A General Multi-Level Evaluation Process for Hybrid MADM With Uncertainty

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***Abstract*— Based on an evidential reasoning framework, a general multi-level evaluation process is developed in this paper for dealing with a multiple attribute decision making (MADM) problem with both quantitative and qualitative attributes. In this new process, a qualitative attribute may be evaluated by uncertain subjective judgments through multiple levels of factors and each of the judgments may be assigned by single or multiple experts in any rational way within the evidential reasoning framework. The qualitative attributes can then be quantified by means of general evaluation analysis and evidential reasoning. A few evaluation analysis models and the corresponding evidential reasoning algorithms are explored for parallel combination and hierarchical propagation of factor evaluations. With all the qualitative attributes being quantified by this rational process, the MADM problem represented by an extended decision matrix is then transformed into an ordinary decision matrix, which can be dealt with using a traditional MADM method. This new general evaluation process and the hybrid decision making procedure are demonstrated using a multiple attribute motor cycle evaluation problem with uncertainty.**

I. Introduction

MULTIPLE attribute decision making problems with both quantitative and qualitative attributes and with uncertainty are common in engineering practice [4], [5], [8], [9], [10], [13], [18], which may simply be called hybrid MADM problems in this paper. To solve a hybrid MADM problem, the first step is to evaluate and quantify the state of a qualitative attribute at each alternative design. To do so, a few evaluation grades may be defined, to which the state of the attribute at each alternative design may be evaluated [9], [24].

It has been realized that multiple factor analysis and reasoning with uncertain subjective judgments are essential for evaluation and quantification of qualitative attributes [10], [20], [24], [28]. The evidential reasoning approach was therefore explored in [24], based upon an evaluation analysis model [21], [28] and the evidence combination rule of the Dempster-Shafer (simply D-S) theory [2], [12]. This approach is different from other MADM approaches using other uncertainty models in that it can deal with incomplete uncertain decision knowledge in a more rational way through multiple factor analysis and evidential reasoning [24]. Such ability for handling incomplete representation of uncertainty will be enhanced in a new process developed in this paper. In the approach reported in [24], it is assumed that factors are directly associated with the evaluations of a qualitative attribute and that each of the factors can be directly evaluated using subjective judgments with the uncertainty being only assigned to two adjacent single evaluation grades simultaneously. In [26], this approach was extended to facilitate hierarchical factor analysis with the factors being of a two-level structure and with uncertainty being assigned to any single evaluation grades.

However, even the two-level factor structure may not always be sufficient to perform preference analysis in more general decision situations as a lower-level factor may still denote too abstract a concept and it may then be evaluated through more detailed sub-factors associated with it. Similarly, upper-level factors may be further aggregated into more abstract factors which may be more suitable for evaluation. Consequently, the set of factors associated with the evaluations of an attribute may constitute a hierarchy. Factors at the top level of the hierarchy are the most abstract ones and are directly connected with the attribute. Their states are determined by more detailed factors in lower levels. Only factors at the bottom level of the hierarchy can be evaluated directly by a single or multiple experts. Based on this view of evaluations by hierarchical aggregation of information [14], [29], this paper is intended to develop a general multi-level evaluation process. In the process, a few hierarchical evaluation analysis models and evidential reasoning algorithms are explored, so that qualitative attributes in a hybrid MADM problem can be evaluated and quantified by means of hierarchical factor analysis and evidential reasoning. As a result, the hybrid MADM problem can be transformed into an ordinary MADM problem and may then be solved using a traditional MADM method.

In the development of the new process, it has been realized that uncertainty may be assigned not only to any single evaluation grades but also to their rational combinations. This new process is so elaborately developed that it can handle any such rational uncertain subjective judgments given by single or multiple experts within the evidential reasoning framework. This enhanced ability for treating incomplete uncertainty probably makes this new process significantly different from other MADM methods using other uncertainty models. In addition, a methodology for the rational transformation of the implied uncertainty contained in uncertain subjective judgments into basic probability assignments required in the D-S theory is put forward.

In section II, a hierarchical analysis for a hybrid MADM problem with uncertainty is presented. A basic evaluation analysis model and a basic factor combination algorithm are then explored. Section III concentrates on the development of the general evaluation process, in which a few hierarchical evalu-

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TABLE I
AN EXTENDED DECISION MATRIX

alternatives	quantitative attributes (y_k)				qualitative attributes (y_k)			
(a_r)	y_1	y_2	...	y_{k_1}	y_{k_1+1}	y_{k_1+2}	...	$y_{k_1+k_2}$
a_1	y_{11}	y_{12}	...	y_{1k_1}	SJ_{11}	SJ_{12}	...	SJ_{1k_2}
a_2	y_{21}	y_{22}	...	y_{2k_1}	SJ_{21}	SJ_{22}	...	SJ_{2k_2}
...	...	...	...	...	...	...	...	...
a_R	y_{R1}	y_{R2}	...	y_{Rk_1}	SJ_{R1}	SJ_{R2}	...	SJ_{Rk_2}

ation analysis models and the corresponding evidential reasoning algorithms are developed. A hierarchical evaluation analysis for a comprehensive multiple attribute motor cycle evaluation problem is then presented to illustrate the new process.

II. BASIC EVALUATION ANALYSIS MODEL AND ALGORITHM

A. Hierarchical Analysis for Hybrid MADM With Uncertainty

A hybrid MADM problem may be expressed by an extended decision matrix, as shown in Table I, where y_{rk} is a numerical value of a quantitative attribute y_k at an alternative design $a_r(r = 1,\ldots,R; k = 1,\ldots,k_1)$ and SJ_{rk} are subjective judgments for evaluation of the state of a qualitative attribute y_k at a_r $(r = 1,\ldots,R; k = k_1+1,\ldots,k_1+k_2)$. The problem is to rank these designs a_r $(r = 1,\ldots,R)$ or select the best compromise design from them with both quantitative and qualitative attributes being simultaneously satisfied to the extent possible.

To deal with such a hybrid decision making problem, a qualitative attribute y_k needs to be measured at first. The following set of evaluation grades may thus be defined for evaluation of y_k

$$H = \{H_1 \cdots H_n \cdots H_N\} \tag{1}$$

where H_n is called an evaluation grade for y_k and N is the number of the evaluation grades in H. H_n represents a grade to which the state of y_k may be evaluated. H_1 and H_N are set to be the worst and the best grades respectively, and H_{n+1} is supposed to be preferred to H_n. It should be noted that different qualitative attributes may have different sets of evaluation grades.

H_n may then be quantified using certain scale. Suppose $p(H_n)$ represents the scale of H_n. Then, if $p(H_n)$ is set to be a real number in the closed interval $[-1\ 1]$ which may be referred to as the preference degree space, the evaluation grade set is quantified by

$$p\{H\} = \left[p(H_1)\cdots p(H_n)\cdots p(H_N)\right]^T \tag{2}$$

where $p(H_n)$ $(n = 1,\ldots,N)$ satisfy the following basic conditions [24]

$$p(H_1) = -1,\ p(H_N) = 1,\ \text{and}\ p(H_{n+1}) > p(H_n) \quad n = 1,\ldots,N-1 \tag{3}$$

Furthermore $p(H_n)(n = 2,\ldots,N-1)$ should be so assigned that the additional consistency condition, defined by (8) in next subsection, can be satisfied.

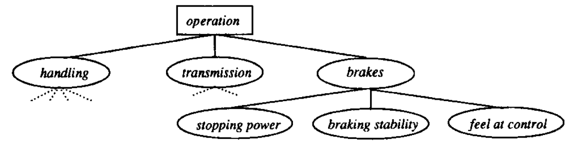

Fig. 1. Evaluation hierarchy for operation.

The state of a qualitative attribute y_k at a design a_r, denoted by $S(y_k(a_r))$, may be evaluated using the defined grades and quantified using the so-called preference degree, which is the function of $p(H_n)(n = 1,\ldots,N)$ and is denoted by $p(y_k(a_r)) \in [-1\ 1]$. If $p(y_k(a_r))$ is obtained for every qualitative attribute at each alternative design in a hybrid MADM problem, then the extended decision matrix representing the problem can be transformed into an ordinary decision matrix which may then be dealt with using some appropriate MADM method.

A simple way of obtaining $p(y_k(a_r))$ is to evaluate $S(y_k(a_r))$ to one of the evaluation grades, say H_n. Then, $p(y_k(a_r))$ is assigned to be $p(H_n)$, i.e. $p(y_k(a_r)) = p(H_n)$. This way is acceptable if the expert is able to evaluate an attribute synthetically and deterministically. Generally speaking, however, that is not the case. First of all, an attribute may represent an aggregated technical or economical concept so that it is comparable with other attributes. Such an attribute may only be evaluated through a set of detailed factors, which are associated with the evaluations of the attribute and which may constitute a hierarchical structure. In addition to this, the expert may not always be one hundred percent sure that the state of a factor at an alternative design is exactly confirmed to one of the evaluation grades. Thus, one or more single evaluation grades or even their combinations may simultaneously be confirmed with total confidence of anything up to one hundred percent.

In a problem of ranking four types of motor cycle [10], for example, both quantitative and qualitative attributes need to be taken into account and the attribute *operation* may be defined as one of the qualitative attributes. To evaluate the *operation* of a motor cycle, the following set of distinct evaluation grades is defined

$$H = \{poor(H_1)\quad indifferent(H_2)\quad average(H_3)\quad good(H_4)\quad excellent(H_5)\} \tag{4}$$

Because *operation* is an abstract technical concept and is not easy to evaluate directly, it is decomposed into three detailed concepts, *handling*, *transmission* and *brakes*, which may be referred to as factors. If a detailed concept is still too abstract to evaluate directly, it may be further decomposed into more detailed concepts. For instance, the concept of *brakes* is measured by *stopping power, braking stability,* and *feel at control,* which can probably be directly evaluated by an expert and may therefore be referred to as basic factors.

Generally speaking, a qualitative attribute may be evaluated through multiple factors which may constitute a hierarchical structure. For instance, the hierarchy for evaluation of the *operation* of a motor cycle can be built as in Fig. 1.

TABLE II
UNCERTAIN SUBJECTIVE JUDGMENTS FOR EVALUATING *BRAKES* OF "YAMAHA"

confidence degrees (β)		evaluation grades				
		poor	*indifferent*	*average*	*good*	*excellent*
factors	*stopping power*			0.3	0.6	
	braking stability				1.0	
	feel at control				0.5	0.5

If the *stopping power*, *braking stability* and *feel at control* of a motor cycle are all *good*, its *brakes* are *good*. Furthermore, its *operation* is regarded to be *good* if its *handling*, *transmission* and *brakes* are all *good*. However, the evaluations provided by an expert or multiple experts for basic factors may not always be so deterministic or consistent. To evaluate the *operation* of "Yamaha" (an alternative motor cycle), for example, an expert may only be able to state that he is

i) 30 percent sure that its *stopping power* is *average* and 60 percent sure that it is *good*,
ii) absolutely sure that its *braking stability* is *good*, and
iii) 50 percent sure that the its *feel at control* is *good* and 50 percent sure that it is *excellent*.

In the statements, the percentage values of 30, 50, 60 and 100 (absolutely sure) may be referred to as the degrees of confidence associated with the *stopping power*, *braking stability* and *feel at control* of "Yamaha" being evaluated to *average*, *good* and *excellent*. The confidence degrees represent uncertainty in the evaluations. These statements may be expressed using a table, such as Table II.

Such uncertain and diverse subjective judgments are often provided for evaluation of other basic factors. The problem is then to synthesize such judgments so as to evaluate and quantify the *operation* of "Yamaha" in a rational manner. In this way, other qualitative attributes may be evaluated and quantified as well. Based on such rational quantification of qualitative attributes, further decision analysis may then be performed so that the motor cycles can be ranked. The rest of the paper is therefore devoted to developing a general evaluation process for dealing with such a hybrid decision making problem by means of hierarchical factor analysis, evidential reasoning and alternative ranking.

B. Basic Evaluation Analysis Model

An evaluation analysis model is used to represent a framework in which multiple factor analysis and reasoning with uncertain decision knowledge can be performed for evaluation and quantification of a qualitative attribute [24], [28].

A basic evaluation analysis model may be constructed as shown in Fig. 2, in which only a single level of basic factors are involved. However, this model is a basic element in a framework for constructing more general evaluation analysis models for hierarchical factor analysis.

In Fig. 2, e_k^j denotes a basic factor such as *stopping power*, which can be directly evaluated for a given design. The set of basic factors for evaluation of y_k is defined by

$$E_k = \{e_k^1 \cdots e_k^j \cdots e_k^{L_k}\} \tag{5}$$

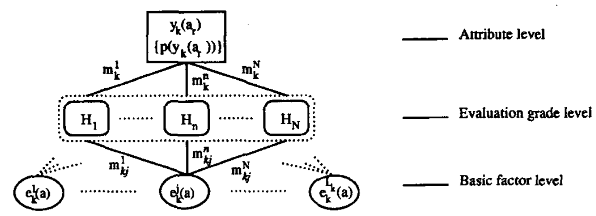

Fig. 2. A basic evaluation analysis model.

$m_{kj}^n = m(H_n/e_k^j(a))$ expresses a basic probability assignment to which a factor e_k^j supports a hypothesis that the state of an attribute y_k at a design a is confirmed to H_n. m_{kj}^n can be generated from the given confidence degree and the normalized relative weight of e_k^j. $m_k^n = m(H_n/E_k(a))$ represents an overall probability assignment to which the state of y_k at a is confirmed to H_n by the whole factor set E_k. m_k^n is obtained by combining all m_{kj}^n ($j = 1, \ldots, L_k$; $n = 1, \ldots, N$).

Suppose $\beta_{kj}^n(a_r)$ denotes a confidence degree associated with the state of a basic factor e_k^j at design a_r being evaluated to H_n. Then, an uncertain subjective judgment for evaluation of the state of $e_k^j(a_r)$, such as statements i), ii) or iii) above, may be expressed by the following expectation

$$S(e_k^j(a_r)) = \left\{(\beta_{kj}^n(a_r), H_n), n = 1, \ldots, N; \text{ and } \sum_{n=1}^N \beta_{kj}^n(a_r) \le 1\right\} \tag{6}$$

which indicates that the state of e_k^j at a design a_r is evaluated to H_n with a confidence degree of $\beta_{kj}^n(a_r)$ for $n = 1, \ldots, N$. In (6), we assume that the state of a basic factor e_k^j at a_r may be evaluated to any single evaluation grade defined in H instead of to two adjacent grades [24]. More general uncertain subjective judgments can be handled as well, as discussed in section III-C of this paper.

$S(e_k^j(a_r))$ may then be quantified using its preference degree, defined as the following expected scale [24]

$$p_{r,kj} = p(e_k^j(a_r)) = \sum_{n=1}^N \beta_{kj}^n(a_r) p(H_n) \tag{7}$$

Thus, the scales $p(H_n)$ ($n = 1, \ldots, N$) must be defined so that in addition to the basic conditions defined by (3) the following consistency condition can be satisfied as well, that is, for two designs a_r and a_h [24]

$$S(e_k^j(a_r)) \text{ is preferred to } S(e_k^j(a_h)) \text{ if and only if } p_{r,kj} > p_{h,kj} \tag{8}$$

Suppose $\lambda_k = [\lambda_k^1 \cdots \lambda_k^j \cdots \lambda_k^{L_k}]^T$ and λ_k^j represents the normalized relative weight of the factor e_k^j in evaluation of y_k where $0 \le \lambda_k^j \le 1$, as discussed in sub-section II-D. m_{kj}^n may then be calculated by

$$m_{kj}^n(a) = \lambda_k^j \beta_{kj}^n(a) \tag{9}$$

Eq. (9) means that the fact that the state of e_k^j is absolutely evaluated to an evaluation grade H_n only supports to the extent of λ_k^j the hypothesis that the state of y_k is confirmed to H_n. From (6), it is obvious that $\sum_{n=1}^N m_{kj}^n \le 1$. Suppose m_{kj}^H is the basic probability assignment to H, which is the remaining belief unassigned after commitment of belief to all $H_n (n = 1, \dots, N)$, that is, $m_{kj}^H = 1 - \sum_{n=1}^N m_{kj}^n$. A basic probability assignment matrix $M(y_k/E_k)$ for evaluation of $y_k(a)$ through $E_k(a)$ may then be formulated by

$$M(y_k/E_k) = \begin{bmatrix} m_{k1}^1 & \cdots & m_{k1}^n & \cdots & m_{k1}^N & m_{k1}^H \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ m_{kj}^1 & \cdots & m_{kj}^n & \cdots & m_{kj}^N & m_{kj}^H \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ m_{kL_k}^1 & \cdots & m_{kL_k}^n & \cdots & m_{kL_k}^N & m_{kL_k}^H \end{bmatrix} \begin{matrix} \{e_k^1(a)\} \\ \cdots \\ \{e_k^j(a)\} \\ \cdots \\ \{e_k^{L_k}(a)\} \end{matrix} \tag{10}$$

Suppose Ψ is a subset of H, that is $\Psi \subseteq H$, and m_k^Ψ is an overall probability assignment to which the state of y_k at a_r is confirmed to Ψ by the factor set E_k, or $m_k^\Psi = m(\Psi/E_k(a_r))$. If m_k^Ψ for all $\Psi \subseteq H$ are generated from $M(y_k/E_k)$, then the state of y_k at a_r may be expressed by the following expectation [24]

$$S(y_k(a_r)) = \{(m(\Psi/E_k(a_r)), \Psi), \text{ for } all\ \Psi \subseteq H\} \tag{11}$$

The preference degree of $y_k(a_r)$, i.e. $p(y_k(a_r))$, is used to quantify $S(y_k(a))$ and may thus be defined as the following expected scale

$$p_{rk} = p(y_k(a_r)) = \sum_{\Psi \subseteq H} m(\Psi/E_k(a_r)) p(\Psi) \tag{12}$$

where $p(\Psi)$ is the scale of Ψ and is defined as the average of $p(H_n)$ for all $H_n \subseteq \Psi$ [24]. A qualitative attribute quantified by (12) possesses the basic property of its marginal utilities being monotonous. In other words, for two designs, a_r and a_h, $S(y_k(a_r))$ is preferred to $S(y_k(a_h))$ if and only if $p_{rk} > p_{hk}$. Such quantification can thus form a rational basis for further decision analysis. In the next-section, a basic evidential reasoning algorithm is developed for generating m_k^Ψ from $M(y_k/E_k)$.

C. Basic Evidential Reasoning Algorithm

Suppose all the evaluation grades in H are defined as distinct grades. In other words, the absolute confirmation can only be given to one subset in H at a time and the total confidence degree of the simultaneous confirmations of Ψ for all $\Psi \subseteq H$ must be one or smaller than one. Suppose the evaluation grades in H cover all possible grades which may be used for evaluation of y_k. Then, the evidence combination rule of the Dempster-Shafer theory may be applied to combine m_{kj}^n $(n = 1, \dots, N)$; $j = 1, \dots, L_k)$.

Suppose Ψ, A and B are any subsets of H (i.e. $\Psi, A, B \subseteq H$), $\emptyset$ is an empty set, and $m(A/e_k^i)$ and $m(B/e_k^j)$ are basic probability assignments to A and B confirmed by e_k^i and e_k^j, respectively, where e_k^i and $e_k^j \in E_k$. Given $m(A/e_k^i)$ and $m(B/e_k^j)$ for all $A, B \subseteq H$, the combination rule is used to calculate the combined probability assignment $m(\Psi/(e_k^i, e_k^j))$, which is defined by [2][12]

$$m(\emptyset/(e_k^i, e_k^j)) = 0 \tag{13}$$

$$m(\Psi/(e_k^i, e_k^j)) = \sum_{A \cap B = \Psi} \frac{m(A/e_k^i) m(B/e_k^j)}{1 - K}, \quad \text{for any } \Psi \subseteq H \text{ other than } \emptyset \tag{14}$$

$$K = \sum_{A \cap B = \emptyset} m(A/e_k^i) m(B/e_k^j) \tag{15}$$

Since the direct use of the combination rule results in exponential increase in computational complexity, a new operational combination algorithm is explored to obtain $m(\Psi/E_k(a))$ from $M(y_k/E_k)$ defined by (10). This new algorithm is an extension of the two algorithms presented in [24].

The "intersection tableau" [2], [24] with values of probability assignments along the rows and columns, respectively, is adopted to develop the new algorithm for factor combination. Define a factor subset $e_{I_k(i)}(a)$ and a combined probability assignment $m_{I_k(i)}^\Psi(a)$ as follows

$$e_{I_k(i)}(a) = \{e_k^1(a) \cdots e_k^i(a)\}, 1 \le i \le L_k;$$
$$m_{I_k(i)}^\Psi(a) = m(\Psi/e_{I_k(i)}(a)) \tag{16}$$

where $m(\Psi/e_{I_k(i)}(a))$ is a combined probability assignment to Ψ confirmed by $e_{I_k(i)}(a)$.

To combine $e_{I_k(2)}(a) = \{e_k^1(a) e_k^2(a)\}$, an intersection tableau is constructed as in Table III. From the combination rule defined by (13) to (15), we have

$$\{H_n\}: \quad m_{I_k(2)}^n = K_{I_k(2)} (m_{k1}^n m_{k2}^n + m_{k1}^n m_{k2}^H + m_{k1}^H m_{k2}^n), \quad n = 1, \dots, N$$

$$\{H\}: \quad m_{I_k(2)}^H = K_{I_k(2)} m_{k1}^H m_{k2}^H$$

$$K_{I_k(2)} = \left[1 - \sum_{\tau=1}^N \sum_{\substack{j=1 \\ j \ne \tau}}^N m_{k1}^\tau m_{k2}^j\right]^{-1}$$

Obviously $m_{I_k(2)}^\Psi = 0$ for any other $\Psi \subseteq H$.

Since $m_{I_k(1)}^n = m_{k1}^n, n = 1, \dots, N$, and $m_{I_k(1)}^H = m_{k1}^H$, we can combine $e_{I_k(i+1)} = \{e_k^1 \cdots e_k^{i+1}\}$ in a similar way and obtain the following recursive algorithm

$$\{H_n\}: m_{I_k(i+1)}^n = K_{I_k(i+1)} (m_{I_k(i)}^n m_{k,i+1}^n + m_{I_k(i)}^n m_{k,i+1}^H + m_{I_k(i)}^H m_{k,i+1}^n), \quad n = 1, \dots, N \tag{17-1}$$

$$\{H\}: m_{I_k(i+1)}^H = K_{I_k(i+1)} m_{I_k(i)}^H m_{k,i+1}^H \tag{17-2}$$

$$K_{I_k(i+1)} = \left[1 - \sum_{\tau=1}^N \sum_{\substack{j=1 \\ j \ne \tau}}^N m_{I_k(i)}^\tau m_{k,i+1}^j\right]^{-1} \tag{17-3}$$
$$i = 1, \dots, L_k - 1$$

(17) is called the basic factor combination algorithm. Obviously, $m_{I_k(i+1)}^\Psi = 0$ for any $\Psi \subseteq H$ other than $\Psi = H_n (n = 1, \dots, N)$ and H.

It can be proved from the combination procedure that $m_{I_k(L_k)}^\Psi$ is the overall probability assignment to $\Psi (\subseteq H)$

TABLE III
INTERSECTION TABLEAU 1

$e_{I_k(2)}$		e_k^2					
		$\{H_1\}$ (m_{k2}^1)	...	$\{H_n\}$ (m_{k2}^n)	...	$\{H_N\}$ (m_{k2}^N)	$\{H\}$ (m_{k2}^H)
e_k^1	$\{H_1\}$ (m_{k1}^1)	$\{H_1\}$ $(m_{k1}^1 m_{k2}^1)$	...	$\{\emptyset\}$ $(m_{k1}^1 m_{k2}^n)$	...	$\{\emptyset\}$ $(m_{k1}^1 m_{k2}^N)$	$\{H_1\}$ $(m_{k1}^1 m_{k2}^H)$
	...	...	...	...	...	...	...
	$\{H_n\}$ (m_{k1}^n)	$\{\emptyset\}$ $(m_{k1}^n m_{k2}^1)$	...	$\{H_n\}$ $(m_{k1}^n m_{k2}^n)$	...	$\{\emptyset\}$ $(m_{k1}^n m_{k2}^N)$	$\{H_n\}$ $(m_{k1}^n m_{k2}^H)$
	...	...	...	...	...	...	...
	$\{H_N\}$ (m_{k1}^N)	$\{\emptyset\}$ $(m_{k1}^N m_{k2}^1)$	...	$\{\emptyset\}$ $(m_{k1}^N m_{k2}^n)$	...	$\{H_N\}$ $(m_{k1}^N m_{k2}^N)$	$\{H_N\}$ $(m_{k1}^N m_{k2}^H)$
	$\{H\}$ (m_{k1}^H)	$\{H_1\}$ $(m_{k1}^H m_{k2}^1)$	...	$\{H_n\}$ $(m_{k1}^H m_{k2}^n)$	...	$\{H_N\}$ $(m_{k1}^H m_{k2}^N)$	$\{H\}$ $(m_{k1}^H m_{k2}^H)$

confirmed by $E_k(a)$ and $m^{\Psi}_{I_k(L_k)} = 0$ for any $\Psi \subseteq H$ other than $\Psi = H_n$ $(n = 1, \dots, N)$ and H, or

$$m_k^n = m(H_n/E_k) = m^n_{I_k(L_k)}, n = 1, \dots, N, \quad \text{and } m_k^H = m(H/E_k) = m^H_{I_k(L_k)} \tag{18}$$

$$m(\Psi/E_k) = m^{\Psi}_{I_k(L_k)} = 0 \text{ for any } \Psi \subseteq H \text{ but } \Psi \neq H_n (n = 1, \dots, N) \text{ and } H \tag{19}$$

Consequently, (11) and (12) can be simplified by

$$S(y_k(a_r)) = \left\{(m^n_{I_k(L_k)}, H_n), n = 1, \dots, N; (m^H_{I_k(L_k)}, H)\right\} \tag{20}$$

$$p_{rk} = p(y_k(a_r)) = \sum_{n=1}^{N} m^n_{I_k(L_k)} p(H_n) + m^H_{I_k(L_k)} p(H) \tag{21}$$

where $p(H) = \sum_{n=1}^{N} p(H_n)/N$.

D. Assignment of Normalized Weights

The normalized weights λ_k of the factors are used to transform the given confidence degrees for evaluation of the single factors into the basic probability assignments (or supports), as shown in (9). Whether or not the transformation is rational is essential for further decision analysis. In [24], it was suggested that λ_k may be obtained from the uniform relative weights of the factors.

Suppose ζ_k^j expresses the relative weight of the factor e_k^j in evaluation of y_k, and ζ_k is defined as a uniform weight vector as follows

$$\zeta_k = [\zeta_k^1 \cdots \zeta_k^j \cdots \zeta_k^{L_k}]^T, \sum_{j=1}^{L_k} \zeta_k^j = 1, 0 \leq \zeta_k^j \leq 1 \tag{22}$$

ζ_k can be readily obtained using any well-known weight assignment method, such as the eigenvector method [14].

Let e_k^I be the most important factor in E_k, called the key factor, that is, $\zeta_k^I = \max_j\{\zeta_k^1, \dots, \zeta_k^j, \dots, \zeta_k^{L_k}\}$. Normalize ζ_k as follows

$$\bar{\zeta}_k^j = \zeta_k^j / \zeta_k^I \quad j = 1, \dots, L_k \tag{23}$$

If for the key factor the following relation is true

$$m(H_n/e_k^I) = \alpha_k \beta_{H_n}(e_k^I), 0 < \alpha_k \leq 1 \tag{24}$$

then, $\lambda_k^j (j = 1, \dots, L_k)$ may be obtained by

$$\lambda_k^j = \alpha_k \bar{\zeta}_k^j \quad j = 1, \dots, L_k \tag{25}$$

α_k may be referred to as a coefficient representing the degree of significance of the role of the most importance factor in the evaluation of the attribute y_k.

The remaining problems include addressing how and why α_k is determined. Let us take as an example the evaluation of the *brakes* of *Yamaha*, as shown in Table II. Let us assume that *stopping power* is the key factor among the three factors, *stopping power, braking stability* and *feel* at *control.* If it is evaluated that the *stopping power* of *Yamaha* is absolutely *good*, is it certain that the *brakes* of *Yamaha* are *good*?

If the answer is yes, α_k should be set to 1 as suggested by (24). This actually means that *stopping power* dominates the other two factors. In other words, the other two factors are only utilized when the support from *stopping power* is uncertain. However, this is generally not the case as the other factors would normally have some role in the evaluation of y_k, no matter what the support from the most important factor. That is, α_k should not be set to 1. Especially, if two or more factors are evaluated to be equally important and significant and if each such factor is absolutely evaluated to a different evaluation grade, then a conflict appears regarding the evaluation grade that the relevant attribute should be evaluated to. In order to resolve such a conflict, some compromise would be necessary. However, α_k=1 means that there is no room left to accommodate any compromise.

Assigning a value smaller than 1 to α_k provides opportunities for resolving the conflict. Furthermore, such an assignment also ensures that less important factors can have a bearing on evaluation of the attribute in any case. The question then naturally arises as to what values of α_k would be appropriate. The common sense answer is that the *brakes* of *Yamaha* are "certainly" *good* if all of its associated factors, *stopping power, braking stability and feel at control,* instead of just one of them, are evaluated to be absolutely *good.* This is generally the case.

Generally, if H_l is an evaluation grade to which all factors are absolutely evaluated, from (9), (23) and (25) we obtain

$$m_{kj}^n = \alpha_k \frac{\zeta_k^j}{\zeta_k^I} \text{ for } n = l; j = 1, \ldots, L_k \quad (26)$$

$$m_{kj}^n = 0 \text{ for all } n \neq l; \quad j = 1, \ldots, L_k \quad (27)$$

From the algorithm (17), we can obtain by combining these L_k factors

$$m(H_l/E_k(a)) \neq 0; m(H/E_k(a)) = \prod_{j=1}^{L_k} \left(1 - \alpha_k \frac{\zeta_k^j}{\zeta_k^I}\right) \quad (28)$$

and any other subsets in H are not confirmed at all. If the word "certainly" used in the above common sense structure means exactly one hundred percent, then $m(H/E_k(a)) = 0$ as well. From (28), this means that $\alpha_k = 1$ as $\zeta_k^j/\zeta_k^I < 1$ for all $j \neq I$, $\zeta_k^j/\zeta_k^I = 1$ for $j = I$ and L_k is a finite integer. This results in a conflict between common sense and the combination rule defined by (13) to (15). To resolve the conflict, it seems necessary to set $m(H/E_k(a)) \leq \delta$ where δ is a sufficiently small non-negative real number, or

$$\prod_{j=1}^{L_k} \left(1 - \alpha_k \frac{\zeta_k^j}{\zeta_k^I}\right) \leq \delta, \qquad \delta \geq 0 \quad (29)$$

In (29), "certainly" used in our common sense structure is modified into "almost certainly" which is explicitly defined by $(1 - \delta) \times 100$ percent. Psychologically, a smaller value of δ is always preferred to a large value. Computationally, δ may be taken so that $1.0 \times 10^{-6} \leq \delta \leq 1.0 \times 10^{-2}$ as "above 99 percent sure" may already mean "almost certain".

α_k may therefore be assigned by satisfying (29), where δ is assigned by the decision maker. It should be noted that δ must not be changed for evaluation of different attributes so that the transformation of the given confidence degrees into the basic probability assignments is consistent for all the attributes.

III. A General Evaluation Process for Hybrid MADM

A. Two-Level Evaluation Analysis Model and Algorithm

An evaluation model with two-levels of factors for evaluation of y_k is shown in Fig. 3, where $f_{1i}(i = 1, \ldots, c_1)$ are called composite factors (such as *brakes*) which are directly associated with the evaluations of the states of $y_k(a)$ and denoted by

$$F_1(a) = \{f_{11}(a) \cdots f_{1i}(a) \cdots f_{1c_1}(a)\} \quad (30)$$

where $f_{1i}(i = 1, \ldots, c_1)$ are evaluated through a set of basic factors, $E_k(a)$, defined by

$$E_k(a) = \left\{e_k^1(a) \cdots e_k^j(a) \cdots e_k^{L_k}(a)\right\} \quad (31)$$

In Fig. 3, $m_{1i,j}^n = m_{1i}(H_n/e_k^j(a))$ denotes a basic probability assignment to which $e_k^j(a)$ supports a hypothesis that the state of f_{1i} at a is confirmed to H_n, and $m_{1i}^\Psi = m_{1i}(\Psi/E_k(a))$

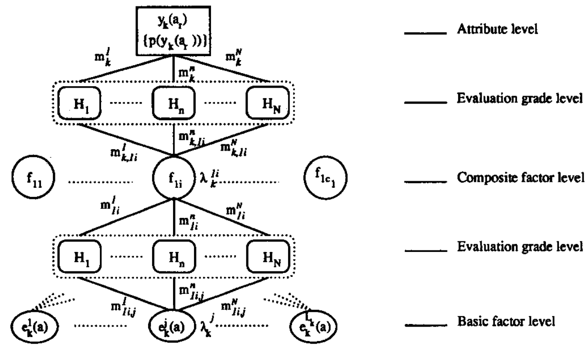

Fig. 3. A hierarchical evaluation analysis model with two levels of factors.

expresses an overall probability assignment to which the state of $f_{1i}(a)$ is confirmed to $\Psi(\subseteq H)$ by the basic factor set $E_k(a)$. If f_{1i} in Fig. 3 is treated as y_k in Fig. 2, we can obtain $m_{1i}(\Psi/E_k(a))$ from $m_{1i}(H_n/e_k^j(a))$ using the basic factor combination algorithm (17) with $m_{1i}(\Psi/E_k(a)) = 0$ for $\Psi \neq H$ and $H_n(n = 1, \ldots, N)$. Then, the state of f_{1i} can be evaluated and quantified through E_k by

$$S(f_{1i}(a)) = \{(m_{1i}(\Psi/E_k(a)), \Psi), \text{ for } \Psi = H_n(n = 1, \ldots, N) \text{ and } H\} \quad (32)$$

$$p(f_{1i}(a)) = \sum_{n=1}^{N} m_{1i}(H_n/E_k(a))p(H_n) + m_{1i}(H/E_k(a))p(H) \quad (33)$$

All the other composite factors in F_1 can be evaluated in the same way.

Suppose $m_k^\Psi = m_k(\Psi/F_1(a))$ expresses a probability assignment to which the state of an attribute y_k at a design a is confirmed to a subset Ψ ($\Psi \subseteq H$) through the composite factor set $F_1(a)$. The state of y_k can then be evaluated and quantified through F_1 by

$$S(y_k(a)) = \{(m_k(\Psi/F_1(a)), \Psi), \text{ for all } \Psi \subseteq H\} \quad (34)$$

$$p(y_k(a)) = \sum_{\Psi \subseteq H} m_k(\Psi/F_1(a))p(\Psi) \quad (35)$$

The remaining problem is how to obtain $m_k(\Psi/F_1(a))$ for all $\Psi \subseteq H$, based on m_{1i}^n ($n = 1, \ldots, N; i = 1, \ldots, c_1$). Let $m_k(H_n/f_{1i}(a))$ be an intermediate probability assignment to which a single composite factor $f_{1i}(a)$ ports a hypothesis that the state of $y_k(a)$ is confirmed to H_n, simply $m_{k,1i}^n = m_k(H_n/f_{1i}(a))$. Then, $m_k(\Psi/F_1(a))$ can be obtained by combining $m_{k,1i}^n$ ($n = 1, \ldots, N$; $i = 1, \ldots, c_1$) while $m_{k,1i}^n$ may be obtained from m_{1i}^n as follows.

Suppose the normalized relative weight of a composite factor f_{1i} in evaluation of y_k is given by $\lambda_k^{1i}, i = 1, \ldots, c_1$. Then $m_{k,1i}^n$ may be calculated in the same way as in (9)

$$m_{k,1i}^n = \lambda_k^{1i} m_{1i}^n, n = 1, \ldots, N; \text{ and } m_{k,1i}^H = 1 - \sum_{n=1}^{N} m_{k,1i}^n \quad (36)$$

An intermediate probability assignment matrix for evaluation of $y_k(a)$ through $F_1(a)$ can then be formulated as follows.

$$M(y_k/F_1) = \begin{bmatrix} m^1_{k,11} & \cdots & m^n_{k,11} & \cdots & m^N_{k,11} & m^H_{k,11} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ m^1_{k,1i} & \cdots & m^n_{k,1i} & \cdots & m^N_{k,1i} & m^H_{k,1i} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ m^1_{k,1c_1} & \cdots & m^n_{k,1c_1} & \cdots & m^N_{k,1c_1} & m^H_{k,1c_1} \end{bmatrix} \begin{matrix} \{f_{11}(a)\} \\ \cdots \\ \{f_{1i}(a)\} \\ \cdots \\ \{f_{1c_1}(a)\} \end{matrix} \tag{37}$$

Let $m^n_{I_k(i)} = m_k(H_n/(f_{11}\dots f_{1i})), n = 1,\dots,N$, and $m^H_{I_k(i)} = m_k(H/(f_{11}\cdots f_{1i}))$. Following the same procedure as in the case of deducing the basic factor combination algorithm, we can obtain the following composite factor combination algorithm

$$\{H_n\}: m^n_{I_k(i+1)} = K_{I_k(i+1)}\big(m^n_{I_k(i)}m^n_{k,1(i+1)} + m^n_{I_k(i)}m^H_{k,1(i+1)} + m^H_{I_k(i)}m^n_{k,1(i+1)}\big), \quad n = 1,\dots,N \tag{38-1}$$

$$\{H\}: m^H_{I_k(i+1)} = K_{I_k(i+1)}m^H_{I_k(i)}m^H_{k,1(i+1)} \tag{38-2}$$

$$K_{I_k(i+1)} = \left[1 - \sum_{\tau=1}^{N}\sum_{\substack{j=1\\ j\neq\tau}}^{N} m^\tau_{I_k(i)}m^j_{k,1(i+1)}\right]^{-1} \tag{38-3}$$

$$i = 1,\dots,c_1 - 1$$

From (38) it is obvious that $m^\Psi_{I_k(i+1)} = 0$ for any $\Psi \subseteq H$ other than $\Psi = H_n (n = 1,\dots,N)$ and H. So

$$m^n_k = m_k(H_n/F_1) = m^n_{I_k(c_1)}, n = 1,\dots,N; \text{ and } m^H_k = m_k(H/F_1) = m^H_{I_k(c_1)} \tag{39}$$

$$m_k(\Psi/F_1) = m^\Psi_{I_k(c_1)} = 0 \text{ for any } \Psi \subseteq H \text{ but } \Psi \neq H_n (n = 1,\dots,N) \text{ and } H \tag{40}$$

Consequently, (34) and (35) can be simplified by

$$S(y_k(a_r)) = \{(m^n_{I_k(c_1)}, H_n), n = 1,\dots,N; (m^H_{I_k(c_1)}, H)\} \tag{41}$$

$$p_{rk} = p(y_k(a_r)) = \sum_{n=1}^{N} m^n_{I_k(c_1)}p(H_n) + m^H_{I_k(c_1)}p(H) \tag{42}$$

B. Multi-Level Multi-Person Evaluation Analysis Models and Algorithms

In last-section, only a single level of composite factors is considered. Fig. 4 shows a more general hierarchical evaluation analysis model with L levels of composite factors.

In Fig. 4, a composite factor at a single level (such as f_{lg} at level l) is associated with factors at a level immediately below (such as $f_{l+1,h}, h = 1,\dots,c_{l+1}$, at level $(l+1)$). Let us assume that the relative importance of the factors at the lower level can be compared for evaluation of the composite factor. The problem is then how to evaluate and quantify the state of the composite factor through the factors at the lower level. A set of composite factors at level l is defined by

$$F_l(a) = \{f_{l1}(a)\cdots f_{lg}(a)\cdots f_{lc_l}(a)\}, l = 1,\cdots,L \tag{43}$$

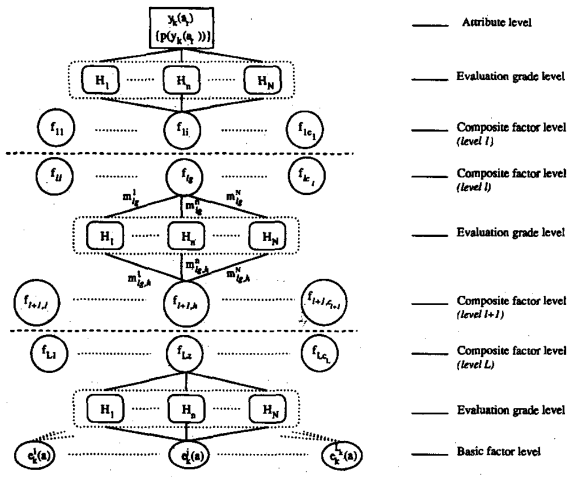

Fig. 4. A hierarchical evaluation analysis model with multiple levels of factors.

Let $m^\Psi_{lg} = m_{lg}(\Psi/F_{l+1}(a))$ be an overall probability assignment to which the state of the gth composite factor f_{lg} at level l at a design a is confirmed to Ψ by the set of factors $F_{l+1}(a)$ at level $(l+1)$. Then, the state of f_{lg} at a is evaluated and quantified by

$$S(f_{lg}(a)) = \{(m_{lg}(\Psi/F_{l+1}(a)), \Psi), \text{for all } \Psi \subseteq H\} \tag{44}$$

$$p(f_{lg}(a)) = \sum_{\Psi\subseteq H} m_{lg}(\Psi/F_{l+1}(a))p(\Psi) \tag{45}$$

Suppose $m^n_{lg,h} = m_{lg}(H_n/f_{l+1,h}(a))$ is an intermediate probability assignment to which a single factor $f_{l+1,h}(a)$ at level $(l+1)$ supports a hypothesis that the state of $f_{lg}(a)$ is evaluated to H_n. As $m^n_{l+1,h}$ is an overall probability assignment to which the state of the hth factor $f_{l+1,h}(a)$ is evaluated to H_n, we may then calculate $m^n_{lg,h}$ as follows

$$m^n_{lg,h} = \lambda^{l+1,h}_{lg}m^n_{l+1,h}, h = 1,\dots,c_{l+1} \tag{46}$$

where $\lambda^{l+1,h}_{lg}$ is a normalized weight representing the relative importance of the role the factor $f_{l+1,h}$ at level $(l+1)$ plays for evaluation of f_{lg} at level l.

Then, the following intermediate probability assignment matrix $M(f_{lg}/F_{l+1})$ for evaluation of $f_{lg}(a)$ at level l through the set of factors $F_{l+1}(a)$ at level $(l+1)$ can be constructed as (see (47) at top of next page) where $m^H_{lg,h} = 1 - \sum_{h=1}^{N} m^n_{lg,h}, h = 1,\dots,c_{l+1}$.

Let $m^n_{I_{lg}(h)} = m_{lg}(H_n/(f_{l+1,1}\cdots f_{l+1,h})), n = 1,\dots,N$ and $m^H_{I_{lg}(h)} = m_{lg}(H/(f_{l+1,1}\cdots f_{l+1,h}))$. Then the hierarchical factor propagation algorithm can be obtained as

$$\{H_n\}: m^n_{I_{lg}(h+1)} = K_{I_{lg}(h+1)}\big(m^n_{I_{lg}(h)}m^n_{lg,h+1} + m^n_{I_{lg}(h)}m^H_{lg,h+1} + m^H_{I_{lg},(h)}m^n_{lg,h+1}\big), \quad n = 1,\cdots,N \tag{48-1}$$

$$\{H\}: m^H_{I_{lg}(h+1)} = K_{I_{lg}(h+1)}m^H_{I_{lg}(h)}m^H_{lg,h+1} \tag{48-2}$$

$$K_{I_{lg}(h+1)} = \left[1 - \sum_{\tau=1}^{N}\sum_{\substack{j=1\\ j\neq\tau}}^{N} m^\tau_{I_{lg}(h)}m^j_{lg,h+1}\right]^{-1}, \quad h = 1,\dots,c_{l+1} - 1 \tag{48-3}$$

$$M(f_{lg}/F_{l+1}) = \begin{bmatrix} m_{lg,1}^1 & \cdots & m_{lg,1}^n & \cdots & m_{lg,1}^N & m_{lg,1}^H \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ m_{lg,h}^1 & \cdots & m_{lg,h}^n & \cdots & m_{lg,h}^N & m_{lg,h}^H \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ m_{lg,c_{l+1}}^1 & \cdots & m_{lg,c_{l+1}}^n & \cdots & m_{lg,c_{l+1}}^N & m_{lg,c_{l+1}}^H \end{bmatrix} \begin{matrix} \{f_{l+1,1}(a)\} \\ \cdots \\ \{f_{l+1,h}(a)\} \\ \cdots \\ \{f_{l+1,c_{l+1}}(a)\} \end{matrix} \tag{47}$$

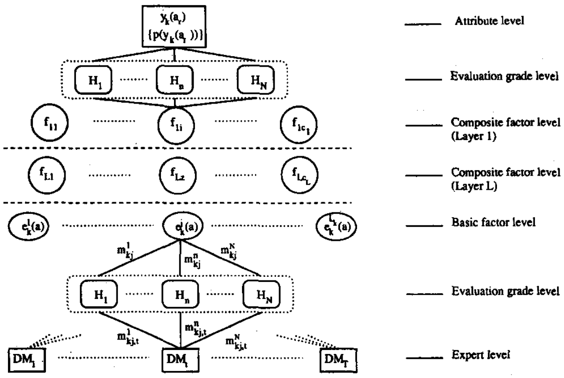

Fig. 5. A hierarchical multi-person evaluation analysis model with multiple levels of factors.

Thus, $m^n_{I_{lg}(c_{l+1})}(n = 1, \ldots, N)$ and $m^H_{I_{lg}(c_{l+1})}$ are non-zero overall probability assignments to which the state of f_{lg} at a is confirmed to $H_n(n = 1, \ldots, N)$ and H by the set of the factors $F_{l+1}(a)$, that is

$$m^n_{lg} = m_{lg}(H_n/F_{l+1}(a)) = m^n_{I_{lg}(c_{l+1})}, \quad n = 1, \ldots, N;$$

$$m^H_{lg} = m_{lg}(H/F_{l+1}(a)) = m^H_{I_{lg}(c_{l+1})} \tag{49}$$

$$m_{lg}(\Psi/F_{l+1}(a)) = 0 \text{ for any } \Psi \subseteq H \text{ but } \Psi \neq H_n n = 1, \ldots, N \text{ and } H \tag{50}$$

So, (44) and (45) can be simplified by

$$S(f_{lg}(a)) = \{(m^n_{I_{lg}(c_{l+1})}, H_n), n = 1, \ldots, N; (m^H_{I_{lg}(c_{l+1})}, H)\} \tag{51}$$

$$p(f_{lg}(a)) = \sum_{n=1}^{N} m^n_{I_{lg}(c_{l+1})} p(H_n) + m^H_{I_{lg}(c_{l+1})} p(H) \tag{52}$$

In the same way, the states of the other factors at level l at a can be evaluated and quantified through the set of factors at the level immediately below, i.e. $F_{l+1}(a)$. Eventually, the state of y_k at a can be evaluated and quantified through the set of factors at level 1, i.e. $F_1(a)$.

So far, we assume that the states of basic factors are evaluated by a single expert. It is possible that multiple experts may be involved for the evaluations, who may have different views about the evaluations. Fig. 5 shows a multi-level & multi-person evaluation analysis model.

In Fig. 5, T experts are involved in the direct evaluation of the basic factors for $y_k(a)$ where the same factor hierarchy is adopted by all experts. It should be noted that if a decision maker wishes to ignore factors at a level, he can simply assign weights of zero to these factors.

The model described in Fig. 5 is different from that described in Fig. 4 only in that an expert level is added in Fig. 5. If in Fig. 5 the expert level is treated as a basic factor level and the basic factor level as a composite factor level, then the different evaluations of a basic factor given by multiple experts can be combined using the basic factor combination algorithm. In Fig. 5, $m^n_{kj,t}$ represents the probability assignment to which the tth expert's evaluation supports a hypothesis that the state of a basic factor e^j_k at a design a is confirmed to H_n, and m^n_{kj} denotes an overall probability assignment (or a confidence degree) to which the state of e^j_k is evaluated to H_n by the T experts while m^n_{kj} can be obtained by combining $m^n_{kj,t}$ $(t = 1, \ldots, T;\ n = 1, \ldots, N)$ using the algorithm (17).

Finally, it should be noted that given the uncertain subjective judgments for evaluation of the basic factors it is possible that different decision makers may obtain different preference degrees of an attribute y_k at an alternative as they may provide different normalized weights λ_k. However, this concerns group decision making rather than the multi-person evaluation analysis discussed above. Although the latter may be regarded as part of the former in some decision situations, the latter doesn't necessarily mean the former. For instance, different scenarios for the evaluation of basic factors may be given by multiple experts. Based on these scenarios, an individual decision maker may then make decision analysis using his judgments over the relative importance of these scenarios as well as the factors.

C. General Evidential Reasoning Algorithms

The evidential reasoning algorithms presented in previous sub-sections are developed based on the assumption that the states of the basic factors are only evaluated to single evaluation grades, as suggested by (6). This assumption can be satisfied in most cases. In some evaluation processes, however, especially when multiple experts are involved, it is quite possible that the states of some basic factors occasionally need to be evaluated to the combinations of the single evaluation grades, such as $\{H_1H_2\}$ and $\{H_3H_4H_5\}$. Fortunately, the combination rule of the D-S theory possesses the ability to handle such uncertainty even though any combination of the single grades may be confirmed. In fact, it is this very ability that makes the D-S theory different from other tools for handling uncertainty.

A general evidential reasoning algorithm is to be developed in this sub-section though it looks more complicated than those presented by (17), (38) and (48). In the development of

$$M(y_k/E_k) = \begin{bmatrix} m_{kl}^1 & \cdots & m_{kl}^N & m_{kl}^{1,2} & \cdots & m_{kl}^{s,t} & \cdots & m_{kl}^{1,N-1} & m_{kl}^{2,N} & m_{kl}^H \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ m_{kj}^1 & \cdots & m_{kj}^N & m_{kj}^{1,2} & \cdots & m_{kj}^{s,t} & \cdots & m_{kj}^{1,N-1} & m_{kj}^{2,N} & m_{kj}^H \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ m_{kL_k}^1 & \cdots & m_{kL_k}^N & m_{kL_k}^{1,2} & \cdots & m_{kL_k}^{s,t} & \cdots & m_{kL_k}^{1,N-1} & m_{kL_k}^{2,N} & m_{kL_k}^H \end{bmatrix} \begin{matrix} \{e_k^1(a)\} \\ \cdots \\ \{e_k^j(a)\} \\ \cdots \\ \{e_k^{L_k}(a)\} \end{matrix} \tag{55}$$

this general algorithm, it has been assumed that only rational combinations (or subsets of H) may be confirmed. A rational subset of H is defined by

$$H_{s,t} = \{H_s \quad H_{s+1} \cdots H_{t-1} \quad H_t\}, \quad 1 \le s \le t \le N \tag{53}$$

In other words, a rational subset is composed of mutually adjacent single evaluation grades. Thus, the sample space H and any single grade defined in H are rational subsets. Altogether, there are $N(N+1)/2$ rational subsets in H.

In the definition given by (4), for example, {*poor, indifferent, average* } and {*good, excellent*} are two rational subsets while {*poor, indifferent, good* } is not regarded as a rational subset in this paper. Although the combination rule of the D-S theory is capable of handling uncertainty assigned to any subsets, only uncertainty assigned to the rational subsets is taken into account in the following algorithm.

Suppose $\beta_{kj}^{s,t}(a_r)$ denotes the confidence degree to which the state of a basic factor e_k^j at a design a_r is evaluated to $H_{s,t}$. Similar to (6), an uncertain subjective judgment for evaluation of the state of $e_k^j(a_r)$ may more generally be expressed as

$$S(e_k^j(a_r)) = \left\{(\beta_{kj}^{s,t}(a_r), H_{s,t}), \text{ for } all\ H_{s,t} \subseteq H\right\} \tag{54}$$

Using (9), we can generate the basic probability assignment matrix as follows, where m_{kj}^H is the remaining probability assignment after commitment of belief to all other rational subsets (see (55) above)

Note that $m_{kj}^{1,N} = m_{kj}^H$ and the intersection of two rational subsets is still a rational subset. Similar to (16), a combined probability assignment $m_{I_k(\gamma)}^{s,t}$ may be defined by

$$m_{I_k(\gamma)}^{s,t} = m(H_{s,t}/e_{I_k(\gamma)}(a)) \tag{56}$$

where $e_{I_k(\gamma)}(a)$ is defined by (16). Following a procedure similar to that for obtaining (17), we can obtain the following general recursive factor combination algorithm where $m_{I_k(\gamma)}^n$ and $m_{k,\gamma+1}^n$ are denoted by $m_{I_k(\gamma)}^{n,n}$ and $m_{k,\gamma+1}^{n,n}$, respectively, for $n = 1, \ldots, N$

$$\begin{aligned} \{H_{s,t}\} : m_{I_k(\gamma+1)}^{s,t} &= K_{I_k(\gamma+1)} \Bigg(m_{I_k(\gamma)}^{s,t} m_{k,\gamma+1}^{s,t} \\ &+ \sum_{j=1}^{s-1} \sum_{i=t}^{N} (m_{I_k(\gamma)}^{s,t} m_{k,\gamma+1}^{j,i} + m_{I_k(\gamma)}^{j,i} m_{k,\gamma+1}^{s,t}) \\ &+ \sum_{i=t+1}^{N} (m_{I_k(\gamma)}^{s,t} m_{k,\gamma+1}^{s,i} + m_{I_k(\gamma)}^{s,i} m_{k,\gamma+1}^{s,t}) \\ &+ \sum_{j=1}^{s-1} \sum_{i=t+1}^{N} (m_{I_k(\gamma)}^{j,t} m_{k,\gamma+1}^{s,i} + m_{I_k(\gamma)}^{s,i} m_{k,\gamma+1}^{j,t}) \Bigg) \\ & 1 \le s \le t \le N \end{aligned} \tag{57-1}$$

$$K_{I_k(\gamma+1)} = \left[1 - \sum_{i=1}^{N} \sum_{j=i}^{N} \sum_{h=j+1}^{N} \sum_{l=h}^{N} (m_{I_k(\gamma)}^{i,j} m_{k,\gamma+1}^{h,l} + m_{I_k(\gamma)}^{h,l} m_{k,\gamma+1}^{i,j})\right]^{-1}$$
$$\gamma = 1, \cdots, L_k - 1 \tag{57-2}$$

with $m_{I_k(\gamma+1)}^{\Psi} = 0$ for any $\Psi \subseteq H$ but $\Psi \ne H_{s,t}$ $(1 \le s \le t \le N)$. It may be noted that (17) is only a special case of (57) when $m_{kj}^{s,t} = 0$ for all $1 \le s < t \le N$ and $s > 1$ if $t = N$.

Noting that (38) and (48) are similar to (17), we can also generate the general recursive composite factor combination algorithm for (38) and the general hierarchical factor propagation algorithm for (48), which have the same structure as (57).

As $m_{I_k(L_k)}^{s,t}$, obtained by (57), is the overall probability assignment to $H_{s,t}$ confirmed by $E_k(a)$ and $m_{I_k(L_k)}^{\Psi} = 0$ for any $\Psi \subseteq H$ other than $\Psi = H_{s,t}$ $1 \le s \le t \le N$, then (11) and (12) can be expressed by

$$S(y_k(a_r)) = \left\{(m_{I_k(L_k)}^{s,t}, H_{s,t}), 1 \le s \le t \le N\right\} \tag{58}$$

$$p_{rk} = \sum_{s=1}^{N} \sum_{t=s}^{N} m_{I_k(L_k)}^{s,t} p(H_{s,t}) \tag{59}$$

where $p(H_{s,t}) = \sum_{i=s}^{t} p(H_i)/(t-s+1)$. Eqs. (34) and (35) as well as (44) and (45) can be expressed in the same way as (58) and (59).

D. A General Evaluation Process for Hybrid MADM with Uncertainty

Based upon the evaluation analysis models and the factor combination and propagation algorithms developed in the previous subsections, the state of a qualitative attribute at each alternative in Table I can be transformed into the preference degree space. The numerical values of a quantitative attribute at each alternative in Table I may also be transformed into the preference degree space as follows [24]

$$p_{rk} = p(y_{rk}) = \frac{2(y_{rk} - y_k^{\min})}{y_k^{\max} - y_k^{\min}} - 1,$$
$$k = 1, \cdots, k_1; r = 1, \ldots, R, \text{ for } all\ benefit\ attributes \tag{60}$$

$$p_{rk} = p(y_{rk}) = \frac{2(y_k^{\max} - y_{rk})}{y_k^{\max} - y_k^{\min}} - 1,$$
$$k = 1, \cdots, k_1; r = 1, \cdots, R, \text{ for } all\ cost\ attributes \tag{61}$$

TABLE IV
THE EVALUATION MATRIX

preference degrees	$p(y_1)$	...	$p(y_{k_1})$	$p(y_{k_1+1})$	...	$p(y_{k_1+k_2})$
a_1	p_{11}	...	p_{1k_1}	p_{1k_1+1}	...	$p_{1k_1+k_2}$
a_2	p_{21}	...	p_{2k_1}	p_{2k_1+1}	...	$p_{2k_1+k_2}$
...	...	...	...	...	...	...
a_R	p_{R1}	...	p_{Rk_1}	p_{Rk_1+1}	...	$p_{Rk_1+k_2}$

$$y_k^{\max} = \max\{y_{1k} \cdots y_{Rk}\}; \quad y_k^{\min} = \min\{y_{1k} \cdots y_{Rk}\} \quad (62)$$

The transformed form of the attribute y_k is denoted by $p(y_k)$, which may be called a marginal preference function of y_k and is a monotonously nondecreasing function. In other words, a_i is preferred to a_j with respect to y_k if and only if $p_{ik} > p_{jk}$. The original extended decision matrix defined by Table I is thus transformed into the following evaluation matrix (Table IV), which is an ordinary decision matrix and in which the states of all attributes, either quantitative or qualitative, are represented in the preference degree space.

Based on Table IV, the alternative designs can then be ranked using an appropriate MADM method. One of the simplest methods, for example, may be the linear additive utility function method (or the simple weighting method) [4], [6], [9], [11]. However, this method assumes linearity of marginal utilities, independence of preferences and direct linear compensation among attributes. These three assumptions may not always be acceptable to the decision maker. The TOPSIS method [4], [9] overcomes some of these demerits although it still requires the direct compensation among attributes. The CODASID method [24], [25] has been developed by integrating the favorable features of the TOPSIS method and the ELECTRE method [9]. CODASID only assumes indirect and limited compensation which does not take place until each alternative design has been compared with the others with respect to every single attribute. Each of these methods can produce a utility value for every alternative design. The designs are then ranked based on the magnitude of the utility values.

As a result of the above discussion, we are now in a position to formulate a general evaluation process for hybrid multiple attribute decision making with uncertainty. Although the following process is based on the assumption that all basic factors are evaluated to single evaluation grades, more general uncertainty can also be handled by replacing the listed algorithms with the general algorithms developed in last subsection. The process may be summarized as the following steps.

Step 1: Define a hybrid MADM problem using an extended decision matrix as in Table I, where a qualitative attribute may be evaluated using uncertain subjective judgments through multiple factors which may constitute a hierarchical structure.

Step 2: Transform the numerical values of a quantitative attribute at each alternative design into the preference degree space using (60) or (61).

Step 3: Evaluate and quantify the state of a qualitative attribute y_k at each alternative design a_r. Let $k = k_1 + 1$ and $r = 1$.

Step 4: Construct a hierarchical evaluation analysis model for evaluation of y_k, where uncertain subjective judgments for evaluation of basic factors are given by single or multiple experts and the number of the levels of composite factors is also determined. Let $l = L$.

Step 5: If $L > 0$, let $g = 1$ and then go to step 7. If $L = 0$, there is no composite factors. Calculate the basic probability assignments for evaluation of $y_k(a_r)$ through basic factors $E_k(a_r)$ from the given confidence degrees by using the formula (9), resulting in the basic probability assignment matrix $M(y_k(a_r)/E_k(a_r))$ defined by (10).

Step 6: Combine the basic probability assignments contained in $M(y_k(a_r)/E_k(a_r))$ using the basic factor combination algorithm described by (17). $p_{rk} = p(y_k(a_r))$ is then calculated using the formula (21). Go to step 13.

Step 7: Calculate the basic probability assignments for evaluation of $f_{lg}(a_r)$ through basic factors $E_k(a_r)$ from the given confidence degrees by using the formula (9), resulting in the basic probability assignment matrix $M(f_{lg}(a_r)/E_k(a_r))$ similar to (10).

Step 8: Combine the basic probability assignments contained in $M(f_{lg}(a_r)/E_k(a_r))$ using the basic factor combination algorithm described by (17), generating the overall probability assignments for evaluation of $f_{lg}(a_r)$. Let $g = g + 1$. If $g \leq c_l$, go to step 7. If $g > c_l$, let $l = l - 1$ and $g = 1$ and go to step 9.

Step 9: If $l = 0$, go to step 11. Otherwise, calculate the intermediate probability assignments for evaluation of $f_{lg}(a_r)$ at level l through factors $F_{l+1}(a_r)$ at level $(l + 1)$ by using the formula (46), resulting in the intermediate probability assignment matrix $M(f_{lg}(a_r)/F_{l+1}(a_r))$ defined by (47).

Step 10: Combine the intermediate probability assignments contained in $M(f_{lg}(a_r)/_{l+1}(a_r))$ using the hierarchical factor propagation algorithm described by (48), generating the overall probability assignments for evaluation of $f_{lg}(a_r)$. Let $g = g + 1$. If $g \leq c_l$, go to step 9. If $g > c_l$, let $l = l - 1$ and $g = 1$ and then go to step 9.

Step 11: Calculate the intermediate probability assignments for evaluation of $y_k(a_r)$ through factors $F_{l+1}(a_r)$ at level $(l + 1)$ by using the formula (36), resulting in the intermediate probability assignment matrix $M(y_k(a_r)/F_{l+1}(a_r))$ defined by (37).

Step 12: Combine the intermediate probability assignments contained in $M(y_k(a_r)/F_{l+1}(a_r))$ using the composite factor combination algorithm described by (38), generating the overall probability assignments for evaluation of $y_k(a_r)$. $p_{rk} = p(y_k(a_r))$ is then calculated using the formula (42).

Step 13: Let $r = r + 1$. If $r \leq R$, let $l = L$ and then go to step 5. If $r > R$ and $k < k_1 + k_2$, let $k = k + 1$ and $r = 1$ and then go to step 4. If $r > R$ and $k \geq k_1 + k_2$, go to step 14.

Step 14: Construct the evaluation matrix as shown in Table IV.

Step 15: Based on Table IV, rank the alternative designs using an appropriate traditional MADM approach such as the CODASID method, the TOPSIS method, or perhaps the simple weighting method.

IV. Hierarchical Analysis for a Hybrid MADM Problem

A. Problem Description

A customer intends to buy a motor cycle. There are four types of motor cycle available for selection, and these are "Kawasaki," "Yamaha," "Honda" and "BMW". The technical and economical performance attributes of the four types of motor cycle are also available [10]. These are represented by either numerical values with appropriate units or subjective judgments with uncertainty.

The customer takes into account seven of the performance attributes, including both qualitative and quantitative ones. These seven attributes are described in Table V. The numerical values of the quantitative attributes and the uncertain subjective judgments for evaluation of the qualitative attributes are discussed in [10].

The uncertain subjective judgments listed in Table V are represented in a compact form. They can also be expressed using tables such as Table II or statements such as statements i>, ii> and iii> listed in subsection II.A. In [24], a simplified version of a similar problem is discussed without considering a hierarchical structure.

To quantify the qualitative attributes, a possible approach could be based on the simple weighting technique, which has often been used by practitioners due to its simplicity. In such a method, there might be two procedures to deal with the hierarchical subjective evaluations. Firstly, basic factors such as *responsiveness, fuel economy, quietness, vibration and starting* could be used as measures to replace a qualitative attribute such as *engine*. The overall weight of a basic factor could be obtained from top to bottom. For instance, suppose the weight of *engine* is ω_5 and the relative weight of *responsiveness* among the five basic factors associated with *engine* is ζ_5^1. Then, the overall weight of *responsiveness* could be calculated by $\omega_5 \times \zeta_5^1$. The subjective judgments about the state of each alternative on every basic factor can be quantified using (7). Thus, the extended decision matrix could be transformed into a traditional decision matrix where the three qualitative attributes are replaced by the nineteen basic factors.

Secondly, each subjective judgment for the evaluation of an alternative on a basic factor can be quantified using (7). Then, the obtained numerical values for the basic factors associated with an upper level factor (or an attribute) could be weighted and summed up to generate a numerical value for evaluation of the upper level factor. In this way, each qualitative attribute could be measured by a numerical value obtained from bottom upwards. The extended decision matrix is thus transformed into a traditional decision matrix where the three qualitative attributes become quantitative ones.

As the two procedures discussed above are based upon the simple weighting technique, however, they inherently suffer from the same disadvantages as mentioned in subsection III.D. It is therefore advisable to be cautious in adopting such procedures for the quantification. This section is intended to illustrate how to use the new general evaluation process to deal with this hybrid decision making problem with uncertainty by means of hierarchical factor analysis, evidential reasoning and alternative ranking.

B. Preference Weight Assignment

As Table V shows that no single motor cycle type dominates or is dominated by the other types, the customer needs to provide his preference information about the relative importance of the seven attributes. He uses a ten-point scale to estimate the relative importance. The weights of the seven attributes are estimated to be as follows

$$\hat{W} = [\hat{\omega}_1\ \hat{\omega}_2\ \hat{\omega}_3\ \hat{\omega}_4\ \hat{\omega}_5\ \hat{\omega}_6\ \hat{\omega}_7]^T = [9\ 5\ 7\ 7\ 7\ 7\ 4]^T$$

$\hat{W}$ is then normalized by

$$W = \hat{W} / \sum_{n=1}^{7} \hat{\omega}_n = [0.1957\ 0.1087\ 0.1522\ 0.1522\ 0.1522\ 0.1522\ 0.087]^T$$

To implement the hierarchical evaluation process, the normalized relative weights of factors at a single level for evaluation of an upper level factor or attribute are also required. The eigenvector method [9], [14] is used to generate the relative weights. In this example, δ is chosen to be 0.03 as "over 97 percent sure" is regarded to be equivalent to "almost certain". All priority coefficients can thus be set to be 0.9, that is, $\alpha_5 = \alpha_6 = \alpha_{61} = \alpha_{62} = \alpha_{63} = \alpha_7 = 0.9$. The relative weights of the factors are thus given by

$$\zeta_5 = [0.222\ 0.333\ 0.111\ 0.111\ 0.222]^T,$$
$$\lambda_5 = \alpha_5 \zeta_5 / \zeta_5^2 = [0.6\ 0.9\ 0.3\ 0.3\ 0.6]^T$$
$$\zeta_6 = [0.5\ 0.167\ 0.333]^T,$$
$$\lambda_6 = \alpha_6 \zeta_6 / \zeta_6^1 = [0.9\ 0.301\ 0.599]^T$$
$$\zeta_{61} = [0.333\ 0.111\ 0.333\ 0.222]^T,$$
$$\lambda_{61} = \alpha_{61} \zeta_{61} / \zeta_{61}^1 = [0.9\ 0.3\ 0.9\ 0.6]^T$$
$$\zeta_{62} = [0.5\ 0.5]^T,$$
$$\lambda_{62} = \alpha_{62} \zeta_{62} / \zeta_{62}^1 [0.9\ 0.9]^T$$
$$\zeta_{63} = [0.5\ 0.25\ 0.25]^T,$$
$$\lambda_{63} = \alpha_{63} \zeta_{63} / \zeta_{63}^1 = [0.9\ 0.45\ 0.45]^T$$
$$\zeta_7 = [0.375\ 0.25\ 0.125\ 0.125\ 0.125]^T,$$
$$\lambda_7 = \alpha_7 \zeta_7 / \zeta_7^1 = [0.9\ 0.6\ 0.3\ 0.3\ 0.3]^T$$

C. Hierarchical Evaluation Analysis

In Table V. Three basic factor sets E_5, E_6 and E_7 for evaluation of y_5, y_6 and y_7 are defined by

$$E_5 = \{e_5^1 e_5^2 e_5^3 e_5^4 e_5^5\} = \{\textit{responsiveness, fuel economy quietness, vibration, starting}\}$$

$$E_6 = \{E_{11} E_{12} E_{13}\} = \{e_6^1 e_6^2 e_6^3 e_6^4 e_6^5 e_6^6 e_6^7 e_6^8 e_6^9\} = \{\textit{steering, bumpy bends, maneuverability, top speed stability, clutch operation, gearbox operation, stopping power braking, stability, feel at control}\}$$

$$E_7 = \{e_7^1 e_7^2 e_7^3 e_7^4 e_7^5\} = \{\textit{quality of finish, seat comfort, headlight, mirrors, horn}\}$$

TABLE V
AN EXTENTED DECISION MATRIX OF FOUR TYPES OF MOTOR CYCLE

types of attributes	definition of attributes	units or factors: composite factors	units or factors: basic factors	types of motor cycle (alternatives): Kawasaki (a_1)	Yamaha (a_2)	Honda (a_3)	BMW (a_4)
quantitative	price (y_1)	pounds		6499	5199	6199	8220
	displacement (y_2)	cc		1052	1188	998	987
	range (y_3)	miles		175	160	170	200
	top speed (y_4)	mph		160	155	160	145
qualitative	engine (y_5)	responsiveness (e_5^1)		E(0.8)	G(0.3) E(0.6)	G(1.0)	I(1.0)
		fuel economy (e_5^2)		A(1.0)	I(1.0)	I(0.5) A(0.5)	E(1.0)
		quietness (e_5^3)		I(0.5) A(0.5)	A(1.0)	G(0.5) E(0.3)	E(1.0)
		vibration (e_5^4)		G(1.0)	I(1.0)	G(0.5) E(0.5)	P(1.0)
		starting (e_5^5)		G(1.0)	A(0.6) G(0.3)	G(1.0)	A(1.0)
	operation (y_6)	handling (f_{11})	steering (e_6^1)	E(0.9)	G(1.0)	A(1.0)	A(0.6)
			bumpy bends (e_6^2)	A(0.5) G(0.5)	G(1.0)	G(0.8) E(0.1)	P(0.5) I(0.5)
			maneuverability (e_6^3)	A(1.0)	E(0.9)	I(1.0)	P(1.0)
			top speed stability (e_6^4)	E(1.0)	G(1.0)	G(1.0)	G(0.6) E(0.4)
		transmission (f_{12})	clutch operation (e_6^5)	A(0.8)	G(1.0)	E(0.85)	I(0.2) A(0.8)
			gearbox operation (e_6^6)	A(0.5) G(0.5)	I(0.5) A(0.5)	E(1.0)	P(1.0)
		brakes (f_{13})	stopping power (e_6^7)	G(1.0)	A(0.3) G(0.6)	G(0.6)	E(1.0)
			braking stability (e_6^8)	G(0.5) E(0.5)	G(1.0)	A(0.5) G(0.5)	E(1.0)
			feel at control (e_6^9)	P(1.0)	G(0.5) E(0.5)	G(1.0)	G(0.5) E(0.5)
	general (y_7)	quality of finish (e_7^1)		P(0.5) I(0.5)	G(1.0)	E(1.0)	G(0.5) E(0.5)
		seat comfort (e_7^2)		G(1.0)	G(0.5) E(0.5)	G(0.6)	E(1.0)
		headlight (e_7^3)		G(1.0)	A(1.0)	E(1.0)	G(0.5) E(0.5)
		mirrors (e_7^4)		A(0.5) G(0.5)	G(0.5) E(0.5)	E(1.0)	G(1.0)
		horn (e_7^5)		A(1.0)	G(1.0)	G(0.5) E(0.5)	E(1.0)

(The evaluation grades for qualitative attributes are defined as $P(\beta)$ — *poor*, $I(\beta)$ — *indifferent*, $A(\beta)$ — *average*, $G(\beta)$ — *good* and $E(\beta)$ — *excellent* where β represents confidence degree [10])

where $E_{11}(a), E_{12}(a)$ and $E_{13}(a)$ are defined by

$$E_{11}(a) = \{e_6^1 e_6^2 e_6^3 e_6^4\}, E_{12}(a) = \{e_6^5 e_6^6\}, E_{13}(a) = \{e_6^7 e_6^8 e_6^9\}$$

The factors $E_{11}(a), E_{12}(a)$ and $E_{13}(a)$ in E_6 are aggregated into the three mutually comparable factors *handling* (f_{11}), *transmission* (f_{12}) and *brakes* (f_{13}), which are closely associated with evaluation of the attribute *operation*. The composite factor set $F_1(a)$ is thus defined by

$$F_1(a) = \{f_{11}(a) f_{12}(a) f_{13}(a)\} = \{\textit{handling, transmission, brakes}\}$$

In reference [10], the same set of evaluation grades was used for evaluation of the three qualitative attributes, as defined by (4). In (4), five distinct evaluation grades are involved. The

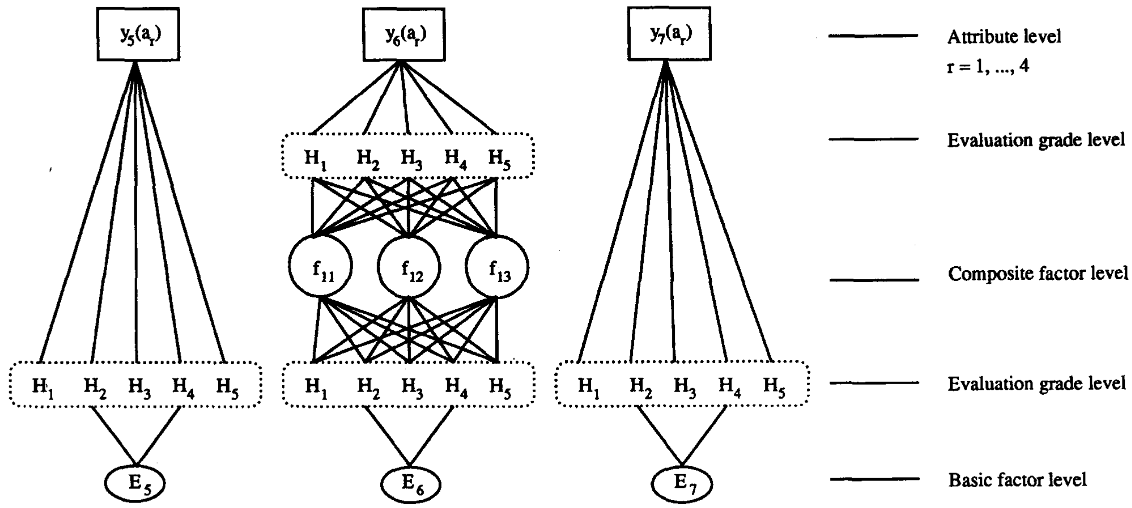

Fig. 6. The general evaluation analysis for the motor cycle evaluation problem.

TABLE VI
PROBABILITY ASSIGNMENTS FOR $f_{11}(a_1)$

basic probability assignments ($\beta\times\lambda_{61}^{i}$)		evaluation grades				
		$P(\beta)$	$I(\beta)$	$A(\beta)$	$G(\beta)$	$E(\beta)$
factors	e_6^1					0.81
	e_6^2			0.15	0.15	
	e_6^3			0.9		
	e_6^4					0.6
total probability assignments $m_{11}^{n}(a_1)$		0.000	0.000	0.455	0.009	0.496

TABLE VII
PROBABILITY ASSIGNMENTS FOR $f_{12}(a_1)$

basic probability assignments ($\beta\times\lambda_{62}^{i}$)		evaluation grades				
		$P(\beta)$	$I(\beta)$	$A(\beta)$	$G(\beta)$	$E(\beta)$
factors	e_6^5			0.72		
	e_6^6			0.45	0.45	
total probability assignments $m_{12}^{n}(a_1)$		0.000	0.000	0.772	0.186	0.000

TABLE VIII
PROBABILITY ASSIGNMENTS FOR $f_{13}(a_1)$

basic probability assignments ($\beta\times\lambda_{63}^{i}$)		evaluation grades				
		$P(\beta)$	$I(\beta)$	$A(\beta)$	$G(\beta)$	$E(\beta)$
factors	e_6^7				0.9	
	e_6^8				0.225	0.225
	e_6^9	0.45				
total probability assignments $m_{13}^{n}(a_1)$		0.053	0.000	0.000	0.855	0.027

TABLE IX
PROBABILITY ASSIGNMENTS FOR $y_6(a_1)$

probability assignments ($m_{1i}^{n}(a_1)\times\lambda_6^{i}$)		evaluation grades				
		$P(\beta)$	$I(\beta)$	$A(\beta)$	$G(\beta)$	$E(\beta)$
factors	$f_{11}(a_1)$			0.41	0.008	0.446
	$f_{12}(a_1)$			0.232	0.056	
	$f_{13}(a_1)$	0.032			0.512	0.016
total probability assignments $m_6^{n}(a_1)$		0.007	0.000	0.42	0.143	0.333
preference degree p_{16}		0.383				

general hierarchical analysis model for evaluation of the three qualitative attributes may then be depicted as in Fig.6.

The customer estimates the following scales for $H_n(n = 1, \ldots, N)$, that is

$$p\{H\} = [p(H_1)\cdots p(H_5)]^T = [-1 - 0.400.41]^T$$

$p(H_n)(n = 1, \ldots, 5)$ assigned above satisfy the basic conditions defined by (3) and also the consistency test defined by (8).

Each of the preference degrees, $p_{rk} = p(y_k(a_r)), (k = 5, 6, 7; r = 1, \ldots, 4)$, is obtained following the steps listed in subsection III.D. The basic probability assignments are generated from the confidence degrees given in Table V and the normalized weights listed in subsection IV.B. The calculation procedure for generating the preference degree for evaluation and quantification of the *operation* (y_6) of "Kawasaki"(a_1) is demonstrated by Tables VI to IX. The completed hierarchical evaluation analysis model for evaluating the *operation* of "Kawasaki" is shown in Fig. 7.

D. Alternative Ranking

The states of the three qualitative attributes $y_k(k = 5, 6, 7)$ at each of the four types of motor cycle $a_r(r = 1, \ldots, 4)$ are therefore evaluated and quantified by their preference degrees.

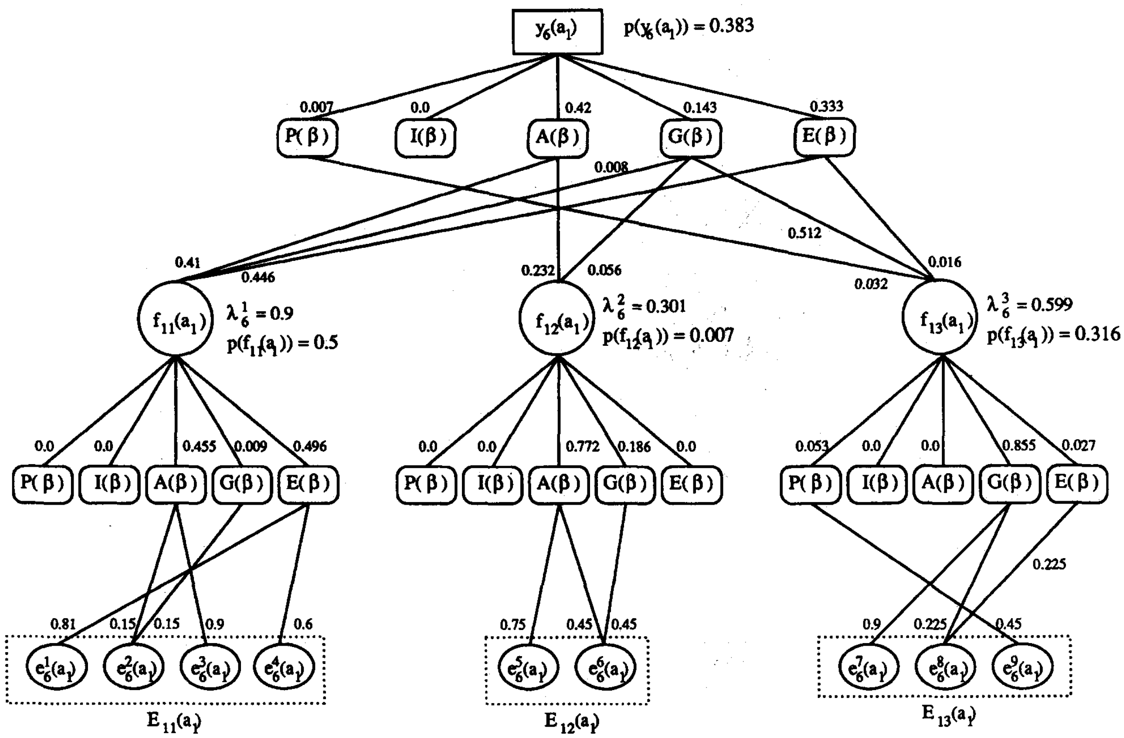

Fig. 7. The completed hierarchical evaluation analysis model for evaluating the operation of Kawasaki.

TABLE X
THE EVALUATION MATRIX

preference degrees	$p(y_1)$	$p(y_2)$	$p(y_3)$	$p(y_4)$	$p(y_5)$	$p(y_6)$	$p(y_7)$
a_1	0.139	-0.353	-0.250	1.000	0.117	0.383	-0.353
a_2	1.000	1.000	-1.000	0.333	-0.237	0.397	0.403
a_3	0.338	-0.891	-0.500	1.000	0.097	0.028	0.940
a_4	-1.000	-1.000	1.000	-1.000	0.692	-0.352	0.811

The quantification may be regarded as the transformation of the subjective judgments with uncertainty into the preference degree space defined by [-1 1]. The four quantitative attributes $y_k (k = 1, \ldots, 4)$ are incommensurate and are also transformed into the preference degree space using (60) for y_2, y_3 and y_4 and (61) for y_1. Table X shows the evaluation matrix obtained by evaluation and quantification of the qualitative attributes and by transformation of the quantitative attributes.

Based on Table X, the candidate motor cycles are then ranked using a MADM method. Using the CODASID method, we can obtain the following utility values for the four alternatives as follows

$$[u^1(a_1) u^1(a_2) u^1(a_3) u^1(a_4)]^T = [0.757\ 0.940\ 0.731\ 0.000]^T$$

The preference order of the four types of motor cycle is therefore given by

$$a_2 \succ a_1 \succ a_3 \succ a_4$$

Hence, "Yamaha" is evaluated to be the best compromise choice in this instance by the customer based on Table V as it is the cheapest, its operation the best and its displacement the largest.

Implementing the TOPSIS method results in the following utility values

$$[u^2(a_1)\, u^2(a_2)\, u^2(a_3)\, u^2(a_4)]^T = [0.5632\ 0.5634\ 0.5281\ 0.4259]^T$$

Thus, "Yamaha" is still ranked to be the best although in this case the utility value of "Kawasaki" is nearly the same as that of "Yamaha" and direct compensation is assumed.

The simple weighting method is also used to generate the following utility values

$$[u^3(a_1)\, u^3(a_2)\, u^3(a_3) u^3(a_4)]^T = [0.5632\ 0.558\ 0.4589\ 0.3827]^T$$

where the attribute values in Table X are normalized so that the best value of an attribute is transformed to 1 and the worst to 0. In this case, "Kawasaki" is evaluated to be slightly better than "Yamaha". However, the difference between the utility values of "Kawasaki" and "Yamaha" is negligible. This means that the weighting method does not differentiate between the two alternatives significantly. Moreover, the three assumptions associated with the method are made implicitly.

V. CONCLUDING REMARKS

The general evaluation process developed in this paper is capable of dealing with a hybrid multiple attribute decision making problem with uncertainty, in which a qualitative attribute may be evaluated using uncertain subjective judgments given by single or multiple experts through detailed factors possibly with a multi-level structure.

This process is basically composed of two main steps for information transformation, aggregation and synthesis. In the first step, a framework is explored for evaluating and quantifying the qualitative attributes of the problem by means of hierarchical factor analysis and evidential reasoning. The new evaluation analysis models and the factor combination and propagation algorithms developed within the framework have extended the evidential reasoning approach of [24] and can be used to handle any rational uncertain subjective data within the evidential reasoning framework. The second main step consists of applying a traditional MADM method to rank alternative designs or to select the best compromise design with both quantitative and qualitative attributes being simultaneously considered. The hierarchical evaluation analysis for the multiple attribute motor cycle evaluation problem has demonstrated the application of the new process.

It may be noted, however, that the exact values of the confidence degrees are given in the uncertain judgments for evaluation of the qualitative attributes. To acquire such uncertain decision knowledge, however, considerable expertise in the problem domain is required and certain techniques for assigning subjective probability need to be used as well [10]. In some decision situations, uncertainty may be associated with the quantitative data as well as the qualitative judgments. Furthermore, a hybrid decision problem with uncertainty may not be adequately represented by a well-structured extended decision matrix. For instance, feasible alternative designs may be implicitly represented by nonlinear (often non-convex and maybe discrete) multiple objective optimization problems [3], [15], [16], [17], [22], [23], [27], or by non-mathematical models using such devices as knowledge-based systems [7], [17], [19], [20]. More effort is therefore required in future research to apply this new process to deal with such decision problems with uncertainty.

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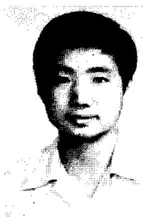

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