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A gas-spring system for optimizing loudspeakers in thermoacoustic refrigerators

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Moving-coil loudspeakers are appropriate drivers for thermoacoustic refrigeration. They are cheap, commercially available, compact, light, and can be adapted to meet specific requirements. This paper deals with the optimization of loudspeakers for thermoacoustic refrigeration. Using an electrical model that describes the refrigerator, it is concluded that the electroacoustic efficiency can be maximized over a wider frequency range by matching the mechanical resonance frequency of the driver to the acoustic resonance frequency of the resonator. A gas-spring system is introduced as a practical tool to shift the mechanical resonance frequency of the driver. An electroacoustic efficiency of 35% is obtained when the mechanical resonance frequency of the driver and the acoustic resonance frequency are equal. Additionally, the efficiency is constant over a relatively wide frequency range. This has advantages for thermoacoustic refrigeration. During cool-down, the operating acoustic frequency decreases so that the refrigerator will keep near the optimum performance. [DOI: 10.1063/1.1492867]

I. INTRODUCTION

For two decades, thermoacoustic cooling has been investigated as a new cooling technology.[1–4] Thermoacoustic coolers are systems which use sound to generate cooling. They can reach temperatures of −70 °C and can have a coefficient of performance of 20% of the Carnot efficiency. These systems are environmentally friendly as they use inert gases instead of CFCs.

A schematic diagram of a thermoacoustic refrigerator is shown in Fig. 1. Typically, a thermoacoustic cooler consists of an acoustic resonator (e.g., tube) filled with an inert gas at some average pressure. A loudspeaker is attached to one end of the resonator. In the resonator a structure with channels, called a stack, is placed. The stack is the heart of the cooler where the heat transfer process takes place. Heat exchangers are installed at both ends of the stack. The temperature of the hot heat exchanger is fixed at room temperature by circulating water. At the cold heat exchanger cooling power is generated. The loudspeaker sustains a standing wave in the resonance tube. This wave causes the gas to oscillate while compressing and expanding. The interaction of the moving gas in the stack with the surface of the channels generates heat transport.[1–3]

Moving-coil loudspeakers have many advantages which make them appropriate drivers for thermoacoustic refrigeration. They are cheap, commercially available, compact, light, and can be adapted to meet specific requirements. However, they have an important drawback: a low electroacoustics efficiency which generally ranges from 3% to 5%. In the following an optimization method for loudspeakers in thermoacoustic refrigerators, using a gas-spring system, is reported. An electrical model is used to simulate the behavior of the loudspeaker in the refrigerator and to determine how the performance of the loudspeaker can be improved. The calculations show that the electroacoustic efficiency, defined as the output acoustic power divided by the input electric power, for a given loudspeaker can be optimized by matching the mechanical resonance frequency of the loudspeaker to the acoustic resonance frequency of the resonator.[5] Since, in general, these two frequencies are different, a method is necessary to make them equal. A concept, using the volume of the gas behind the cone of the loudspeaker as an adjustable additional spring, will be presented along with the experimental results.

II. ELECTRICAL MODEL FOR THE REFRIGERATOR

The aim of this section is to derive the equivalent electrical circuit for the refrigerator system. Once this circuit is determined, straightforward electrical circuit analysis yields the behavior of the system, so that one can determine how the performance of the loudspeaker can be improved.

An illustration of our thermoacoustic refrigerator is shown in Fig. 1.[6,7] A modified loudspeaker (driver) is attached to one end of an acoustic resonator which is filled with helium gas at 10 bar. A stack and two heat exchangers are placed in the resonator. The loudspeaker sustains an acoustic standing wave in the gas, at the fundamental resonance frequency of the resonator. The other end of the resonator terminates in a buffer volume which simulates an open end. The system is essentially a quarter-wavelength system, with a pressure antinode at the loudspeaker end and pressure node at the buffer volume end. The background for the choice of the geometry of the resonator is explained elsewhere.[6,7] The acoustic driver consists of a modified moving-coil loudspeaker, from which the fabric dome was cut off near the voice coil and replaced by a thin walled light

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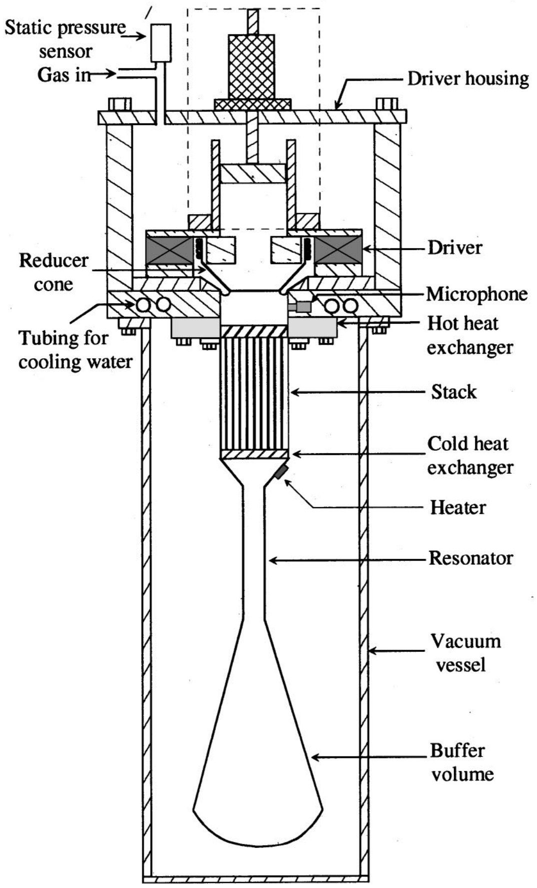

FIG. 1. Schematic diagram of the thermoacoustic refrigerator, showing the different parts. The back volume system is indicated by the dashed line.

aluminum cone glued onto the voice coil. A rolling diaphragm is used to seal the resonator from the driver housing, as shown in Fig. 1. The driver is placed in a housing so that the pressurized working helium gas is confined. The word driver will be used in the following to refer to the modified loudspeaker. A detailed description of the refrigerator can be found elsewhere.[6,7]

The electromechanicoacoustical expressions needed for the electrical model describing the refrigerator will be given. To simplify the discussion, the combined system of driver and resonator shown in Fig. 2 will be considered instead of the refrigerator depicted in Fig. (1). This configuration, in which a loudspeaker is attached to a straight tube, will be used to develop the electrical model for the dynamic study of moving-coil loudspeakers in thermoacoustic refrigerators.

The basic operational principle of moving-coil loudspeakers is well known and has been well-described in many textbooks.[8,9] The moving-coil loudspeaker is an electromechanicoacoustical transducer that converts electrical energy into sound. The loudspeaker consists of a cylindrical coil placed in an annular gap between the poles of a permanent magnet, as shown in Fig. 2. The magnet is shaped so that the lines of the magnet field are radially directed and cut across every part of the coil at right angles. When an alternating current i flows through the wire, a alternating Lorentz force F on the coil results. To avoid ejection of the coil out of the gap, it is suspended by a ring-shaped diaphragm which pro-

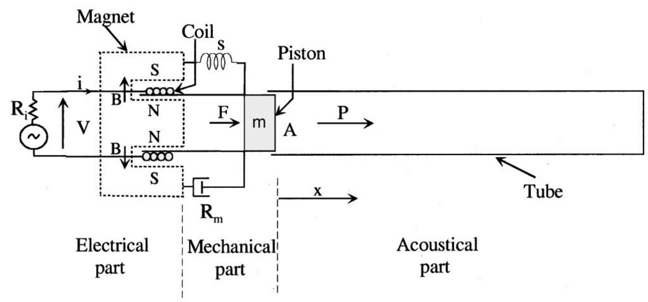

FIG. 2. A schematic illustration of the loudspeaker-tube combined system. An amplifier, of internal resistance R_i, drives a current i through the coil. This produces a force $F=Bli$ on the coil and piston, with resulting motion u. The motion of the piston is transmitted to the gas in the tube via the area of the piston A. The frictional force is represented by a dashpot (R_m), and stiffness is represented by the spring (s).

vides a restoring force. The current is provided by a power amplifier by sustaining a voltage V across the coil, as shown in Fig. 2. The electrical resistance R_e and inductance L_e of the coil form the electrical part of the loudspeaker. The moving mass m, the mechanical resistance R_m and stiffness of the suspension s, combined in series, form the mechanical part. From a mechanical point of view the loudspeaker can be seen as a damped harmonic oscillator. The tube attached to the loudspeaker forms the acoustic part. A piston (cone) of an effective area A attached to the moving coil transforms the alternating force F to pressure oscillations p in the tube. Furthermore, Faraday's law states that if a conductor is moving in a magnetic field, a potential difference will be induced. For a more detailed outline of the expressions given in this section, we refer to Ref. 6. The electrical equation describing the driver-loudspeaker combined system is given by

$$V=\left[Z_e+\frac{(Bl)^2}{Z_m}\right]i=(Z_e+Z_{\mathrm{mot}})i=Z_{\mathrm{et}}i, \tag{1}$$

where Bl is the product of the magnetic-flux density B and the total length of the coil l. The electrical impedance of the coil is given by

$$Z_e=R_e+\mathrm{j}\omega L_e, \tag{2}$$

where j is the complex unit, and ω is the angular frequency. The impedance Z_{mot} is the equivalent electrical impedance of the total mechanical impedance (Z_m) of the driver–resonator combined system given by

$$Z_m=\left[R_m+\mathrm{j}\left(m\omega-\frac{s}{\omega}\right)\right]+\frac{Ap}{u}=Z_{\mathrm{md}}+Z_{\mathrm{ma}}. \tag{3}$$

Without the acoustic part (tube), the mechanical impedance reduces to the mechanical impedance of the driver alone given by

$$Z_{\mathrm{md}}=R_m+\mathrm{j}\left(m\omega-\frac{s}{\omega}\right). \tag{4}$$

The attachment of the acoustic resonator to the loudspeaker thus adds an extra mechanical impedance

$$Z_{\mathrm{ma}}=\frac{Ap}{u}=\frac{p}{Au}A^2=Z_aA^2, \tag{5}$$

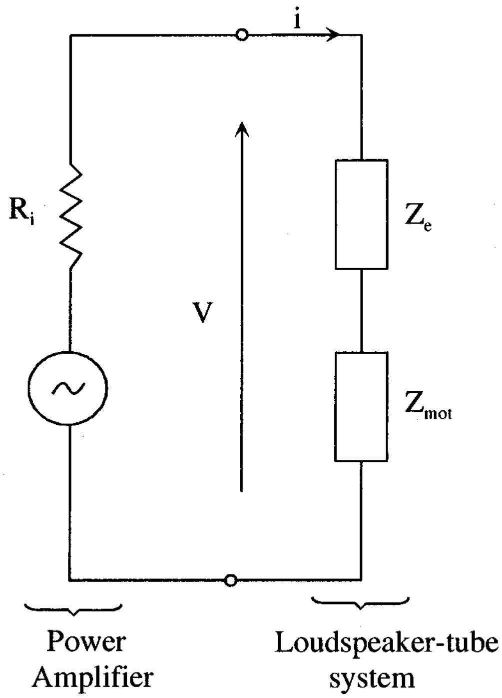

FIG. 3. The equivalent electrical circuit representing the driver-tube coupled system. The mobility impedance Z_{mot} is the electrical equivalent impedance of the mechanical and acoustical parts.

where u and A are the velocity and the area of the piston, respectively. The quantity Z_{ma} is the equivalent mechanical impedance of the acoustical impedance Z_a of the tube. The acoustic impedance Z_a is defined as the ratio of the acoustic pressure p to the volume velocity $U=Au$.

Once Z_{mot} is determined, the behavior of the loudspeaker in the refrigerator can be studied by analyzing the equivalent electrical circuit illustrated in Fig. 3. The loudspeaker resonates when the reactance part of its mechanical impedance is zero, thus

$$\omega_d=\sqrt{\frac{s}{m}}. \tag{6}$$

The tube resonates when

$$\mathrm{Im}(Z_{ma})=0. \tag{7}$$

The resonance frequencies of the combined driver-tube system are given by

$$\mathrm{Im}(Z_{md}+Z_{ma})=0, \tag{8}$$

which can be different from those of the loudspeaker and tube acting alone. This expression will be used later to optimize the performance of the driver. In the following section, the expression of Z_a of our resonator will be derived, so that we can proceed with the performance analysis.

III. PERFORMANCE CALCULATIONS

The derivation of the acoustic impedance of the system, shown in Fig. 1, is given in the Appendix. Such a resonator can be approximately represented by the resonator shown in Fig. 4(b), and the acoustic impedance is given by

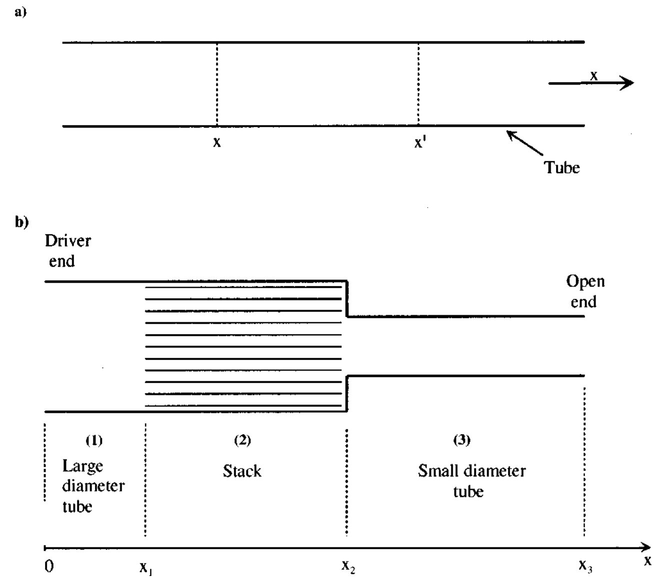

FIG. 4. Schematic illustrations used in the derivation of the acoustic impedance. (a) An acoustic tube of constant cross-section where plane waves propagate. (b) Simple illustration of the acoustic resonator used in the thermoacoustic refrigerator.

$$Z_a(0)=\frac{Z_a(x_1)\cos(k_1x_1)+\mathrm{j}Z_{c_1}\sin(k_1x_1)}{\cos(k_1x_1)+\mathrm{j}\dfrac{Z_a(x_1)}{Z_{c_1}}\sin(k_1x_1)}. \tag{9}$$

Substituting Eq. (9) into Eq. (3) and then Z_m into Eq. (1) gives the total equivalent electrical impedance Z_{et} of the refrigerator system. The expression of Z_{et} is long and complicated. It contains the driver, tube, and gas parameters.

Referring to Fig. 3, the calculation procedure is as follows: the starting point is at the power amplifier, which sustains a voltage with amplitude V across the total electrical impedance Z_{et}. The current i that flows through Z_{et} is obtained by dividing the voltage V by Z_{et} [Eq. (1)]. The Lorentz force F is then obtained by multiplying i by the force factor Bl. After that the velocity u is obtained by dividing the force F by the mechanical impedance Z_m (driver+tube). The dynamic pressure p in the tube is obtained by multiplying the volume velocity $U=Au$ by the acoustic impedance $Z_a(0)$. Then, the input electrical power, mechanical power, and acoustical power output can be calculated. Finally, the efficiencies for the different energy conversion processes in the system can be evaluated.

By varying the system parameters, the influence on the performance of the driver in the combined system driver–resonator can be studied. The electroacoustic efficiency η_{ea} is defined as the ratio of acoustic power output P_a to the electric power input P_e:

$$\eta_{ea}=\frac{P_a}{P_e}. \tag{10}$$

The two powers are given by

$$P_e=\tfrac{1}{2}\mathrm{Re}(Vi^*),$$
$$P_a=\tfrac{1}{2}\mathrm{Re}(pU^*), \tag{11}$$

TABLE I. Electromechanical parameters of the driver.

Parameter	Value
Bl-factor	12.8 Tm
Moving-mass m	13.16 g
Mechanical resistance R_m	3 Ns/m
Stiffness s	93 kN/m
Electrical resistance R_e	5 Ω
Self-inductance L_e	1.3 mH
Piston area A	7 cm^2

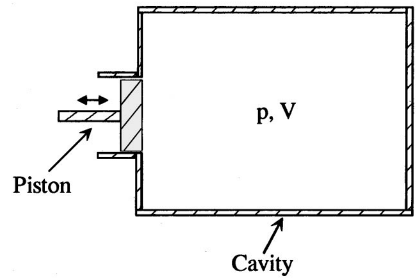

FIG. 5. Cavity of volume V containing a gas at pressure p, and driven by a piston of area A.

where p and U are the magnitude of the acoustic pressure and volume velocity, respectively; and V and i are the voltage across the driver and current into the coil, respectively. The star denotes complex conjugation.

The input parameters are:

(1) The voltage amplitude of the amplifier: V.
(2) Working gas parameters: speed of sound a, density ρ, viscosity μ, and thermal conductivity K.
(3) Resonator parameters: the length $L_3=(x_3-x_2)$ and radius r_3 of the small tube; length x_1 and radius r_1 of the large diameter tube, and the length $L_s=(x_2-x_1)$ and spacing y_0 of the stack.
(4) Electromechanical driver parameters: Electrical resistance R_e, self-inductance L_e, mechanical resistance R_m, moving mass m, stiffness constant s, the force factor *Bl*, and the area of the piston A.

The quantities that can be calculated can be any of the following quantities: current i, any impedance (acoustical, mechanical, or electrical), force F, velocity u, dynamic pressure p, electrical power P_e, mechanical power P_m, acoustical power P_a, electromechanical performance η_{em}, mechanoacoustical performance η_{ma}, and electroacoustic performance η_{ea}. In Fig. 6, a computation example is illustrated. The electromechanical parameters, which are used for the driver, are given in Table I. The parameters for the resonator are given in Table II. The calculations show that the electroacoustic efficiency can be optimized by matching the mechanical resonance frequency of the loudspeaker to the acoustic resonance frequency of the resonator.

In the next section we will discuss a practical tool that can be used to match the mechanical resonance of the driver to the acoustical resonance of the tube. In thermoacoustics it is desirable to use relatively high frequencies, because the energy density in the system is proportional to the frequency. However, the operating frequency must not be too large because the thermal penetration depth δ_k is proportional to the inverse of the square root of the frequency. Since the spacing in the stack depends on δ_k, a large frequency will result in very small spacing which makes the engineering of the stack difficult. Frequencies between 200 and 600 Hz are typical. Most midrange moving-coil loudspeakers have mechanical resonances of roughly 100 Hz. It is desirable to shift the mechanical resonance of the driver to that of the resonator. From Eq. (6), it can be seen that this can be done by lowering the moving mass, which is already kept as low as possible, or by increasing the stiffness of the driver. The latter solution is the one of interest here.

TABLE II. Parameters of the resonator and stack.

Parameter	Value
Length of the large tube x_1	3.7 cm
Radius of the large tube r_1	1.9 cm
Length of the stack L_s	9.3 cm
Spacing in the stack $2y_0$	3 mm
Length of the small diameter tube L_3	24 cm
Radius of the small diameter tube r_3	1 cm

To match the mechanical resonance of the driver to the acoustical resonance of the tube one can use a material for the suspension with a high stiffness or add a mechanical spring system to the driver to increase its stiffness. But these solutions are inflexible. Changing the resonator or the gas, which is the case during experimental work, changes the acoustical resonance, and hence the two resonances no longer match. A solution to this problem is to use the volume of gas behind the loudspeaker piston as an extra gas spring whose spring constant is inversely proportional to the volume of gas.[6] The volume can be externally and continuously adjusted. The working principle of this system will be explained in the next section.

IV. THE GAS-SPRING SYSTEM

The stiffness of a volume of gas contained in a sealed cavity of volume V, at a pressure p, and closed by a piston of area A (Fig. 5) is given by[6]

$$s_v=\gamma\frac{p}{V}A^2, \tag{12}$$

where γ is the ratio of the specific heats at constant pressure and constant volume. Expression 12 shows that the stiffness of the volume of gas contained in the cavity is inversely proportional to the volume. This idea forms the basis for the gas-spring principle that will be used to shift the mechanical resonance of the driver towards that of the resonator. In the electrical model simulating the refrigerator system, the total stiffness of the driver is obtained by adding the mechanical stiffness of the driver s and the stiffness of the back volume s_v.

As can be seen from Fig. 1, the volume of the gas in the back of the driver can be varied from the outside of the housing by using a cylinder and a piston. In Fig. 1 this system is referred to as the back volume system. The gas-spring system consists of a brass cylinder mounted on the back of the driver. By turning a crank fixed to the lid of the housing, a screw system varies the height of the piston. The cylinder has an inner diameter of 8 cm, a height of 15 cm and accepts a piston system mounted on the lid of the driver housing. The back volume can be varied between 1100 and 38 cc. This has the advantage that the resonance of the driver can be adjusted from the outside in a continuous manner.

V. RESULTS

The measured acoustic resonance frequency of the resonator is 425 Hz, and the mechanical resonance frequency of the driver measured in vacuum is 133 Hz.[6] Fig. 6 shows the measured electroacoustic efficiencies along with the calculated efficiencies using the model described previously. In total six plots are shown corresponding to six different back volumes. During the measurements the back volume can been continuously changed and all other parameters kept constant.

The calculations show a maximal efficiency of over 50%. This result is in agreement with the results of Wakeland[10] who has shown that the maximum efficiency is given by $\eta_{\max}=(\sqrt{m'+1}-1)/(\sqrt{m'+1}+1)$. For the driver parameters in Table I, $m'=10.9$, so that $\eta_{\max}=55\%$.

As can be seen from Fig. 6, the measured and calculated efficiencies show two peaks as indicated by the arrows: one peak at the acoustical resonance f_a and a second one at the driver resonance f_d. The latter can be increased by decreasing the back volume [Eq. (12)]. As the mechanical resonance frequency of the driver is shifted towards the acoustical resonance frequency, the efficiency peak, related to the driver, shifts towards the fixed efficiency peak related to the resonator. An overlap of the two peaks with a large efficiency is obtained when f_a is nearly equal to f_d, as shown in Fig. 6(d). When the driver resonance frequency is larger than the tube resonance, the efficiency decreases again for both peaks, as depicted in Figs. 6(e) and 6(f). If the two frequencies nearly match, the efficiency is constant over a wide frequency range. This has advantages for thermoacoustic refrigeration. During cool-down, the operating acoustic frequency decreases and so the refrigerator will keep near the optimum performance. In all graphs the agreement between experiment and model is quite satisfactory. As can be seen from Fig. 6(d), an efficiency of about 35% is measured over a relatively wide frequency range. The discrepancy between the magnitude of the calculated and measured efficiencies may be caused by the extra mechanical damping of the rolling membrane used to seal the resonator from the driver housing, by the eddy current losses in the magnet structure, by the viscous losses due to the flow of the gas through the small halls on the back of the driver, or to a combination of these effects. Another cause my be the area of the piston, which may not be optimal.[10]

VI. CONCLUSIONS

Using an electrical model that simulates the refrigerator, we showed that for a given driver, a broad efficiency maximum is obtained by matching the mechanical resonance of the driver to the acoustical resonance of the tube. An analytic expression for the acoustic impedance of the refrigerator is derived, using the Rott's equation. We showed that the model describing the refrigerator is a valid tool which can be used to evaluate, design, or optimize the behavior of loudspeakers in thermoacoustic refrigerators.

From the measurements, we conclude that our gas-spring system works very well. An electroacoustic efficiency of 35% is measured. Additionally, the efficiency is constant over a wide frequency range. This is advantageous from the point of view of thermoacoustic refrigeration because the operating acoustic frequency is temperature dependent. A flat efficiency peak provides the refrigerator with a temperature independent performance.

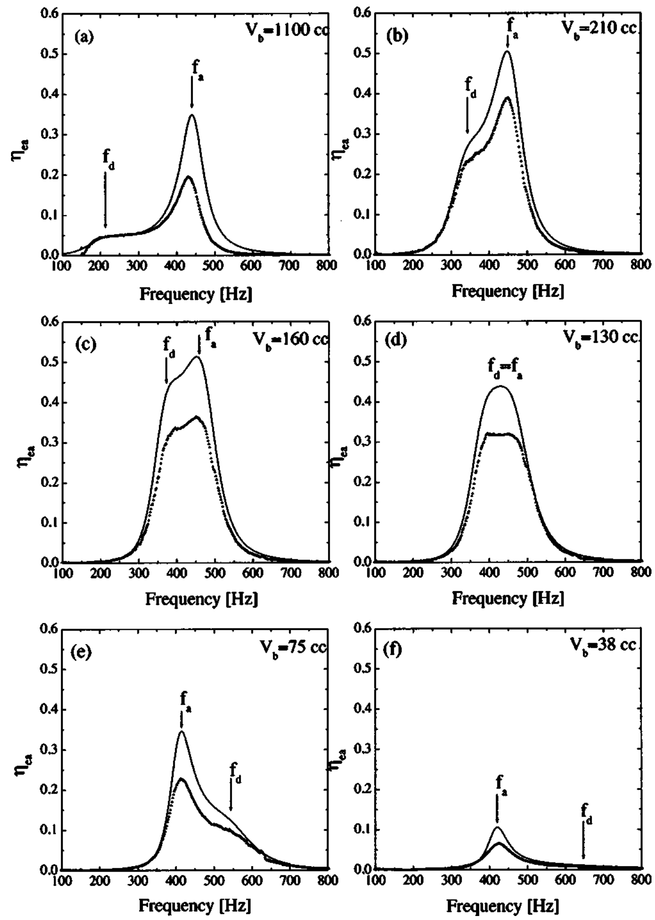

FIG. 6. Calculated (solid line) and measured (dots) electroacoustic efficiency plotted against the frequency for different back volumes V_b. The efficiency has a broad maximum when the mechanical resonance frequency of the driver is equal to the acoustical resonance of the tube [graph (d)].

APPENDIX: THE ACOUSTIC IMPEDANCE OF THE RESONATOR

In acoustics the parameters of concern are the dynamic pressure p and volume velocity U. The acoustic impedance is given by

$$Z_a=\frac{P}{U}. \tag{A1}$$

As can be seen from Fig. 1, the resonator has a complicated shape and consists of various parts. To derive the acoustic impedance Z_a, we will first derive the acoustic transfer function, which gives the acoustic impedance at a given position x as function of the impedance at an other position x' in a tube.

Fig. 4(a) shows an acoustic tube of cross-sectional area A through which a plane wave propagates. Such a geometry will be used to derive the acoustic transfer function. The starting point for our derivation is Rott's wave equation.[2,3] If we assume that the mean temperature T_m in the tube is uniform, Rott's wave equation reduces to

$$[1+(\gamma-1)f_k]p+\frac{a^2}{\omega^2}(1-f_\nu)\frac{d^2p}{dx^2}=0. \tag{A2}$$

The functions f_k and f_ν are Rott's functions.[2,3] The acoustic pressure is solution of Eq. (A2). The spatial average gas velocity is given by[2,3]

$$\langle u\rangle=\frac{j}{\rho_m\omega}\frac{dp}{dx}(1-f_\nu). \tag{A3}$$

If the time dependence is of the form $e^{j\omega t}$, then the solution of Eq. (A2) in the geometry illustrated in Fig. 4(a) is given by

$$p(x,t)=(Be^{-jkx}+Ce^{jkx})e^{j\omega t}. \tag{A4}$$

The volume velocity U can be written as

$$U=A\langle u(x,t)\rangle=A[De^{-jkx}+Ee^{jkx}]e^{j\omega t}, \tag{A5}$$

where k is the wave number and A is the cross-sectional area of the tube. The parameters B, C, D, and E are complex to account for both the magnitude and phase of oscillation at angular frequency ω. By using Eqs. (A4), (A5), and (A3) the acoustic impedance at any position x in the tube is obtained:

$$Z_a(x)=\frac{p}{U}=Z_c\left[\frac{Be^{-jkx}+Ce^{jkx}}{Be^{-jkx}-Ce^{jkx}}\right], \tag{A6}$$

where Z_c is given by

$$Z_c=\frac{\rho_m\omega}{Ak[1-f_\nu]}. \tag{A7}$$

Expression (A6) represents the equivalent impedance of the complete subsystem downstream of the point x [Fig. 4(a)]. Splitting the exponential factors into cosine and sine and combining terms, Eq. (A6) can be rewritten as

$$Z_a(x)=Z_c\frac{\dfrac{B+C}{B-C}-j\tan kx}{1-j\dfrac{B+C}{B-C}\tan kx}. \tag{A8}$$

The impedance at x' is given by Eq. (A8), where x is replaced by x'. By using $Z_a(x')$, the ratio $(B+C)/(B-C)$ can be eliminated to obtain the transfer function

$$Z_a(x)=\frac{Z_a(x')\cos[k(x'-x)]+jZ_c\sin[k(x'-x)]}{\cos[k(x'-x)]+j\dfrac{Z_a(x')}{Z_c}\sin[k(x'-x)]}. \tag{A9}$$

This important expression shows how the acoustic impedance at a point x is related to that at point x' [Fig. 4(a)]. Now we will apply expression (A9) to our resonator shown in Fig. (1), which can be considered to consist of three parts: one part with a large diameter, the stack and heat exchangers, and a small diameter tube terminating in a buffer volume which simulates an open end. Such a resonator can be approximately represented by the resonator shown in Fig. 4(b).

Using the continuity condition of p and U at the interfaces between the three different parts of the resonator and applying Eq. (A9) three times at x_2, x_1, and 0, we obtain the expressions of the impedances at these positions:

$$Z_a(x_2)=jZ_{c3}\tan(k_3L_3), \tag{A10}$$

$$Z_a(x_1)=\frac{Z_a(x_2)\cos(k_2L_s)+jZ_{c_2}\sin(k_2L_s)}{j\dfrac{Z_a(x_2)}{Z_{c_2}}\sin(k_2L_s)+\cos(k_2L_s)}, \tag{A11}$$

and

$$Z_a(0)=\frac{Z_a(x_1)\cos(k_1x_1)+jZ_{c_1}\sin(k_1x_1)}{j\dfrac{Z_a(x_1)}{Z_{c_1}}\sin(k_1x_1)+\cos(k_1x_1)}, \tag{A12}$$

where L_3 is the length of the small diameter tube and L_s is the length of the stack. The impedance Z_{ci} is given by

$$Z_{ci}=\frac{\rho_m\omega}{A_ik_i[1-f_{\nu i}]}, \tag{A13}$$

where A_i is the cross-sectional area of part i of the tube (Fig. 4), k_i and $f_{\nu i}$ are the wave number and Rott's function, respectively, in part i. The viscous and thermal damping effects are included in k_i and $f_{\nu i}$, which are given by[6]

$$k_i^2=\frac{\omega^2}{a^2}\frac{1+(\gamma-1)f_{ki}}{1-f_{\nu i}} \tag{A14}$$

and

$$f_{ki}=\frac{\tanh(\alpha_k)}{\alpha_k},\quad f_{\nu i}=\frac{\tanh(\alpha_\nu)}{\alpha_\nu}, \tag{A15}$$

where

$$\alpha_k=\frac{(1+j)y_0}{\delta_k},\quad \alpha_\nu=\frac{(1+j)y_0}{\delta_\nu}.$$

The quantities δ_ν and δ_k are the viscous and thermal penetration depths, respectively, and r_i is the radius of part i. In parts 1 and 3 the boundary layer approximation ($\delta_\nu,\delta_k \ll r_i$) is valid and f_ν and f_k can be simplified.[2,3]

Inserting Eqs. (A10) and (A11) into Eq. (A12) yields the acoustic impedance of the resonator as seen by the driving piston. The resultant expression is too long and complicated to be given explicitly here, but it is appropriate for computer applications.

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