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A General Computer Program to Determine the Perturbation of Alternating Electric Currents in a Two-Dimensional Model of a Region of Uniform Conductivity with an Embedded Inhomogeneity

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Summary

A computer program to calculate the perturbation of alternating electric currents in a two-dimensional Earth model with a conductivity inhomogeneity is presented. The program provides for an inhomogeneity of arbitrary shape surrounded by a region of different conductivity. The equations and boundary conditions are solved by a numerical method for both E-polarization and H-polarization. The computer program allows for the solution over a grid of variable mesh dimensions and for a general model which consists of several conductivities. The program is given in detail and an example for a particular model is illustrated.

1. Introduction

There is considerable interest at present in electromagnetic induction in the Earth and the solution of the induction problem for a surface or buried region of conductivity different from its surroundings.

Many observational studies have been made in recent years of the effects of vertical discontinuities in electrical conductivity of the Earth on geomagnetic variations. Several mathematical approaches have been taken with respect to these problems. D'Erceville & Kunetz (1962), Rankin (1962) and Weaver (1963) have approached the problem analytically, while Wright (1970) employed a transmission line analogy with a numerical approach.

Price (1964) pointed out that the problem to be considered is one of determining the local perturbations of a given alternating system of induced currents by given abrupt changes of conductivity. Unifoım currents are induced in a conductor and are perturbed locally by 'local' variations in conductivity.

Jones & Price (1970) discussed the equations and boundary conditions for a two-dimensional problem in which the conducting region is a semi-infinite half-space made up of two quarter spaces of different conductivity. This problem was solved by a numerical technique to obtain the field distributions within the conductor and the surface values of the various components along the surface of the conducting half-space. Both the E-polarization (E parallel to the strike) and the H-polarization (H parallel to the strike) were considered. Jones & Price (1971a) extended this to a comparison of three models with different contact geometry between the two conducting regions. Also, Jones & Price (1971b) considered a model with one region

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surrounded by a region of different conductivity, and Jones (1971) investigated a two-layered structure of general contact topography.

In the work by Jones & Price (1971b) a surface or buried rectangular region of one conductivity surrounded by a region of different conductivity was considered. Both the E-polarization and H-polarization cases were solved for a given frequency, and surface values as a function of conductivity, depth of overburden and dimensions of the anomaly were considered. From the foregoing work it has become clear that there is a need for a general computer program to deal with a two-dimensional anomaly of arbitrary shape. The present work illustrates a flexible method of dealing with such a problem. The method allows for a region of arbitrary shape made up of one or more regions of different conductivity and gives the solution in terms of field distributions and surface values of the components. Also, the method includes a provision for a variable grid size in order to remove some of the limitations encountered by using a square grid.

2. The general model

The general model is illustrated in Fig. 1 along with the co-ordinate system. The interface between the anomalous region and the surrounding region is of arbitrary shape and can be adjusted. The grid size is variable, and the anomalous region can be composed of several different conductivities as represented by the different letters. The conductivity A represents free space.

An alternating current, of circular frequency ω, flows in the model. This current is parallel to the surface at $y = \pm\infty$.

3. The differential equations and boundary conditions

For the two polarization cases the equations, in electromagnetic units, to be solved in the various regions are identical and are given by Jones & Price (1970) as:

E-polarization:

$$\frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} = i\eta^2 E_x \tag{1}$$

H-polarization:

$$\frac{\partial^2 H_x}{\partial y^2} + \frac{\partial^2 H_x}{\partial z^2} = i\eta^2 H_x \tag{2}$$

where $\eta^2 = 4\pi\sigma\omega$.

These equations must be solved in each region with the appropriate conductivity (σ) inserted and with the appropriate boundary conditions. The usual boundary conditions exist between the media at internal points of the mesh as explained by Jones & Price (1970). The boundary conditions on the outer boundaries of the mesh ($y \to \pm\infty$, $z \to \pm\infty$) will be discussed for E-polarization and H-polarization separately.

E-polarization

In the case of E-polarization, the only non-zero field components are E_x, H_y and H_z. E_x satisfies equation (1) with the appropriate value of η inserted for each region. At large distances from any discontinuity in σ it is assumed that the field behaves like that for a uniform conductor. Hence as $y \to +\infty$ or $-\infty$, E_x within the conductor is of the form (Jones & Price 1970)

$$E_x = E_0 \exp\{\eta\sqrt{[(i)]}z\}, \tag{3}$$

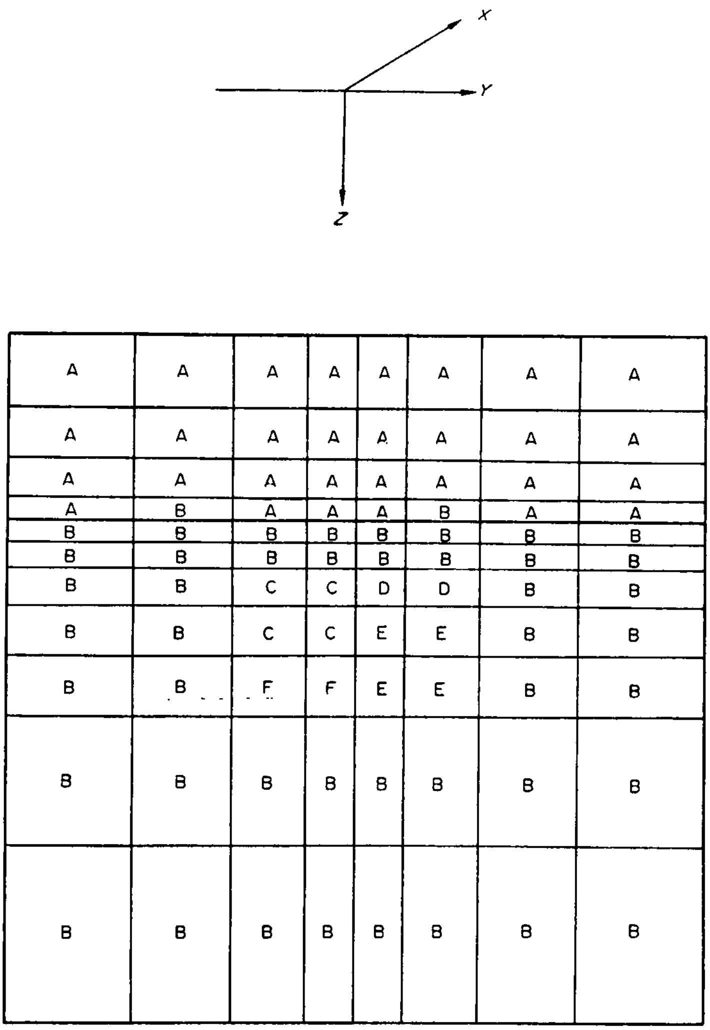


FIG. 1. The co-ordinate system and the general model. The different letters indicate regions of different conductivity. Regions lettered A constitute the non-conducting region.

where E_0 is the value of E_x at the surface and η depends on σ. When the region surrounding the conductivity anomaly is uniform, as in the case we are considering here, E_0 is the same for $y = +\infty$ and $y = -\infty$.

Within the conductor the field components tend to zero as $z \to \infty$, and in particular we require that the perturbation effect of the anomalous structure be negligible at the lower boundary. In the computational method used the field components can be made to approach zero on the lower boundary by choosing vertical grid dimensions such that the lower boundary ($z = d$) is several skin depths from the surface. It is then possible to set the value of E_x at the lower boundary constant and equal to

$$E_x|_{y=\infty,\, z=d}.$$

Outside the conductor ($z < 0$) for $|y|$ large we have

$$E_x = E_0\{1+\eta\sqrt{[(i)]}z\} \qquad (4)$$

as shown by Jones & Price (1970) and so is a linear function increasing with $-z$ $z \to -\infty$. Jones & Price (1970) have shown that the horizontal component of magnetic field (H_y) is the same at $y \to \pm\infty$ for all negative values of z. H_y can then be taken equal to a constant value (say H_0) on finite boundaries corresponding to $z = -h_0$ at $y = \pm k$ and all along the boundary $z = -h_0$ provided that this boundary is far enough away to make the local perturbation in **H** negligible there. Since, in this particular problem, $E = E_0$ for $y = \pm\infty$ (or in fact for $y = \pm k$), then we may take

$$E_x = E_0\{1+\eta\surd[(i)]\,h_0\} \tag{5}$$

along the upper boundary of the grid as long as the above conditions on **H** are met.

H-polarization

For the H-polarization case the components H_x, E_y and E_z are involved. Also, for the H-polarization case the magnetic field is constant in $z < 0$ (Jones & Price 1970). H_x is therefore constant and equal to H_0, say, along the surface of the conductor as well. It is therefore only necessary to consider the region $z > 0$.

At large distances ($y \to \pm\infty$) from the anomaly we again assume a uniform conductor as in the E-polarization case. The solution is then similar to the E-polarization case and so for $|y|$ large and $z > 0$,

$$H_x = H_0 \exp\{-\eta\surd[(i)]z\} \tag{6}$$

where H_0 is the value of H_x at the surface and η depends on σ.

Within the conductor ($z > 0$), the field components vanish as $z \to \infty$, and we assume a similar boundary condition on the lower boundary of the mesh as we did in the E-polarization case. We choose the lower boundary constant and equal to the value at $|y|$ large. It should be emphasized that this lower boundary must be at large enough z so that the fields approach zero.

4. The numerical formulation

The method of solution involves the solution of the appropriate finite difference equations over a mesh of grid points by the Gauss–Seidel iterative method. The equation to be solved in all regions for both the E-polarization and H-polarization cases is of the form

$$\nabla^2 F = i\eta^2 F, \quad \text{where} \quad \eta^2 = 4\pi\sigma\omega \tag{7}$$

and F is either E_x or H_x, depending upon the case we are considering. If we let $F = f+ig$ then

$$\nabla^2 f + i\nabla^2 g = i\eta^2 f - \eta^2 g$$

and equating real and imaginary parts we obtain

$$\nabla^2 f = -\eta^2 g \tag{8}$$

$$\nabla^2 g = \eta^2 f. \tag{9}$$

If a small region of the mesh is considered as illustrated in Fig. 2, equations (8) and (9) must be satisfied at each point and in particular point '0'. Four conductivities occupy the quadrants surrounding the point '0'. Also, the mesh sizes about the point '0' vary and in general $d_1 \neq d_2 \neq d_3 \neq d_4$. Equations (8) and (9) become:

$$(\nabla^2 f)_0 = (-\eta^2 g)_0, \tag{10}$$

$$(\nabla^2 g)_0 = (\eta^2 f)_0. \tag{11}$$

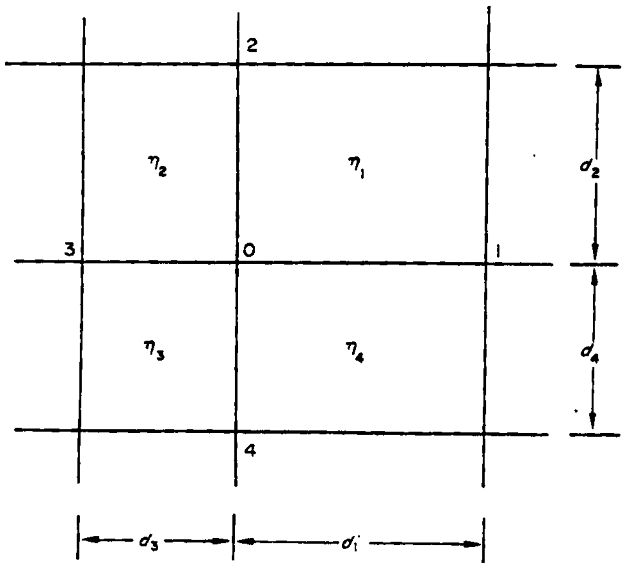


FIG. 2. Notation used for grid points, dimensions and conductivities of the regions surrounding point ' 0 '.

To obtain a pair of finite difference equations we make use of Taylor's Theorem which yields

$$f_1 = f_0 + \left(\frac{\partial f}{\partial y}\right)_0 d_1 + \frac{1}{2}\left(\frac{\partial^2 f}{\partial y^2}\right)_0 d_1^2 + \ldots$$

$$f_2 = f_0 + \left(\frac{\partial f}{\partial z}\right)_0 d_2 + \frac{1}{2}\left(\frac{\partial^2 f}{\partial z^2}\right)_0 d_2^2 + \ldots$$

$$f_3 = f_0 - \left(\frac{\partial f}{\partial y}\right)_0 d_3 + \frac{1}{2}\left(\frac{\partial^2 f}{\partial y^2}\right)_0 d_3^2 + \ldots$$

$$f_4 = f_0 - \left(\frac{\partial f}{\partial z}\right)_0 d_4 + \frac{1}{2}\left(\frac{\partial^2 f}{\partial z^2}\right)_0 d_4^2 + \ldots$$

and similar equations for g_1, g_2, g_3, g_4.

If we neglect higher order terms we can express equations (10) and (11) as a pair of finite difference equations:

$$f_0\left(\frac{1}{d_1^2} + \frac{1}{d_2^2} + \frac{1}{d_3^2} + \frac{1}{d_4^2}\right) - \eta^2 g_0$$

$$= f_1\left[\frac{1}{d_1^2} + \frac{1}{(d_1+d_3)}\left(\frac{1}{d_3} - \frac{1}{d_1}\right)\right] + f_2\left[\frac{1}{d_2^2} + \frac{1}{(d_2+d_4)}\left(\frac{1}{d_4} - \frac{1}{d_2}\right)\right]$$

$$+ f_3\left[\frac{1}{d_3^2} + \frac{1}{(d_1+d_3)}\left(\frac{1}{d_1} - \frac{1}{d_3}\right)\right] + f_4\left[\frac{1}{d_4^2} + \frac{1}{(d_2+d_4)}\left(\frac{1}{d_2} - \frac{1}{d_4}\right)\right] \quad (12)$$

or

$$f_0\left(\Sigma\frac{1}{d_i^2}\right)-\eta^2 g_0=f_1\,D_1+f_2\,D_2+f_3\,D_3+f_4\,D_4 \tag{12'}$$

$$\begin{aligned} &g_0\left(\frac{1}{d_1^2}+\frac{1}{d_2^2}+\frac{1}{d_3^2}+\frac{1}{d_4^2}\right)+\eta^2 f_0 \\ &\quad=g_1\left[\frac{1}{d_1^2}+\frac{1}{(d_1+d_3)}\left(\frac{1}{d_3}-\frac{1}{d_1}\right)\right]+g_2\left[\frac{1}{d_2^2}+\frac{1}{(d_2+d_4)}\left(\frac{1}{d_4}-\frac{1}{d_2}\right)\right] \\ &\qquad+g_3\left[\frac{1}{d_3^2}+\frac{1}{(d_1+d_3)}\left(\frac{1}{d_1}-\frac{1}{d_3}\right)\right]+g_4\left[\frac{1}{d_4^2}+\frac{1}{(d_2+d_4)}\left(\frac{1}{d_2}-\frac{1}{d_4}\right)\right] \end{aligned} \tag{13}$$

or

$$g_0\left(\Sigma\frac{1}{d_i^2}\right)+\eta^2 f_0=g_1\,D_1+g_2\,D_2+g_3\,D_3+g_4\,D_4. \tag{13'}$$

Equations (12′) and (13′) must be satisfied at each interior point of each region. In particular, these two equations can be solved simultaneously at point '0' for f_0 and g_0 where up-to-date values of f_i and g_i are obtained from the previous iteration.

In the following, the first subscript indicates the conductive region considered (1, 2, 3 or 4) and the second subscript refers to the particular point of interest. Equations (12′) and (13′) must hold for each of the surrounding regions. That is:

$$f_{10}\left(\Sigma\frac{1}{d_i^2}\right)-\eta_1^2 g_{10}=f_{11}\,D_1+f_{12}\,D_2+\underline{f_{13}}\,D_3+\underline{f_{14}}\,D_4 \tag{14}$$

$$f_{20}\left(\Sigma\frac{1}{d_i^2}\right)-\eta_2^2 g_{20}=\underline{f_{21}}\,D_1+f_{22}\,D_2+f_{33}\,D_3+\underline{f_{24}}\,D_4 \tag{15}$$

$$f_{30}\left(\Sigma\frac{1}{d_i^2}\right)-\eta_3^2 g_{30}=\underline{f_{31}}\,D_1+\underline{f_{32}}\,D_2+f_{33}\,D_3+f_{34}\,D_4 \tag{16}$$

$$f_{40}\left(\Sigma\frac{1}{d_i^2}\right)-\eta_4^2 g_{40}=f_{41}\,D_1+\underline{f_{42}}\,D_2+\underline{f_{43}}\,D_3+f_{44}\,D_4 \tag{17}$$

$$g_{10}\left(\Sigma\frac{1}{d_i^2}\right)+\eta_1^2 f_{10}=g_{11}\,D_1+g_{12}\,D_2+\underline{g_{13}}\,D_3+\underline{g_{14}}\,D_4 \tag{18}$$

$$g_{20}\left(\Sigma\frac{1}{d_i^2}\right)+\eta_2^2 f_{20}=\underline{g_{21}}\,D_1+g_{22}\,D_2+g_{23}\,D_3+\underline{g_{24}}\,D_4 \tag{19}$$

$$g_{30}\left(\Sigma\frac{1}{d_i^2}\right)+\eta_3^2 f_{30}=\underline{g_{31}}\,D_1+\underline{g_{32}}\,D_2+g_{33}\,D_3+g_{34}\,D_4 \tag{20}$$

$$g_{40}\left(\Sigma\frac{1}{d_i^2}\right)+\eta_4^2 f_{40}=g_{41}\,D_1+\underline{g_{42}}\,D_2+\underline{g_{43}}\,D_3+g_{44}\,D_4 \tag{21}$$

where the underlined values are 'fictitious' values. The boundary conditions for the interfaces allow these values to be expressed in terms of known values. We consider first the E-polarization case and then the H-polarization case.

(a) *Internal boundaries*

E-polarization. The boundary conditions are that both the tangential and normal components of **H** are continuous across any interface. These two components may be expressed in terms of E_x as (Jones & Price 1970)

$$H_y = \frac{i}{\omega}\frac{\partial E_x}{\partial z}$$

$$= \frac{i}{\omega}\left(\frac{\partial f}{\partial z}\right) - \frac{1}{\omega}\left(\frac{\partial g}{\partial z}\right)$$

$$H_z = \frac{-i}{\omega}\frac{\partial E_x}{\partial y}$$

$$= \frac{-i}{\omega}\left(\frac{\partial f}{\partial y}\right) + \frac{1}{\omega}\left(\frac{\partial g}{\partial y}\right).$$

The condition for continuity of the tangential components applied to each boundary lead to the finite difference equations:

$$\begin{aligned}
\underline{f_{13}}-f_{10} &= f_{23}-f_{20} & \underline{g_{13}}-g_{10} &= g_{23}-g_{20}\\
\underline{f_{21}}-f_{20} &= f_{11}-f_{10} & \underline{g_{21}}-g_{20} &= g_{11}-g_{10}\\
\underline{f_{31}}-f_{30} &= f_{41}-f_{40} & \underline{g_{31}}-g_{30} &= g_{41}-g_{40}\\
\underline{f_{43}}-f_{40} &= f_{33}-f_{30} & \underline{g_{43}}-g_{40} &= g_{33}-g_{30}\\
\underline{f_{14}}-f_{10} &= f_{44}-f_{40} & \underline{g_{14}}-g_{10} &= g_{44}-g_{40}\\
\underline{f_{24}}-f_{20} &= f_{34}-f_{30} & \underline{g_{24}}-g_{20} &= g_{34}-g_{30}\\
\underline{f_{32}}-f_{30} &= f_{22}-f_{20} & \underline{g_{32}}-g_{30} &= g_{22}-g_{20}\\
\underline{f_{42}}-f_{40} &= f_{12}-f_{10} & \underline{g_{42}}-g_{40} &= g_{12}-g_{10}.
\end{aligned}$$

These equations allow us to express the fictitious values of equations (14) to (21) in terms of known values. Adding equations (14), (15), (16), (17) and making use of the fact that

$$f_{ab} = f_b$$

$$g_{ab} = g_b,$$

we obtain

$$Af_0 + Bg_0 = f_1 C_1 + f_2 C_2 + f_3 C_3 + f_4 C_4. \tag{22}$$

Similarly, adding (18), (19), (20), (21) we obtain

$$-Bf_0 + Ag_0 = g_1 C_1 + g_2 C_2 + g_3 C_3 + g_4 C_4, \tag{23}$$

where in these two equations

$$\begin{aligned}
A &= 4\Sigma\frac{1}{d_i^2}\\
B &= -\Sigma\eta_i^2\\
C_1 &= 4D_1\\
C_2 &= 4D_2\\
C_3 &= 4D_3\\
C_4 &= 4D_4.
\end{aligned}$$

Equations (22) and (23) are simultaneous equations which must be solved for f_0 and g_0.

H-polarization. As in the *E*-polarization case, continuity of the tangential components of **E** allows the fictitious values of equations (14) to (21) to be expressed in terms of known values. From Jones & Price (1970), the electric field components may be written:

$$E_y = \frac{\omega}{\eta^2}\frac{\partial H_x}{\partial z}$$

$$= \frac{\omega}{\eta^2}\left(\frac{\partial f}{\partial z}\right) + i\frac{\omega}{\eta^2}\left(\frac{\partial g}{\partial z}\right)$$

$$E_z = \frac{-\omega}{\eta^2}\frac{\partial H_x}{\partial y}$$

$$= \frac{-\omega}{\eta^2}\left(\frac{\partial f}{\partial y}\right) - i\frac{\omega}{\eta^2}\left(\frac{\partial g}{\partial y}\right).$$

When the condition that the tangential components of **E** must be continuous is applied the following finite difference equations are obtained for f:

$$\underline{f_{13}} - f_{10} = \frac{{\eta_1}^2}{{\eta_2}^2}(f_{23} - f_{20})$$

$$\underline{f_{14}} - f_{10} = \frac{{\eta_1}^2}{{\eta_4}^2}(f_{44} - f_{40})$$

$$\underline{f_{21}} - f_{20} = \frac{{\eta_2}^2}{{\eta_1}^2}(f_{11} - f_{10})$$

$$\underline{f_{24}} - f_{20} = \frac{{\eta_2}^2}{{\eta_3}^2}(f_{34} - f_{30})$$

$$\underline{f_{31}} - f_{30} = \frac{{\eta_3}^2}{{\eta_4}^2}(f_{41} - f_{40})$$

$$\underline{f_{32}} - f_{30} = \frac{{\eta_3}^2}{{\eta_2}^2}(f_{22} - f_{20})$$

$$\underline{f_{43}} - f_{40} = \frac{{\eta_4}^2}{{\eta_3}^2}(f_{33} - f_{30})$$

$$\underline{f_{42}} - f_{40} = \frac{{\eta_4}^2}{{\eta_1}^2}(f_{12} - f_{10}).$$

A similar set of equations is obtained for g.

These equations allow us to express the fictitious values of equations (14)–(21) in terms of known values. Adding equations (14), (15), (16), (17) we obtain

$$Af_0 + Bg_0 = f_1\, C_1 + f_2\, C_2 + f_3\, C_3 + f_4\, C_4 \qquad (24)$$

and adding (18), (19), (20), (21) we obtain

$$-Bf_0 + Ag_0 = g_1\, C_1 + g_2\, C_2 + g_3\, C_3 + g_4\, C_4 \qquad (25)$$

where, in these two equations

$$A = \frac{4}{d_1^2} + D_1\left(\frac{\eta_2^2}{\eta_1^2} + \frac{\eta_3^2}{\eta_4^2} - 2\right) + \frac{4}{d_2^2} + D_2\left(\frac{\eta_3^2}{\eta_2^2} + \frac{\eta_4^2}{\eta_1^2} - 2\right)$$

$$+ \frac{4}{d_3^2} + D_3\left(\frac{\eta_4^2}{\eta_3^2} + \frac{\eta_1^2}{\eta_2^2} - 2\right) + \frac{4}{d_4^2} + D_4\left(\frac{\eta_1^2}{\eta_4^2} + \frac{\eta_2^2}{\eta_3^2} - 2\right)$$

$$B = -(\eta_1^2 + \eta_2^2 + \eta_3^2 + \eta_4^2)$$

$$C_1 = D_1\left(2 + \frac{\eta_2^2}{\eta_1^2} + \frac{\eta_3^2}{\eta_4^2}\right)$$

$$C_2 = D_2\left(2 + \frac{\eta_3^2}{\eta_2^2} + \frac{\eta_4^2}{\eta_1^2}\right)$$

$$C_3 = D_3\left(2 + \frac{\eta_4^2}{\eta_3^2} + \frac{\eta_1^2}{\eta_2^2}\right)$$

$$C_4 = D_4\left(2 + \frac{\eta_1^2}{\eta_4^2} + \frac{\eta_2^2}{\eta_3^2}\right)$$

and D_1, D_2, D_3, D_4 are the same as for the E-polarization case.

(b) *External boundaries*

E-polarization. For E-polarization, on the boundary between the non-conducting region and the conductor ($z = 0$) and for $y \to +\infty$, $y \to -\infty$, we set $E_x = E_0 = 1$, that is, $f = 1$, $g = 0$. In the non-conducting region ($z < 0$) and for $|y|$ large, we have from equation (4),

$$E_x = E_0\{1 - \eta\sqrt{[(i)]}z\}$$

$$= 1 - \frac{\eta z}{\sqrt{2}} - \frac{i\eta z}{\sqrt{2}}$$

and so:

$$f = 1 - \frac{\eta z}{\sqrt{2}}$$

and

$$g = -\frac{\eta z}{\sqrt{2}}.$$

In the conductor ($z > 0$) and for $|y|$ large, from equation (3) we have

$$E_x = E_0 \exp\{-\eta\sqrt{[(i)]}z\}$$

$$= \exp\{-\eta\sqrt{(i)}z\}.$$

Therefore:

$$f = \exp\left(\frac{-\eta}{\sqrt{2}}z\right)\cos\frac{\eta}{\sqrt{2}}z$$

and

$$g = -\exp\left(\frac{-\eta}{\sqrt{2}}z\right)\sin\frac{\eta}{\sqrt{2}}z.$$

The lower boundary ($z = d$) is assumed to be far enough away from the perturbation that it can be made constant. Then

$$f = \exp\left(-\frac{\eta}{\sqrt{2}}d\right)\cos\frac{\eta}{\sqrt{2}}d$$

and

$$g = -\exp\left(\frac{-\eta}{\sqrt{2}}d\right)\sin\frac{\eta}{\sqrt{2}}d$$

on that boundary. Along the upper boundary ($z = -h_0$) E_x is constant, and so:

$$f = 1 - \frac{\eta}{\sqrt{2}}(-h_0)$$

and

$$g = -\frac{\eta}{\sqrt{2}}(-h_0)$$

on this boundary.

H-polarization. Along the surface of the conductor ($z = 0$)

$$H_x = H_0 = 1$$

and therefore $$f = 1, \quad g = 0.$$

Above the surface of the conductor ($z < 0$), H_x is constant and equal to the value at the surface. This means that $f = 1$, $g = 0$ in the non-conducting region and it is not necessary to solve for f and g there. However, in our programs we have had occasion to compare E-polarization and H-polarization problems and we have provided for a variable placement of the surface of the conductor in both programs. Hence, we initially set the E-polarization and H-polarization grids the same, place the surface of the conductor along the same row of grid points for E-polarization and H-polarization cases, and then solve for the whole grid in the E-case, but only for the grid corresponding to the conducting regions for the H-case. In the conducting region ($z > 0$) and for $|y|$ large

$$H_x = H_0 \exp\{-\eta\sqrt{[(i)]}z\}$$

$$= H_0 \exp\left(-\frac{\eta}{\sqrt{2}}z\right)\left(\cos\frac{\eta z}{\sqrt{2}} - i\sin\frac{\eta z}{\sqrt{2}}\right).$$

Therefore

$$f = \exp\left(\frac{-\eta}{\sqrt{2}}z\right)\cos\frac{\eta}{\sqrt{2}}z$$

and

$$g = -\exp\left(-\frac{\eta}{\sqrt{2}}z\right)\sin\frac{\eta}{\sqrt{2}}z.$$

The values of f and g on the lower boundary of the model ($z = d$) are the same as for the E-case:

$$f = \exp\left(-\frac{\eta}{\sqrt{2}} d\right) \cos \frac{\eta}{\sqrt{2}} d$$

$$g = -\exp\left(-\frac{\eta}{\sqrt{2}} d\right) \sin \frac{\eta}{\sqrt{2}} d.$$

5. Calculation of components

In general

$$F = (f + ig) \exp(i\theta)$$

where $F = H_x$ or E_x and $\theta = \omega t$ is a function of time.

E-polarization

In this case, the value of E which is actually observed may be written

$$E_{x_{obs}} = Re\,[(f+ig)\exp(i\theta)] = f\cos\theta - g\sin\theta.$$

Similarly for the magnetic field components:

$$H_{y_{obs}} = Re\left[\frac{i}{\omega}\frac{\partial E_x}{\partial z}\right] = \frac{-1}{\omega}\left(\frac{\partial f}{\partial z}\sin\theta + \frac{\partial g}{\partial z}\cos\theta\right),$$

$$H_{z_{obs}} = Re\left[\frac{-i}{\omega}\frac{\partial E_x}{\partial y}\right] = \frac{1}{\omega}\left(\frac{\partial f}{\partial y}\sin\theta + \frac{\partial g}{\partial y}\cos\theta\right).$$

The phases of these three components may be calculated as follows:

$$(\text{Phase } E_x)_{obs} = \text{Arctan}\left(\frac{f\sin\theta + g\cos\theta}{f\cos\theta - g\sin\theta}\right),$$

$$(\text{Phase } H_y)_{obs} = \text{Arctan}\left\{\frac{\dfrac{\partial f}{\partial z}\cos\theta - \dfrac{\partial g}{\partial z}\sin\theta}{-\dfrac{\partial f}{\partial z}\sin\theta - \dfrac{\partial g}{\partial z}\cos\theta}\right\},$$

$$(\text{Phase } H_z)_{obs} = \text{Arctan}\left\{\frac{-\dfrac{\partial f}{\partial y}\cos\theta + \dfrac{\partial g}{\partial y}\sin\theta}{\dfrac{\partial f}{\partial y}\sin\theta + \dfrac{\partial g}{\partial y}\cos\theta}\right\}.$$

In the computer program the relative phase between each point on the surface and the end point is calculated. This is independent of the time (i.e. θ), since if at a particular point we have

$$E_x = f + ig$$

and if we write

$$\phi = \tan^{-1}\frac{g}{f}$$

then $f = \cos\phi$ and $g = \sin\phi$, so that at this point the phase calculation for a given θ gives

$$\begin{aligned}\Phi &= \tan^{-1}\left[\frac{f\sin\theta+g\cos\theta}{f\cos\theta-g\sin\theta}\right]\\ &= \tan^{-1}\left[\frac{\sin(\phi+\theta)}{\cos(\phi+\theta)}\right]\\ &= \phi+\theta.\end{aligned}$$

Similarly, at some other point the phase calculation will give

$$\Phi' = \phi'+\theta.$$

Hence the difference in phase between these two points will be

$$\Omega = \Phi'-\Phi = \phi'-\phi,$$

and so in general the phase difference between any two points will be independent of θ and so constant with respect to time. It is therefore sufficient to calculate the phase shift across the surface with respect to an end point for only one value of θ.

H-polarization

In this case similar expressions are obtained for the components:

$$H_{x_{obs}} = Re\,[(f+ig)\exp(i\theta)] = f\cos\theta-g\sin\theta,$$

$$E_{y_{obs}} = Re\left[\frac{\omega}{\eta^2}\frac{\partial H_x}{\partial z}\right] = \frac{\omega}{\eta^2}\left\{\frac{\partial f}{\partial z}\cos\theta-\frac{\partial g}{\partial z}\sin\theta\right\},$$

$$E_{z_{obs}} = Re\left[\frac{-\omega}{\eta^2}\frac{\partial H_x}{\partial y}\right] = \frac{-\omega}{\eta^2}\left\{\frac{\partial f}{\partial y}\cos\theta-\frac{\partial g}{\partial y}\sin\theta\right\},$$

$$(\text{Phase } H_x)_{\text{obs}} = \text{Arctan}\left(\frac{f\sin\theta+g\cos\theta}{f\cos\theta-g\sin\theta}\right)$$

$$(\text{Phase } E_y)_{\text{obs}} = \text{Arctan}\left\{\frac{\dfrac{\partial f}{\partial z}\sin\theta+\dfrac{\partial g}{\partial z}\cos\theta}{\dfrac{\partial f}{\partial z}\cos\theta-\dfrac{\partial g}{\partial z}\sin\theta}\right\}$$

$$(\text{Phase } E_z)_{\text{obs}} = \text{Arctan}\left\{\frac{\dfrac{\partial f}{\partial y}\sin\theta+\dfrac{\partial g}{\partial y}\cos\theta}{\dfrac{\partial f}{\partial y}\cos\theta-\dfrac{\partial g}{\partial y}\sin\theta}\right\}$$

The same comments about the phase calculations apply for this case as for the E-polarization case.

6. The computer programs

The computer programs are written in FORTRAN IV, and the development of the program and the solution for the example illustrated have been done on the University of Alberta IBM 360/67. The program for the H-polarization case is given in detail in Figs 3–10. Comment statements are included in the program for guidance. The E-polarization program is similar to that for the H-polarization. However, three sub-routines are slightly different, the subroutine for calculating the boundary values (BYCOND), the iteration subroutine (ITERE), and the subroutine for calculating the surface values of the components (SURFVL). These subroutines are given in Figs 11, 12 and 13. Also, the notation throughout differs for the two cases.

Both the input and output data are in electromagnetic units. The same data can be used as input for either the E or H case. The programs compute the amplitudes

```
FORTRAN IV G COMPILER        MAIN             03-16-71            12:20.40         PAGE 0001

      C                                                                                   C
      C                                                                                   C
      C                                                                                   C
      C     H-POLARISATION PROGRAM                                                        C
      C                                                                                   C
      C                                                                                   C
      C                                                                                   C
      C     PURPOSE                                                                       C
      C                                                                                   C
      C     TO SOLVE FOR THE MAGNETIC FIELD FOR A TWO DIMENSIONAL MODEL OF A              C
      C     CONDUCTIVE CONFIGURATION ON A 41 X 41 SET OF GRID POINTS                      C
      C                                                                                   C
      C                                                                                   C
      C                                                                                   C
      C     REMARKS                                                                       C
      C                                                                                   C
      C     AN ITERATIVE METHOD IS USED TO COMPUTE THE REAL AND IMAGINARY                 C
      C     PARTS OF THE MAGNETIC FIELD                                                   C
      C                                                                                   C
      C                                                                                   C
      C                                                                                   C
      C     SUBROUTINES REQUIRED                                                          C
      C                                                                                   C
      C     BYCOND (N) SETS THE BOUNDARY VALUES ON THE 41X41 GRID WITH                    C
      C     THE SURFACE OF THE EARTH ON THE N'TH ROW OF THE GRID                          C
      C                                                                                   C
      C     ITERH (EPS,MAXIT,N) ITERATES UP TO MAXIT TIMES OVER THE GRID IN               C
      C     THE REGION BELOW THE EARTH'S SURFACE UNTIL THE CHANGE IN                      C
      C     BOTH F AND G IS LESS THAN EPS                                                 C
      C                                                                                   C
      C     SURFVL (N) CALCULATES THE ELECTRIC AND MAGNETIC COMPONENTS AT                 C
      C     THE SURFACE OF THE EARTH                                                      C
      C                                                                                   C
      C     HFIELD PRINTS OUT THE MAGNETIC FIELD AS CALCULATED AT EACH GRID               C
      C     POINT FOR ANY DESIRED PHASE OF THE CYCLE                                      C
      C                                                                                   C
      C                                                                                   C
      C                                                                                   C
      C     METHOD                                                                        C
      C                                                                                   C
      C     A 41 X 41 VARIABLE SIZED GRID IS SUPERIMPOSED ON THE TWO-                     C
      C     DIMENSIONAL MODEL OF INTEREST.  THE GRID STEP SIZES ARE READ                  C
      C     FOR THE HORIZONTAL AND VERTICAL AXES AS WELL AS THE SCALE (CM.)               C
      C     AND THE FREQ (SEC**(-1)) OF THE APPLIED SOURCE FIELD USED FOR                 C
      C     THE MODEL.  THE CONDUCTIVE CONFIGURATION (40X40) IS READ NEXT ---             C
      C     THE DATA FOR THIS CONSISTS OF THE INDEX OF THE CONDUCTIVITY                   C
      C     DESIRED FOR ANY PARTICULAR REGION.  THERE MAY BE UP TO 15                     C
      C     CONDUCTIVITIES IN THE MODEL (READ INTO THE VECTOR CONDUC(15)).                C
      C                                                                                   C
      C     ONCE THE DATA FOR ANY MODEL HAS BEEN READ BY THE PROGRAM,                     C
      C     THE BOUNDARY VALUES ARE SET BY A CALL TO BYCOND (N). THE                      C
      C     ITERATION IS PERFORMED BY A CALL TO ITERH (EPS,MAXIT,N). SURFACE              C
      C     VALUES OF INTEREST ARE CALCULATED BY THE SUBROUTINE SURFVL (N),               C
      C     AND THE MAGNETIC FIELD IS PRINTED OUT BY HFIELD.                              C
      C                                                                                   C
```

FIG. 3. H-polarization program (main).

```
FORTRAN IV G COMPILER          MAIN              03-16-71          12:20.40          PAGE 0002

       C                                                                     C
0001              REAL K
0002              DIMENSION ALPHA(15), CONDUC(15), SKIDE(15)
0003              COMMON F(41,41),G(41,41),H(40),K(40),SCALE,FREQ,REGION(40,40)
0004              DATA ALPHA/'A  ','B  ','C  ','D  ','E  ','G  ','H  ','K
                 1'M  ','L  ','S  ','U  ','W  ','X  ','Z  '/
0005              DATA CONDUC/15*0.0/
0006              READ (5,200) H
0007              READ (5,200) K
0008              READ (5,210) SCALE,FREQ
0009              READ (5,200) ((REGION(I,J),J=1,40),I=1,40)
0010              READ (5,220,END=115) CONDUC
0011      115     WRITE (6,230)
          C
          C
0012              DO 110 L=1,40
0013              H(L)=H(L)*SCALE
0014      110     K(L)=K(L)*SCALE
0015              PI=4.0*ATAN(1.0)
0016              OMEGA=2.0*PI*FREQ
0017              DO 120 I=1,40
0018              DO 120 J=1,40
0019      120     REGION(I,J)=4.0*PI*CONDUC(IFIX(REGION(I,J)))*OMEGA
          C
          C       TO SET BOUNDARY VALUES OF F AND G
0020              CALL HYCOND (6)
          C
          C       TO PERFORM THE ITERATION WITH EPS=.0001 AND MAXIT=500 AND SURF=6
0021              CALL ITERH (.0001,500,6)
          C
          C
          C       TO CALCULATE VALUES AT THE SURFACE
0022              CALL SURFVL (6)
          C
          C       NOW PRINT OUT THE CONDUCTIVE CONFIGURATION BY PLACING ALPHA DATA
          C       INTO REGION
0023              DO 140 I=1,40
0024              DO 140 J=1,40
0025              DO 130 L=1,15
0026              IF (REGION(I,J).NE.(4.0*PI*CONDUC(L)*OMEGA)) GO TO 130
0027              REGION(I,J)=ALPHA(L)
0028              GO TO 140
0029      130     CONTINUE
0030      140     CONTINUE
0031              WRITE (6,240)
0032              DO 150 I=1,40
0033      150     WRITE (6,250) (REGION(I,J),J=1,40)
0034              DO 160 L=1,40
0035              K(L)=K(L)/SCALE
0036      160     H(L)=H(L)/SCALE
0037              WRITE (6,260)
0038              DO 190 I=1,15
0039              IF (CONDUC(I).EQ.0.0) GO TO 170
0040              SKIDE(I)=(1.0)/(2.0*PI*SQRT(CONDUC(I)*FREQ)*SCALE)
0041              GO TO 180
```

FIG. 4. *H*-polarization program (main).

```
FORTRAN IV G COMPILER          MAIN             03-16-71         12:20.40        PAGE 0003

 0042          170    SKIDE(I)=999999999.
 0043          180    WRITE (6,270) ALPHA(I),CONDUC(I),SKIDE(I)
 0044          190    CONTINUE
 0045                 WRITE (6,280) H
 0046                 WRITE (6,290) K
 0047                 WRITE (6,300) SCALE,FREQ
               C
               C      TO CALCULATE THE MAGNETIC FIELD FOR THE REGION
 0048                 CALL HFIELD
               C
               C
 0049                 STOP
               C
               C
 0050          200    FORMAT (40F2.0)
 0051          210    FORMAT (2F10.0)
 0052          220    FORMAT (E10.5)
 0053          230    FORMAT (1H1/////,60X,20H/* H-POLARISATION */// )
 0054          240    FORMAT (1H1/////,30X,34H/* THE CONDUCTIVE CONFIGURATION *//)
 0055          250    FORMAT (1H ,20X,40A2)
 0056          260    FORMAT (1H0,46X,5HSIGMA,5X,10HSKIN DEPTH/)
 0057          270    FORMAT (1H ,40X,A2,E13.4,F8.2)
 0058          280    FORMAT (1H0,8HH VALUES,40F3.0)
 0059          290    FORMAT (1H0,8HK VALUES,40F3.0)
 0060          300    FORMAT (1H0,7HSCALE =,F10.0,7H FREQ =,F10.6)
 0061                 END

  TOTAL MEMORY REQUIREMENTS 000822 BYTES
```

FIG. 5. *H*-polarization program (main).

```
FORTRAN IV G COMPILER          BYCOND           03-16-71         12:20.51        PAGE 0001

 0001                 SUBROUTINE BYCOND (N)
 0002                 REAL K
 0003                 COMMON F(41,41),G(41,41),H(40),K(40),SCALE,FREQ,REGION(40,40)
 0004                 DIMENSION DIST(41)
 0005                 FACTOR=SQRT(REGION(40,40)/2.0)
 0006                 DIST(1)=0.0
 0007                 DO 110 I=2,41
 0008          110    DIST(I)=K(I-1)+DIST(I-1)
 0009                 DISP=DIST(N)
 0010                 DO 120 I=1,41
 0011          120    DIST(I)=DIST(I)-DISP
 0012                 DO 130 I=1,N
 0013                 F(I,1)=1.0
 0014          130    G(I,1)=0.0
 0015                 M=N+1
 0016                 DO 140 I=M,41
 0017                 F(I,1)=EXP(-DIST(I)*FACTOR)*COS(DIST(I)*FACTOR)
 0018          140    G(I,1)=-EXP(-DIST(I)*FACTOR)*SIN(DIST(I)*FACTOR)
 0019                 DO 150 I=1,41
 0020                 DO 150 J=2,41
 0021                 F(I,J)=F(I,1)
 0022          150    G(I,J)=G(I,1)
 0023                 RETURN
 0024                 END

  TOTAL MEMORY REQUIREMENTS 00045A BYTES
```

FIG. 6. *H*-polarization boundary condition subroutine.

```
FORTRAN IV G COMPILER          ITERH          03-16-71          12:21.01          PAGE 0001

0001              SUBROUTINE ITERH (EPS,MAXIT,N)
0002              REAL K
0003              COMMON F(41,41),G(41,41),H(40),K(40),SCALE,FREQ,REGION(40,40)
0004              DIMENSION A(40,40), B(40,40), C1(40,40), C2(40,40), C3(40,40), C4(
                 140,40)
0005              M=N+1
0006              DO 110 I=M,40
0007              DO 110 J=2,40
0008              D1=1./H(J)**2+(1./(H(J)+H(J-1)))*(1./H(J-1)-1./H(J))
0009              D2=1./K(I-1)**2+(1./(K(I)+K(I-1)))*(1./K(I)-1./K(I-1))
0010              D3=1./H(J-1)**2+(1./(H(J)+H(J-1)))*(1./H(J)-1./H(J-1))
0011              D4=1./K(I)**2+(1./(K(I)+K(I-1)))*(1./K(I-1)-1./K(I))
0012              A(I,J)=4./H(J)**2+D1*(REGION(I-1,J-1)/REGION(I-1,J)+REGION(I,J-1)/
                 1REGION(I,J)-2.)+4./K(I-1)**2+D2*(REGION(I,J-1)/REGION(I-1,J-1)+REG
                 2ION(I,J)/REGION(I-1,J)-2.)+4./H(J-1)**2+D3*(REGION(I,J)/REGION(I,J
                 3-1)+REGION(I-1,J)/REGION(I-1,J-1)-2.)+4./K(I)**2+D4*(REGION(I-1,J)
                 4/REGION(I,J)+REGION(I-1,J-1)/REGION(I,J-1)-2.)
0013              B(I,J)=-(REGION(I,J)+REGION(I-1,J-1)+REGION(I-1,J)+REGION(I,J-1))
0014              C1(I,J)=D1*(REGION(I-1,J-1)/REGION(I-1,J)+REGION(I,J-1)/REGION(I,J
                 1)+2.)
0015              C2(I,J)=D2*(REGION(I,J-1)/REGION(I-1,J-1)+REGION(I,J)/REGION(I-1,J
                 1)+2.)
0016              C3(I,J)=D3*(REGION(I,J)/REGION(I,J-1)+REGION(I-1,J)/REGION(I-1,J-1
                 1)+2.)
0017        110   C4(I,J)=D4*(REGION(I-1,J)/REGION(I,J)+REGION(I-1,J-1)/REGION(I,J-1
                 1)+2.)
0018              ITER=0
0019              WRITE (6,150) EPS,MAXIT
            C
            C
0020              DO 130 L=1,MAXIT
0021              ITER=ITER+1
0022              BIGF=0.0
0023              BIGG=0.0
0024              DO 120 I=M,40
0025              DO 120 J=2,40
0026              C=F(I,J+1)*C1(I,J)+F(I,J-1)*C3(I,J)+F(I+1,J)*C4(I,J)+F(I-1,J)*C2(I
                 1,J)
0027              P=G(I,J+1)*C1(I,J)+G(I,J-1)*C3(I,J)+G(I+1,J)*C4(I,J)+G(I-1,J)*C2(I
                 1,J)
0028              TEMPF=(C*A(I,J)-B(I,J)*P)/(A(I,J)**2+B(I,J)**2)
0029              TEMPG=(A(I,J)*P+C*B(I,J))/(A(I,J)**2+B(I,J)**2)
0030              RESIDF=ABS(TEMPF-F(I,J))
0031              RESIDG=ABS(TEMPG-G(I,J))
0032              IF (RESIDF.GT.BIGF) BIGF=RESIDF
0033              IF (RESIDG.GT.BIGG) BIGG=RESIDG
0034              F(I,J)=TEMPF
0035        120   G(I,J)=TEMPG
0036              IF ((BIGF.LT.EPS).AND.(BIGG.LT.EPS)) GO TO 140
0037        130   CONTINUE
0038              WRITE (6,160) BIGF,BIGG
0039              RETURN
0040        140   WRITE (6,170) ITER
0041              RETURN
            C
```

FIG. 7. *H*-polarization iteration subroutine.

```
FORTRAN IV G COMPILER          ITERH              03-16-71            12:21.01           PAGE 0002

              C
              C
 0042         150    FORMAT (9H0/* EPS =,F9.6,28H MAXIMUM NO. OF ITERATIONS =,I5,2H*/)
 0043         160    FORMAT (1H ,45H/* STOPPED ON MAX. NO. OF ITERATIONS, FDIFF =,F10.6
                    1,11H AND GDIFF=,F10.6,3H */)
 0044         170    FORMAT (1H0,23H/* STOPPED ON ITERATION,I6,3H */)
 0045                END

 TOTAL MEMORY REQUIREMENTS 009EDE BYTES
```

FIG. 8. *H*-polarization iteration subroutine (contd.)

```
FORTRAN IV G COMPILER          SURFVL             03-16-71            12:21.16           PAGE 0001

 0001                SUBROUTINE SURFVL (L)
 0002                REAL K
 0003                DIMENSION AMH(41), AMEY(41), AMEZ(41), DPHASH(41), DPHAEY(41), DPH
                    1AEZ(41), APPRES(41)
 0004                COMMON F(41,41),G(41,41),H(40),K(40),SCALE,FREQ,REGION(40,40)
              C
              C
 0005                PI=4.0*ATAN(1.0)
 0006                OMEGA=2.0*PI*FREQ
 0007                WRITE (6,140)
 0008                WRITE (6,150)
 0009                I=L
              C
 0010                DO 110 J=2,40
 0011                DPHASH(J)=ATAN2(G(I,J),F(I,J))
 0012                DPHAEY(J)=ATAN2((G(I+1,J)-G(I-1,J)),(F(I+1,J)-F(I-1,J)))
 0013                DPHAEZ(J)=0.0
 0014                AMH(J)=SQRT(F(I,J)**2+G(I,J)**2)
 0015                AMEY(J)=((2.*OMEGA)/(REGION(I,J)+REGION(I,J-1)))*SQRT(((F(I,J)-F(I
                    1+1,J))/(K(I)))**2+((G(I,J)-G(I+1,J))/(K(I)))**2)
 0016                AMEZ(J)=((2.*OMEGA)/(REGION(I,J)+REGION(I,J-1)))*SQRT(((F(I,J+1)-F
                    1(I,J-1))/(H(J)+H(J-1)))**2+((G(I,J+1)-G(I,J-1))/(H(J)+H(J-1)))**2)
 0017         110    APPRES(J)=(2.0/FREQ)*((AMEY(J)/AMH(J))**2)
              C
              C      THE COMPONENTS AMEY AND AMEZ ARE NORMALIZED WITH RESPECT TO
              C      THE FIELD AT INFINITY (POINT 2) AND PHASE DIFFERENCES ARE
              C      CALCULATED RELATIVE TO POINT 40
 0018                AME=SQRT(AMEY(2)**2+AMEZ(2)**2)
 0019                DO 120 J=2,40
 0020                AMEY(J)=AMEY(J)/AME
 0021                AMEZ(J)=AMEZ(J)/AME
 0022                DPHASH(J)=DPHASH(J)-DPHASH(40)
 0023                DPHAEY(J)=DPHAEY(J)-DPHAEY(40)
 0024         120    DPHAEZ(J)=DPHAEZ(J)-DPHAEZ(40)
              C
 0025                DO 130 J=2,40
 0026         130    WRITE (6,160) J,AMH(J),AMEY(J),AMEZ(J),DPHASH(J),DPHAEY(J),DPHAEZ(
                    1J),APPRES(J)
 0027                RETURN
              C
              C
              C
 0028         140    FORMAT (1H0,40X,20H/* SURFACE VALUES *///)
 0029         150    FORMAT (1H0,T9,'AMH',T21,'AMEY',T33,'AMEZ',T45,'DPHASH',T57,'DPHAE
                    1Y',T69,'DPHAEZ',T81,'APPRES'//)
 0030         160    FORMAT (1H ,I2,6(2X,F10.3),E12.3)
 0031                END

 TOTAL MEMORY REQUIREMENTS 000AFE BYTES
```

FIG. 9. *H*-polarization surface values subroutine.

```
FORTRAN IV G COMPILER          HFIELD            03-16-71          12:21.26          PAGE 0001

0001                SUBROUTINE HFIELD
0002                REAL K
0003                COMMON F(41,41),G(41,41),H(40),K(40),SCALE,FREQ,REGION(40,40)
0004                DIMENSION FIELD(41,41)
          C
          C
0005                DO 140 L=32,64,32
0006                THETA=(FLOAT(L))*(4.*ATAN(1.0))/64.
0007                DO 110 I=1,41
0008                DO 110 J=1,41
0009          110   FIELD(I,J)=F(I,J)*COS(THETA)-G(I,J)*SIN(THETA)
0010                WRITE (6,150) L
0011                DO 120 I=1,41
0012          120   WRITE (6,160) (FIELD(I,J),J=1,21)
0013                DO 130 I=1,41
0014          130   WRITE (6,160) (FIELD(I,J),J=21,41)
0015          140   CONTINUE
0016                RETURN
          C
          C
          C
0017          150   FORMAT (1H1/////,21H/* PRINT OF HFIELD AT,I3,17H/64 PI RADIANS */)
0018          160   FORMAT (1H0,21F6.3)
0019                END

  TOTAL MEMORY REQUIREMENTS 001E2E BYTES
```

FIG. 10. *H*-polarization field print-out subroutine.

```
FORTRAN IV G COMPILER          BYCOND            03-16-71          13:03.53          PAGE 0001

0001                SUBROUTINE BYCOND (N)
0002                REAL K
0003                COMMON F(41,41),G(41,41),H(40),K(40),SCALE,FREQ,REGION(40,40)
0004                DIMENSION DIST(41)
0005                FACTOR=SQRT(REGION(40,40)/2.0)
0006                DIST(1)=0.0
0007                DO 110 I=2,41
0008          110   DIST(I)=K(I-1)+DIST(I-1)
0009                DISP=DIST(N)
0010                DO 120 I=1,41
0011          120   DIST(I)=DIST(I)-DISP
0012                DO 130 I=1,N
0013                F(I,1)=1.0-DIST(I)*FACTOR
0014          130   G(I,1)=-DIST(I)*FACTOR
0015                M=N+1
0016                DO 140 I=M,41
0017                F(I,1)=EXP(-DIST(I)*FACTOR)*COS(DIST(I)*FACTOR)
0018          140   G(I,1)=-EXP(-DIST(I)*FACTOR)*SIN(DIST(I)*FACTOR)
0019                DO 150 I=1,41
0020                DO 150 J=2,41
0021                F(I,J)=F(I,1)
0022          150   G(I,J)=G(I,1)
0023                RETURN
0024                END

  TOTAL MEMORY REQUIREMENTS 000472 BYTES
```

FIG. 11. *E*-polarization boundary condition subroutine.

```
FORTRAN IV G COMPILER        ITERE            03-16-71          13:03.56        PAGE 0001

0001               SUBROUTINE ITERE (EPS,MAXIT)
0002               REAL K
0003               COMMON F(41,41),G(41,41),H(40),K(40),SCALE,FREQ,REGION(40,40)
0004               DIMENSION A(40,40), B(40,40), C1(40,40), C2(40,40), C3(40,40), C4(
                  140,40)
0005               DO 110 I=2,40
0006               DO 110 J=2,40
0007               A(I,J)=4.*(1./H(J)**2+1./H(J-1)**2+1./K(I)**2+1./K(I-1)**2)
0008               B(I,J)=-(REGION(I,J)+REGION(I-1,J-1)+REGION(I-1,J)+REGION(I,J-1))
0009               C1(I,J)=4.*(1./H(J)**2+(1./(H(J)+H(J-1)))*(1./H(J-1)-1./H(J)))
0010               C2(I,J)=4.*(1./K(I-1)**2+(1./(K(I)+K(I-1)))*(1./K(I)-1./K(I-1)))
0011               C3(I,J)=4.*(1./H(J-1)**2+(1./(H(J)+H(J-1)))*(1./H(J)-1./H(J-1)))
0012         110   C4(I,J)=4.*(1./K(I)**2+(1./(K(I)+K(I-1)))*(1./K(I-1)-1./K(I)))
0013               ITER=0
0014               WRITE (6,150) EPS,MAXIT
             C
             C
0015               DO 130 L=1,MAXIT
0016               ITER=ITER+1
0017               BIGF=0.0
0018               BIGG=0.0
0019               DO 120 I=2,40
0020               DO 120 J=2,40
0021               C=F(I,J+1)*C1(I,J)+F(I,J-1)*C3(I,J)+F(I+1,J)*C4(I,J)+F(I-1,J)*C2(I
                  1,J)
0022               P=G(I,J+1)*C1(I,J)+G(I,J-1)*C3(I,J)+G(I+1,J)*C4(I,J)+G(I-1,J)*C2(I
                  1,J)
0023               TEMPF=(C*A(I,J)-B(I,J)*P)/(A(I,J)**2+B(I,J)**2)
0024               TEMPG=(A(I,J)*P+C*B(I,J))/(A(I,J)**2+B(I,J)**2)
0025               RESIDF=ABS(TEMPF-F(I,J))
0026               RESIDG=ABS(TEMPG-G(I,J))
0027               IF (RESIDF.GT.BIGF) BIGF=RESIDF
0028               IF (RESIDG.GT.BIGG) BIGG=RESIDG
0029               F(I,J)=TEMPF
0030         120   G(I,J)=TEMPG
0031               IF ((BIGF.LT.EPS).AND.(BIGG.LT.EPS)) GO TO 140
0032         130   CONTINUE
0033               WRITE (6,160) BIGF,BIGG
0034               RETURN
0035         140   WRITE (6,170) ITER
0036               RETURN
             C
             C
             C
0037         150   FORMAT (9H0/* EPS =,F9.6,28H MAXIMUM NO. OF ITERATIONS =,I6,2H*/)
0038         160   FORMAT (1H0,45H/* STOPPED ON MAX. NO. OF ITERATIONS, FDIFF =,F10.6
                  1,11H AND GDIFF=,F10.6,3H */)
0039         170   FORMAT (1H0,23H/* STOPPED ON ITERATION,I6,3H */)
0040               END

 TOTAL MEMORY REQUIREMENTS 0009D84 BYTES
```

FIG. 12. *E*-polarization iteration subroutine.

```
FORTRAN IV G COMPILER        SURFVL          03-16-71          13:04.03          PAGE 0001

 0001             SUBROUTINE SURFVL (L)
 0002             REAL K
 0003             DIMENSION AME(41), AMHY(41), AMHZ(41), DPHASE(41), DPHAHY(41), DPH
                 1AHZ(41), APPRES(41)
 0004             COMMON F(41,41),G(41,41),H(40),K(40),SCALE,FREQ,REGION(40,40)
            C
            C
 0005             PI=4.0*ATAN(1.0)
 0006             OMEGA=2.0*PI*FREQ
 0007             WRITE (6,140)
 0008             WRITE (6,150)
 0009             I=L
            C
 0010             DO 110 J=2,40
 0011             DPHASE(J)=ATAN2(G(I,J),F(I,J))
 0012             DPHAHY(J)=ATAN2((G(I+1,J)-G(I-1,J)),(F(I+1,J)-F(I-1,J)))
 0013             DPHAHZ(J)=ATAN2((G(I,J+1)-G(I,J-1)),(F(I,J+1)-F(I,J-1)))
 0014             AME(J)=SQRT(F(I,J)**2+G(I,J)**2)
 0015             AMHY(J)=(1./OMEGA)*(SQRT(((F(I-1,J)-F(I+1,J))/(K(I)+K(I-1)))**2+((
                 1G(I-1,J)-G(I+1,J))/(K(I)+K(I-1)))**2))
 0016             AMHZ(J)=(1./OMEGA)*(SQRT(((F(I,J+1)-F(I,J-1))/(H(J)+H(J-1)))**2+((
                 1G(I,J+1)-G(I,J-1))/(H(J)+H(J-1)))**2))
 0017       110   APPRES(J)=(2.0/FREQ)*((AME(J)/AMHY(J))**2)
            C
            C     THE COMPONENTS AMHY AND AMHZ ARE NORMALIZED WITH RESPECT TO
            C     THE FIELD AT INFINITY (POINT 2) AND PHASE DIFFERENCES ARE
            C     CALCULATED RELATIVE TO POINT 40
 0018             AMH=SQRT(AMHY(2)**2+AMHZ(2)**2)
 0019             DO 120 J=2,40
 0020             AMHY(J)=AMHY(J)/AMH
 0021             AMHZ(J)=AMHZ(J)/AMH
 0022             DPHASE(J)=DPHASE(J)-DPHASE(40)
 0023             DPHAHY(J)=DPHAHY(J)-DPHAHY(40)
 0024       120   DPHAHZ(J)=DPHAHZ(J)-DPHAHZ(40)
            C
 0025             DO 130 J=2,40
 0026       130   WRITE (6,160) J,AME(J),AMHY(J),AMHZ(J),DPHASE(J),DPHAHY(J),DPHAHZ(
                 1J),APPRES(J)
 0027             RETURN
            C
            C
            C
 0028       140   FORMAT (1H0,40X,20H/* SURFACE VALUES *///)
 0029       150   FORMAT (1H0,T9,'AME',T21,'AMHY',T33,'AMHZ',T45,'DPHASE',T57,'DPHAH
                 1Y',T69,'DPHAHZ',T81,'APPRES'//)
 0030       160   FORMAT (1H ,I2,6(2X,F10.3),E12.3)
 0031             END

  TOTAL MEMORY REQUIREMENTS 000A08 BYTES
```

FIG. 13. *E*-polarization surface value subroutine.

of the surface values for the components and normalize them with respect to the field over a uniform conducting region. Also, the phases of the components and the apparent resistivities are calculated. The programs can easily be altered to compute further ratios of interest or other relative phases. The surface values are printed out and a representation of the conductivity distribution is also exhibited.

The programs calculate the field distributions throughout the mesh, and print out two instantaneous field values, ($\theta = \pi/2$, which corresponds to a field value of—g and $\theta = \pi$, which corresponds to a field value of $-f$). The program can be modified to calculate and print out the field distributions for any instant during the cycle.

The program illustrated is for a mesh of 1681 grid points (41×41), although it can be adapted for any grid size.

7. Computed example

The model used to illustrate the program is one with an anomaly of several conductivities. Fig. 14 gives the conductive configuration which is printed out in both programs. The anomaly consists of four conductivities. The conductivities used are shown in Fig. 14. The frequency employed in this example was 0·000333 Hz (approximately 50-min period) and is also given in Fig. 14. The skin depths for the various conductivities are calculated and shown in Fig. 14 as well. The product $\sigma\omega$ only is required in the calculations, and so it follows that the same solution will apply if both conductivities and the period are decreased in the same ratio, with suitable adjustment of the grid size. The horizontal and vertical grid sizes are also given in Fig. 14, and in this example the vertical grid sizes (K) vary, while the horizontal grid sizes (H) are equal. Fig. 15 is the H-polarization printer output for the computed surface values. Fig. 16 illustrates these surface values graphically. For this polarization $|E_z|$ (AMEZ), phase of E_z (DPHAEZ) and phase of H_x (DPHASH) are zero, while the amplitude of H_x along the surface (AMH) has been set constant and equal to one. The normalized amplitude of E_y (AMEY) is shown along with its phase (DPHAEY). Also the apparent resistivity (APPRES) profile is given.

Fig. 17 gives the computed surface values for the E-polarization, and Fig. 18 illustrates them graphically.

8. Conclusions

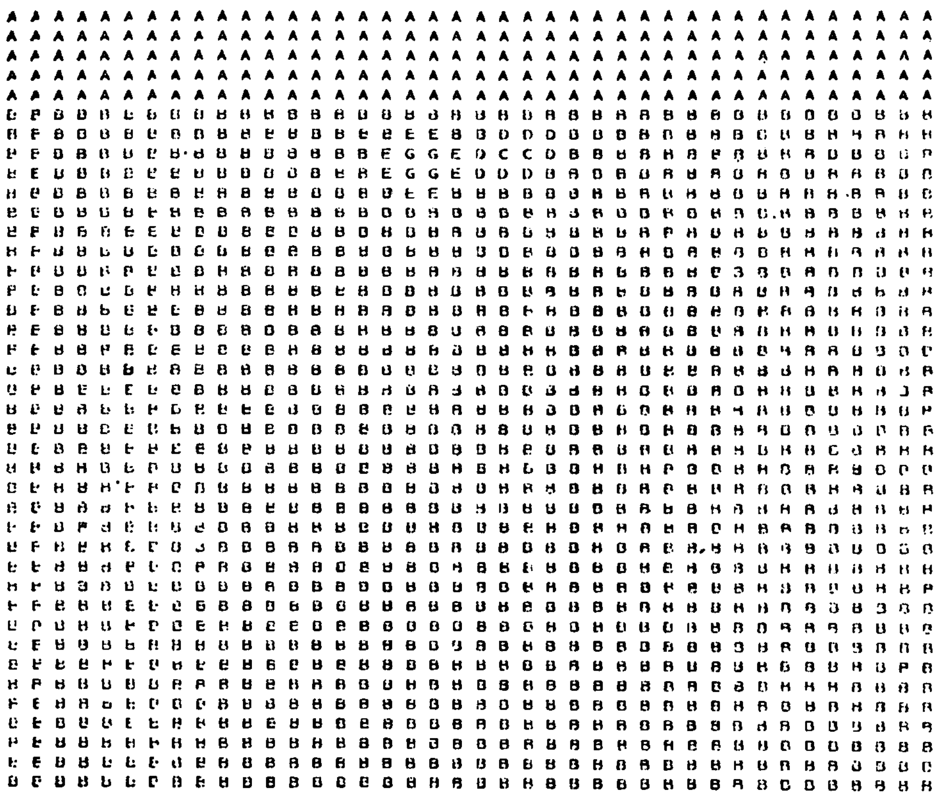
For the model illustrated the computation time for the H-polarization case was 89·7 s, and for the E-polarization was 115·3 s. The computation time depends on the grid size, conductivity contrasts and the frequency. Also, the time depends upon the convergence criterion imposed (value of EPS). The initial values for f and g at interior points are set to values corresponding to a uniform conductor.

In the present programs the surface values are approximated by finite differences. This leads to error in the surface values which is evident in the apparent resistivity curve. The position of the curve is displaced from the true apparent resistivity values over the uniform regions.

In the E-polarization case the graph of DPHAHZ (the phase H_z) as shown in Fig. 18 exhibits two jump discontinuities of order 2π. This is because of the limited range of the ATAN2 function of FORTRAN IV. The graph can be made to appear continuous by shifting the displaced portion of the curve by 2π.

It should be noted that the programs solve the problem of an isolated inhomogeneity and so the anomaly should be far away from the boundaries of the grid so that the assumption of uniform conductivity as $y \to \pm\infty$ will be valid.

/* THE CONDUCTIVE CONFIGURATION */



	SIGMA	SKIN DEPTH
A	0.0	********
B	0.1000E-09	8.72
C	0.1000E-08	2.76
D	0.5000E-09	3.90
E	0.5000E-10	12.33
G	0.1000E-10	27.58
H	0.0	********
K	0.0	********
M	0.0	********
C	0.0	********

FIG. 14i. Conductive configuration.

```
                         S     0.0        ********
                         U     0.0        ********
                         W     0.0        ********
                         X     0.0        ********
                         Z     0.0        ********

H VALUES10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.

K VALUES90.50.10. 5. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 2. 3. 4. 5. 6. 7. 8. 9.10.11.12.13.14.15.16.17.18.19.20.

SCALE =   100000. FREC =  0.000333
```

Fig. 14 ii.

/* H-POLARISATION */

/* EPS = 0.000100 MAXIMUM NO. OF ITERATIONS = 500*/

/* STOPPED ON ITERATION 283 */

/* SURFACE VALUES */

	AMH	AMEY	AMEZ	DPHASH	DPHAEY	DPHAEZ	APPRES
2	1.000	1.000	0.0	0.0	0.0	0.0	0.893E 10
3	1.000	1.000	0.0	0.0	-0.000	0.0	0.894E 10
4	1.000	1.000	0.0	0.0	-0.000	0.0	0.894E 10
5	1.000	1.000	0.0	0.0	-0.000	0.0	0.894E 10
6	1.000	1.000	0.0	0.0	-0.000	0.0	0.894E 10
7	1.000	1.000	0.0	0.0	-0.000	0.0	0.894E 10
8	1.000	1.000	0.0	0.0	-0.000	0.0	0.894E 10
9	1.000	1.000	0.0	0.0	-0.000	0.0	0.894E 10
10	1.000	1.000	0.0	0.0	-0.000	0.0	0.894E 10
11	1.000	1.000	0.0	0.0	-0.000	0.0	0.894E 10
12	1.000	1.000	0.0	0.0	-0.000	0.0	0.894E 10
13	1.000	1.000	0.0	0.0	-0.000	0.0	0.894E 10
14	1.000	1.000	0.0	0.0	-0.000	0.0	0.894E 10
15	1.000	1.001	0.0	0.0	0.003	0.0	0.896E 10
16	1.000	1.002	0.0	0.0	0.008	0.0	0.897E 10
17	1.000	1.056	0.0	0.0	0.028	0.0	0.995E 10
18	1.000	1.266	0.0	0.0	0.066	0.0	0.143E 11
19	1.000	1.359	0.0	0.0	0.059	0.0	0.165E 11
20	1.000	1.270	0.0	0.0	0.049	0.0	0.144E 11
21	1.000	0.906	0.0	0.0	0.018	0.0	0.733E 10
22	1.000	0.503	0.0	0.0	0.100	0.0	0.226E 10
23	1.000	0.445	0.0	0.0	0.062	0.0	0.177E 10
24	1.000	0.505	0.0	0.0	-0.042	0.0	0.228E 10
25	1.000	0.769	0.0	0.0	-0.069	0.0	0.52[illegible]E 10
26	1.000	1.006	0.0	0.0	-0.031	0.0	0.704E 10
27	1.000	0.998	0.0	0.0	-0.004	0.0	0.890E 10
28	1.000	1.000	0.0	0.0	-0.000	0.0	0.893E 10
29	1.000	1.000	0.0	0.0	-0.000	0.0	0.894E 10
30	1.000	1.000	0.0	0.0	-0.000	0.0	0.894E 10
31	1.000	1.000	0.0	0.0	-0.000	0.0	0.894E 10
32	1.000	1.000	0.0	0.0	-0.000	0.0	0.894E 10
33	1.000	1.000	0.0	0.0	-0.000	0.0	0.894E 10
34	1.000	1.000	0.0	0.0	-0.000	0.0	0.894E 10
35	1.000	1.000	0.0	0.0	-0.000	0.0	0.894E 10
36	1.000	1.000	0.0	0.0	-0.000	0.0	0.894E 10
37	1.000	1.000	0.0	0.0	-0.000	0.0	0.894E 10
38	1.000	1.000	0.0	0.0	-0.000	0.0	0.894E 10
39	1.000	1.000	0.0	0.0	-0.000	0.0	0.894E 10
40	1.000	1.000	0.0	0.0	0.0	0.0	0.893E 10

FIG. 15. Line printer output of *H*-polarization surface values. Amplitudes of components normalized, phase differences in radians, apparent resistivity in emu.

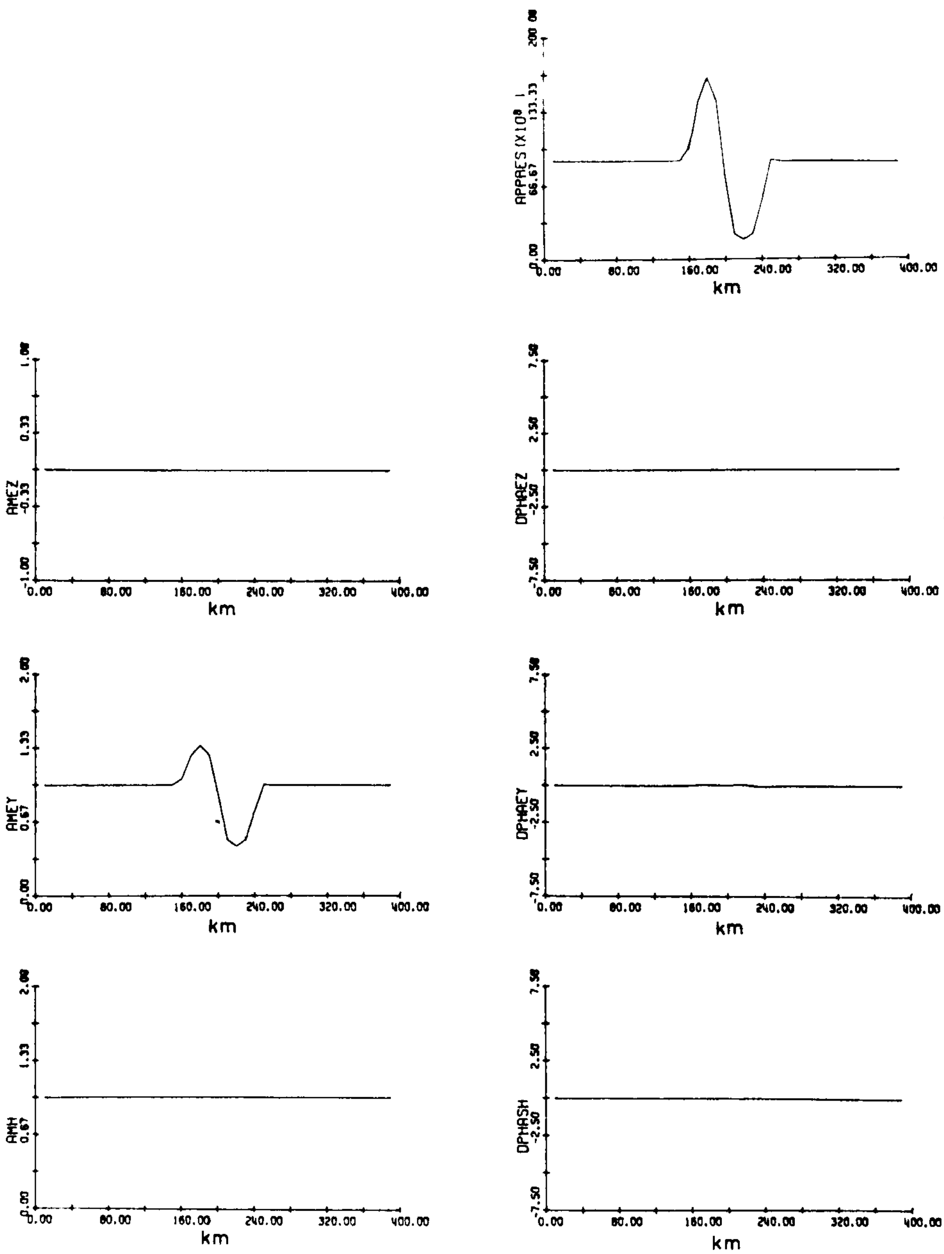


FIG. 16. Graphs of surface values in Fig. 15.

/* E-POLARISATION */

/* EPS = 0.000100 MAXIMUM NO. OF ITERATIONS = 500*/

/* STOPPED ON ITERATION 327 */

/* SURFACE VALUES */

	AME	AMHY	AMHZ	DPHASE	DPHAHY	DPHAHZ	APPRES
2	1.000	1.000	0.001	-0.000	-0.000	-3.074	0.106E 11
3	0.999	1.000	0.000	-0.001	-0.000	-3.762	0.106E 11
4	0.999	1.000	0.000	-0.001	-0.001	-4.048	0.106E 11
5	0.999	1.000	0.000	-0.001	-0.001	-4.126	0.106E 11
6	0.999	1.000	0.000	-0.001	-0.001	-4.328	0.106E 11
7	0.999	1.000	0.000	-0.001	-0.001	1.553	0.106E 11
8	0.999	1.000	0.000	-0.001	-0.001	1.284	0.106E 11
9	0.999	1.000	0.000	-0.001	-0.000	1.087	0.106E 11
10	0.999	1.000	0.000	-0.001	-0.000	0.832	0.106E 11
11	0.999	1.000	0.000	-0.000	-0.000	0.288	0.106E 11
12	0.999	1.000	0.000	-0.000	-0.000	-1.103	0.106E 11
13	0.999	1.001	0.000	-0.000	-0.001	-1.701	0.105E 11
14	1.000	1.002	0.002	-0.001	-0.001	-1.687	0.105E 11
15	1.004	1.005	0.005	-0.002	-0.000	-1.408	0.105E 11
16	1.017	1.011	0.018	-0.002	0.004	-0.953	0.107E 11
17	1.057	1.001	0.042	0.018	0.012	-0.619	0.118E 11
18	1.119	0.953	0.044	0.075	0.013	-0.312	0.146E 11
19	1.138	0.913	0.024	0.122	0.006	1.442	0.164E 11
20	1.052	0.925	0.104	0.106	0.016	-4.087	0.137E 11
21	0.826	1.025	0.150	0.017	0.011	-4.210	0.687E 10
22	0.589	1.102	0.102	-0.023	-0.044	-4.417	0.302E 10
23	0.505	1.103	0.012	-0.010	-0.067	-3.882	0.222E 10
24	0.557	1.103	0.079	-0.064	-0.042	-1.447	0.270E 10
25	0.754	1.034	0.118	-0.044	0.015	-1.137	0.562E 10
26	0.920	0.953	0.071	0.040	0.011	-1.071	0.987E 10
27	0.971	0.975	0.022	0.026	0.014	-1.688	0.105E 11
28	0.986	0.988	0.009	0.013	0.010	-2.076	0.105E 11
29	0.991	0.994	0.004	0.006	0.006	-2.209	0.105E 11
30	0.994	0.996	0.002	0.003	0.003	-2.169	0.105E 11
31	0.996	0.997	0.001	0.001	0.002	-2.068	0.105E 11
32	0.997	0.998	0.001	0.001	0.001	-1.969	0.105E 11
33	0.997	0.999	0.000	0.000	0.001	-1.884	0.106E 11
34	0.998	0.999	0.000	-0.000	0.000	-1.808	0.106E 11
35	0.998	0.999	0.000	-0.000	-0.000	-1.735	0.106E 11
36	0.999	0.999	0.000	-0.000	-0.000	-1.651	0.106E 11
37	0.999	0.999	0.000	-0.001	-0.000	-1.518	0.106E 11
38	0.999	1.000	0.000	-0.001	-0.000	-1.257	0.106E 11
39	0.999	1.000	0.000	-0.000	-0.000	-0.805	0.106E 11
40	1.000	1.000	0.001	0.0	0.0	0.0	0.106E 11

FIG. 17. Line printer output of *E*-polarization surface values. Amplitudes of components normalized, phase differences in radians, apparent resistivity in emu.

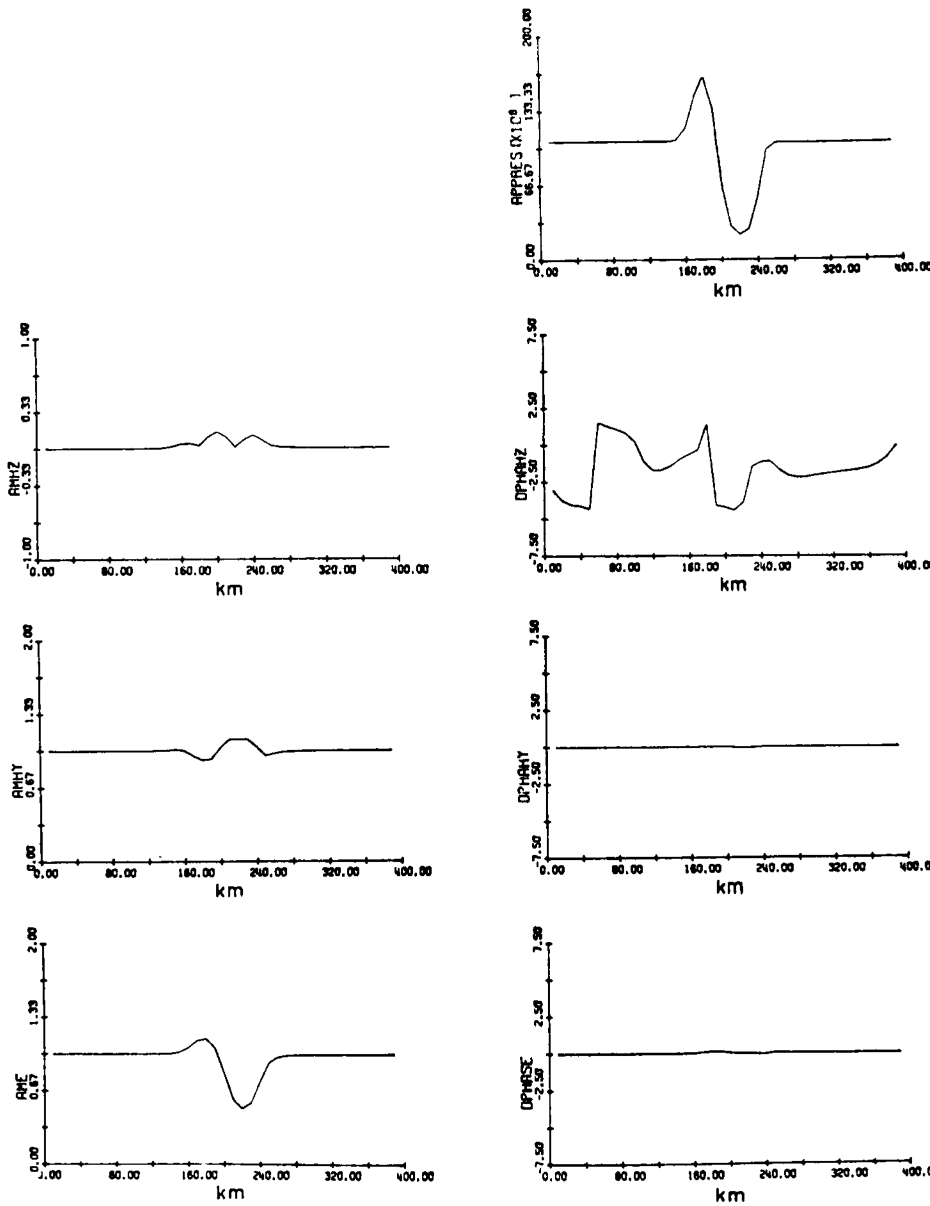


FIG. 18. Graphs of surface values in Fig. 17.

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Edmonton, Canada.

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