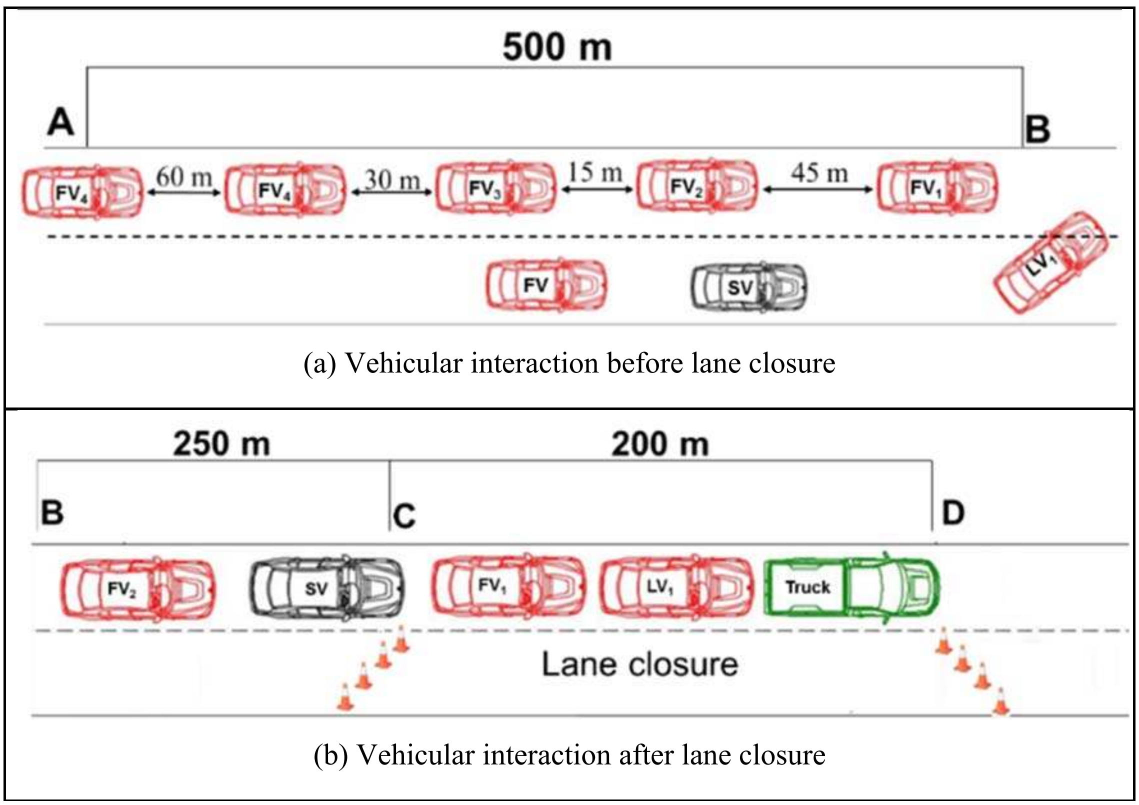
2) Data collected from the advanced driving simulator experiment

As a powerful tool for collecting data in a controlled environment, driving simulators are regularly used to study traffic-related road issues. In this study, an advanced high-fidelity driving simulator experiment was designed and conducted for connected environment consisting of a mandatory lane-changing scenario. Seventy-eight participants of diverse backgrounds were recruited to drive in two randomised driving conditions: baseline (i.e., without driving aids), and connected environment (i.e., with driving aids). The mean age of the participants was 30.8 years, and 64.1% were male. Data were collected in the form of vehicle trajectories and advisory information at every 0.05 s. (More details of the participants and the advanced driving simulator that were presented at the Centre for Accident Research and Road Safety-Queensland [CARRS-Q] can be seen in Ali et al. (2018).)

a) Experiment design

A four-lane motorway with two lanes in each direction was designed. It had a posted speed limit of 100 km/h, and was about 1 km in length. The experiment consisted of a mandatory lane-changing scenario where the current driving lane was closed. Following the game theory approach, SV needed to change lanes, whilst FVs were scripted to accelerate, decelerate, or remain unaffected by SV's merging attempts. (Note that FVs are programmed for data collection purpose and their behaviour is treated as the same observed in NGSIM database and no prior information of programmed vehicles is used for LCD modelling.) Since the roadway segment consisted of two lanes in each direction to avoid complexity in designing vehicular interactions, the lane-changing manoeuvre of FV, when SV is merging, is very unlikely. Thus, this strategy is not observed in simulator data. The vehicular interaction and the design of connectivity are explained below.

Baseline scenario: Each participant drove the simulator vehicle without driving aids, and faced a lane closure 750 m from the start of the scenario (Figure 4). A lead vehicle (LV_1) in front of SV, and five FVs on the adjacent lane, surround SV. At point B (Figure 4a), LV_1 changed lane due to the lane closure and, at this instant, following the game theory approach, SV faces five mandatory lane-changing opportunities. SV can choose any gap between FVs in the target lane to avoid the lane closure (Figure 4b), and FVs will then follow SV with a predefined speed.

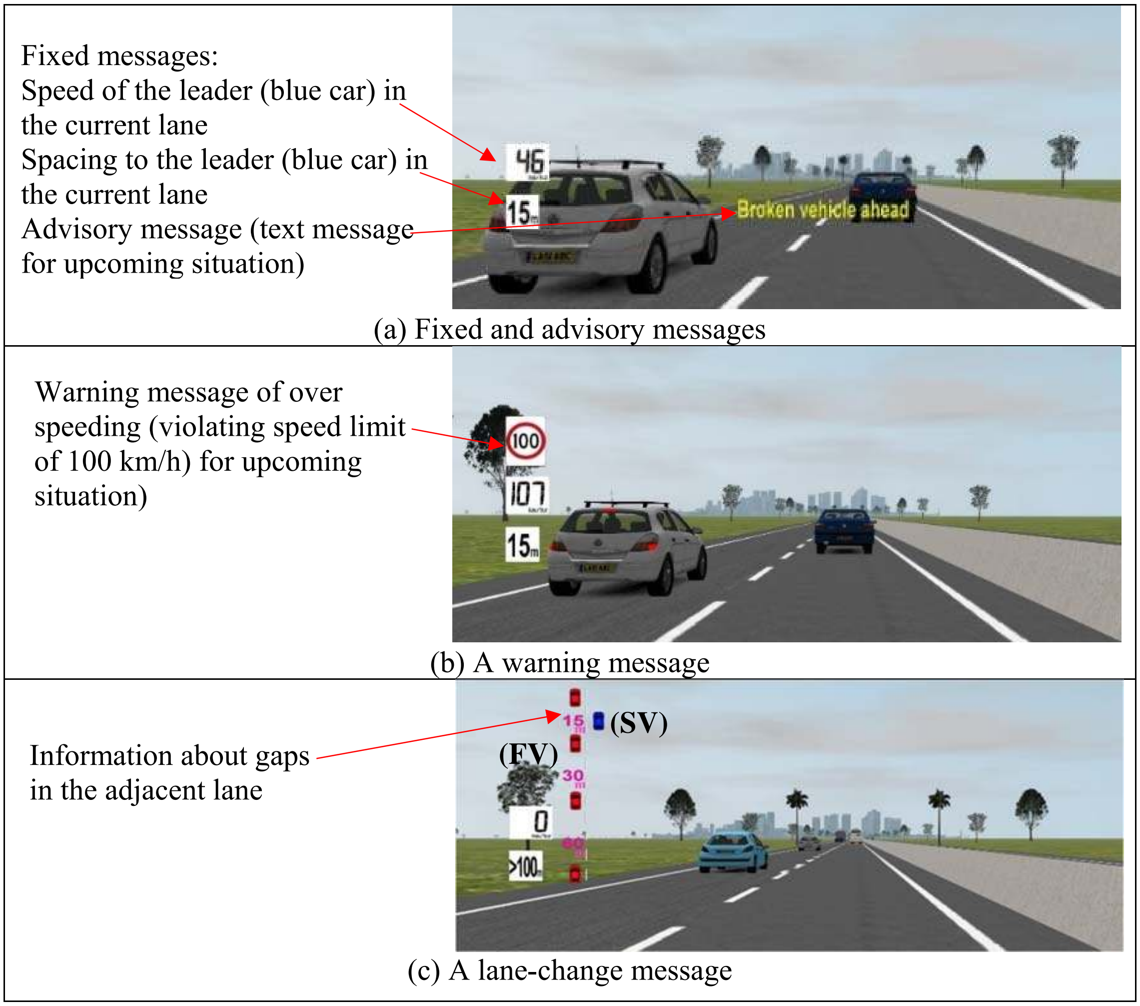


(a) Vehicular interaction before lane closure

(b) Vehicular interaction after lane closure

Fig. 4. Design of mandatory lane-changing events (not to scale)

Connected environment scenario: The vehicular interactions in the connected environment scenario were the same as those in the baseline scenario; however, in the former scenario, participants were assisted with information about upcoming situations. After comprehensively reviewing the literature on in-vehicle information systems and how driving aids are currently provided by major car manufacturers, participants in the connected environment scenario were provided with four types of assisted driving aids: fixed messages, warning messages, advisory messages, and lane-changing messages. (The design of some driving aids related to mandatory lane-changing is presented in Figure 5.)



(a) Fixed and advisory messages
(b) A warning message
(c) A lane-change message

Fig. 5. Driving aids presented on the windscreen

Fixed messages continuously appeared on the left corner of the driving screen, and informed the driver of the speed and distance to LV on the current driving lane (Figure 5a; the leader is the blue car on the right lane). An advisory message, in text form – such as "Broken vehicle ahead" – was presented at the bottom of the screen (with a beep sound) to warn of any upcoming situation (Figure 5a). Warning messages flashed up (with three beep sounds) to flag a hazardous or critical situation, such as over-speeding (Figure 5b) of SV. To assist in mandatory lane-changing decision-making, a lane-changing image appeared on the left corner of the driving screen, and was accompanied by a beep sound (Figure 5c) whenever a lane-changing opportunity was available in the adjacent lane. The suitability of these messages had been tested and confirmed during the pilot study. Furthermore, prior to driving in a research drive, the participants were briefed about the design of messages, and performed a practice drive to become familiar with the vehicle, the driving environment, and the information design. (The participant experiment protocol can be found in Ali et al. (2018).)

To minimise the effect of any learning, the study implemented several strategies, such as randomising the order of the scenarios, and changing the surrounding environment (e.g., colour of FVs, vehicle type, and nature of the road blockage) for each drive. In addition, after each drive, the participants were required to take a short break. (See Ali et al. (2018), for more information on this strategy.)

4.1.1. Balance between merging and non-merging events

To develop a mandatory lane-changing decision model, the data need to include both merging and non-merging events. The decision horizon for non-merging events needs to be carefully selected to avoid the dominance of non-merging events. Note that the decision horizon refers to the portion of trajectory prior to merging in which the merge decision process likely starts. For example, the decision horizon of 2 s indicates one merging and one non-merging events are selected, and as the decision horizon increases, more and more non-merging events are included. A balance between merging and non-merging events in the data is necessary for evaluating the behavioural soundness and consistency of mandatory lane-changing decision models because this balance impacts the model calibration and validation results. However, with the exception of Kang and Rakha (2017), existing studies ignore this important aspect. Thus, this study uses the first 5 s data immediately before the merging to maintain a reasonable proportion of both events (i.e., merging and non-merging). A sensitivity analysis is performed by varying the waiting period prior to the merging as 2 s, 5 s, 10 s, and the entire waiting period. Results show that 5 s data before the merging give reasonable prediction for both merging and non-merging events. It also reveals that: (i) a driver started to actively seek a merging opportunity in this 5 s period; and (ii) a driver's decision time window was approximately 2 s. In modelling driving behaviour, this (latter) measure is widely adopted as the reaction time (Sagberg and Bjørnskau, 2006, Rakha et al., 2008), and is also consistent with the Australian Road Standards (AUSTROADS, 1993). See Section 7.1 for more detail.

It is worth mentioning here that there may exist strong correlations between non-merging events and merging events simply because of the time series nature of the data. However, there are two types of such correlation: genuine and false. The genuine correlation is the inherent similarity (to some extent) that may exist between how a non-merging decision and how a merging decision are made, because mandatory lane-changing is a sequential process where the decision at one time interval is likely to be influenced by the preceding decision. The false correlation is the correlation falsely created in the data by mislabelling a traffic situation responsible for merging as one for non-merging. This could be a consequence of selecting a decision horizon too short. The former should be considered and captured by a mandatory lane-changing model, while the latter should be avoided because it would only confound the analysis and lead to ambiguity.

4.1.2. Decision of players in the game

The decision of SV (i.e., to merge or wait) can be directly obtained from the trajectory data; however, to obtain data on FV's decision, previous studies adopt subjective methods (including visual observation for selection of strategies, which could be tricky) that can induce a significant error, and produce biased results (Liu et al., 2007, Talebpour et al., 2015). To prevent such error and bias, this study utilises the speed segmentation algorithm (i.e., the Bottom-Up algorithm) to extract FV's response/decision. The Bottom-Up algorithm uses a piecewise linear approximation of a time series data (Keogh and Pazzani, 1998), and often outperforms its counterparts (that are, Sliding window and Top-down) (Keogh et al., 2004). This algorithm has been successfully used to segment traffic data in the literature (Zheng et al., 2011).

The Bottom-Up algorithm results in segmented speed profiles, and in a matrix containing segment numbers and corresponding slopes. Ozaki (1993) proposes an empirical definition for the steady-state regime: if the acceleration or deceleration rate is within 0.05g (“g” is acceleration due to gravity), then it can be termed as “the steady-state regime” (in our study, it is termed as the “doing nothing” response). Based on this definition (Ozaki, 1993), the obtained slopes are divided into three categories of FV responses, namely: acceleration (positive slope > 0.05g), deceleration (negative slope < -0.05g), and doing nothing (slope between 0.05g to -0.05g). Meanwhile, the changing lane strategy is traced by plotting the trajectory of FV in two lanes (i.e., the first lane where FV is following SV, and the second lane where FV takes lane-changing).

As the merging scenario is a typical example of a mandatory lane-changing, the drivers’ intention is to merge into mainline traffic as soon as possible. However, there is no ground truth available about when SV wants to (or starts thinking about) merge into the mainline traffic. In an ideal situation, we require high-quality trajectory data along with drivers’ intentions informed by the driver him/herself in order to accurately decide the time when the decision-making process starts. Obviously, obtaining such merging intention is extremely challenging, if not impossible at all. Thus, we solely rely on the high-resolution trajectories to extract information about when merging action took place. At each decision interval (which is 2 s, the lane-changing frequency or resolution) prior to the merging point, SV makes a non-merging decision and the corresponding decision of the following vehicle is obtained using the segmentation algorithm. Similarly, at the last decision interval, the SV decides to merge (considered as the merging point), which is obtained from NGSIM database, the corresponding action of the following vehicle is extracted using the aforementioned approach.

4.2. Empirical evidence of the strategies

This study first formulates strategies based on theoretical knowledge, and then verifies them using field observations and the Bottom-Up segmentation algorithm. The strategies in the field observations are formulated on the basis of slopes obtained from the Bottom-Up algorithm. Consider the merging strategy as an example. The last point in SV’s trajectory (i.e., the merging point) is used to identify the corresponding point in FV’s trajectory. At the corresponding time, if the slope is positive/negative, FV’s strategy is classified as accelerating/decelerating; if the slope is between 0.05g and -0.05g, the FV strategy is doing nothing. FV’s strategies are similarly determined when SV is waiting.

Table 5 shows empirical evidence of the strategies extracted from the I-80-F and simulator data. Note that the empirical evidence of the I-80-R data is not shown here to avoid redundancy with the I-80-F data. The data for the model evaluation (either NGSIM or simulator) properly indicates the game between the players. It can be seen that about 5.3% (4.1%) of the following vehicles (FVs) in the field (NGSIM data) accelerated (decelerated) in response to the merging action of the subject vehicle (SV), whilst about 14.3% of FVs remained unaffected by the merging attempt of SV. Similar proportions of FVs’ actions have been observed in the simulator data. The corresponding proportions of FVs’ actions (in NGSIM data) when SV is waiting for another gap are respectively about 4.5%, 8%, and 63.7%, corresponding to acceleration, deceleration, and doing nothing strategies. This clearly indicates that there exists a game between SV and the immediate FV. Table 5 also shows that merging/waiting and changing lane (the shaded rows) are rarely selected in the field for various reasons such as the high density in the road segment; hence, the changing lane strategy is not considered for further evaluation. Another reason for leaving this strategy out is that changing lane altogether becomes a new game for FV. Hence, our final (parsimonious) model contains three strategies for FV: acceleration, deceleration, and doing nothing. The revised payoffs for SV and FV are presented in Table 4 (the shaded rows of Table 4 are excluded from the original payoff matrix).

Table 5. Empirical evidence of strategies extracted from the I-80-F and the simulator data.

Strategy	I-80-F (NGSIM)		Baseline (Simulator data)		CE (Simulator data)	
	Count	Percentage	Count	Percentage	Count	Percentage
S–1	126	5.28	7	2.27	0	0
S–2	98	4.11	20	6.49	17	5.47
S–3	340	14.24	51	16.56	61	19.61
S–4	0	0	0	0	0	0
S–5	109	4.57	4	1.3	2	0.64
S–6	190	7.96	18	5.84	20	6.43
S–7	1522	63.76	208	67.53	211	67.85
S–8	2	0.08	0	0	0	0

S–1= Accelerating: Merging; S–2 = Decelerating: Merging; S–3 = Doing nothing: Merging; S–4 = Changing lane: Merging; S–5 = Accelerating: Waiting; S–6 = Decelerating: Waiting; S–7 = Doing nothing: Waiting; S–8 = Changing lane: Waiting; CE: Connected environment

Note that the data processing procedure is the same for both sets of data (i.e., NGSIM and the simulator data).

5. Model calibration and validation

5.1. Calibration approach

The calibration process determines a set of model parameters that can minimise the difference between the observed and the predicted merging decisions. For this purpose, this study adopts the calibration framework proposed by Liu et al. (2007), as shown in Figure 6. In this framework, which has also recently been used in Kang and Rakha (2017), the parameters are estimated by solving a bi-level programming problem. The upper level is a non-linear programming problem, to minimise the squared difference between the observed and the predicted actions as follows:

$$min \sum_{i=1}^{n}[\left(F_i - \hat{F}_i\,(y_i)\right)^2 + (M_i - \widehat{M}_i(z_i))^2] \quad (6)$$

Where, i is an index of an observation; F_i& $\hat{F}_i$ are the observed and the predicted actions for FV (acceleration, deceleration, and doing nothing); M_i& $\widehat{M}_i$ are the observed and the predicted actions of SV (i.e., merging and waiting); y_i & z_i are the probabilities of FV's and SV's choices, respectively, and the optimisers for the upper level programming.

This study adopts the gradient descent method (Spiess, 1990) to minimise the objective function shown in Equation (6). The gradient descent, also known as "the steepest descent method", is an iterative search algorithm that searches the optimal solution proportional to the negative of the gradient of the function at the current point. The convergence of gradient descent algorithm is compared with genetic algorithm, which is widely used for calibrating microscopic models, as shown in Figure 7. It can be seen that both algorithms perform reasonably well for the developed game theory-based mandatory lane-changing model. Genetic algorithm, due to its heuristic nature, takes a longer time to converge compared to gradient descent method. Given the model's complexity, and the higher number of parameters to estimate, the gradient descent method is simple and computationally efficient (Mok et al., 2005) and thus, adopted in this study for calibration purpose.

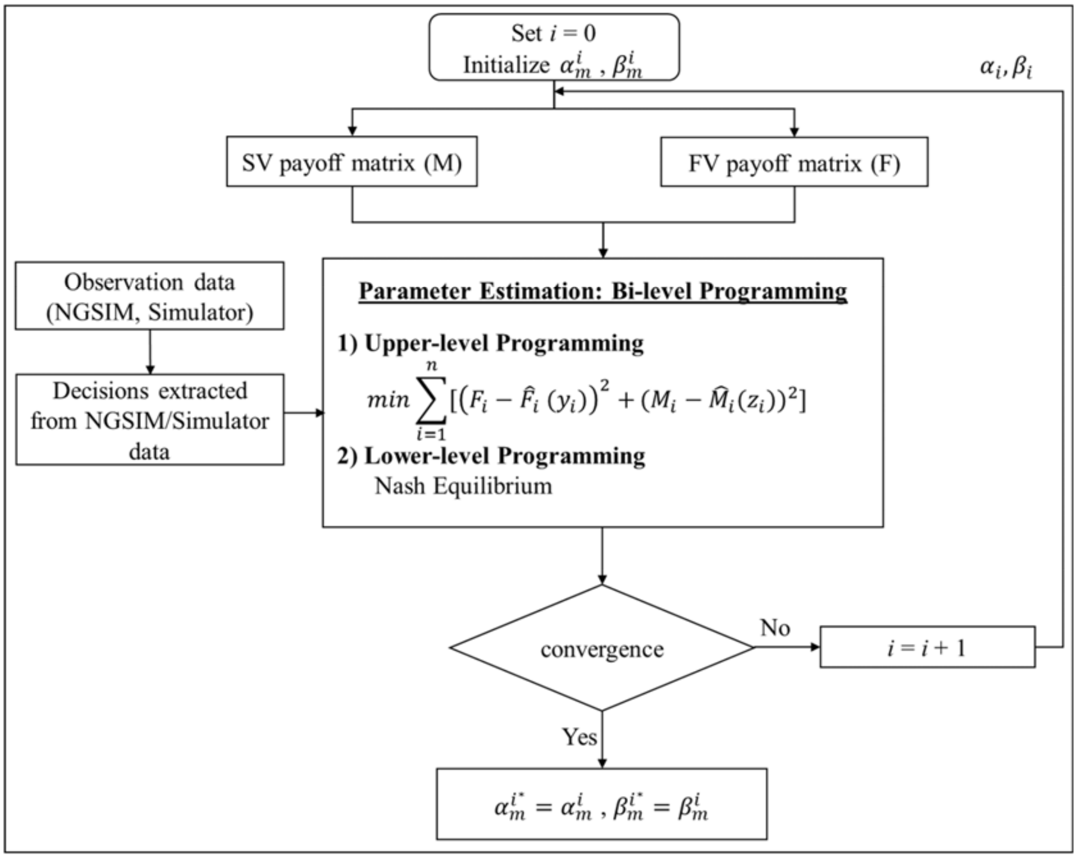


Fig. 6. Calibration framework for this study

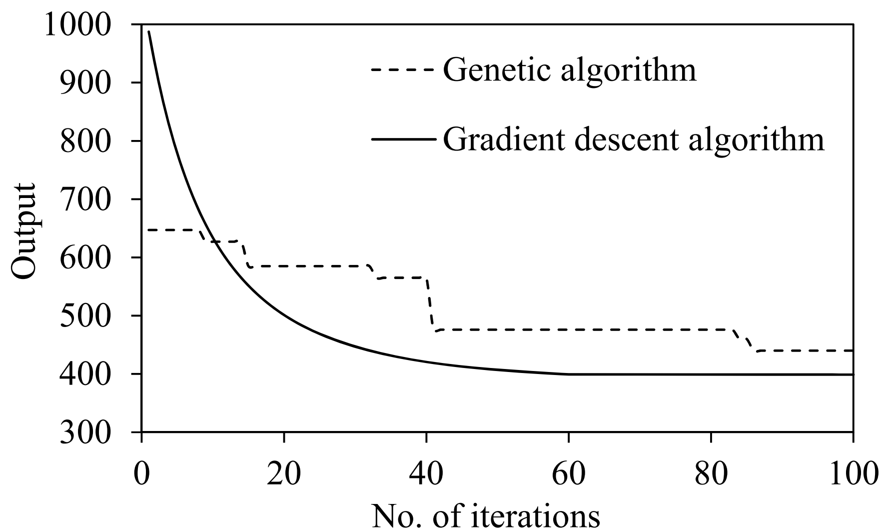


Fig. 7. Comparison of the convergences of the genetic and gradient decent algorithms

The lower level programming seeks the solution for the Nash equilibrium. Obtaining the entire set of Nash equilibria is generally challenging (Talebpour et al., 2015), and the non-uniqueness of Nash equilibrium makes it even more difficult (Liu et al., 2007). This study adopts the support enumeration method (Dickhaut and Kaplan, 1993) to determine the entire set of Nash equilibria. This approach uses graph theory (i.e., the homeomorphic nature of graph) to determine Nash equilibria, and solves a system of linear equations corresponding to a set of strategies with a positive selection probability (Talebpour et al., 2015). The adopted calibration framework jointly estimates the parameters of payoffs and the probability of equilibrium selection to accommodate multiple equilibria. This framework is consistent with the probability of equilibrium selection method, originally developed by Kita et al. (2002). Table 1 defines the probabilities of different equilibrium strategies and Equation (6) shows the objective function of the game theory-based model incorporating these probabilities. This method does not require any priori selection criteria from their resultant actions (Kita et al., 2002). In other words, this method does not require the realised equilibrium and the corresponding parameter estimates.

The entire set of Nash equilibria are obtained from the *nashpy* package of python (Nisan et al., 2007), which is integrated with MATLAB to solve the bi-level optimisation problem.

5.1.1. Calibration results

a) NGSIM data

For calibration of the LCD_TE model, the NGSIM I-80-R and I-80-F data are used. However, to avoid confusion between I-80-F and I-80-R, the calibration results for the I-80-R data are not presented in this paper. A total of 2385 observations were obtained for the I-80-F data. As a common practice in the literature (Liu et al., 2007, Talebpour et al., 2015), 70% of the data were randomly selected for calibration, while the remaining data were used for validation purposes. Three hundred and ninety-five (out of 564) merging events, and 1305 (out of 1821) non-merging events were used for calibration. Table 6 shows the calibration results for the LCD_TE model. The mean absolute error (MAE) for calibration is calculated using Equation (7). The MAE for the I-80-F data is 0.15—an error that implies that, on average, the developed model can accurately capture 85% of mandatory lane-changing decisions. A lower MAE was obtained for the I-80-R data.

$$MAE = \frac{1}{n}\sum_{i=1}^{n}|x_i - \hat{x}_i| \quad (7)$$

where x represents the actual observation; $\hat{x}$ is the model predicted decision; n is the number of observations; and i is an index of the observations.

Table 6. Model calibration results

Strategy	Payoff	Parameter	I-80-F	Baseline	CE	Parameter	I-80-F	Baseline	CE
S–1	FV	α_{11}^{0}	-0.65	-1.60	0	α_{11}^{1}	0.29	0.58	0
S–2		α_{21}^{0}	0.13	-1.78	-0.61	α_{21}^{1}	-0.57	-1.63	-0.6
S–3		α_{31}^{0}	0.47	1.87	1.30	α_{31}^{1}	3.36	3.87	1.71
S–5		α_{12}^{0}	2.91	1.78	1.55	α_{12}^{1}	0.48	2.27	1.76
S–6		α_{22}^{0}	3.34	1.57	1.36	α_{22}^{1}	3.68	1.08	1.07
S–7		α_{32}^{0}	0.19	1.05	1.16	α_{32}^{1}	0.01	1.25	1.11
S–1	SV	β_{11}^{0}	0.80	6.39	0	β_{11}^{1}	1.27	2.64	0
S–2		β_{21}^{0}	0.44	4.02	6.38	β_{21}^{1}	-1.19	-4.65	-0.82
S–3		β_{31}^{0}	3.20	5.91	2.18	β_{31}^{1}	-1.17	-5.99	-7.51
S–5		β_{12}^{0}	0.79	2.25	2.73	β_{12}^{1}	0.36	2.16	2.96
S–6		β_{22}^{0}	0.96	2.89	0.88	β_{22}^{1}	2.46	4.20	2.18
S–7		β_{32}^{0}	0.20	1.78	1.79	β_{32}^{1}	0.08	6.75	0.64
MAE (I-80-F) = 0.15; MAE (Baseline) = 0.15; MAE (CE) = 0.11									

For a description of strategies (S–1, S–2, etc.), refer to Table 4; Baseline and CE data are from the advanced driving simulator; CE: Connected environment

b) Advanced driving simulator data

The data collected from the advanced driving simulator includes both the baseline (i.e., without driving aids, and similar to NGSIM) and connected environment scenarios (i.e., with driving aids). (See Section 4 for more details.) Note that the data used in this study are the same as in Ali et al. (2018). In the baseline scenario, 78 merging events and 230 non-merging events were obtained. Respectively, there were 55 and 162 merging events and non-merging events used for calibration purpose. Table 6 summarises the parameter estimates for the baseline scenario with the MAE of 0.15. In the connected environment scenario, the number of observations is the same as in the baseline, and a similar proportion of observations was used for calibration. However, in the connected environment scenario, the participants (i.e., SVs) avoided using the merging and acceleration strategy (see Table 5 for more details), perhaps considering it as an unsafe manoeuvre. Similar and consistent findings are reported in Ali et al. (2018) where drivers also tend to avoid selecting risky gaps in the connected environment. The parameter estimates are presented in Table 6. The MAE for this model is 0.11.

5.2. Model validation

This section presents the mandatory lane-changing predictive capability of the model based on the parameter estimates obtained from calibration. To assess the performance of the model, this study adopts the confusion matrix (Sun et al., 2018). This matrix consists of various performance indicators that provide valuable insights into a model's predictive capability of mandatory lane-changing behaviour. (These indicators are highlighted by Zheng [2014] in his review study.) The adopted performance indicators include: *true positive* (cases where the model's predicted decision matches the observed decision); *false positive* (cases where the model predicates a merging event, but the observed decision is a non-merging event); *detection rate* (the percentage of merging events that are correctly predicted by the model); and *false alarm rate* (the percentage of merging events that are falsely predicted by the model). The model's performance is also assessed for each strategy so as to gain a more complete understanding of the performance of the proposed modelling approach.

As Zheng (2014) notes, as well as using the confusion matrix, the performance of a mandatory lane-changing decision model can be further evaluated at a finer level by using the time and the location errors of a merging event that are predicted by the mandatory lane-changing decision model. The *time error* is the time difference between the observed merging events and the model's predicted merging events; the *location error* is the spatial difference between the observed merging events and the model's predicted merging events. A mandatory lane-changing decision model's time and location errors are two important performance indicators, as they directly indicate the readiness and suitability of a mandatory lane-changing decision model for integration into a car-following model in a microsimulation framework.

Note that in some cases (although rare) the developed model is unable to predict a merging event during the entire simulation period. In such cases, a pragmatic yet reasonable strategy is adopted. At the end of the acceleration lane, all the merging vehicles would have to force their way into the through traffic; that is, we override the model's decision to "merge" at the end of the acceleration lane, and then calculate these time and location errors accordingly. While there is no perfect solution to this problem, this approach seems more realistic than simply removing these vehicles from the simulation in a brute-force manner, as is done in some microsimulation packages (Zheng, 2014).

5.2.1. Validation results

a) NGSIM data

The LCD_TE model is validated using the NGSIM I-80-R and I-80-F. (Again, because of confusion, results are presented only for the I-80-F.) Table 7 summarises the validation results using the confusion matrix that provides information about overall predictive capability of the model; the prediction for mandatory lane-changing (merging) events and non-merging (waiting) events; and for each strategy separately. Note that Table 7 also shows the total number of events/instances that were validated. The overall detection rate of the LCD_TE model is 88%. The model correctly predicts 114 mandatory lane-changing events and 489 non-merging events. The results imply that overall the LCD_TE model performs well in predicting the observed merging behaviour, and shows a good predictive capability for each strategy.

Table 7. Model validation results using the confusion matrix

Cases	I-80-F					Baseline data from simulator					CE data from simulator				
	N	TP	FP	DR (%)	FAR (%)	N	TP	FP	DR (%)	FAR (%)	N	TP	FP	DR (%)	FAR (%)
Overall	685	603	82	88	12	91	81	10	89	11	91	82	9	90	10
Merging	169	114	55	67	33	23	19	4	83	17	23	18	5	78	22
Non-merging	516	489	27	95	5	68	62	6	91	9	68	64	4	94	6
S–1	42	25	17	60	40	5	2	3	40	60	0	0	0	0	0
S–2	28	10	18	36	64	2	2	0	100	0	4	2	2	50	50
S–3	99	79	20	80	20	16	15	1	94	6	19	16	3	84	16
S–5	29	26	3	90	10	1	1	0	100	0	1	1	0	100	0
S–6	31	29	2	94	6	6	6	0	100	0	5	4	1	80	20
S–7	456	434	22	95	5	61	55	6	90	10	62	59	3	95	5

TP: true positive; FP: false positive; DR: detection rate; FAR: false alarm rate; CE: Connected environment

To gain more insights into the model's performance against different data and quality of data, the time and the location errors are calculated. The mean time and location errors for the I-80-F (Table 8) are 9.3 s and 155.4 m, respectively. The mean time error of 9.3 s implies that, on average, the time difference between the observed and the predicted merging decisions varies by 9.3 s. Similarly, the mean location error of 155.4 m indicates that the difference in location of the observed and the predicted merging decisions differs, on average, by 155.4 m.

Table 8. The time and the location errors

Error	I-80-F	Baseline	CE
	Mean (SD)	Mean (SD)	Mean (SD)
Time (s)	9.3 (5.87)	0.42 (0.72)	0.011 (0.016)
Location (m)	155.4 (47.97)	6.23 (4.93)	1.002 (1.37)
Paired *t*-test for the time error	p-value = 0.01		
Paired *t*-test for the location error	p-value <0.001		

Using the I-80-R data, consistent results have been found for the confusion matrix and the time and the location errors. The time and the location errors are lower in the I-80-R than the I-80-F. At a 95% confidence level, this difference is also statistically significant.

b) Advanced driving simulator data

The baseline scenario data were utilised to validate the LCD_TE model, and the results are presented in Table 7. The model shows an overall detection rate of approximately 89%, with

83% and 89% detection rates respectively for merging events and non-merging events. The model has successfully predicted a higher number of mandatory lane-changings. The predictive capability of the model against each strategy is also higher, showing the capability of the developed model to replicate the observed merging behaviour.

The data from the connected environment scenario were used for the LCD_CE model validation, and results are summarised in Table 7. Notably, the drivers have avoided the merging and acceleration strategy, which can be unsafe. The predictive capability of this model is relatively higher than the LCD_TE model using the baseline data. The model predicts the merging events and non-merging events, with the detection rate of 78% and 95%, respectively. Although the detection rate for merging events is lower in the LCD_CE model than in the LCD_TE model, it should be noted that there are no cases related to the merging and acceleration strategy in this data. This can be a reason for a higher detection rate in the LCD_TE model for the baseline. The predictive power of the model for validating each strategy is also found to be reasonable.

The time and the location errors are also calculated for the baseline and connected environment data, and results show that the errors (i.e., time and location) in the LCD_CE model are lower than in the LCD_TE model. These errors are significantly different (*p*-value <0.001). Notably, the time error in the LCD_TE model is about 38 times higher than in the LCD_CE model, while the location error in the LCD_TE model is about 6 times higher than the LCD_CE model. These results imply that the LCD_CE model is able to more accurately capture the merging behaviour in the connected environment.

In addition to validating the merging vehicle's actions, this study also validates FV's actions. The observed FV behaviour (in terms of acceleration) is compared to the model predicted actions of FV (accelerating/decelerating/doing nothing). Table 9 shows confusion matrix for FV. For NGSIM data (I-80-F), approximately 51% actions are successfully predicted by the model; notably 72% of FV actions during merging scenario are predicted by the model. The overall detection rates for the baseline and connected environment scenarios are respectively about 54% and 62%. Prediction accuracy of the model for FV's action can be similarly interpreted as in the case of SV. For instance, 60% of FV's decisions to accelerate during merging scenario in baseline condition (S-1) are successfully predicted by the model. The developed model shows a reasonable prediction accuracy in validating the FV actions during the merging event.

Table 9. Confusion matrix for validating FV actions

Cases	I-80-F					Baseline data from simulator					CE data from simulator				
	N	TP	FP	DR (%)	FAR (%)	N	TP	FP	DR (%)	FAR (%)	N	TP	FP	DR (%)	FAR (%)
Overall	685	348	337	51	49	91	48	33	53	37	91	56	39	62	38
Merging	169	121	48	72	28	23	11	12	48	52	23	17	6	74	26
Non-merging	516	227	289	44	56	68	37	31	54	46	68	38	30	56	44
S–1	42	12	30	29	71	5	3	2	60	40	0	0	0	0	0
S–2	28	14	14	50	50	2	1	1	50	50	4	2	2	50	50
S–3	99	95	4	96	4	16	7	9	44	56	19	15	4	79	21
S–5	29	22	7	76	24	1	0	1	0	100	1	0	1	0	100
S–6	31	25	6	81	19	6	2	4	33	67	5	3	2	60	40
S–7	456	180	279	39	61	61	35	26	57	43	62	35	27	56	44

6. Comparison of the models

This section compares the developed LCD_TE and LCD_CE models (collectively referred to as AZHW models) with the two existing game theory-based mandatory lane-changing models: Liu's model for traditional environment (Liu et al., 2007), and Talebpour's model for connected environment (Talebpour et al., 2015). In both these models, the two SV strategies are: merging (or changing lane) and waiting (or not changing lane). Two common strategies for FV are: yield (i.e., decelerate), and do not yield (i.e., accelerate). Changing lane is an additional strategy considered by Talebpour et al. (2015).

To compare these existing models with our AZHW models, we remove the third strategy (i.e., doing nothing) from the AZHW models as it is not considered by the other two models. We also remove the changing lane strategy of FV in Talebpour's model, as this strategy is not observed in the simulator data.

Hence, the previous models and the AZHW models contain two strategies for SV (merging, and waiting), and two strategies for FV (acceleration, and deceleration). Furthermore, the payoffs for Liu's model are calculated based on the equations provided in the original work (Liu et al., 2007). However, the original work of Talebpour et al. (2015) does not provide a detailed information about the formulation of payoffs and how accelerations corresponding to different payoffs were calculated. For a fair comparison of the models, we need to calculate payoffs for the Talebpour's model. A simple and pragmatic strategy is to calculate payoffs using Newtonian equations both for the Talebpour's and our models. For example, the payoff for the subject vehicle when it is merging, and the following vehicle is accelerating in the Talebpour's model consists of accelerations: (1) with respect to the leading vehicle in the target lane; (2) with respect to the following vehicle in the target lane; and (3) change in speed. It is unclear in the original work that how variables like acceleration with respect to the leader and the follower within the payoffs are calculated whether they are directly observed from the data or derived using basic variables. Thus, using the Newtonian equations, we determined above accelerations and used for the model comparison purpose. (Details of these calculations are presented in Appendix C.)

To fully assess the performance of the AZHW models, we also compare a three-strategy and a two-strategy AZHW model for FV. (Note that the three-strategy and two-strategy are two variations of the AZHW models.)

6.1. Comparison results for the traditional environment

For calibration using the I-80-F data, 153 (out of 223) merging events and 140 (out of 199) non-merging events were utilised. Table 10 shows the calibration results for both models. Note that we recalibrated the AZHW model by removing the doing nothing strategy for FV. The MAEs of the AZHW model and Liu's model are respectively 0.19 and 0.21.

Table 11 presents validation results for both models, using the confusion matrix. It can be observed that the detection rate of the Liu's model is 35%, while the corresponding rate for the AZHW model is 71%. This shows that the AZHW model predicts the mandatory lane-changing actions significantly more accurately than Liu's model.

Table 12 shows the time and the location errors calculated for both models. The results depict that the time and the location errors are lower in the AZHW model than in Liu's model. More specifically, the time and the location errors in Liu's model are respectively 3 and 2.5 times higher than in the AZHW model. This indicates the behavioural soundness and consistency of the AZHW model in predicting the observed merging behaviour. The statistical analysis (a paired *t*-test) further confirms that these errors are statistically significant. Consistent results have been found for the I-80-R data.

Table 10. Calibration results for the Liu, Talebpour, and AZHW models

Model	Data source	Player	Strategies				MAE
			S-1	S-2	S-5	S-6	
AZHW	I-80-F (NGSIM)	FV	α_{11}^0 = -3.41 α_{11}^1 = 1.28	α_{21}^0 = 5.19 α_{21}^1 = -0.43	α_{12}^0 = 1.85 α_{12}^1 = 3.70	α_{22}^0 = 1.93 α_{22}^1 = 0.82	0.19
		SV	β_{11}^0 = 2.04 β_{11}^1 = 2.01	β_{21}^0 = 4.33 β_{21}^1 = -2.52	β_{12}^0 = 2.25 β_{12}^1 = -2.47	β_{22}^0 = 2.01 β_{22}^1 = 0.70	
Liu	I-80-F (NGSIM)	FV	β_1 = -1.83 β_2 = 0.82	β_3 = 3.63			0.21
		SV	β_4 = 3.25 β_5 = 0.96	β_6 = -1.31 β_7 = 4.45	β_8 = 2.80 β_9 = 2.81 β_{10} = 0.73	β_{11} = -1.36 β_{12} = -2.17 β_{13} = 3.07	
AZHW	CE (Simulator data)	FV	α_{11}^0 = 0 α_{11}^1 = 0	α_{21}^0 = 4.32 α_{21}^1 = -2.80	α_{12}^0 = 0.81 α_{12}^1 = 5.29	α_{22}^0 = 2.80 α_{22}^1 = 0.63	0.14
		SV	β_{11}^0 = 0 β_{11}^1 = 0	β_{21}^0 = 3.25 β_{21}^1 = -2.44	β_{12}^0 = 2.60 β_{12}^1 = -3.77	β_{22}^0 = 1.20 β_{22}^1 = 0.38	
Talebpour	CE (Simulator data)	FV	α_{11}^0 = 0 α_{11}^1 = 0	α_{21}^0 = 0.62 α_{21}^1 = -0.53	α_{12}^0 = 0.81 α_{12}^1 = 1.66	α_{22}^0 = 0.36 α_{22}^1 = 0.75	0.19
		SV	β_{11}^0 = 0 β_{11}^1 = 0 β_{11}^2 = 0 β_{11}^3 = 0	β_{12}^0 = 1.09 β_{12}^1 = -0.34 β_{12}^2 = 0.53 β_{12}^3 = 0.44	β_{21}^0 = 0.83 β_{21}^1 = -3.77	β_{22}^0 = 0.04 β_{22}^1 = 0.38	

Table 11. Model comparison using the confusion matrix

Data	I-80-F									CE								
PI	N	TP		FP		DR (%)		FAR (%)		N	TP		FP		DR (%)		FAR (%)	
Model		A	L	A	L	A	L	A	L		A	T	A	T	A	T	A	T
Overall	129	92	45	37	84	71	35	29	65	12	10	7	2	5	83	58	17	42
Merging	70	55	17	15	53	79	24	21	76	5	3	2	2	3	60	40	40	60
Non-merging	59	37	28	22	31	63	47	37	53	7	4	5	2	2	100	71	0	29
S–1	33	23	9	10	24	70	27	30	73	0	0	0	0	0	0	0	0	0
S–2	37	32	8	5	29	86	22	14	78	5	3	2	2	3	60	67	40	33
S–5	30	17	15	13	15	57	50	43	50	1	1	0	0	1	100	0	0	100
S–6	29	20	13	9	16	69	45	31	55	6	3	5	0	1	100	83	0	17

PI: Performance indicator; A: the AZHW model; *L: the Liu's model; T: the Talebpour's model*

Table 12. Comparison of the time and the location errors

Data source	I-80-F		CE from the advanced driving simulator	
Error	AZHW model Mean (SD)	Liu's model Mean (SD)	AZHW model Mean (SD)	Talebpour's model Mean (SD)
Time (s)	2.3 (4.17)	6.69 (7.55)	0.004 (0.008)	0.012 (0.015)
Location (m)	14.31 (21.86)	34.54 (22.44)	0.569 (1.073)	1.51 (1.38)
Paired *t*-test for the time error	*p*-value <0.001		*p*-value <0.001	
Paired *t*-test for the location error	*p*-value <0.001		*p*-value <0.001	

6.2. Comparison results for the connected environment

Table 10 also presents calibration results for the selected strategies using the connected environment scenario data. Note that 12 (out of 17) merging events, and 15 (out of 22) non-merging events were used for calibration purpose. The MAEs for the AZHW and Talebpour's models are respectively 0.14 and 0.19. Table 11 shows validation results for both models. The overall detection rates for the AZHW and Talebpour models are respectively 83% and 58%. The AZHW model also shows a high accuracy in validating each strategy.

Table 12 also presents the time and the location errors of both models, and it is observed that both the errors (i.e., time and location) are lower in the AZHW model than in Talebpour's model. The comparison analysis indicates that the time and the location errors in Talebpour's model are respectively approximately 3 and 2.65 times higher than in the AZHW model, indicating that the AZHW model predicts the observed merging behaviour more accurately than the Talebpour's model. The differences in the time and the location errors are also found to be statistically significant (Table 12).

6.3. The three-strategy AZHW model and the two-strategy AZHW model: A comparison

Since earlier LCD_TE models formulate the mandatory lane-changing game with two strategies for FV, this study extends the strategy space to capture the actual driving behaviour; therefore, using NGSIM and the simulator data, we compare the AZHW model with three strategies for FV with a two-strategy AZHW model. Using the I-80-F data, the detection rates for the model's overall performance (88% versus 82%), merging events (77% versus 61%), and non-merging events (99% versus 95%) were higher for the three-strategy AZHW model than for the two-strategy AZHW model. This suggests that the three-strategy AZHW model more accurately captures the driving behaviour. In addition, the time and the location errors are significantly less in the three-strategy AZHW model; a paired *t*-test further confirms that this difference is statistically significant.

More importantly, the third strategy – that is, doing nothing – cannot be ignored because it has a high percentage occurrence in the real-world (see Table 5 for empirical evidence). Moreover, the advanced driving simulator data were also used to justify the need for the three-strategy AZHW model. In terms of the detection rate, and the time and the location errors, results confirm the better performance of the three-strategy AZHW model when compared with its two-strategy counterpart. With the exception of the baseline, the time and the location errors are also found to be statistically different. Consistent results have also been found when using the I-80-R data.

7. Discussion and Conclusion

7.1 Discussion

The AZHW models address the shortcomings of existing game theory-based mandatory lane-changing models such as improperly defined strategies, no empirical evidence of strategies, evaluating model performance using conventional measures and etc (Liu et al., 2007, Talebpour et al., 2015, Kang and Rakha, 2017). In a game theory-based mandatory lane-changing model, the consideration of strategies for each player and their presence in the field/real data play a significant role in replicating the observed mandatory lane-changing behaviour. Although frequently observed in the field, the 'doing nothing' strategy was ignored by prior research in mandatory lane-changing models, which can lead to unrealistic estimates of observed mandatory lane-changing behaviour. To overcome this issue, this study considers a comprehensive set of strategies for FV based on a thorough literature review and theoretical knowledge. Such theoretical strategies are carefully defined using Newtonian equations of motion to mathematically translate the real driving behaviour. The considered strategies are then verified using the real data to determine whether all of the strategies exist in the real data and how frequent drivers adopt such strategies. Using the Bottom-Up segmentation algorithm, empirical evidence for different mandatory lane-changing decision strategies was extracted, and the results suggested that changing lane strategy is rarely selected by drivers in response to drivers' merging actions. In addition, changing lane action of FV would become a new game for FV, which requires separate formulation of discretionary lane-changing by FV in the adjacent lanes. Thus, changing lane strategy was not further considered.

Calibration of lane-changing (more specifically, mandatory) models is an important step to assess the performance of the developed models. The selection of lane-changing frequency is critical in model calibration. This is a common challenge existing in the lane-changing modelling literature, that is, "the lane-changing frequency depends on the number of times that the decision-making process has been evaluated; this indicates that the duration of the time step becomes a parameter of the model" (Zheng, 2014). Due to rare merging events, without a careful and a proper consideration the number of non-merging events can easily become dominant in the data. We implemented a strategy to minimise its consequence in mandatory lane-changing modelling. More specifically, since there is no guideline in the literature on how to tackle this problem and select the appropriate proportion of merging and non-merging events, a sensitivity analysis has been carried out by varying the decision horizon (i.e., increasing number of non-merging events). Figure 8 shows the results of sensitivity analysis of the LCD_TE model using NGSIM data (i.e., I-80-F). It can be seen that when the decision horizon is 2 s prior to merging, the model shows approximately equal detection rates for each of the cases (i.e., overall model performance, merging, and non-merging). Three noteworthy observations in our sensitivity analysis, when the decision horizon increases from 2 s to the entire trajectory, are: (a) the overall detection rate tends to increase; (b) the detection rate of merging events drastically decreases; and (c) the detection rate of non-merging events increases. However, the detection rate, when the entire trajectory is considered, does not truly reflect the model's performance because the sample is dominated by non-merging events, and consequently the model tends to over-emphasise non-merging events in order to increase its detection rate. As such, considering the entire trajectory results in biased estimates of the model.

Furthermore, it can be seen that when the decision horizon is about 5 s prior to merging, the detection rate for each case (that is, overall model performance, merging, and non-merging) are well above than 50%, which is reasonable. Meanwhile, the decision horizon of 2 s shows better results compared to other decision horizons. But we prefer the decision horizon of 5 s over 2 s mainly for three reasons: (1) the decision horizon of 2 s gives a 1:1 merging vs non-merging ratio, which is the fewest decision events we can get, and thus contains less information useful for distinguishing these two types of events; (2) as the 5 s situation before the merging may have a strong correlation with the situation that suits for merging decision, traffic conditions within 2 s prior to the merging event may strongly resemble those that lead to merging events, thus, it would be difficult for a model to meaningfully distinguish these two types of events using the data in the 2 s decision horizon; and (3) the decision horizon of 5 s is also consistent with many studies in the literature (Thiemann et al., 2008, Doshi and Trivedi, 2008, Doshi and Trivedi, 2009, Beck et al., 2017).

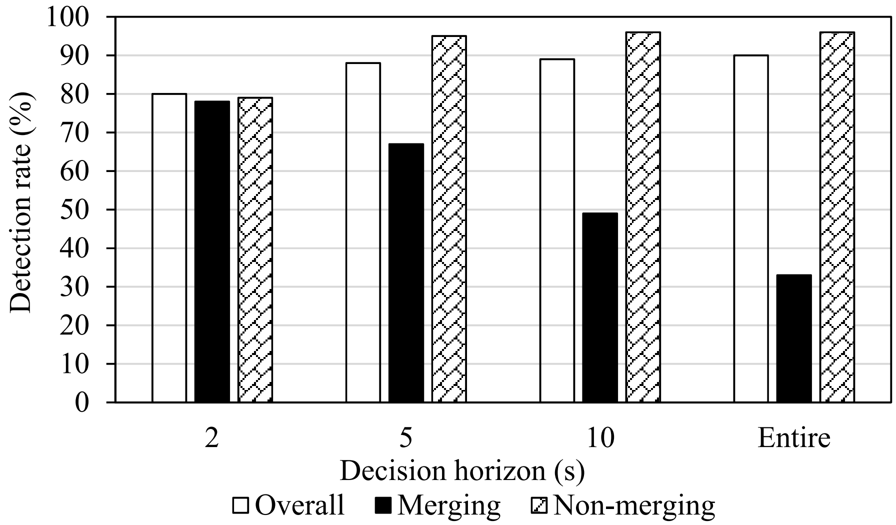


Fig. 8. Results of sensitivity analysis of LCD_TE model using NGSIM

Empirical evidence shows that observations for different strategies obtained from either NGSIM or simulator data are unbalanced (refer to Table 5). Such unbalanced representation across strategies may have some implications on the model performance in the calibration process, which is a topic for future research.

To assess the mandatory lane-changing models' performance that generates discrete outcome, prior research has used conventional measures such as mean absolute error or root mean square error, which provide little or no information into predictive power and behavioural soundness of models. As such, this study used the confusion matrix to assess the model's performance consisting of the true and false positive, and the detection and false alarm rates. The confusion matrix is an excellent tool for rigorously and objectively assessing a decision model's performance. To further evaluate models' performance at a micro level, the time and locations errors are calculated to measure temporal and spatial difference between the observed and predicted outcomes. The time and the location errors were also used to assess the model's ability to estimate the merging occurrence time and location. This ability can be helpful in improving the realism of microsimulation tools where a mandatory lane-changing decision model is one of the building blocks. Using these performance indicators, the predictive capability of the model in general, and for each strategy in particular, has been thoroughly examined.

As game theory incorporates decisions of two players, all of the existing studies (to the best of authors' knowledge) only validated the actions of merging vehicles while ignoring the following vehicle actions. This is understandable because the focus of a mandatory lane-changing model is to replicate mandatory lane-changing decision-making behaviour, however, it does not fully utilise the game theory approach's efficacy in describing actions of both players in a merging scenario. Thus, this study also validates FV actions by using confusion matrix and results show a satisfactory performance of the developed model. A lower prediction capability of model for FV actions can be attributed to discrete nature of the game theory-based model whereas FV actions are continuous (such as acceleration, deceleration, etc.).

One of the issues with game theory-based models is the large number of parameters, which can make model calibration challenging. As such, a simple minimisation algorithm, i.e., gradient descent method, was used for calibrating all the parameters in this study. However, the performance of gradient descent method has been questioned in the literature. Thus, genetic algorithm was also used for comparison purpose. Both the algorithms showed similar performance, however, gradient descent method was selected and used in this study due to its simplicity and computational efficiency.

Another issue with game theory-based models is its scalability. It is already very complicated to formulate the game for two players. In reality, there can be interaction with more than two players, especially in the connected environment. Such work is left for future research.

Since connected vehicular data are not readily available, researchers mainly rely either on NGSIM or numerical simulations to investigate driving behaviour in a connected environment. However, neither NGSIM data nor numerical simulations have the realism of connected environment, and the impact of connected environment on human factors is unlikely to be represented or approximated by such data. Thus, the data collected from the advanced driving simulator in this study can be a valuable data source for evaluating and driving behaviour under a connected environment. Such (simulator) data has been used previously to record the car-following behaviour of distracted drivers (Saifuzzaman et al., 2015). Furthermore, the model developed using the driving simulator data has been used to successfully extract traffic characteristics (e.g., hysteresis) that are observed in NGSIM data (Saifuzzaman et al., 2017).

The current simulator system is deterministic in nature, and all the information are programmed. In our study, we intentionally controlled the complexity of the connected environment for the purpose of ensuring that the workload of a participant is reasonable, the collected data are reliable, and also the simulated connected environment is consistent with the state-of-the-practice in the automobile industries on how major car manufacturers have designed their driving aids (e.g., Adell et al. (2011); Saffarian et al. (2013)). The connected environment has the potential to provide more dynamic information. However, to ensure connected environment's safety, security, and public acceptance (and user friendliness), it is very unlikely the connected environment in the real life would adopt any complicated information dissemination strategy, especially for safety-critical events like lane-changing.

Finally, in the advanced driving simulator experiment, we hired a professional programmer to carefully program FVs' movements by considering car-following, safety rule, and SV's movement. FV's actions were triggered corresponding to the action of SV by tracking the steering wheel angle of SV. In our experiment design, FVs were programmed to maintain the same speed as SV's to ensure that all the participants face similar vehicular interactions at the same simulation point. Due to driver heterogeneity, it is difficult to define a representative speed for FVs in the simulator. As such and to realistically mimic the field conditions, FVs were scripted to accelerate, decelerate or remain unaffected by the mandatory lane-changing's action of SV; these actions mimic how drivers react to mandatory lane-changing attempt in the real data or NGSIM. Such information of experiment design was not used during the data processing and game theory model evaluation to avoid favourable but biased evaluation results of our model. In contrast, we employ the segmentation algorithm to extract the strategies from the simulator data rather than using the designed interactions.

7.2 Conclusion

As one of the first studies, this study has developed comprehensive mandatory lane-changing models (i.e., the AZHW models) for modelling drivers' merging behaviour (a typical type of mandatory lane-changing) both for the traditional environment and the connected environment. The connected environment provides information about surrounding traffic conditions that can be useful for efficient mandatory lane-changing decision-making, and in assisting drivers to avoid hazards caused by inaccurate mandatory lane-changing decisions. However, the mandatory lane-changing decision modelling in the connected environment is still in its early stages. Thus, by focusing on merging behaviour, this paper develops comprehensive mandatory lane-changing decision models for the traditional environment and for the connected environment. The developed models show a high accuracy in replicating observed mandatory lane-changing behaviour and outperforms the existing models.

The study also compared the developed LCD_TE and LCD_CE models with the Liu's model (Liu et al., 2007) and the Talebpour's model (Talebpour et al., 2015). Using the confusion matrix, the comparison analyses indicate that our models (the AZHW models) have consistently performed better than the Liu and the Talebpour models. It has also more accurately predicted the merging occurrence time and location. This implies that it is more consistent with the observed merging behaviour, and more suitable for integration into a microscopic simulation package. Furthermore, the two-strategy AZHW model was compared with the three-strategy AZHW model to justify the inclusion of the 'doing nothing strategy'. This result was consistent with the empirical evidence from the field observations.

Note that the driving behaviour in the real world can be different from that observed in the simulated environment. Thus, in this study, the LCD_TE model using NGSIM is not compared with the LCD_CE model using the advanced driving simulator data. Instead, to capture the relative behavioural differences between traditional and connected environment, the baseline and connected environment scenario data collected from the advanced driving simulator experiment are utilised to compare the performance of the LCD_TE and LCD_CE models. This is because both the driving conditions and the environment are the same in both scenarios.

This study focusses on modelling mandatory lane-changing behaviour in the connected environment using game theory approach, and solves the game by using the Nash equilibrium concept; however, it would be interesting to analyse the impact of different equilibria concepts on the game outcome, as suggested by Dixit and Denant-Boemont (2014). A similar modelling framework could be developed for discretionary lane-changing decision-making in a connected environment. Furthermore, Sharma et al. (2018) highlight the importance of human factors in microscopic driving behaviour. With the emergence of connectivity, the urgency to incorporate human factors into LCD models increases; such enhancement/extension of the models will make them more realistic.

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Appendix A. Calculation of the payoffs for FV

To calculate the projected accelerations, basic equations of motion are utilized to predict future states. Initial states at the decision time are: v_{SV} = Initial speed (m/s) of SV; v_{FV} = Initial speed (m/s) of FV; a_{SV} = Initial acceleration (m/s^2) of SV; a_{FV} = initial acceleration (m/s^2) of FV; RD = remaining distance (m) on the acceleration lane for FV; X = initial gap (distance in m) between SV and FV.

The definition of projected time is adopted from Liu et al. (2007) i.e., "*the time at which FV anticipates SV to enter in through traffic*". The projected/final states from FV's perspective are: v'_{SV} = projected speed (m/s) of SV; v'_{FV} = projected speed (m/s) of FV; t'_{SV} = the time duration (s) that FV anticipates SV would need to complete the remaining distance (RD) on the acceleration lane; *X'* = gap distance (m) between FV and SV when SV joins freeway.

$$v'_{SV} = \sqrt{(v_{SV})^2 + 2a_{SV}RD}\ ;\ t'_{SV} = \frac{v'_{SV} - v_{SV}}{a_{SV}}\ ;\ v'_{FV} = v_{FV} + a_{FV}t'_{SV}$$

Calculation of gap (distance between the front bumper of the leader to the front bumper of the follower) between SV and FV is based on the difference of speed and RD. L_n indicates the length of vehicle (m) under consideration.

$$X' = RD + X - L_n - \frac{(v'_{Fv})^2 - (v_{FV})^2}{2a_{FV}}\ ;\ Acc_{D-M} = \frac{v_{SV} - v_{FV}}{t'_{SV}}\ ;\ Acc_{FV}^{LV\,TL} = \frac{v_{LV\,TL} - v_{FV}}{t'_{SV}}\ ;$$

$Acc_{FV}^{FV\,TL} = \frac{v_{FV} - v_{FV\,TL}}{t'_{SV}}$, $v_{LV\,TL}$ is the speed of LV in the target lane, $v_{FV\,TL}$ is speed of FV in the target lane; $\Delta V = v_{LV\,in\,current\,lane} - v_{LV\,in\,target\,lane}$, which is change in speed; $G = Lead\ gap + Lag\ gap$, which is the available gap.

Appendix B. Calculation of the payoffs for SV

$Acc_{M-A} = \frac{2(RD - v_{SV}t_{M-A})}{t_{M-A}^2}$; $t_{M-A} = \frac{\sqrt{v_{SV}^2 + 2Acc_{max}RD} - v_{SV}}{Acc_{max}}$, which is the waiting time that SV has to wait on the acceleration lane before merging; $Acc_{M-D} = \frac{2(RD - v_{SV}t'_{M-D})}{t'^2_{M-D}}$; $t_{M-D} = \frac{\sqrt{v_{SV}^2 + 2Acc_{comfort}RD} - v_{SV}}{Acc_{comfort}}$, which is waiting time that SV has to wait on the acceleration lane before merging and comprehend the signal of merging from FV.

$Acc_{M-LC} = \frac{2(RD - v_{SV}t'_{SV})}{t'^2_{SV}}$; $Acc_{W-A} = \frac{v'_{SV} - v_{SV}}{t_{W-A}}$; $t_{W-A} = \frac{(v_{SV} - v_{FV}) + \sqrt{(v_{SV} - v_{FV})^2 + 2a_{FV}X}}{a_{FV}}$, which is the waiting time SV has to wait till FV overpasses it; $Acc_{W-D} = \frac{2(RD - v_{SV}(t'_{SV} + t_{W-D}))}{t'^2_{SV}}$; $t_{W-D} = \frac{\sqrt{v_{SV}^2 + 2Acc_{comfort}(RD - v_{SV}t'_{SV})} - v_{SV}}{Acc_{comfort}}$, which is the waiting time SV has to wait to recognize the invitation of FV; $Acc_{W-DN} = \frac{v'_{SV} - v_{SV}}{t_{W-DN}}$; $t_{W-DN} = \frac{(v_{SV} - v_{FV}) + \sqrt{(v_{SV} - v_{FV})^2 + 2a_{FV}X}}{a_{FV}}$, which is the waiting time for SV till FV overpasses it; $Acc_{W-LC} = \frac{v'_{SV} - v_{SV}}{t'_{SV}}$

Appendix C. The payoffs for the Talebpour's model

$$Acc_{Target}^{C} = \frac{v'_{SV} - v'_{FV\,TL}}{t'_{SV}};\quad Acc_{Lead}^{C} = \frac{v'_{LV\,TL} - v'_{SV}}{t'_{SV}}\ ;\ \Delta V = v_{LV\,in\,current\,lane} - v_{LV\,in\,target\,lane}$$

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