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A general approach to tune the vibration properties of the mounting system in the high-speed and heavy-duty engine

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Abstract

Majority of the current research on the mounting system has emphasised on the low/medium power engine, rare work has been reported for the high-speed and heavy-duty engine, the vibration characteristics of which exhibits significantly increased complexity and uncertainty. In this work, a general dynamics model was firstly established to describe the dynamic properties of a mounting system with various numbers of mounts. Then, this model was employed for the optimization of the mounting system. A modified Powell conjugate direction method was developed to improve the optimization efficiency. Basing on the optimization results obtained from the theoretical model, a mounting system was constructed for a V6 diesel engine. The experimental measurement of the vibration intensity of the mounting systems shows excellent agreement with the theoretical calculations, indicating the validity of the model. This dynamics model opens a new avenue in assessing and designing the mounting system for a high-speed and heavy-duty engine. On the other hand, the delineated dynamics model, and the optimization algorithm should find wide applications for other mounting systems, such as the power transmission system which usually has various uncertain mounts.

Keywords: mounting system, vibration, inertia matrix, stiffness matrix

1. Introduction

For a long period of time, the engine research were majorly focused on dynamics issues, such as how to increase the power-to-weight ratio, decrease the specific fuel consumption and specific weight, or improve the reliability and durability of engine. Owing to the increasing requirement of the energy conservation and environment protection, a great deal of research effort has been drawn on the exhaust gas emission (Zheng et al., 2004, Abdelaal and Hegab, 2012), vibration and noise control and other environmental and riding comfort related issues. It is witnessed that almost all recent major technical progress of engine are focused on these areas (Williams et al., 2012, Salvi et al., 2012). Specifically, the mounting system is the principal research field for the vibration and noise control of engine, which could ensure the stable operation of the engine by adjusting the vibration intensity on the one hand, and on the other hand, could alleviate the actuation from the engine that exerted on the vehicle (Shangguan, 2009, Yu et al., 2001).

A great amount of research has been conducted on the control of engine vibration. As is well known, the vibration of an internal combustion engine is originated from two aspects, one is the pressure combustion and the other one is the inertia force and torque caused by the rotating and reciprocating parts (Periyasamy and Alwarsamy, 2012, Sigmund et al., 2012). Therefore, a most effective and typical way to reduce the vibration is to control and moderate the vibration excitations from the inner part of the engine. The balancing schemes are the popular method that

being used to reduce the inner inertia force and torque of engine, which have been applied in various engines (Baocheng et al., 2002), e.g., Cummins 4TB, CAT 3304, DEUTZ BFM1015CP, EQB 140-11 and others. Recent development of the new combustion technique (like the moderate combustion) has also greatly benefited the alleviation of engine vibration. Another usual way is by the aid of the external mounting system. The concepts of "centre of percussion", "torque roll axis" (Hu and Singh, 2011), "elastic centre" and others that proposed during the 70-80th last century are still acting critically in the design of the mounting system, especially when these concepts are combined with the modern optimization (Ahn et al., 2003), signal processing, simulation and modelling techniques (Park and Singh, 2010, Christopherson and Jazar, 2006, Barszcz et al., 2012). Besides, the design of new shock-absorber or damper is also a promising way to control the engine vibration, which has become an active research area owing to the advancement of techniques and materials. For instance, the hydraulic engine mounts has been widely used in the intermediate cars due to its compact structure and adjustable damping property (Wu and Shangguan, 2010a). In addition, researchers have also proposed various novel active or semi-active techniques to tune the performance of the mounting system and thus control the engine vibration (Fakhari and Ohadi, 2012, Wu and Shangguan, 2010b, Truong and Ahn, 2010, Lee and Lee, 2009), though the popularization of these techniques are still requiring wide investigation.

It is noticed that majority of previous researches or techniques are devoted to the engine with low/medium power, and rare work has been reported for the high-speed and heavy-duty diesel engines (Wu et al., 2005). For example, the wire rope vibration isolator (Tinker and Cutchins, 1992), knitted wire mesh shock-absorber and other metallic elastic mounts are reported with good oil and aging resistance, stable performance under a wide range of temperature, whereas seldom of them has been successfully utilized in the high-speed and heavy-duty diesel due to their large volume, installation inconvenience and lacking of displacement limiting capacity. Restricted by the limitation of affordable load, the application of hydraulic elastic mounts for the heavy-duty engine is still requesting extensive investigation. Therefore, along with the increasing requirement of the environmental protection and riding comfort for the vehicles with high-speed and heavy-duty diesel engines, such as engineering vehicles and heavy-duty trucks, the investigation of the mounting system shows great significances and contributions to the current knowledge body, which is the major aim of this paper.

Conclusively, although the high-speed and high engine power do not play influential role in the mounting design process, they usually requires a mounting system with more mounts that is different from a typical 3 or 4 mounts system. In this paper, we will establish a general dynamics

model to describe the dynamic properties of such a mounting system, basing on the assumption that the mounting system is supported by the normal rubber elastic shock absorber, which is usually the case for the high-speed and heavy-duty diesel engine. The outline of the present paper is as follows: in Sec. 2.1 below, the free vibration analysis of the engine mounting system will be detailed to derive the system's natural frequencies and their corresponding vibration modes. In Sec. 2.2, the forced vibration analysis will be conducted which is the actual vibration states when engine works. The optimization as well as the design of the engine mounting system will be followed in Sec. 3. Finally, we shall construct the mounting system for a V6 diesel engine, in situ vibration experiments will be carried out to validate the theoretical model in Sec. 4.

2. Dynamics model of the engine mounting system

From the viewpoint of the vibration isolation, the natural frequency of the engine mounting system ranges from 6 to 30 Hz (which is far below the first natural frequency of the engine (usually hundreds of Hz), thereby the engine can be simplified as spatial rigid body that is comprised of by many separate rigid components, such as cylinder head, cylinder block, lower crankcase, crankshaft, turbo and others. The mounting system can then be described by the generalized coordinates vector $\{q\} = \{x, y, z, \theta_x, \theta_y, \theta_z\}^T$ with six degrees of freedom. Here x, y and z denote the axe of the global coordinates system. As schematically shown in Figure 1, we consider n mounts, each of which has stiffness in three principal directions. The origin of the global coordinates is located at the balance position of the centre of mass, while the x axis is along the axe of the crankshaft and z axis is along the vertical direction. It is noteworthy to mention that, the usual treatment is adopted to establish the theoretical model in this paper, i.e., the installation foundation is considered as a rigid body (Yu et al., 2001). This assumption is valid, as the highest natural frequency of the engine mounting system is usually far below the first natural frequency of the chassis that is installed with high-speed and heavy-duty engine.
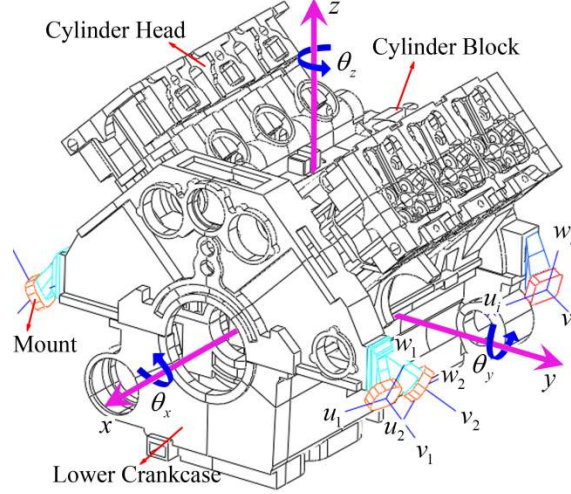
Figure 1. Schematic view of the engine mounting system.

2.1 Free vibration of the engine mounting system

For a conservative mechanical system, the Lagrangian equation is

$$\frac{d}{dt}\left(\frac{\partial T}{\partial q_j}\right) - \frac{\partial T}{\partial q_j} = -\frac{\partial V}{\partial q_j} \tag{1}$$

where T and V represent the kinetic and potential energy of the system, respectively. q_j is the generalized coordinates of the system. Hence, the differential equation for the free vibration of the mounting system can be expressed as

$$[M]\{\ddot{q}\} + [K]\{q\} = 0 \tag{2}$$

Here, $[M]$ and $[K]$ are the inertial and stiffness matrices of the system, respectively. $\{\ddot{q}\}$ denotes the generalized acceleration vector.

2.1.1 Kinetic energy of the engine mounting system

During the vibration of the engine mounting system, the kinetic energy of the whole system T is composed by contributions from two aspects, i.e., the translational kinetic energy T_t and the rotational kinetic energy T_r of the system regarding to the mass centre. Basically, T_t is estimated by considering the engine as a whole rigid body, while T_r is calculated from a summation over all rigid components, i.e.,

$$T = T_t + T_r = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + \frac{1}{2}\sum_{i=1}^{n} I_i \dot{\theta}_i^2 \tag{3}$$

where m is the mass of the whole system. I_i and ω_i are the moment of inertia and the angular velocity of the ith component around the rotation axis, respectively. n is the total number of engine components.

For simplicity, the unit vector along x, y and z directions are denoted as $\vec{i}$, $\vec{j}$ and $\vec{k}$, respectively. Assuming slight vibration of the system, the angular displacement vector as well as the angular velocity vector of the system with respect to the mass centre can be obtained as below

$$\vec{\theta} = \theta_x \vec{i} + \theta_y \vec{j} + \theta_z \vec{k} , \ \dot{\vec{\theta}} = \dot{\theta}_x \vec{i} + \dot{\theta}_y \vec{j} + \dot{\theta}_z \vec{k} \tag{4}$$

According to Eq. (4), for any rigid component i with a radius vector of $\vec{r}_i = x_i \vec{i} + y_i \vec{j} + z_i \vec{k}$ with respect to the mass centre, its rotational kinetic energy is given as

$$T_{ri} = \frac{1}{2} m_i \left[(-y_i \dot{\theta}_z + z_i \dot{\theta}_y) \vec{i} + (-z_i \dot{\theta}_x + x_i \dot{\theta}_z) \vec{j} + (-x_i \dot{\theta}_y + y_i \dot{\theta}_x) \vec{k} \right] \tag{5}$$

Hence, the total rotational kinetic energy T_r can be expressed as below

$$T_r = \frac{1}{2} J_x \dot{\theta}_x^2 + \frac{1}{2} J_y \dot{\theta}_y^2 + \frac{1}{2} J_z \dot{\theta}_z^2 - (J_{xy} \dot{\theta}_x \dot{\theta}_y + J_{yz} \dot{\theta}_y \dot{\theta}_z + J_{zx} \dot{\theta}_z \dot{\theta}_x) \tag{6}$$

Here, J_x, J_y and J_z are the system's moment of inertia, and J_{xy}, J_{yz} and J_{zx} are the system's product of inertia, they are given as $J_x = \sum_{i=1}^{n} m_i (y_i^2 + z_i^2)$, $J_y = \sum_{i=1}^{n} m_i (z_i^2 + x_i^2)$,

$J_z = \sum_{i=1}^{n} m_i (x_i^2 + u_i^2)$, $J_{xy} = \sum_{i=1}^{n} m_i x_i y_i$, $J_{yz} = \sum_{i=1}^{n} m_i y_i z_i$ and $J_{zx} = \sum_{i=1}^{n} m_i z_i x_i$, respectively.

Substituting Eq. (6) into Eq. (3), the total kinetic energy has the following matrix form

$$T = \frac{1}{2} \{\dot{q}\}^T [M] \{\dot{q}\} \tag{7}$$

where $[M]$ represents the inertial matrix of the system, which equals

$$[M] = \begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & J_x & -J_{xy} & -J_{xz} \\ 0 & 0 & 0 & -J_{xy} & J_y & -J_{yz} \\ 0 & 0 & 0 & -J_{xz} & -J_{yz} & J_z \end{bmatrix} \tag{8}$$

Apparently, by considering x, y and z as the engine's principal axe of inertia, we have $J_{xy} = J_{yz} = J_{zx} = 0$. Under such circumstance, $[M]$ becomes a diagonal matrix, indicating the system has no inertia coupling.

2.1.2 Potential energy of the engine mounting system

As usual, we assume the potential energy of the system at static balance states equals zero and ignore the gravitational potential energy during engine vibration. Hence, the potential energy of

the engine mounting system can be calculated through the summation of the potential energy of each mount that is induced by the elastic deformation during vibration. As aforementioned, we consider the mounting system has n mounts, each of which possesses stiffness in three principal directions as k_{xj}, k_{yj} and k_{zj}. To derive the potential energy, a local coordinates system is established for each mount with u_j, v_j and w_j representing the unit vector along the three principal axe. According to Figure 1, the relation between the local coordinates system and the global coordinates system can be described by the angular values as listed in Table 1.

Table 1. The angles between the local coordinates system and global coordinates system.

	ox	oy	oz
u_j	α_{1j}	β_{1j}	γ_{1j}
v_j	α_{2j}	β_{2j}	γ_{2j}
w_j	α_{3j}	β_{3j}	γ_{3j}

Suppose the displacement vector of the jth mount equals $\Delta \vec{q}_j = \Delta x_j \vec{i} + \Delta y_j \vec{j} + \Delta z_j \vec{k}$, which can be decomposed along the local coordinates system as

$$\{\Delta U_j\} = \begin{Bmatrix} \Delta u_j \\ \Delta v_j \\ \Delta w_j \end{Bmatrix} = [F_j]\{q_j\}, j=1, 2, \ldots, n \tag{9}$$

where $[F_j]$ is determined by the location and orientation of the mount, which is derived as

$$[F_j] = \begin{bmatrix} \cos\alpha_{1j} & \cos\beta_{1j} & \cos\gamma_{1j} \\ \cos\alpha_{2j} & \cos\beta_{2j} & \cos\gamma_{2j} \\ \cos\alpha_{2j} & \cos\beta_{3j} & \cos\gamma_{3j} \end{bmatrix}$$

$$\left. \begin{matrix} -z_j\cos\alpha_{1j}+y_j\cos\gamma_{1j} & -x_j\cos\gamma_{1j}+z_j\cos\alpha_{1j} & -y_j\cos\alpha_{1j}+x_j\cos\beta_{1j} \\ -z_j\cos\alpha_{2j}+y_j\cos\gamma_{2j} & -x_j\cos\gamma_{2j}+z_j\cos\alpha_{2j} & -y_j\cos\alpha_{2j}+x_j\cos\beta_{2j} \\ -z_j\cos\alpha_{3j}+y_j\cos\gamma_{3j} & -x_j\cos\gamma_{3j}+z_j\cos\alpha_{3j} & -y_j\cos\alpha_{3j}+x_j\cos\beta_{3j} \end{matrix} \right] \tag{10}$$

Therefore, the total potential energy of the vibration system can be written as

$$V = \frac{1}{2}\sum_{j=1}^{n}\left(k_{uj}\Delta u_j^2 + k_{vj}\Delta v_j^2 + k_{wj}\Delta w_j^2\right) = \frac{1}{2}\{q\}^T[K]\{q\} \tag{11}$$

Here, $[K]$ represents the stiffness matrix of the system, which equals

$$[K] = \sum_{j=1}^{n}[F_j]^T[K_j][F_j], \; [K_j] = \begin{bmatrix} k_{uj} & 0 & 0 \\ 0 & k_{vj} & 0 \\ 0 & 0 & k_{wj} \end{bmatrix} \tag{12}$$

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From Eq. (12), the system stiffness matrix is determined by the mount's number, stiffness, as well as its location and orientation. It is noteworthy to mention that, the stiffness matrix can be greatly simplified when the local coordinates system u, v and w are parallel to the global coordinates system, under which condition, $[F_j]$ is only related to the mount's location, i.e.,

$$[F_j] = \begin{bmatrix} 1 & 0 & 0 & 0 & z_j & -y_j \\ 0 & 1 & 0 & -z_j & 0 & x_j \\ 0 & 0 & 1 & y_j & -x_j & 0 \end{bmatrix} \tag{13}$$

2.1.3 Solving the differential equation for the free vibration of the mounting system

Since we get the inertia matrix $[M]$ and the stiffness matrix $[K]$, the differential equation for the free vibration of the engine mounting system can be solved by substituting Eqs. (8) and (12) into Eq. (2). The solution can be formally given as

$$\{q\} = \{X\}\sin(\omega t + \alpha) \tag{14}$$

Simplify Eq. (2) by using Eq. (14), we get

$$[K]\{X\} = \omega^2 [M]\{X\} \tag{15}$$

Let $[M]^{-1}[K] = [A]$, then Eq. (15) can be expressed as

$$[A]\{X\} = \omega^2 \{X\} \tag{16}$$

It is evident from Eq. (16) that ω^2 is the eigenvalue of the matrix $[A]$, and $\{X\}$ is its corresponding eigenvector. In particular, $[A]$ is an asymmetric matrix that has real eigenvalue and eigenvector, for which, the common solving methods will inevitably induce round-off error. To improve the calculation precision, Eq. (16) is solved by the procedure described below. According to Eq. (8), the inertia matrix $[M]_{6\times6}$ is a symmetric positive definite matrix, which can be decomposed according to Cholesky factorization as $[M] = [D][D]^T$, where $[D]$ is a low triangular matrix. Let

$$[P] = [D]^{-1}[K]([D]^{-1})^T, \quad \{Y\} = [D]^T\{X\} \tag{17}$$

Eq. (16) is changed as

$$[P]\{Y\} = \omega^2 \{Y\} \tag{18}$$

Apparently, $[P]$ and $[K]$ are similar matrices, and $[P]$ is a symmetric matrix. According to the Jacobi method, the eigenvalues and their corresponding eigenvectors can be easily calculated. These eigenvalues are actually the square values of the natural frequencies of the mounting system. Through Eq. (17), the eigenvector $\{Y\}$ can be converted to the system eigenvector $\{X\}$.

Including all eigenvectors in a matrix, the modal vector matrix of the mounting system $[\Phi]$ can be obtained. Conclusively, through the analysis of the free vibration of the mounting system, we obtained the system's natural frequencies, and their corresponding vibration modes.

2.2 Forced vibration of the engine mounting system

2.2.1 Differential equation of the structural damping vibration system

The vibration excitations that exerted on the engine are usually periodic, and each excitation can be decomposed into several harmonic components. According to the principle of linear superposition, each harmonic component can be treated separately, and the overall response of the system can be obtained by the summation of all individual response. Particularly, for a periodic harmonic excitation load, the differential equation of a linear vibration system is given as

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} = \{f(t)\} \tag{19}$$

where $[M]$ and $[K]$ are the inertia and stiffness matrices that deduced from Sec. 2.1, $\{\dot{q}\}$ is the generalized velocity vector, $\{f(t)\}$ is the generalized load vector and $[C]$ is the damping matrix that is obtained by assembling the damping characteristics from the mounting system. To mention that, the generalized load vector includes the two major aspects of vibration excitations, i.e., the overturning moment due to the combustion pressure, and the unbalanced inertia forces and torques of the moving parts.

As mentioned before, for the high-speed and heavy-duty diesel engine, the mount is usually rubber elastic component which can be treated as structural damping as it is working under low frequency status. Hence the damping matrix is given as

$$[C] = j\eta[K] \tag{20}$$

where $j = \sqrt{-1}$, η is the structural loss factor which is between 0.05 ~ 0.3 for the rubber elastic component. For the natural butadiene nitrile rubber, $\eta = 0.075$ (Yang and Bai, 2002). Substituting Eq. (20) into Eq. (19), the differential equation is then given as

$$[M]\{\ddot{q}\} + (1 + j\eta)[K]\{q\} = \{f(t)\} \tag{21}$$

For the above equation, the stiffness and damping characteristics of the mount are represented by the complex stiffness matrix. $j\eta[K]\{q\}$ is the imaginary restoring force of the system's complex stiffness matrix, which represents the amplitude and phase of the damping force $[C]\{\dot{q}\}$. Particularly, the vibration isolation performance of the mounting system is determined by the real part $[K]$ of the complex stiffness matrix. In other words, we can adjust the installation location and orientation of the rubber elastic component, as well as change the stiffness of the component to avoid the system from falling in the resonance region. We should note that the damping effect

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of the rubber component exert ignorable influence to the isolation of the system vibration, while it does benefit the reduction of the vibration amplitude during the acceleration or deceleration of the engine when it passes the resonance domain.

2.2.2 Solving the differential equation for the forced vibration of the mounting system

Since the principal axe of inertia of the mounting system is usually unparallel to the axis of the crankshaft, thereby, the off-diagonal elements J_{xy}, J_{yz} and J_{zx} in the inertia matrix Eq. (8) are generally not all zero, i.e., the vibration system is inertial coupled. In the meanwhile, the stiffness matrix also possesses many nonzero off-diagonal elements, i.e., the vibration system is also elastic coupled. Thus, Eq. (21) is actually an equation set that being comprised by a group of non-independent equations, to solve which the decoupling is required.

Assume one of the vibration excitation equals $\{f(t)\} = \{F\}e^{j\omega t}$, according to the orthogonal property of both the inertia matrix $[M]$ and stiffness matrix $[K]$ with the system model matrix $[\Phi]$, Eq. (21) can be termed as

$$[M_m]\{\ddot{q}_m\} + (1+j\eta)[K_m]\{q_m\} = [\phi]^T\{F\}e^{j\omega t} \tag{22}$$

where $[M_m]$ and $[K_m]$ are the diagonalized inertia and stiffness matrices under the modal coordinates, respectively. $[q_m]$ is the modal coordinates, and $\{q\} = [\phi]\{q_m\}$. Thus, the inter-coupled vibration equation Eq. (21) is changed to the six independent single-degree-of-freedom vibration equation under the modal coordinates through the orthogonal transformation. The decoupled ith equation is then expressed as

$$m_{mi}\ddot{q}_{mi} + K_{mi}(1+j\eta)q_{mi} = F_{mi}e^{j\omega t}, \ (i = 1, 2, \ldots, 6) \tag{23}$$

Here, m_{mi} is the ith modal mass, $K_{mi}(1+j\eta)$ is the ith modal stiffness and q_{mi} is the ith modal coordinate of the system. F_{mi} is the force that exerted on the ith modal coordinate, which is given as $F_{mi} = \sum_{j=1}^{6}\phi_{ji}F$ (the overturning moment is applied along the θ_x direction for $i = 4$ and the unbalanced inertia forces and torques are implemented along their corresponding directions). By solving Eq. (23), following result is obtained

$$q_{mi} = \frac{F_{mi}/K_{mi}}{\sqrt{\left(1 - \frac{\omega^2}{p_i^2}\right)^2 + \eta^2}} \sin\left(\omega t - \theta_{mi}\right) \tag{24}$$

where $p_i^2 = K_{mi}/m_{mi}$ and $\theta_{Pi} = arctg(\eta p_i^2 / (p_i^2 - \omega^2))$. Hence, the displacement response under the global coordinates equals

$$\{q\} = [\varphi]\{q_m \sin(\omega t - \theta_m)\} \tag{25}$$

From above equation, the corresponding velocity $\{\dot{q}\}$ and acceleration $\{\ddot{q}\}$ can be obtained. Similarly, we can calculate the response of the system under a group of excitations, and then obtain the overall response of the system through the principle of linear superposition. Since we get the overall system response, the response of any locations α with the coordinates of x_α, y_α and z_α can be easily estimated from the flowing equation

$$\{X_\alpha, Y_\alpha, Z_\alpha\}^T = [E]\{q\} \tag{26}$$

where $[E]$ is the coordinates transformation matrix, which is given as

$$[E] = \begin{bmatrix} 1 & 0 & 0 & 0 & z_\alpha & -y_\alpha \\ 0 & 1 & 0 & -z_\alpha & 0 & x_\alpha \\ 0 & 0 & 1 & y_\alpha & -x_\alpha & 0 \end{bmatrix} \tag{27}$$

3. Optimization of the engine mounting system

Sec. 2 has developed a theoretical model that is capable of describing any kind of engine mounting system, which could not only be used to analyse the vibration characteristics of the existing mounting system, but also be applied to guide the design of the mounting system. In this section, we will show how to optimize the mounting system by using this model which will be achieved by following a typical optimization procedure.

3.1 Design parameters and objective functions

According to the governing equations of the mounting system given by Eq. (2) and (21), the vibration characteristics of the mounting system is determined by the inertia matrix $[M]$, stiffness matrix $[K]$ and damping matrix $[C]$. Hence the design parameters can be categorized into two groups for a mounting system with n mounts. The first group is the characteristic parameters, i.e., the stiffness of the mount. The second one is the geometry parameters, which includes the installation location and orientation of the mount (see Table 1).

To monitor the vibration of the mounting system, we can either measure the vibration intensity of the engine itself or the excitations that it exerts on the foundation. In this paper, the minimum equivalent vibration intensity of the engine V_s is chosen as the objective function, which is defined as below. For a working circulation, the effective vibration velocity V_{rms} of the engine is defined as

$$V_{rms} = \sqrt{\frac{1}{T} \int_0^T v^2(t) dt} \tag{28}$$

where $v(t)$ is the vibration velocity and T is the vibration period. Suppose $v(t)$ is comprised by n known harmonic components, then V_{rms} can be calculated either through the known effective vibration velocity component V_{rmsi},

$$V_{rms} = \sqrt{\sum_{i=1}^{n} V_{rmsi}^2} \tag{29}$$

or the known vibration peak of each velocity component $\hat{v}_i$,

$$V_{rms} = \sqrt{\frac{1}{2} \sum_{i=1}^{n} \hat{v}_i^2} \tag{30}$$

Hence, the equivalent vibration intensity equals

$$V_s = \sqrt{\left(\frac{\sum_{i=1}^{N_x} V_{xi}}{N_x}\right)^2 + \left(\frac{\sum_{i=1}^{N_y} V_{yi}}{N_y}\right)^2 + \left(\frac{\sum_{i=1}^{N_z} V_{zi}}{N_z}\right)^2} \tag{31}$$

where V_x, V_y and V_z are the effective vibration velocity along the three orthogonal directions, with N_x, N_y and N_z as the corresponding inspection points that being considered along the three principal directions. For instance, the objective function can be termed as below when nine inspection points of the engine are considered along each principal direction.

$$\min V_x(\vec{X}) = \sqrt{\left(\frac{\sum_{i=1}^{9} V_{xi}}{9}\right)^2 + \left(\frac{\sum_{i=1}^{9} V_{yi}}{9}\right)^2 + \left(\frac{\sum_{i=1}^{9} V_{zi}}{9}\right)^2} \tag{32}$$

3.2 Constraints and optimization algorithm

To conduct the optimization, following aspects of constraints should be considered:

i. Geometrical: the location and orientation of the mounts are restricted by the available engine and foundation compartment space;

ii. Stiffness: the mount should have high stiffness to ensure a low shake level at low frequency and support the whole engine, while it also requires low stiffness to ensure low noise levels;

iii. Natural frequency of the mounting system: the natural frequency of the mounting system should avoid the resonance and shock excitation;

iv. Vibration transmissibility, upper limit around 25%;

v. Decoupling of the vibration system.

According to Eq. (32), the objective function as well as some of the constraints (such as constraints iii, iv and v) is a complex implicit function of the design parameters. Therefore, the traditional gradient-based optimization algorithm is not suitable, while the non-gradient-based optimization algorithms, such as the Complex Method are unable to treat problems with multiple design parameters. Hence, we developed a new optimization algorithm as detailed below.

Firstly, the constraint mathematic model is converted to the unconstraint model according to the augmented Lagrange multiplier method (Deb and Srivastava, 2012). Then, a new algorithm that requires no calculation of gradient is employed, which is based on the Powell conjugate direction method (PCDM). Since it is uneasy to obtain the global optimal solution for a medium or large optimization problem for the traditional PCDM, several modifications have been made to the PCDM. More specifically, the initial iteration point is chosen by following the idea of interior point penalty function, which could ensure it satisfies all the constraints. Afterwards, a set of orthogonal directions is adopted as the conjugate searching reference direction. Furthermore, the traditional PCDM iteration step selection is replaced by the one dimensional optimal selection along the reference direction that based on the Fibonacci method. Such step selection scheme could avoid the PCDM method from falling in an infinite loop when it comes across the local optimal solution, thus enable the fast locating of the global optimal solution from the feasible domain.

4. Experimental validation

To show the validity of the theoretical model, in situ experiments were conducted. The mounting system was constructed according to the optimization results that obtained following certain specific requirements. Comparisons between the experimental measurements and theoretical calculations were made.

4.1 Design of the mounting system

The considered target is a V6 diesel engine that used in a heave-duty vehicle, which uses the reciprocating piston and side-by-side connecting rods. The cylinder diameter is 155 mm, with the crank radius as 80 mm and the connecting rod's length as 300 mm. The mass of the engine is estimated as 1286.31 Kg, with its overall dimension as $1 \times 1.05 \times 1.02$ m^3. According to the dynamics calculation and equilibrium analysis of the inertial force, the vibration excitations are basically originated from the overturning moment and the secondary reciprocating inertia moment (Deng and Zhang, 2012). Recall Eq. (8), the inertial matrix is calculated as below, which incorporates contributions from totally 88 different components, including cylinder head, cylinder block, lower crankcase, crankshaft, turbo and others. To note that, to best mimic a real situation, the lubrication oil and cooling liquid are also considered during the calculation of $[M]$.

$$[M] = \begin{bmatrix} 1286.31 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1286.31 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1286.31 & 0 & 0 & 0 \\ 0 & 0 & 0 & 109.45 & -1.39 & -17.02 \\ 0 & 0 & 0 & -1.39 & 139.35 & 2.08 \\ 0 & 0 & 0 & -17.02 & 2.08 & 132.77 \end{bmatrix} \tag{33}$$

According to the actual engineering requirement, the mounting system of this V6 diesel engine has four mounts, as schematically shown in Figure 2. Specifically, the left and right mounts are symmetric along the axis of crankshaft, and the u-axis of the local coordinate system of the mount is parallel to the global x-axis, which eliminates two angular degrees of freedom. According to Figure 2, each mount has seven design mounts, and there are only fourteen independent parameters due to the symmetric configuration of the mount as denoted below

$$\{X\} = \left(x_f, y_f, z_f, \theta_f, k_{uf}, k_{vf}, k_{wf}, x_r, y_r, z_r, \theta_r, k_{ur}, k_{vr}, k_{wr}\right) \tag{34}$$

The first seven parameters are for the right front mount M_f, with the rest seven parameters for the right left mount M_r as illustrated in Figure 2.
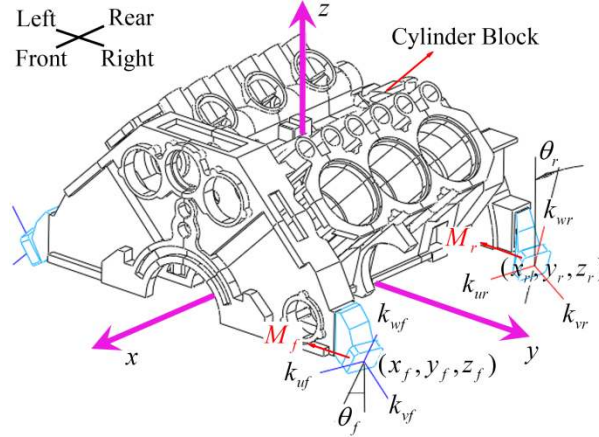
Figure 2. Schematic view of an engine mounting system with four mounts. x_f, y_f and z_f represent the coordinates of the right front mount M_f, θ_f is the angle between the local w axis and the global z axis, k_{uf}, k_{vf} and k_{wf} are the stiffness along the three local principal axe. Similar definitions are applied to the right rear mount M_r.

Since the confirmation of the design parameters, we then consider the constraints. Firstly, the installation space of the four mounts is restricted not only by the geometry of the diesel engine but also by the installation space of the foundation. The stiffness of the mount should be high enough to support the whole engine, while higher stiffness will lead to worse vibration isolation. For each vibration mode, the frequency of the mounting system is $1/\sqrt{2}$ of the lowest excitation frequency along the corresponding frequency direction (Kelly, 2007). To keep the elastic centre of the front and rear mounts locating at the torque axis, and thus make the horizontal vibration

along y direction, vertical vibration along z direction as well as torsional vibration along x axis decoupled, following two constraints are applied (Zhang et al., 2004) (see Ref. 25 for the derive details of these two constraints)

$$\begin{cases} tg\theta_f = \dfrac{\left(1-k_{uf}/k_{wf}\right)-\sqrt{\left(1-k_{uf}/k_{wf}\right)^2-4\left(|z_f|/|y_f|\right)\left(k_{uf}/k_{wf}\right)}}{2\left(|z_f|/|y_f|\right)} \\[4mm] tg\theta_r = \dfrac{\left(1-k_{ur}/k_{wr}\right)-\sqrt{\left(1-k_{ur}/k_{wr}\right)^2-4\left(|z_r|/|y_r|\right)\left(k_{ur}/k_{wr}\right)}}{2\left(|z_r|/|y_r|\right)} \end{cases} \qquad (35)$$

By considering nine inspection locations, substituting all parameters and constraints to the objective function Eq. (32), the following optimization results were obtained for the fourteen design parameters $\{X\}$ as listed in Table 2.

Table 2 Optimization results of the mounting system with four mounts for a V6 diesel engine.

	x (mm)	y (mm)	z (mm)	θ	k_u (N/mm)	k_v (N/mm)	k_w (N/mm)
Front mount	491.9	410.4	-48.1	18.5°	1265.3	214.6	214.6
Rear mount	-623.5	378.5	-110	37.5°	1050.4	146.3	146.3

Under these optimization results, the modal frequencies of the mounting system, as well as their corresponding normalized vibration modes were calculated as listed in Table 3. As is seen, almost all of off-diagonal elements of the normalized vibration mode table are very close to zero, suggesting vibration along the six directions are well decoupled. Specifically, the vibration transmissibility along all three principal directions is estimated below 10%.

Table 3 Six vibration modes of the mounting system and their corresponding normalized modes of vibration.

| Modal frequency (Hz)	f_x	f_y	f_z	$f_{\theta x}$	$f_{\theta y}$	$f_{\theta z}$
	4.04	3.98	9.34	14.27	13.99	7.96
Corresponding normalized vibration mode	1	-0.00006	-0.00861	0.00189	-0.05723	-0.00071
	-0.00018	1	-0.00203	0.05821	0.00120	0.01346
	0.00059	-0.00006	1	0.00177	-0.05199	-0.00055
	-0.00033	-0.00641	-0.00124	1	0.02764	0.00229
	0.00571	0.00047	0.04462	-0.03088	1	0.02012
	-0.00087	-0.09290	-0.00247	-0.36652	-0.00044	1

4.2 Establishment and testing of the real mounting system

According to the above optimization, we build the mounting system for the V6 diesel engine, basing on which several vibration intensity measurements were conducted.

The experiments were carried out according to the Chinese national standard GB/T 12779-91, i.e., the method of measurement and evaluation of mechanical vibration for reciprocating machines. The measurement scheme is presented in Figure 3, where a Schenck D-1200 dynamometer was used to apply the load to the engine, and the B&K 4321 acceleration sensor

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was utilized to collect the vibration acceleration signals along three principal directions. After that, the signal was converted and amplified by the B&K 2635 charge amplifier before sending to the HP 3565S/I-DEAS^{TM} 4.0 signal collection and analysis system. In the end, the signals were recorded by the SGI workstation.
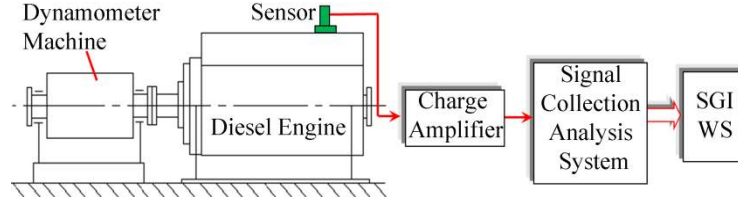

Figure 3. Schematic view of the experimental set up for the vibration tests.

We tested the three typical working status of the diesel engine, which includes I: idling condition, with the rotation speed of 800 rpm and torque of 163 N·m; II: external characteristic peak torque condition, with the rotation speed of 1500 rpm and torque of 1930 N·m; III: standard running condition, with the rotation speed of 2200 rpm and torque of 1681 N·m. During each testing, nine B&K 4321 acceleration sensors were assigned to the engine which corresponds to the nine inspection points that being considered during the optimization. Figure 4 shows the actual locations of these nine inspection points. All experiments were conducted under the temperature of 20 ℃, with the humidity as 30% and the pressure of 675 mmHg.
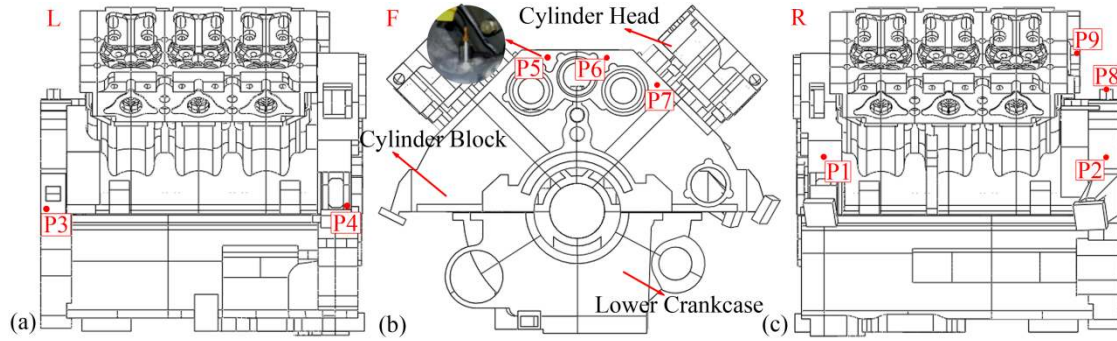
Figure 4. Arrangement of the nine acceleration sensors on the V6 diesel engine: (a) left view; (b) front view; (c) right view. P1-P9 represent the nine inspection locations. Inset shows the actual sensor that assigned to the P5 location.

Figure 5 presents the time history of the velocity derived from the acceleration signals that detected from inspection point P3 under the standard running condition (status III). By incorporating the velocity data into Eqs. (28)-(30), the effective vibration velocity of each inspection point can be obtained, basing on which the equivalent vibration intensity of the engine can be acquired from Eq. (31).
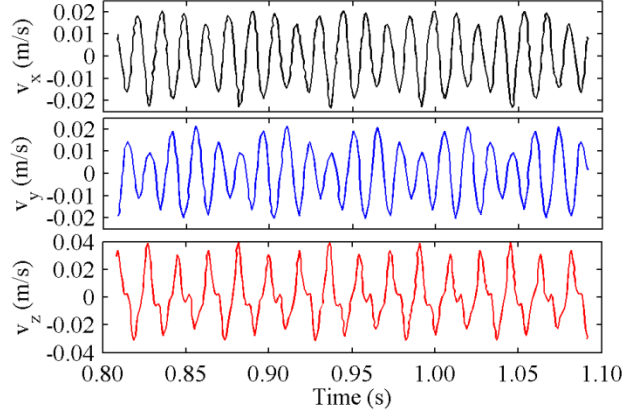
Figure 5. Time history of the velocity for inspection point P3 along three orthogonal directions under the standard running condition (status III). The velocity is obtained from the integration of the acceleration signals.

To aware that, during the test, the power and the speed of the engine vary with time. In order to minimize the influence from this fact, a number of testing results were recorded for each working status. The vibration intensity is then obtained by using the regression analysis basing on these experimental measurements. As is seen in Table 4, under the three typical working statuses, the calculation results show excellent agreement with the experimental measurements. For example, the vibration intensity under the external characteristic peak torque condition (status II) is around 21.04 mm·s^{-1}, which only appears a 3.91% diverge comparing with that calculated from the theoretical model. These results indicate the validity of our model.

Table 4 Comparisons of the vibration intensity between the experimental measurements and theoretical calculations.

Working status	I	II	III
Experimental (mm·s^{-1})	10.08	22.04	27.86
Theoretical (mm·s^{-1})	9.56	21.21	26.77
Relative error (%)	5.43	3.91	4.07

5. Conclusions

A general dynamics model was established to depict the vibration properties of a mounting system for a high-speed and heavy-duty engine. This model was used to guide the configuration of the mounts for a V6 diesel engine. In situ experiments were conducted to verify the validity of the model. Major conclusions include:

1) The inertia matrix of the mounting system was determined by the mass and mass centre of each engine component, while the stiffness matrix was decided by mount's number, location, orientation, and the stiffness;

2) To treat the complex optimization problem herein, where the objective function as well as constraints is an implicit function of the design parameters, the Powell conjugate direction method was modified. The modified algorithm is able to avoid optimization from being

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trapped by the local optimal value with infinite loop, and could lead to fast locating of the global optimal value from the feasible domain;

3) An in situ vibration test was conducted based on a V6 diesel engine mounting system that was constructed according to the theoretical optimization results. It was found that the theoretical calculations exhibit excellent agreement with the experimental measurements, which indicates the validity of the current dynamics model.

Conclusively, this paper has established a general approach for the engine mounting system, which is able to not only assess the existing mounting system, but also guide the design of the mounting system for certain specific requirements. Though this model was established basing on the high-speed and heavy-duty diesel engine, it should also be suitable to treat a mounting system with low/medium power engine. Particularly, the delineated dynamics model, optimization algorithms should find wide applications for other different mounting systems, such as power transmission system.

Acknowledgement

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