# A General Framework for Optimum Iterative Blockwise Equalization of Single Carrier MIMO Systems and Asymptotic Performance Analysis

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***Abstract*—The paper proposes a general framework for both time-domain (TD) and frequency-domain (FD) iterative blockwise equalization in single carrier (SC) wideband multiple-input multiple-output (MIMO) channels. First, a novel turbo blockwise operating equalizer structure is proposed by jointly optimizing the feed-forward and feedback filters at each iteration based on the minimum mean squared error (MMSE) criterion. Optimization of the filter coefficients, utilized for feed-forward equalization and decision feedback, is performed in both time and frequency domains, and the equivalence between them is established by unifying the iterative block equalization with the feed-forward and feedback filters in various domains. The corresponding filters are obtained analytically in a closed form as a solution of the constrained Wiener-Hopf equation with the use of average reliability information. Second, asymptotic performance and diversity analysis of the proposed single carrier frequency domain equalizer (SC-FDE) is carried out that leads to insights on the structures of the filters in the high SNR regime. Furthermore, we elaborate on the rate and diversity tradeoff of the space-time coded SC-FDE based on the proposed equalizer structure and the space-frequency coded OFDM systems. Performance comparison between the proposed SC-FDE scheme and OFDM is made. The proposed SC-FDE scheme performs very close to the genie-aided performance bounds and better than OFDM based transmission.**

***Index Terms*—MIMO ISI channels, optimal blockwise equalization, decision feedback, MMSE, SC-FDE, OFDM, rate-diversity tradeoff, diversity order, matched filter bound (MFB)**

## I. Introduction

Multiple-input multiple-output (MIMO) systems have received much attention due to their multiplexing and diversity capabilities and potentially can offer much higher rates in future wireless systems. However, sophisticated equalization and decoding schemes are required for reliable communication at high rates. While OFDM based schemes are well recognized candidates as a broadband wireless technology, single-carrier (SC) transmission has also received considerable attention due its comparable complexity with OFDM. It has been shown in [1] that frequency domain equalization (FDE) can be readily applied to SC transmission to yield similar performance as OFDM. Since OFDM suffers from the peak-to-average power ratio (PAPR) problem, SC techniques leading to more efficient use of power amplifiers are more suitable for uplink channels [1], [2]. It is known that OFDM and SC techniques are similar in terms of spectral efficiency and that OFDM only shifts the multipath fading problem from the time domain to the frequency domain. Actually, OFDM breaks the frequency diversity and channel coding is needed to reclaim it as opposed to the SC based systems where multipath diversity can be attained without channel coding [3].

Iterative blockwise equalization for the SC-FDE scheme was studied for single-input single-output (SISO) systems in [4], [5], [6]. Average reliability information obtained from detectors or channel decoders are utilized in feed-forward and feedback filters in those studies, where feedback filters were designed in a domain of choice, either time or frequency. Blockwise equalization was applied to MIMO ISI channels in studies such as [7]–[14] and references in [15], [16]. In this paper, we set up a general framework for optimal iterative blockwise equalization in SC MIMO ISI channels where feed-forward and feedback filters are jointly optimized in time and frequency domains by taking the reliability of the decision feedback into account. Previous studies cited in [16] do not consider a general structure as laid out here, since they either employ suboptimal approaches to mitigating interference or use decision feedback in time domain.

The contribution of the paper is twofold. We first derive the closed form solutions for the optimal filter coefficients in both time domain (TD) and frequency domain (FD) that take the reliability information into account at each iteration, and the equivalence between filters in different domains is established. The resultant blockwise equalizer in FD can be seen as the MIMO extension of the SC-FDE scheme for SISO channels in [5]. To the authors' knowledge, this extension and the relationships between filters in different domains are novel. In our formulation, optimal filter coefficients are found by solving Wiener-Hopf equations with a feedback-related constraint in TD first. Then, its equivalent counterpart is obtained in FD. Hence, decision feedback complexity is much reduced by operating in FD. In a related paper [17], SC-FDE with FD decision feedback is applied to space-time coded MISO systems with a different filter formulation than ours.

As a second contribution, asymptotic performance and diversity analysis of the proposed SC-FDE scheme is car-

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ried out by assuming perfect decision feedback. Furthermore, the rate-diversity tradeoff of the proposed space-time coded MIMO SC-FDE and space-frequency coded MIMO OFDM are obtained for fixed constellations. The tradeoff between diversity and multiplexing gain for flat and frequency selective fading MIMO systems with the use of Gaussian codebooks were evaluated in [18] and in [19] respectively. Since we deal here with finite constellations, we adopt the approach in establishing the tradeoff between rate and diversity for coded block fading channels in [20] and space-time coded MIMO channels in [21]. It is known for some time that SC based systems can achieve full multipath diversity with maximum-likelihood (ML) detectors [3]. The rate-diversity tradeoffs for the SC-FDE scheme with the use of linear MMSE receivers have been recently obtained for SISO ISI channels in [22] and for MIMO ISI channels in [23]. Although those studies show that SC-FDE may fail to capture the full multipath diversity of the ISI channel with the use of linear MMSE filtering, we show that the proposed SC-FDE scheme with iterative frequency domain decision feedback is able to attain full multipath diversity. This stems from the fact that MMSE detectors with decision feedback equalization (DFE) can achieve near-capacity performance for MIMO multipath channels [24], [25] and the resultant receiver coincides with the information theoretically optimum structure developed in [24], [25].

As a result of our derivations and analysis, we observe that the proposed SC-FDE receiver is composed of a channel matched filter (CMF) and an interference canceller. Simulation results reveal that the proposed SC-FDE scheme performs very close to the genie-aided matched filter bound (MFB) [14], [26]. It is also observed that the proposed SC-FDE scheme outperforms OFDM based transmission. The paper is organized as follows. In Section II, the system model is described. In Section III, iterative block equalization and decoding structures are investigated both in time and frequency domain. In Section IV, asymptotic performance analysis for the proposed SC-FDE scheme is provided and optimal rate-diversity tradeoffs for SC-FDE and OFDM based MIMO systems are obtained. Finally, code construction techniques, simulation results and concluding remarks are presented in Section V and Section VI, respectively.

## II. System Model

The following notation is used throughout the paper. Boldface upper-case letters denote matrices, boldface lower-case letters denote column vectors, and scalars are denoted by plain lower-case letters. The superscript $(\cdot)^*$ denotes the complex conjugate for scalars and $(\cdot)^H$ denotes the conjugate transpose for vectors and matrices. The $n \times n$ identity matrix is shown with $\mathbf{I}_n$. The correlation matrix for two random vectors $\mathbf{a}$ and $\mathbf{b}$ is defined as $\mathbf{R_{ab}} = E\{\mathbf{ab}^H\}$ where $E\{\cdot\}$ stands for the expected value operator. The $(i,j)^{th}$ element of a matrix $\mathbf{A}$ is denoted by $\mathbf{A}(i,j)$ and the $i^{th}$ element of a vector $\mathbf{a}$ is denoted by $a^i$.

Some operations to be used in the paper are defined as follows. If $\mathbf{A}_i$'s for $i = 1, \ldots, N$ are $m \times n$ matrices, $\text{circ}\left\{ \mathbf{A}_1 \quad \cdots \quad \mathbf{A}_N \right\}$ operation constructs an $Nm \times Nn$ circulant matrix. If $\mathbf{A}$ is an $m \times m$ square matrix, $\text{diag}\{\mathbf{A}\}$ constructs a $m \times 1$ vector by using the diagonal elements of $\mathbf{A}$. The operation $\text{diag}[a_1, \ldots, a_N]$ constructs an $N \times N$ diagonal matrix where scalar $a_i$'s are on the main diagonal. A block diagonal matrix where $\mathbf{A}_i$'s are on the main diagonal is denoted by $\text{bdiag}\{\mathbf{A}_i\}_{i=1}^N$.

Assuming symbol rate sampling, the discrete time baseband equivalent model of the point-to-point MIMO wideband channel with $n_r$ receive antennas and $n_t$ transmit antennas can be written as [16],

$$\mathbf{y}_k = \sum_{l=0}^{L-1} \mathbf{H}_l \mathbf{x}_{k-l} + \mathbf{n}_k, \qquad k = 0, 1, \ldots, N-1, \tag{1}$$

where $\mathbf{H}_l$'s, $l = 0, \ldots, L-1$, are complex channel matrices comprised of independent zero-mean circularly symmetric complex Gaussian (ZMCSCG) random variables with variance given by the power delay profile of each channel. Noise vectors $\mathbf{n}_k$ are also taken as ZMCSCG white (spatially and temporally) noise with variance $N_0$. Transmitted symbols $x_k(i)$'s, $i = 1, \ldots, n_t$, are selected from a signal constellation $S \in \mathbb{C}$ with $|S| = M$ and $E\{|x_k(i)|^2\} = E_s$. The codewords $[\mathbf{x}_0, \ldots, \mathbf{x}_{N-1}]_{n_t \times N}$ form a coded modulation scheme $\chi \subset \mathbb{C}^{n_t \times N}$. A cyclic prefix (CP) is used to prevent inter-block interference and create a circulant channel matrix with length larger than or equal to the maximum channel length $(L)$ as explained in [11]. The signal for a transmitted block with CP is a sequence of vectors: $[\mathbf{x}_{N-(L-1)}, \ldots, \mathbf{x}_{N-1}, \mathbf{x}_0, \mathbf{x}_1, \ldots, \mathbf{x}_{N-1}]$.

This paper considers block based transmission with length $N$ as in [4] and [5]. A block fading model is considered and the channel state information at transmitter (CSIT) is not available. Since channel estimation is beyond the scope of this paper, the channel is assumed to be perfectly known at each block transmission at the receiver.

The DFT matrix for a vector sequence $\{\mathbf{x}_n\}_{n=0}^{N-1}$ with vectors of size $m \times 1$ can be defined as

$$\mathbf{Q_m} = \mathbf{Q} \otimes \mathbf{I}_m \tag{2}$$

where $\mathbf{Q}$ is the regular $N \times N$ normalized DFT matrix with the $(m,n)^{th}$ element $q_n^m = \frac{1}{\sqrt{N}} e^{-j2\pi mn/N}$ and $\otimes$ denotes the Kronecker product operation ($\mathbf{Q_m}^H \mathbf{Q_m} = \mathbf{I}_{Nm}$). Then, one can write the vector DFT operation for an extended vector $\mathbf{x} = [\mathbf{x}_0^T, \ldots, \mathbf{x}_{N-1}^T]^T$ obtained from the vector sequence $\{\mathbf{x}_n\}_{n=0}^{N-1}$ as $\mathbf{X} = \mathbf{Q_m x}$.

After the DFT operation is applied to the sequence of received vectors $\{\mathbf{y}_k\}_{k=0}^{N-1}$ in (1), one can obtain the following expression in the frequency domain as done in [11]

$$\mathbf{Y}_k = \overline{\mathbf{\Lambda}}_k \mathbf{X}_k + \mathbf{N}_k, \qquad k = 0, \ldots, N-1 \tag{3}$$

where $\overline{\mathbf{\Lambda}}_k$ is an $n_r \times n_t$ matrix representing the channel frequency response at the $k^{th}$ frequency bin with the entries

$$\overline{\mathbf{\Lambda}}_k(i,m) = \sum_{l=0}^{L-1} \mathbf{H}_l(i,m) e^{-j2\pi kl/N}, \tag{4}$$

for $i = 1, \ldots, n_r$ and $m = 1, \ldots, n_t$. In (3), $\mathbf{X}_k$ is the DFT of a vector sequence $\{\mathbf{x}_k\}$ with $\mathbf{X}_k = [X_k^1, \ldots, X_k^{n_t}]^T$ for $k = 0, \ldots, N-1$. Similarly, $Y_k^i$ and $N_k^i$ are the DFT of the corresponding received and noise sequences at the $i^{th}$ receive antenna at the $k^{th}$ frequency bin for $i = 1, \ldots, n_r$. The expression in (3) is the frequency domain equivalent of the channel in (1) and will be frequently used in the remainder of the paper.

## III. Iterative Block Equalization for Wideband MIMO Channels

We consider iterative block equalization techniques with decision feedback both in time and frequency domains. Both the equalization and channel decoding processes are performed in each iteration, and the turbo principle can be applied as in [5], [27]. The signal streams from the multiple transmit antennas are transmitted at the same time and frequency, thus they introduce both inter-symbol-interference (ISI) and co-channel interference (CCI) in wideband MIMO communication. The feed-forward and feedback filters in the equalizer will be jointly optimized according to the minimum mean square error (MMSE) criterion to mitigate the effects of ISI and CCI by taking the reliability of decoded symbols at each iteration into account.

### A. Iterative Block Decision Feedback Equalization (IB-DFE)

By defining the overall received signal vector $\mathbf{y} = [\mathbf{y}_0^T, \mathbf{y}_1^T, \ldots, \mathbf{y}_{N-1}^T]^T$ and similarly for $\mathbf{x}$ and $\mathbf{n}$, one can rewrite the equivalent model of (1) as

$$\begin{aligned} \mathbf{y} &= \mathbf{H}_c\mathbf{x} + \mathbf{n}, \text{ where} \\ \mathbf{H}_c &= \left[\text{circ}\left\{ \mathbf{H}_0 \quad \mathbf{0} \quad \cdots \quad \mathbf{0} \quad \mathbf{H}_{L-1} \quad \cdots \quad \mathbf{H}_1 \right\}\right] \end{aligned} \tag{5}$$

$\mathbf{H}_c$ is a $(Nn_r) \times (Nn_t)$ block circulant matrix and can be block diagonalized by DFT vectors to produce (3).

One can write the output of DFE at the $i^{th}$ equalizer update iteration using the whole received signal and soft feedback decisions obtained from the decoder as

$$\tilde{\mathbf{x}}^{(i)} = (\mathbf{F}^{(i)})^H\mathbf{y} - (\mathbf{B}^{(i)})^H\hat{\mathbf{x}}^{(i-1)} \tag{6}$$

where $\mathbf{F}^{(i)}$ is an $(Nn_r) \times (Nn_t)$ feed-forward matrix, and $\mathbf{B}^{(i)}$ is an $(Nn_t) \times (Nn_t)$ feedback matrix, and $\tilde{\mathbf{x}}^{(i)} = \left[(\tilde{\mathbf{x}}_0^{(i)})^T, \ldots, (\tilde{\mathbf{x}}_{N-1}^{(i)})^T\right]^T$. In (6), $\hat{\mathbf{x}}_k^{(i-1)}$'s are the channel decoder's soft decisions from the previous iteration, and they are utilized at the feedback filtering process to improve the estimate of $\mathbf{x}_k$. One can refer to this equalizer structure as the time domain equalizer with time domain decision feedback (TDE-TDDF). For the iterative TDE-TDDF operation, we consider feeding back the entire block of interfering vectors. In other words, our scheme here is operating on a block basis, therefore it cancels both the pre-cursor and the post-cursor ISI while eliminating interference from other streams.

The feed-forward and feedback filter matrices will now be jointly optimized according to the MMSE criterion presented in [4], [5]. The total mean square error (MSE) in one block at the $i^{th}$ iteration, conditioned on the channel matrices and the results of the previous iteration, can be expressed as

$$MSE^{(i)} = E\left\{||\tilde{\mathbf{x}}^{(i)} - \mathbf{x}||^2\right\} = E\left\{\sum_{k=0}^{N-1} ||\tilde{\mathbf{x}}_k^{(i)} - \mathbf{x}_k||^2\right\} \tag{7}$$

where expectations are taken with respect to the transmitted data, the noise, and the received data. The constraint on the feedback filter matrix is established as

$$\text{diag}\{\mathbf{B}\} = \mathbf{0}_{(Nn_t)\times 1} \tag{8}$$

where the diagonal elements of $\mathbf{B}$ are set to zero. By imposing this constraint, one can avoid self-subtraction of the desired symbol by its previous estimate. In other words, the estimation of the $k^{th}$ symbol transmitted at the $m^{th}$ antenna does not use its self soft estimate from the previous iteration, but it can utilize the soft estimate of other coded symbols.

The Lagrange multiplier method can be used to obtain the optimal filter coefficients. Lagrangian coefficients and the corresponding cost function can be written at the $i^{th}$ equalizer update iteration as in

$$\begin{aligned} J^{(i)} &= MSE^{(i)} + \sum_{m=1}^{Nn_t} (\mathbf{B}^{(i)}(m,m))^*\Gamma_m^{(i)} \quad \text{for} \\ \mathbf{\Gamma}^{(i)} &= \text{diag}\left[\Gamma_1^{(i)}, \ldots, \Gamma_{Nn_t}^{(i)}\right]_{(Nn_t)\times(Nn_t)}. \end{aligned} \tag{9}$$

By taking the gradient of the cost function $J^{(i)}$ with the Lagrangian, namely $\mathbf{\Gamma}^{(i)}$ in (9), with respect to rows of $(\mathbf{F}^{(i)})^H$ and $(\mathbf{B}^{(i)})^H$, and equating the gradients to zero vectors, the following Wiener-Hopf equation (Chapter 6 in [28]) with the constraint (8) in time domain is obtained

$$\begin{bmatrix} \mathbf{R}_{\mathbf{yy}} & -\mathbf{R}_{\mathbf{y}\hat{\mathbf{x}}^{(i-1)}} \\ -\mathbf{R}_{\mathbf{y}\hat{\mathbf{x}}^{(i-1)}}^H & \mathbf{R}_{\hat{\mathbf{x}}^{(i-1)}\hat{\mathbf{x}}^{(i-1)}} \end{bmatrix} \begin{bmatrix} \mathbf{F}^{(i)} \\ \mathbf{B}^{(i)} \end{bmatrix} = \begin{bmatrix} \mathbf{R}_{\mathbf{yx}} \\ -\mathbf{R}_{\hat{\mathbf{x}}^{(i-1)}\mathbf{x}} - \mathbf{\Gamma}^{(i)} \end{bmatrix} \tag{10}$$

where the details of the derivation can be found in our work (Chapter 4 in [29]).

Due to an interleaving operation both in time and space, we can assume that symbols transmitted from different antennas or at different time epochs are uncorrelated so that

$$E\{\mathbf{x}_k(\mathbf{x}_l)^H\} = E_s\mathbf{I}_{n_t}\delta_{kl}, \text{ for } k, l = 0, \ldots, N-1. \tag{11}$$

Some important correlation matrices used for the calculation of feed-forward and feedback filters in (10) are defined at the $i^{th}$ iteration as

$$\mathbf{P}_1^{(i)} = E\{\mathbf{x}_k(\hat{\mathbf{x}}_k^{(i-1)})^H\}, \quad \mathbf{P}_2^{(i)} = E\{\hat{\mathbf{x}}_k^{(i-1)}(\hat{\mathbf{x}}_k^{(i-1)})^H\} \tag{12}$$

for $k = 0, \ldots, N-1$. Furthermore, due to the interleaving operation on the coded symbols, feedback decisions are assumed to be uncorrelated with the symbols transmitted at different antennas or symbol times. It is further assumed that the reliability matrices of the decision feedback are the same for all $k$, i.e.,

$$E\{\mathbf{x}_k(\hat{\mathbf{x}}_l^{(i-1)})^H\} = \mathbf{0}, \quad E\{\hat{\mathbf{x}}_k^{(i-1)}(\hat{\mathbf{x}}_l^{(i-1)})^H\} = \mathbf{0}, \ \forall\, k \neq l \tag{13}$$

$$E\{x_k^m(\hat{x}_k^n)^*\} = \rho_m\delta_{mn}, \ E\{\hat{x}_k^m(\hat{x}_k^n)^*\} = \beta_m\delta_{mn} \tag{14}$$

for $m, n = 1, \ldots, n_t$ and the expectations are independent of symbol index $k$. Then, we can write

$$\mathbf{P}_1^{(i)} = \text{diag}\left[\rho_1, \ldots, \rho_{n_t}\right], \; \mathbf{P}_2^{(i)} = \text{diag}\left[\beta_1, \ldots, \beta_{n_t}\right]. \quad (15)$$

These are standard and reasonable assumptions as stated in [5], [27], since the average symbol error probability is approximately the same for each symbol in a large block with quasi-static fading. Forward and feedback filters are jointly updated at each iteration as a solution to (10) with (8). At each equalizer update, reliability of the decisions are taken into account by using $\mathbf{P}_1^{(i)}$ and $\mathbf{P}_2^{(i)}$.

The direct computation of the filter coefficients requires an $(N(n_r+n_t)) \times (N(n_r+n_t))$ matrix inversion, but the block circulant structure of $\mathbf{H}_c$ allows low-complexity inversions in frequency domain as will be seen in Section III-B.

### *B. Frequency Domain Equalization with Frequency Domain Decision Feedback (FDE-FDDF)*

DFT vectors are actually the eigenvectors of circulant matrices, and thus the correlation matrices of channel outputs and feedback decisions in time domain can be block diagonalizable by using vector DFT operations defined in Section II. In frequency domain, one can write the vector DFTs of transmitted, received and feedback signal vectors at the $i^{th}$ iteration as

$$\mathbf{X} = \mathbf{Q_{n_t}}\mathbf{x}, \quad \mathbf{Y} = \mathbf{Q_{n_r}}\mathbf{y}, \quad \hat{\mathbf{X}}^{(i)} = \mathbf{Q_{n_t}}\hat{\mathbf{x}}^{(i)}. \quad (16)$$

By using (16), the Wiener-Hopf equation in time domain in (10) can be expressed in the frequency domain as

$$\begin{bmatrix} \mathbf{R_{YY}} & -\mathbf{R}_{\mathbf{Y}\hat{\mathbf{X}}^{(i-1)}} \\ -\mathbf{R}^H_{\mathbf{Y}\hat{\mathbf{X}}^{(i-1)}} & \mathbf{R}_{\hat{\mathbf{X}}^{(i-1)}\hat{\mathbf{X}}^{(i-1)}} \end{bmatrix} \begin{bmatrix} \mathbf{Q_{n_r}} & 0 \\ 0 & \mathbf{Q_{n_t}} \end{bmatrix} \begin{bmatrix} \mathbf{F}^{(i)} \\ \mathbf{B}^{(i)} \end{bmatrix} = \begin{bmatrix} \mathbf{R_{YX}} \\ -\mathbf{R}_{\hat{\mathbf{X}}^{(i-1)}\mathbf{X}} - \mathbf{\Gamma}_F^{(i)} \end{bmatrix} \mathbf{Q_{n_t}} \quad (17)$$

where $\mathbf{\Gamma}_F^{(i)}$ is the Lagrangian matrix in frequency domain such that $\mathbf{\Gamma}_F^{(i)} = \mathbf{Q_{n_t}}\mathbf{\Gamma}^{(i)}\mathbf{Q_{n_t}}^H$. One can write the equivalent feed-forward and feedback filters in frequency domain as in

$$\mathbf{W}^{(i)} = \mathbf{Q_{n_r}}\mathbf{F}^{(i)}\mathbf{Q_{n_t}}^H, \text{ and } \mathbf{C}^{(i)} = \mathbf{Q_{n_t}}\mathbf{B}^{(i)}\mathbf{Q_{n_t}}^H. \quad (18)$$

Since the correlation matrices in frequency domain have a block diagonal structure due to the DFT operation and correlation assumptions, $\mathbf{W}^{(i)}$ and $\mathbf{C}^{(i)}$ possess a block diagonal structure:

$$\mathbf{W}^{(i)} = \text{bdiag}\left\{\mathbf{W}_j^{(i)}\right\}_{j=0}^{N-1}, \text{ and } \mathbf{C}^{(i)} = \text{bdiag}\left\{\mathbf{C}_j^{(i)}\right\}_{j=0}^{N-1}. \quad (19)$$

The block diagonal structure of the frequency domain filter matrices impose a block circulant structure for the time domain feed-forward and feedback filters. The constraint on time domain feedback filters in (8) corresponds to the following constraint on the frequency domain filter due to (18):

$$\sum_{j=0}^{N-1} \mathbf{C}_j^{(i)}(n,n) = 0, \; n = 1, \ldots, n_t. \quad (20)$$

The equation in (17) can be decomposed into $N$ parallel subequations with reduced matrix dimensions to produce the optimal feed-forward and feedback filter matrices in the frequency domain by using (3) and (12) as

$$\begin{aligned} \mathbf{R}_{\mathbf{Y}_j\mathbf{Y}_j}\mathbf{W}_j^{(i)} &= \overline{\mathbf{\Lambda}}_j\left[E_s\mathbf{I}_{n_t} + \mathbf{P}_1^{(i)}\mathbf{C}_j^{(i)}\right], \\ \mathbf{P}_2^{(i)}\mathbf{C}_j^{(i)} &= (\mathbf{P}_1^{(i)})^H\left[\overline{\mathbf{\Lambda}}_j^H\mathbf{W}_j^{(i)} - \mathbf{I}_{n_t}\right] - \mathbf{\Delta}^{(i)} \end{aligned} \quad (21)$$

for $j = 0, \ldots, N-1$, and $\mathbf{R}_{\mathbf{Y}_k\mathbf{Y}_k} = \left(\overline{\mathbf{\Lambda}}_k\overline{\mathbf{\Lambda}}_k^H E_s + N_0\mathbf{I}_{n_r}\right)$. Also, $\mathbf{\Delta}^{(i)} = \text{diag}\left[\Delta_1^{(i)}, \ldots, \Delta_{n_t}^{(i)}\right]_{(n_t\times n_t)}$ can be obtained from the constraint in (20).

By substituting $\mathbf{W}_j^{(i)}$'s into the second eqn. of (21) and using the constraint in (20), the Lagrangian terms and feedback filter matrices can be readily found after some calculus as

$$\begin{aligned} \mathbf{C}_j^{(i)} &= \mathbf{A}_j^{(i)}\left[\mathbf{D}_j^{(i)} - \mathbf{\Delta}^{(i)}\right], \quad (22) \\ \Delta_n^{(i)} &= \frac{\left[\sum_{j=0}^{N-1}\mathbf{A}_j^{(i)}(n,:)\mathbf{D}_j^{(i)}(:,n)\right]}{\left[\sum_{j=0}^{N-1}\mathbf{A}_j^{(i)}(n,n)\right]}, \; n = 1, \ldots, n_t \end{aligned}$$

where

$$\begin{aligned} \mathbf{A}_j^{(i)} &= \left[\mathbf{P}_2^{(i)} - (\mathbf{P}_1^{(i)})^H\overline{\mathbf{\Lambda}}_j^H\mathbf{R}^{-1}_{\mathbf{Y}_j\mathbf{Y}_j}\overline{\mathbf{\Lambda}}_j\mathbf{P}_1^{(i)}\right]^{-1}, \\ \mathbf{D}_j^{(i)} &= (\mathbf{P}_1^{(i)})^H\overline{\mathbf{\Lambda}}_j^H\mathbf{R}^{-1}_{\mathbf{Y}_j\mathbf{Y}_j}\overline{\mathbf{\Lambda}}_jE_s - (\mathbf{P}_1^{(i)})^H, \end{aligned} \quad (23)$$

$\mathbf{A}_j^{(i)}(n,:)$ is the $n-$th row of $\mathbf{A}_j^{(i)}$, $\mathbf{D}_j^{(i)}(:,n)$ is the $n-$th column of $\mathbf{D}_j^{(i)}$. Then, $\mathbf{W}_j^{(i)}$'s can be found based on the first equation of (21) with $\mathbf{C}_j^{(i)}$ in (22) for $j = 0, \ldots, N-1$.

The equivalence between the block TDE-TDDF and FDE-FDDF can be established as follows. The output of TDE-TDDF at the $i^{th}$ equalizer update can be written as

$$\begin{aligned} \tilde{\mathbf{x}}^{(i)} &= (\mathbf{F}^{(i)})^H\mathbf{y} - (\mathbf{B}^{(i)})^H\hat{\mathbf{x}}^{(i-1)} \quad (24) \\ &= \mathbf{Q_{n_t}}^H\mathbf{Q_{n_t}}(\mathbf{F}^{(i)})^H\mathbf{Q_{n_r}}^H\mathbf{Y} \\ &\quad -\mathbf{Q_{n_t}}^H\mathbf{Q_{n_t}}(\mathbf{B}^{(i)})^H\mathbf{Q_{n_t}}^H\hat{\mathbf{X}}^{(i-1)} \\ &= \mathbf{Q_{n_t}}^H\left[(\mathbf{W}^{(i)})^H\mathbf{Y} - (\mathbf{C}^{(i)})^H\hat{\mathbf{X}}^{(i-1)}\right] = \mathbf{Q_{n_t}}^H\tilde{\mathbf{X}}^{(i)} \end{aligned}$$

Then, the output of the FDE-FDDF for the $k^{th}$ vector in the block (for the $i^{th}$ iteration) can be expressed as

$$\begin{aligned} \tilde{\mathbf{x}}_k^{(i)} &= \sum_{j=0}^{N-1}(q_j^k)^*\left[(\mathbf{W}_j^{(i)})^H\mathbf{Y}_j - (\mathbf{C}_j^{(i)})^H\hat{\mathbf{X}}_j^{(i-1)}\right] \\ &= \sum_{j=0}^{N-1}(q_j^k)^*\tilde{\mathbf{X}}_j^{(i)} \end{aligned} \quad (25)$$

for $k = 0, \ldots, N-1$. It can be seen that the direct estimate in frequency domain $\tilde{\mathbf{X}}^{(i)}$ can be converted to the desired time domain estimates by a simple inverse vector DFT operation. The frequency domain estimate of the transmitted sequence at the $j^{th}$ frequency bin, namely $\tilde{\mathbf{X}}_j^{(i)}$, is obtained by using only the corresponding frequency domain observations and feedback at the same frequency bin. The structure of FDE-FDDF in (25), which is equivalent to TDE-TDDF in (6), is shown in Fig. 1.

The structure of the frequency domain equalizer in (25) is the direct consequence of the optimal filter derivation. This is different than the previous structure based approaches in the literature, which initially impose an equalizer structure.

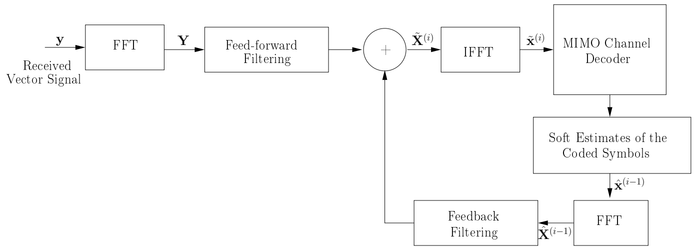


Fig. 1. FDE-FDDF: Iterative Block FDE with frequency domain decision feedback (FDDF)

The total MSE in one block can be written as $MSE^{(i)} = E\left\{||\tilde{\mathbf{x}}^{(i)} - \mathbf{x}||^2\right\} = E\left\{||\tilde{\mathbf{X}}^{(i)} - \mathbf{X}||^2\right\}$ from (24), thus the same MSE for frequency domain estimates is achieved with the use of filter coefficients $\mathbf{W}^{(i)}$ and $\mathbf{C}^{(i)}$. Then, the equivalence relations between optimal feed-forward and feedback filter coefficients in different domains can be expressed as a result of the solution of Wiener-Hopf equation in (10). This equivalence can be summarized in Fig. 2.

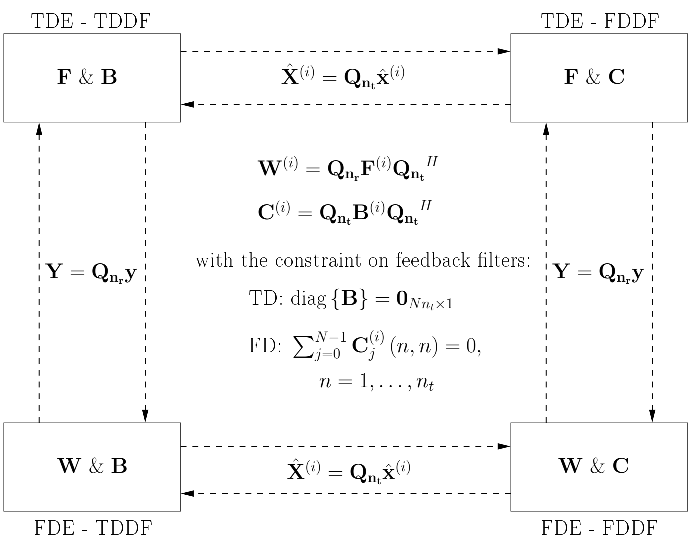


Fig. 2. Equivalence Relation between Time Domain (TD) and Frequency Domain (FD) Block Equalizers: (1) Forward Time Domain Equalization with Time Domain Decision Feedback (TDE-TDDF) (2) Forward Time Domain Equalization with Frequency Domain Decision Feedback (TDE-FDDF) (3) Forward Frequency Domain Equalization with Frequency Domain Decision Feedback (FDE-FDDF) (4) Forward Frequency Domain Equalization with Time Domain Decision Feedback (FDE-TDDF)

It is seen that the computational complexity to obtain feed-forward and feedback filters is considerably reduced for SC FDE-FDDF implementation in comparison to TDE-TDDF, TDE-FDDF and FDE-TDDF since only $n_r \times n_r$ and $n_t \times n_t$ matrix inversions are needed as can be seen from (22)-(23) and size of these matrices is independent of block length $N$ just like OFDM based systems. It is seen that two FFT operations per iteration is required for the proposed SC FDE-FDDF implementation. This makes the complexity of SC-FDE scheme approximately twice that of the OFDM based transmission.

Soft feedback decisions of coded symbols can be obtained by using the information given by the channel decoder. Using these soft decisions, it is possible to approximate correlation matrices $\mathbf{P}_1^{(i)}$ and $\mathbf{P}_2^{(i)}$ in (15) as done for the SISO case in [5]. Correct estimation of $\mathbf{P}_1^{(i)}$ and $\mathbf{P}_2^{(i)}$'s are important since the proposed structure takes into account the reliability of the feedback decisions and therefore alleviates the error propagation problem. In the first iteration, $\mathbf{P}_1^{(i)}$ and $\mathbf{P}_2^{(i)}$ can be taken as $\mathbf{0}_{n_t\times n_t}$, i.e., reliable feedback decisions are not available. In this case, the equalizer is composed of only linear feed-forward block MMSE filter. As the number of iterations increase, both metrics are expected to approach to the asymptotic value: $E_s\mathbf{I}_{n_t}$ (perfect decisions). Calculation of these correlation matrices will be explained in Section III-C.

### *C. Iterative MAP Decoding*

We consider a coded modulation scheme by careful coding across transmit antennas as in [20], [30] and D-BLAST [31] based structures, whereas in V-BLAST [32] type schemes each stream is decoded separately without paying any regard to the possible transmit diversity gains. In this section, we will calculate the log-likelihood ratios (LLR) and soft decisions of the coded symbols for FDE-FDDF. BPSK modulation is assumed for simplicity, but the extension to other modulations is straightforward in principle. At each iteration, extrinsic information is extracted from detection and decoding stages and is then used as a priori information in the next iteration. The soft output from the FDE-FDDF in the $i^{th}$ iteration after (25) can be written as,

$$\tilde{x}_k^{m\ (i)} = \mu_m^{(i)} x_k^m + \eta_k^{m\ (i)} \tag{26}$$

for $k = 0,\ldots,N-1$ and $m = 1,\ldots,n_t$. In this case, the equalized MIMO channel in (26) can be considered as a quasi-parallelized block fading channel. The equivalent complex amplitude, $\mu_m^{(i)}$ of the symbol transmitted from the $m^{th}$ antenna and the residual interference power, $E\{|\eta_k^{m\ (i)}|^2\}$ can be computed by using (25) as follows,

$$\begin{aligned}
\mu_m^{(i)} &= E\{\tilde{x}_k^{m\ (i)}(x_k^m)^*\}/E_s \\
&= \sum_{j=0}^{N-1}\frac{1}{N}\left[(\mathbf{W}_j^{(i)}(m))^H\overline{\mathbf{\Lambda}}_j - \frac{1}{E_s}(\mathbf{C}_j^{(i)}(m))^H(\mathbf{P}_1^{(i)})^H\right]\mathbf{e}_m \\
&= \sum_{j=0}^{N-1}\frac{1}{N}(\mathbf{W}_j^{(i)}(m))^H\overline{\mathbf{\Lambda}}_j\mathbf{e}_m \qquad (27)
\end{aligned}$$

by noting that $\sum_{j=0}^{N-1}\mathbf{C}_j^{(i)}(m,m) = 0$, and $\mathbf{e}_m$ is the $n_t \times 1$ unit vector whose $m^{th}$ entry is one for $m = 1,\ldots,n_t$. $E\{|\eta_k^{m\ (i)}|^2\} = E\{|\tilde{x}_k^{m\ (i)}|^2\} - E_s|\mu_m^{(i)}|^2$ where

$$\begin{aligned}
E\{|\tilde{x}_k^{m\ (i)}|^2\} &= \sum_{j=0}^{N-1}\frac{1}{N}(\mathbf{W}_j^{(i)}(m))^H\mathbf{R}_{\mathbf{Y}_j\mathbf{Y}_j}\mathbf{W}_j^{(i)}(m) \\
&\quad + \sum_{j=0}^{N-1}\frac{1}{N}(\mathbf{C}_j^{(i)}(m))^H\mathbf{P}_2^{(i)}\mathbf{C}_j^{(i)}(m) \qquad (28) \\
&\quad - \sum_{j=0}^{N-1}\frac{2}{N}\mathrm{Re}\left\{(\mathbf{W}_j^{(i)}(m))^H\overline{\mathbf{\Lambda}}_j\mathbf{P}_1^{(i)}\mathbf{C}_j^{(i)}(m)\right\}
\end{aligned}$$

for $m = 1,\ldots,n_t$, where $\mathbf{W}_j^{(i)}(m)$ and $\mathbf{C}_j^{(i)}(m)$ are the $m^{th}$ column of $\mathbf{W}_j^{(i)}$ and $\mathbf{C}_j^{(i)}$ respectively.

It is important to note that $\mu_m^{(i)}$ and $E\{|\eta_k^{m\ (i)}|^2\}$ values do not depend on symbol time index $k$, so these values are calculated only once for the decoding of one block in each iteration, which reduces the complexity significantly. The inputs to the decoder in terms of the LLR for each coded stream can be calculated by knowing the optimal filter coefficients. While computing the LLRs, we resort to simplification of the decoding algorithm by neglecting the correlation existing between the residual noise terms, i.e., $\eta_k^m$'s are taken as uncorrelated for $m = 1, \ldots, n_t$ as done in the decoding stage of [33] for flat fading MIMO channel. The residual interference is further approximated by a Gaussian distribution as in [5], [27], [34]. The LLR for the $k^{th}$ symbol transmitted at the $m^{th}$ antenna can be written as

$$\lambda_k^{m\ (e)} = \log_e \frac{P(\tilde{x}_k^{m\ (i)} | x_k^m = +1)}{P(\tilde{x}_k^{m\ (i)} | x_k^m = -1)} = \frac{4\mathrm{Re}\{(\mu_m^{(i)})^* \tilde{x}_k^{m\ (i)}\}}{E\{|\eta_k^m|^2\}} \tag{29}$$

which is obtained from the equalizer output and will be used by the channel decoder. An a priori probability ratio $\lambda_k^{m\ (p)}$ $(\log_e \frac{P(x_k^m = +1)}{P(x_k^m = -1)})$ is given by the decoder from the previous iteration [5], [27] and used to construct a soft estimate of the coded symbol transmitted at the $m^{th}$ antenna for $k^{th}$ vector. Soft feedback decisions for the FDE-FDDF can be expressed in terms of the extrinsic information provided by the decoder as follows [27], [34]: $\hat{x}_k^m = E\{x_k^m\} = \tanh\left(\frac{1}{2}\lambda_k^{m\ (p)}\right)$ for $E_s = 1$, $m = 1, \ldots, n_t$ and $k = 0, \ldots, N-1$. The non-zero diagonal entries of the correlation matrices $\mathbf{P}_1^{(i)}$ and $\mathbf{P}_2^{(i)}$ in (12) used by the feed-forward and feedback filters can be calculated by using the following approximation as done in [12],

$$\begin{aligned} \rho_{k,m} &\triangleq E\{x_k^m (\hat{x}_k^m)^*\} = E\{E\{x_k^m\}(\hat{x}_k^m)^*\} \approx |\hat{x}_k^m|^2, \text{ and} \\ \rho_m &= \beta_m = \frac{1}{N}\sum_{k=0}^{N-1} \rho_{k,m}. \end{aligned} \tag{30}$$

$E\{x_k^m\}$ was taken as $\hat{x}_k^m$ and this is a common assumption in various turbo detection techniques [5], [12], [27], [34]. For higher order constellations, soft feedback decisions can be calculated as in [35] and [36] by using log−likelihoods of the coded bits. Reliability matrices can be calculated in a similar manner.

The mean of the soft feedback decisions depend on symbol time $k$ and are not equal to each other in general. In that case, block diagonalization of the Wiener-Hopf equation in (10) cannot be carried out due to time dependent correlation matrices $\mathbf{P}_1^{(i)}$ and $\mathbf{P}_2^{(i)}$. However, with the use of averaged reliability matrices (constant over the entire block) in (30), FDE operation brings significant complexity advantage and it is observed, although not presented, that FDE-FDDF shows a very close performance to that of the time-varying equalizer.

## IV. Asymptotic Performance Analysis and Rate-Diversity Tradeoff

### A. *Asymptotic Calculation of Filter Coefficients and SINR for the SC-FDE Scheme*

At each iteration, feed-forward and feedback filters approach the optimal coefficients in case of perfect feedback with the help of improved log a posteriori probability (APP) ratio of each coded symbol obtained from the decoder. As the iterations carry on, feedback decisions become more reliable and correlation matrices approach the asymptotic values if the channel is not in outage: $\mathbf{P}_1^{(i)} \to E_s \mathbf{I}_{n_t}$ and $\mathbf{P}_2^{(i)} \to E_s \mathbf{I}_{n_t}$. The feed-forward and feedback frequency domain block filters are jointly updated at each equalizer iteration according to the reliability information of the decoded symbols and $SNR = \frac{E_s}{N_0}$ values. If the feedback decisions are assumed to be perfect, one can find the equalizer coefficients depending on SNR as proved in Appendix

$$\begin{aligned} \mathbf{W}_j &= N\frac{E_s}{N_0}\mathbf{\Sigma}^{-1}\overline{\mathbf{\Lambda}}_j, \\ \mathbf{C}_j &= -\mathbf{I}_{n_t} + N\mathbf{\Sigma}^{-1}\left[\mathbf{I}_{n_t} + \frac{E_s}{N_0}\overline{\mathbf{\Lambda}}_j^H \overline{\mathbf{\Lambda}}_j\right] \end{aligned} \tag{31}$$

for each frequency bin $j = 0, \ldots, N-1$, where $\mathbf{\Sigma}$ is properly defined in Appendix.

The signal-to-interference-noise-ratio (SINR) of each parallelized channel in (26) after equalization is evaluated in Appendix for the asymptotic case and given as

$$SINR_m = \sum_{l=0}^{L-1}\sum_{i=1}^{n_r} |\mathbf{H}_l(i,m)|^2 \frac{E_s}{N_0} \quad \text{for } m = 1, \ldots, n_t\ . \tag{32}$$

The equivalent counterparts of FD filters in (31) can be obtained in TD at high $SNR$ by using the equivalence relation (18) in different domains and letting $SNR$ to go to infinity after some straightforward but tedious algebraic steps as

$$\mathbf{F} = \mathbf{H}_c \mathbf{\Phi}^{-1}, \quad \mathbf{B} = -\left[\mathbf{I}_{Nn_t} - \mathbf{R}_c \mathbf{\Phi}^{-1}\right] \text{ where} \tag{33}$$

$$\begin{aligned} \mathbf{R}_c = \mathbf{H}_c^H \mathbf{H}_c = \big[\mathrm{circ}\{\ & \mathbf{R}_h(0) \ \cdots \ \mathbf{R}_h(-(L-1)) \ \ \mathbf{0} \\ & \cdots \ \mathbf{0} \ \ \mathbf{R}_h(L-1) \ \cdots \ \mathbf{R}_h(1)\ \}\big]_{(Nn_t)\times(Nn_t)} \end{aligned} \tag{34}$$

and $\mathbf{\Phi}$ is a $(Nn_t) \times (Nn_t)$ diagonal matrix with the diagonal elements of $\mathbf{R}_c$. $\mathbf{R}_h(n)$ is the discrete time autocorrelation function of the MIMO channel response such that $\mathbf{R}_h(n) = \mathbf{H}_n * \mathbf{H}_{-n}^*$, where $*$ is the matrix convolution operator, and $\mathbf{R}_h(n)$ is nonzero only for $|n| < L$. To derive (33), we use the fact that channel matrix of MIMO ISI channel, namely, $\mathbf{H}_c$ in (5) is block circulant so that it can be block diagonalizable by using vector DFT operations. Space limitations prevents providing the detailed steps in the paper.

The equivalent receiver in time domain for the high SNR case is shown in Fig. 3. As can be seen from the structure in Fig. 3, the feed-forward filter is actually a vector type channel matched filter (CMF) behaving as the maximal ratio combiner (MRC) of the multipath components coming from different receive antennas. The feedback filter is an ideal interference canceller of the CCI and ISI components left after the CMF operation. These asymptotic filters in TD appear as the optimal processing in terms of attaining full diversity and the mitigation of both ISI and CCI.

It is seen from (32) that one can achieve the full diversity gain $(n_r \times L)$ at each of the parallelized channels. If transmit diversity schemes in the form of coding across antennas such as [30] are utilized, the maximum potential diversity gain of

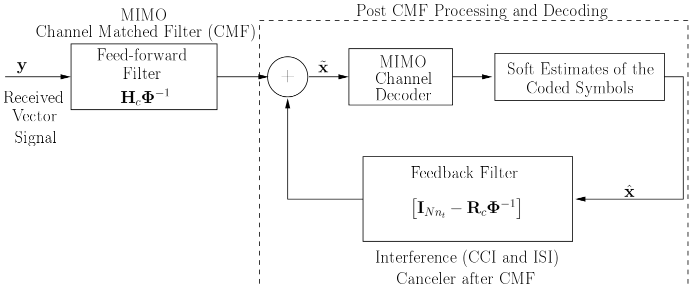


Fig. 3. Time Domain Equivalent of the FDE-FDDF for $SNR \rightarrow \infty$ case

$(n_r \times n_t \times L)$ can be achieved by the proposed equalization scheme here.

In our case, we have used simple coding structures that achieve the optimal rate-diversity tradeoff given by the singleton bound for block-fading channels in [30] and as it will be seen in Section V, one can get a very close performance to the outage probability for fixed constellations. In the next section, the rate-diversity tradeoffs of the proposed space-time coded SC-FDE and space-frequency coded OFDM based MIMO schemes are investigated for a fixed constellation.

### *B. Rate-Diversity Tradeoff of SC-FDE and OFDM with Fixed Constellation in MIMO channels*

*1) MIMO SC-FDE Scheme:* The space-time coded modulation scheme $\chi \subset \mathbb{C}^{n_t \times N}$ over $n_t$ transmit antennas and a duration of $N$ symbol transmission is considered for the construction of one packet. We consider that $k$ bits are desired to be transmitted by each codeword. The coded modulation scheme $\chi$ is constructed such that $2^k$ different codewords are generated. A total of $N_{total} = n_t \times N$ complex dimensions are utilized for the transmission of one codeword. The codeword matrix for the $i^{th}$ codeword can be defined as $\mathbf{X}^i = [\mathbf{x}_0^i, \mathbf{x}_1^i, \cdots, \mathbf{x}_{N-1}^i]_{n_t \times N}$ where $\mathbf{x}_n^i \subset \mathbb{C}^{n_t}$ is the transmitted vector symbol with each element selected from a fixed channel input alphabet $S \in \mathbb{C}$ with cardinality $M$ for $i = 1, \ldots, 2^k$.

Although alternative definitions may exist [21], the desired bit rate, $R_b = \frac{k}{T_0}$ bit/s and $D = \frac{N_{total}}{T_0}$ dimensions/s are defined for the packet duration $T_0$. Then, $R = \frac{R_b}{D} = \frac{k}{n_t N}$ is the ratio showing the communication rate in bits per complex dimension (or channel use). The rate $R$ can be seen as the desired spectral efficiency in bit/s/Hz or transmission rate in bit/dimension. In particular, we can consider that $\chi$ is obtained as the concatenation of a binary code of rate $r$ and a modulation over the signal constellation $S$ with $|S| = M$. Then, the rate in bits per channel use of this scheme is $R = r\log_2(M)$.

It is known that when the signal constellation set $S$ is finite, there is a trade-off between the rate and diversity order (maximum reliability exponent) of a space-time coded systems. This trade-off is given by the singleton bound for block fading channels (BFC) in [20], and MIMO channels in [21].

Since the SC-FDE scheme with proper equalization and decoding can achieve the total antenna receive diversity $(n_r)$ and multipath diversity $(L)$ as predicted by the asymptotic SINR expression in (32), one can write the maximum diversity order based on (32). This asymptotic SINR at each parallelized channel is shown to be achieved in simulations by the proposed iterative FDE-FDDF architecture. The maximum diversity order for the proposed SC-FDE-FDDF scheme based on (32) is given by

$$d^{\chi}_{MIMO\ SC-FDE} = Ln_r\left(1 + \lfloor n_t(1-r)\rfloor\right) \qquad (35)$$

where $r = \frac{R}{\log_2(M)}$ by using the results in [37]. [1] This result is reasonable since we know that uncoded SC-FDE scheme can obtain the receive antenna diversity $(n_r)$ and multipath diversity $(L)$ by optimal CMF type processing explained in Section IV-A. For SISO systems, it is known for some time that SC based systems can achieve full multipath diversity with the use of ML detectors [3]. The rate-diversity tradeoffs for SC-FDE scheme with the use of linear MMSE receivers have been recently obtained for SISO ISI channels in [22] and for MIMO ISI channels in [23] respectively. Although those studies show that SC-FDE may fail to achieve the full multipath diversity of the ISI channel with the use of linear MMSE filtering, we showed that the proposed SC-FDE-FDDF scheme is able to capture full multipath diversity. Space-time coding over transmit antennas brings the transmit diversity gain $\lfloor n_t(1-r)\rfloor$ in addition to the receive antenna and multipath diversity brought by the proposed optimal SC-FDE processing.

*2) MIMO OFDM Scheme:* As for the MIMO-OFDM scheme, one needs channel coding in order to attain multipath and transmit antenna diversity. If one chooses the size of the DFT matrix in OFDM as $L$, the frequency bins have independent channel gains. Then, OFDM can be thought as a BFC, where each frequency bin can be seen as one block. The equivalent problem is to find the maximum diversity order for the following MIMO BFC with $L$ fading blocks each with $K$ symbols:

$$\mathbf{Y}_{i,k} = \overline{\mathbf{\Lambda}}_i\mathbf{X}_{i,k} + \mathbf{N}_{i,k},\ i = 0, \ldots, L-1,\ k = 0, \ldots, K-1, \qquad (36)$$

where $\mathbf{X}_{i,k}$ is the $n_t \times 1$ $k^{th}$ symbol vector for the $i^{th}$ frequency bin in the codeword. $\overline{\mathbf{\Lambda}}_i$ is an $n_r \times n_t$ matrix denoting the gain of the MIMO channel at the $i^{th}$ frequency bin. $\overline{\mathbf{\Lambda}}_i$'s are independent with i.i.d. Rayleigh entries. The space-frequency code $\chi^F$ is constructed such that

$$\begin{bmatrix} \mathbf{X}_{0,0} & \cdots & \mathbf{X}_{0,K-1} \\ \vdots & \ddots & \vdots \\ \mathbf{X}_{L-1,0} & \cdots & \mathbf{X}_{L-1,K-1} \end{bmatrix}_{(Ln_t)\times K} \subset \mathbb{C}^{(Ln_t)\times K} \qquad (37)$$

In this system, the maximum transmit diversity gain $\nu$ is determined by the minimum rank of all possible codeword differences which is given as $\nu = 1 + \lfloor Ln_t(1-r)\rfloor$ in [21],

[1] In [37], the maximum diversity order for transmission over $B$ fading blocks is found for the Nakagami$-m$ distribution at each fading block. The maximum diversity order is given as $m\nu$ where $\nu$ is the transmit diversity gain determined by $\nu \leq 1 + \lfloor B(1-r)\rfloor$. In our case, each parallelized channel has $\chi_2$ distribution with order $2Ln_r$ and the transmission is over $n_t$ blocks. $\chi_2$ is a special case of Nakagami$-m$, thus the result in (35) can be obtained by taking $m = Ln_r$ and $B = n_t$.

[38]. The maximum achievable diversity is $n_r \nu$, since $n_r$ diversity coming from the receive antennas can always be achieved without the space-frequency coding structure and the resultant diversity order is the product of number of receive antennas $n_r$ and the transmit diversity gain $\nu$ [38]. Then, the maximum diversity order for the MIMO OFDM system can be written as

$$d^{\chi}_{MIMO\ OFDM} = n_r \left(1 + \lfloor L n_t (1 - r) \rfloor\right) \tag{38}$$

for $r = \frac{R}{\log_2(M)}$. Full rate-full diversity attaining space-frequency code design criterion is depicted in [39], [40], and the explicit rate-diversity tradeoff expression, depending on data rate and block length, is obtained for OFDM systems in [41] for SISO channels by taking the correlation between frequency bins into account recently.

As it is well known, OFDM needs channel coding over frequency bins (space-frequency coding) to attain multipath diversity, whereas SC-FDE scheme, in which space-time code brings only transmit diversity from the multiple use of transmit antennas, achieves full multipath diversity gain asymptotically regardless of whether channel coding is employed or not. One can show $d^{\chi}_{MIMO\ OFDM} \le d^{\chi}_{MIMO\ SC-FDE}$ and thus there may be great differences between the achievable diversity order of the MIMO OFDM and MIMO SC-FDE schemes depending on the system parameters (length of the channel $(L)$, $n_t$ and the code rate $r$ with a finite constellation $S$). Although OFDM is the capacity achieving strategy for the Gaussian alphabet with which the maximum diversity order is always achievable, the last result suggests that it is not so for fixed constellations. Simulation based findings in [16], [42] support this result.

## V. Simulation Results

### A. *Outage Probability and MFB Calculations*

The capacity of the SC scheme for frequency selective channels is difficult to obtain for finite constellations [43] and hence the capacity of MIMO OFDM is instead evaluated. The constrained MIMO-OFDM capacity can be found for the system model in (3) given the complex vector set $\chi$ of cardinality $M^{n_t}$ similar to the derivations for block fading channels in [30] as

$$\begin{aligned} C^{\chi}_{MIMO-OFDM} &= \frac{1}{N}\sum_{j=0}^{N-1} I(\mathbf{X}_j;\mathbf{Y}_j|\overline{\mathbf{\Lambda}}_j) = \\ n_t \log_2(M) &- \frac{1}{N}\sum_{j=0}^{N-1} E_{\mathbf{N}_j}\left\{\sum_{\mathbf{X}_k\in\chi}\frac{1}{M^{n_t}}\log_2\left[\sum_{\mathbf{X}_i\in\chi}\right.\right. \\ &\left.\left.\exp\left(\frac{-\|\overline{\mathbf{\Lambda}}_j(\mathbf{X}_k-\mathbf{X}_i)+\mathbf{N}_j\|^2+\|\mathbf{N}_j\|^2}{N_0}\right)\right]\right\} \end{aligned} \tag{39}$$

and the corresponding outage probability is $P_{out}^{MIMO-OFDM,\,\chi}(R) = \mathbb{P}\left\{C^{\chi}_{MIMO-OFDM} < R\right\}$. Matched filter bound (MFB) corresponds to the genie aided case in which all transmitted symbols leading to ISI and CCI are available at the receiver and canceled perfectly after the CMF operation [14], [26], [27]. One can observe that the SINR obtained after the proposed SC-FDE scheme in (32) coincides with MFB in case of perfect decision feedback. The constrained outage probability and MFB will be used for performance evaluation in subsequent subsections.

### B. *Code Construction and Performance Results*

The code construction used in our work is similar to the structure for random-like codes adapted to the block-fading channel based on blockwise concatenation and on bit-interleaved coded modulation (BICM) in [30]. The presented coded modulation construction in [30] systematically yields maximum diversity order achieving turbo-like codes defined over an arbitrary signal set. As such, any other coding architecture that performs well in parallel block fading channels can be used in our system. We have used the same encoding and decoding structures as in [30] in simulations.

For producing the results in Fig. 4, the outer code used is a simple repetition code of rate $r = 1/n_t$ and the inner codes are rate-1 accumulators, which is referred to as the repeat and blockwise accumulate (RBA) code [30]. The performance of the proposed FDE-FDDF is shown for a $4 \times 4$ MIMO system with the use of full block diversity attaining RBA code of rate $r = 1/4$. The channel model described in Section II is assumed and the COST207 channel with exponential power delay profile for suburban and urban areas (Chapter 14 in [44]) is used with a 7 $\mu$sec delay spread. BPSK modulation is used for simplicity, but other modulations combined with BICM [45] can be applied to the proposed structure. The symbol duration is taken as 1 $\mu$sec, and the channel length $L$ equals 8. The first channel tap has unitary power. The information block length, i.e., the information bits entering the outer encoder is taken as $K = 250$, then the block length $N$ is equal to $K/(r \cdot n_t)+1 = 251$ including termination bits. The number of iterations inside the RBA decoder is set to 10 and the number of equalizer iterations at which the feed-forward and feedback filters are updated by using the reliability matrices is taken as 3.

It is seen from Fig. 4 that the performance of FDE-FDDF is 0.3 dB away from MFB. There is approximately 1.5 dB difference between the outage probability of the MIMO-OFDM at rate $R = r = 0.25$ bit/s/Hz and this gap from the outage is similar to the gaps obtained with RBA in parallel block fading channels in [30]. Then, one can say that ISI, spatial interference, and the error propagation problem in decision feedback are almost eliminated, since the perfect decision feedback performance (MFB) is approximately achieved. Moreover, it is seen that the performance of FDE-FDDF shows the same slope as the MIMO-OFDM outage probability. It can be concluded that the maximum diversity of the MIMO broadband channel can be attained by using the proposed space-frequency equalizer and coding across transmit antennas.

In Fig. 5, simulation results are depicted for a code rate of $r = 1/2$. A full block diversity attaining blockwise concatenated code (BCC) is used for encoding as adapted from [30]. The outer code is a rate$-\frac{1}{2}$ convolutional code and the inner codes are $n_t$ rate$-1$ accumulators. The information block length $K$ is taken as 248. Similar results are obtained

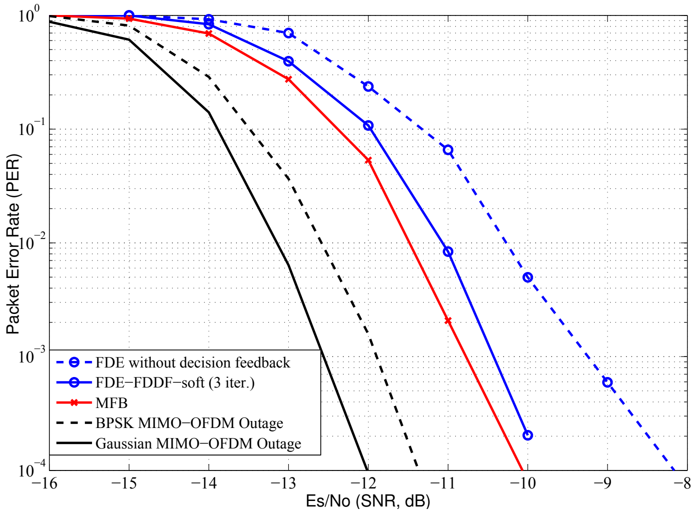


Fig. 4. Performance comparison of the proposed SC-FDE scheme (FDE-FDDF) with MFB and outage for $4 \times 4$ MIMO, $T_s = 1$ $\mu$sec and COST 207 suburban channel with a 7 $\mu$sec delay spread, $L = 8$. The first channel tap has unitary power. BPSK is used with code rate of $r = 1/4$. Spectral efficiency $R = r \log_2(M) = 1/4$ bit/dimension.

and a close performance to MIMO-OFDM outage at rate $R = r = 0.5$ bit/s/Hz is achieved within 2 dB.

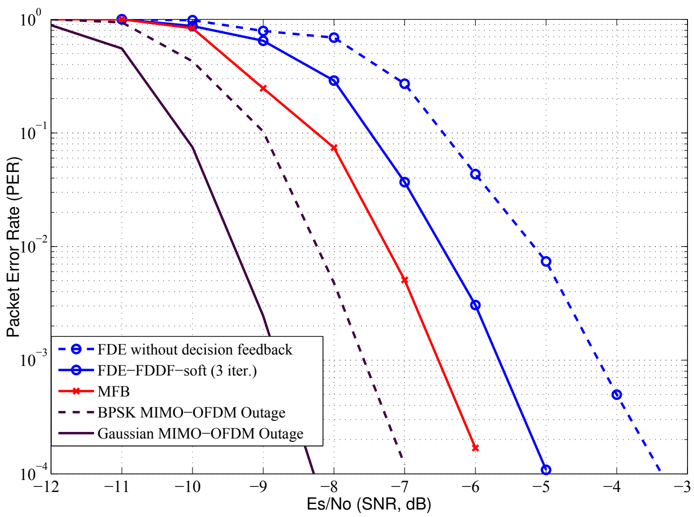


Fig. 5. Performance comparison of the proposed SC-FDE scheme (FDE-FDDF) with MFB and outage for $4 \times 4$ MIMO, $T_s = 1$ $\mu$sec and COST 207 suburban channel with a 7 $\mu$sec delay spread, $L = 8$. The first channel tap has unitary power. BPSK is used with code rate of $r = 1/2$. Spectral efficiency $R = r \log_2(M) = 1/2$ bit/dimension.

The proposed SC-FDE can also be applied to SISO ISI channels. In Fig. 6, we compared the performance of the proposed iterative SC-FDE-FDDF-soft feedback system with the outage probability of an OFDM scheme. A convolutional encoder with $r = 1/2$ serially concatenated with a rate$-1$ accumulator is used for an information block length $K = 123$. At first glance, it is surprising to note that the constrained OFDM outage probability is surpassed by the iterative FDE-FDDF. As stated in [43], the capacity of wideband channels under non-Gaussian alphabets is an open problem and OFDM is not the capacity achieving scheme for non-Gaussian input alphabets. It is also interesting to note that the performance improvement of the FDE-FDDF scheme over FDE without decision feedback is about 2 dB at PER=$10^{-4}$ for all simulation results. There is also a loss in diversity as observed in the reduced PER slope without decision feedback. One can say that the proposed space-frequency equalizer gains more diversity in comparison to FDE without feedback by a careful design of both the feed-forward and feedback filters.

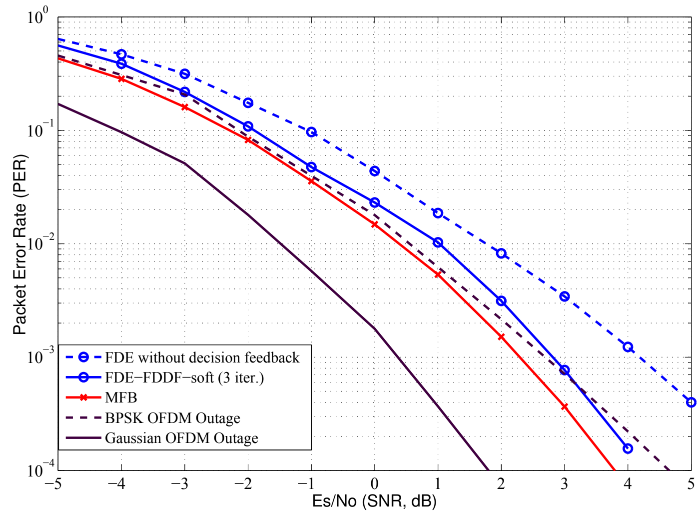


Fig. 6. Performance comparison of the proposed SC-FDE scheme (FDE-FDDF) with MFB and outage for SISO system, $T_s = 0.5$ $\mu$sec and COST 207 suburban channel with a 7 $\mu$sec delay spread, $L = 15$. The first channel tap has unitary power. BPSK is used with code rate of $r = 1/2$. Spectral efficiency $R = r \log_2(M) = 1/2$ bit/dimension.

In Fig. 7, the performance of the proposed MIMO SC-FDE-FDDF is compared with that of MIMO OFDM for QPSK. Also, the corresponding outage probabilities for SC-FDE based on MFB in (32) and for OFDM are obtained. MIMO OFDM receiver is constructed such that the optimal $n_r \times n_t$ and $n_t \times n_t$ feedforward and feedback filtering with decision feedback is utilized to eliminate CCI for each frequency bin separately in an iterative fashion. This is similar to the iterative receiver structure for MIMO OFDM systems in [13]. The results are depicted for a $2 \times 2$ MIMO system and $R = 1.5$ bit/dimension. A full block diversity attaining blockwise concatenated code (BCC) is used for encoding as adapted from [30]. The outer code is a rate$-\frac{3}{4}$ convolutional code and the inner codes are rate$-1$ accumulators. The information block length $K$ is taken as 248. The channel is composed of 3 taps with equal power for better visualization of diversity orders. It is seen that the diversity order predicted by the Singleton bound in (35) is achieved by the FDE-FDDF structure, since FDE-FDDF has the same error rate slope as the outage probability based on MFB. The Constrained outage probability based on MFB corresponds to the outage probability calculation based on the channel model in (32) where all multipath and receive antenna diversity are obtained at each parallel stream after CMF and perfect interference cancellation. Compared to previous scenarios, the diversity difference between SC-FDE and OFDM is more apparent in this case due to the smaller values of channel length ($L$) and larger transmission rate.

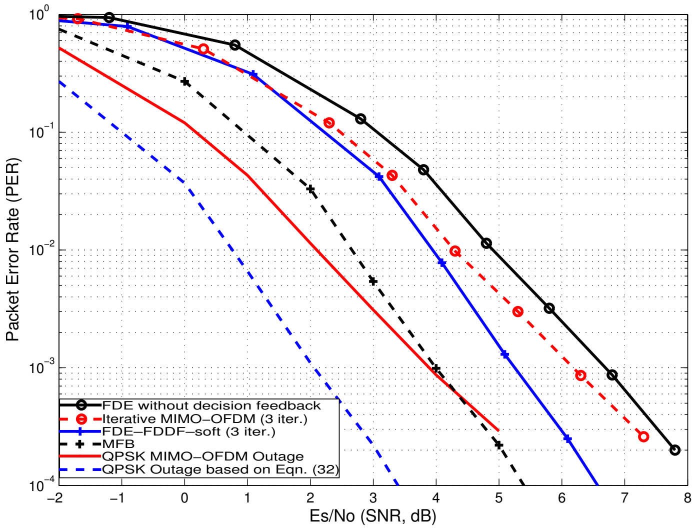


Fig. 7. Performance comparison of the proposed SC-FDE scheme (FDE-FDDF) with MFB and outage for $2 \times 2$ MIMO. The channel is composed of 3 taps with equal power and unity gain, $L = 3$. QPSK is used with code rate of $r = 3/4$. Spectral efficiency $R = r \log_2(M) = 1.5$ bit/dimension.

The diversity orders for $r = \frac{3}{4}$ is given as $d^{\chi}_{MIMO\ SC-FDE} = 6$ for SC-FDE and $d^{\chi}_{MIMO\ OFDM} = 4$ for MIMO OFDM ($L = 3,\ n_r = n_t = 2$), and there is a difference between the achievable diversity orders of SC-FDE and OFDM as can be seen from the slopes of error rate curves. Moreover, the advantage of SC-FDE over OFDM will disappear if one uses FDE without decision feedback. The linear MMSE equalizer without feedback no longer behaves as CMF and the maximal ratio combiner of the multipaths and it fails to achieve the diversity order predicted by $d^{\chi}_{MIMO\ SC-FDE}$ as can be seen from the figure.

## VI. Conclusion

Iterative block equalization with decision feedback is investigated both in frequency and time domains for wideband MIMO systems. Optimal feed-forward and feedback filter coefficients are obtained by establishing the equivalence of time and frequency domain implementations. It is shown through analysis and simulations that the proposed SC-FDE type receiver has achieved the full multipath diversity and performed better than OFDM for fixed rate values and constellations when CSIT is not available. These findings further emphasize the SC-FDE's strong position as an alternative to OFDM in wideband MIMO systems.

## Appendix: Asymptotic Calculation of Filter Coefficients and SINR of FDE-FDDF

If $\mathbf{P}_1^{(i)} = \mathbf{P}_2^{(i)} = E_s \mathbf{I}_{n_t}$, one can write $\mathbf{A}_j$ in (23) as $\mathbf{A}_j = \left[E_s \mathbf{I}_{n_t} - E_s^2 \overline{\mathbf{\Lambda}}_j^H \mathbf{R}_{\mathbf{Y}_j}^{-1} \overline{\mathbf{\Lambda}}_j\right]^{-1}$. By using Matrix Inversion Lemma, one can get

$$\begin{aligned}\mathbf{A}_j &= \frac{1}{E_s}\mathbf{I}_{n_t} + \mathbf{I}_{n_t}\overline{\mathbf{\Lambda}}_j^H \left[\mathbf{R}_{\mathbf{Y}_j} - \overline{\mathbf{\Lambda}}_j \frac{1}{E_s}\mathbf{I}_{n_t} E_s^2 \overline{\mathbf{\Lambda}}_j^H\right]^{-1} \overline{\mathbf{\Lambda}}_j \\ &= E_s^{-1}\mathbf{I}_{n_t} + \overline{\mathbf{\Lambda}}_j^H N_0^{-1} \mathbf{I}_{n_r} \overline{\mathbf{\Lambda}}_j \end{aligned} \quad (40)$$

and $\mathbf{D}_j = -(\mathbf{A}_j)^{-1}$ is written from (23). Lagrangian terms in (21) can be found as

$$\Delta_n = \frac{-N}{\sum_{j=0}^{N-1} \mathbf{A}_j(n,n)} = \frac{-E_s N}{\sum_{j=0}^{N-1}\left[1 + \frac{E_s}{N_0}\overline{\mathbf{\Lambda}}_j^H(n)\overline{\mathbf{\Lambda}}_j(n)\right]} \quad (41)$$

for $n = 1, \ldots, n_t$ by using (22) and (40) and further noting that $\mathbf{A}_j^{(i)}(n,:)\mathbf{D}_j^{(i)}(:,n) = (\mathbf{A}_j^{(i)}\mathbf{D}_j^{(i)})(n,n) = -1$. Defining $\Sigma_n = \sum_{j=0}^{N-1}\left[1 + \frac{E_s}{N_0}\overline{\mathbf{\Lambda}}_j^H(n)\overline{\mathbf{\Lambda}}_j(n)\right]$, feedback filter matrices can be obtained as

$$\mathbf{C}_j = -\mathbf{I}_{n_t} - \mathbf{A}_j \mathbf{\Delta} \quad (42)$$

from (22) and noting that $\mathbf{A}_j \mathbf{D}_j = -\mathbf{I}_{n_t}$. One can obtain the columns of feed-forward and feedback filter matrices by putting (40) and (41) into (42) such that

$$\begin{aligned}\mathbf{C}_j(n) &= -\mathbf{e}_n + \frac{N}{\Sigma_n}\left[\mathbf{e}_n + \frac{E_s}{N_0}\overline{\mathbf{\Lambda}}_j^H \overline{\mathbf{\Lambda}}_j(n)\right], \\ \mathbf{W}_j(n) &= \frac{\frac{E_s}{N_0} N \overline{\mathbf{\Lambda}}_j(n)}{\Sigma_n} \end{aligned} \quad (43)$$

for $j = 0, \ldots, N-1$, $n = 1, \ldots, n_t$ from (21). Defining the following diagonal matrix, $\mathbf{\Sigma} \triangleq \text{diag}\left[\Sigma_1, \ldots, \Sigma_{n_t}\right]_{(n_t \times n_t)}$, one can obtain the block feed-forward and feedback filters in (31) for each frequency bin $j = 0, \ldots, N-1$.

In this case, the soft estimate of $x_k^m$ is a scaled version of the matched filter output after ideal interference cancellation in the frequency domain. SINR's of parallelized channels after equalization can be found after some manipulation by using (27) and (28) for the asymptotic case as follows,

$$\mu_n = \frac{\sum_{j=0}^{N-1}\left[\frac{E_s}{N_0}\overline{\mathbf{\Lambda}}_j^H(n)\overline{\mathbf{\Lambda}}_j(n)\right]}{\Sigma_n} \quad (44)$$

and

$$\begin{aligned}E\{|\tilde{x}_k^n|^2\} &= \frac{E_s}{N}\sum_{j=0}^{N-1}\left[1 + \frac{N^2}{\Sigma_n^2} - 2\frac{N}{\Sigma_n}\right] \\ &\quad + \frac{\sum_{j=0}^{N-1}\left[\frac{E_s^2}{N_0} N \overline{\mathbf{\Lambda}}_j^H(n)\overline{\mathbf{\Lambda}}_j(n)\right]}{\Sigma_n^2} \end{aligned} \quad (45)$$

$$\begin{aligned}E\{|\eta_k^n|^2\} &= \frac{E_s\left[\Sigma_n - N\right]^2}{\Sigma_n^2} + \frac{\sum_{j=0}^{N-1}\left[\frac{E_s^2}{N_0} N \overline{\mathbf{\Lambda}}_j^H(n)\overline{\mathbf{\Lambda}}_j(n)\right]}{\Sigma_n^2} \\ &- E_s|\mu_n|^2 = \frac{\sum_{j=0}^{N-1}\left[\frac{E_s^2}{N_0} N \overline{\mathbf{\Lambda}}_j^H(n)\overline{\mathbf{\Lambda}}_j(n)\right]}{\Sigma_n^2} \end{aligned} \quad (46)$$

for $n = 1, \ldots, n_t$. The SINR can then be evaluated as

$$SINR_m = \frac{|\mu_m|^2 E_s}{E\{|\eta_k^m|^2\}} = \frac{1}{N}\sum_{j=0}^{N-1}\overline{\mathbf{\Lambda}}_j^H(m)\overline{\mathbf{\Lambda}}_j(m)\frac{E_s}{N_0} \quad (47)$$

from (26), and one can obtain (32) by using the Parseval's relation and (4) as

$$SINR_m = \frac{1}{N}\sum_{j=0}^{N-1}\frac{E_s}{N_0}\sum_{i=1}^{n_r}|\overline{\mathbf{\Lambda}}_j(i,m)|^2 = \frac{E_s}{N_0}\sum_{i=1}^{n_r}\sum_{j=0}^{N-1}|\mathbf{H}_j(i,m)|^2 \quad (48)$$

for $m = 1, \ldots, n_t$.

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