2012 IEEE 13th International Workshop on Signal Processing Advances in Wireless Communications (SPAWC)

A General Power System Control Technique based on Lyapunov's Function

Francesco Torelli, Federico Milano, *Senior Member, IEEE* and Antonio De Bonis

***Abstract*— This paper presents a general adaptive control scheme for regulating power system devices. The main idea of the paper is to build a fictitiou set of differential equations starting from a function that satisfie Lyapunov's second stability criterion. In such a way, the fictitiou set of differential equations surely converge to a stable equilibrium point. We propose to use this mathematical tool to control arbitrary power system devices and to improve the transient behavior of the whole network. We apply the proposed technique to stabilize synchronous machine oscillations and show that the proposed controller provides better results than the conventional power system stabilizer. Finally, the paper provides several remarks on the applicability of the proposed technique.**

***Index Terms*— Lyapunov's function, power system stability, automatic voltage regulator (AVR), power system stabilizer (PSS), Hopf bifurcation (HB), limit cycle.**

I. Introduction

A. Motivation and Proposed Approach

SINCE the translation into English in 1966 [1], Lyapunov's second stability method, or *direct method*, has been widely exploited for the stability analysis of nonlinear systems. In its simplest form, this method attempts to infer the stability of an autonomous ODE system of the form:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) \tag{1}$$

where $\boldsymbol{f}(\boldsymbol{x})$ is a set of n ordinary smooth differential equations (ODE). The Lyapunov's direct method consists in building a function $V(\boldsymbol{x}) : \mathbb{R}^n \mapsto \mathbb{R}$ called *Lyapunov's function* able to "measure" the stability of the system. In mathematical terms, the Lyapunov's theory shows that $\boldsymbol{x}_0$ is a stable equilibrium point if $\boldsymbol{f}(\boldsymbol{x}_0) = \boldsymbol{0}$ and if there exists $V(\boldsymbol{x})$ such that:

1) $V(\boldsymbol{x}) \geq 0$ (with $V(\boldsymbol{x}_0) = 0$), and
2) $\dot{V}(\boldsymbol{x}) = \frac{d}{dt}V(\boldsymbol{x}) \leq 0$.

Similar conditions can be also defined for non-autonomous systems of the form $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, t)$, but this generalization is out of the scope of this paper.

The main advantage of the Lyapunov's direct method is that the large disturbance stability of a multi-variable system is reduced to the study of a scalar function. Thus no numerical integration is needed. However, Lyapunov's theory does not provide the tools to build the Lyapunov's function for a generic system (1). This is actually one of the two most binding drawbacks of Lyapunov's stability theory. The other one is that, for lossy systems (i.e., for any physical system), the instability test provided by the Lyapunov's function is a necessary but not sufficient condition.

The most straightforward application of Lyapunov's theory is to try to define a Lyapunov's function for the system (1) under study. However, since there is no simple way to define the Lyapunov's function for a generic ODE system, to apply Lyapunov's direct method is generally a challenge.

In this paper, we propose to take advantage of Lyapunov's direct method the other way round. In other words, we propose to build a fictitious ODE starting from a well-formed Lyapunov's function. In this way, the fictitious ODE is certainly stable and its trajectories end up into a stable equilibrium point. Then, we use such artificial ODE as a nonlinear transfer function to control power system devices. The nonlinearity and the adaptivity of the resulting control system makes it particularly efficacious. Moreover, since the regulator is based on the general Lyapunov's theory, it can be applied to control any device without worrying about the particular scope and properties of the device itself.

B. Literature review

With regard to power system analysis, Lyapunov theory has been mostly applied to transient stability analysis. This has been a natural application since the total energy of a power system is a good and meaningful Lyapunov function. For this reason, the Lyapunov's function is often called *Transient Energy Function* (TEF) in most publications on transient stability.

The applications to power system analysis of Lyapunov's direct methods have been basically twofold: (i) to try to define the exact Lyapunov's function of a generic power system, no matter how complex and large the electric system is; and (ii), since the Lyapunov's function is known for a simple one-machine infinite-bus (OMIB) system, to define the equivalent OMIB for a given power system. Both approaches are approximated: the former because the Lyapunov's function can be defined only in form of a series that has to be truncated at a certain order; the latter because the OMIB is a drastic approximation of the power system behavior.

Despite the intrinsic difficulty in applying the Lyapunov's direct method, Lyapunov's theory is so intriguing that one can hardly resist to the temptation of trying to solve mathematical issues. As a matter of fact, for over three decades there have been attempts to provide suitable procedures for building the Lyapunov's function [2]–[8].

Of all methods, the most successful application of the OMIB-based approach is likely the SIME method developed

Francesco Torelli and Antonio De Bonis are with the Department of Electrical Engineering, Politecnico di Bari, Italy. E-mail: torelli@poliba.it.

F. Milano is with the Department of Electrical Engineering, University of Castilla-La Mancha, 13071 Ciudad Real, Spain. E-mail: Federico.Milano@uclm.es.

Federico Milano is partly supported by the Ministry of Science and Education of Spain through CICYT Project ENE-2009-07685 and by Junta de Comunidades de Castilla - La Mancha through Project POI11-0148-1022.

978-1-4673-0971-4/12/$31.00 © 2012 IEEE

by University of Liège [9]. SIME is actually an hybrid method since applies the OMIB and the Lyapunov stability method at each step of the time domain integration of the full power system. In this way, it is possible to overcome the approximation introduced by the OMIB and, based on the Lyapunov criterion, one can decide if a given trajectory is stable or not well before performing the full time domain integration.

C. Contributions

The contributions of this paper are twofold:

1) A non-conventional control strategy based on Lyapunov's function. The proposed technique is aimed to define a general approach for controlling physical system, not necessarily limited to power systems.
2) An application of the proposed control technique to power system regulation and stability improvement. In particular, we focus on the damping of oscillations through proper regulation of the synchronous machine AVR reference voltage.

D. Paper organization

The paper is organized as follows. Section II poses the theoretical basis and provides the detailed formulation of the proposed control technique. Applicability limits are also discussed in this section. Section III describes a simple yet complete case study that explains how to practically implement the proposed control scheme and illustrates results through the well-known benchmark IEEE 14-bus system. Section III also gives relevant remarks on the proposed technique. Finally, Section IV draws conclusions and anticipates future work.

II. Proposed Control Scheme

Let consider the typical differential algebraic model of power systems:

$$\begin{aligned}\dot{\boldsymbol{x}} &= \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{u}) \\ \boldsymbol{0} &= \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{u})\end{aligned} \tag{2}$$

where $\boldsymbol{f}$ are the differential equations ($\boldsymbol{f} : \mathbb{R}^{n+m+p} \mapsto \mathbb{R}^n$, $\boldsymbol{g}$ are the algebraic equations ($\boldsymbol{g} : \mathbb{R}^{n+m+p} \mapsto \mathbb{R}^m$, $\boldsymbol{x}$ are the state variables ($\boldsymbol{x} \in \mathbb{R}^n$), $\boldsymbol{y}$ are the algebraic variables ($\boldsymbol{x} \in \mathbb{R}^m$), and $\boldsymbol{u}$ are the controllable input variables ($\boldsymbol{u} \in \mathbb{R}^p$).

Let assume to know the vector of controllable inputs $\boldsymbol{u} = \boldsymbol{u}_0$. This is typically the case as $\boldsymbol{u}_0$ can be defined through an optimal power flow or based on system knowledge. Then, the equilibrium point $(\boldsymbol{x}_0, \boldsymbol{y}_0)$ must satisfy:

$$\begin{aligned}\boldsymbol{0} &= \boldsymbol{f}(\boldsymbol{x}_0, \boldsymbol{y}_0, \boldsymbol{u}_0) \\ \boldsymbol{0} &= \boldsymbol{g}(\boldsymbol{x}_0, \boldsymbol{y}_0, \boldsymbol{u}_0)\end{aligned} \tag{3}$$

The first step of the proposed control scheme is to define a *virtual* set of equations that reproduce the physical DAE system (2):

$$\dot{\tilde{\boldsymbol{x}}} = \tilde{\boldsymbol{f}}(\tilde{\boldsymbol{x}}, \tilde{\boldsymbol{y}}, \tilde{\boldsymbol{u}}) \tag{4}$$

$$\boldsymbol{0} = \tilde{\boldsymbol{g}}(\tilde{\boldsymbol{x}}, \tilde{\boldsymbol{y}}, \tilde{\boldsymbol{u}}) \tag{5}$$

It is important note that (4) is just an emulation of the physical system that is used as part of the proposed control scheme, hence both $\tilde{\boldsymbol{x}}$ and $\tilde{\boldsymbol{y}}$ are fully measurable. The determination of the equilibrium point of (4) can be viewed as the minimization of the following error functions:

$$\begin{aligned}\boldsymbol{e}_f &= \tilde{\boldsymbol{f}}(\tilde{\boldsymbol{x}}, \tilde{\boldsymbol{y}}, \tilde{\boldsymbol{u}}) \\ \boldsymbol{e}_g &= \tilde{\boldsymbol{g}}(\tilde{\boldsymbol{x}}, \tilde{\boldsymbol{y}}, \tilde{\boldsymbol{u}})\end{aligned} \tag{6}$$

where $\tilde{\boldsymbol{x}}$, $\tilde{\boldsymbol{y}}$, and $\tilde{\boldsymbol{u}}$ indicate the "optimal" or "desired" values that we want $\boldsymbol{x}$, $\boldsymbol{y}$, and $\boldsymbol{u}$ to follow.

To simplify the notation, let define:

$$\boldsymbol{\phi} \equiv [\tilde{\boldsymbol{f}}^T, \tilde{\boldsymbol{g}}^T]^T, \quad \tilde{\boldsymbol{z}} \equiv [\tilde{\boldsymbol{x}}^T, \tilde{\boldsymbol{y}}^T]^T, \quad \boldsymbol{e} \equiv [\boldsymbol{e}_f^T, \boldsymbol{e}_g^T]^T \tag{7}$$

Hence (6) becomes:

$$\boldsymbol{e} = \boldsymbol{\phi}(\tilde{\boldsymbol{z}}, \tilde{\boldsymbol{u}}) \tag{8}$$

Then, define the scalar function:

$$V_z(\tilde{\boldsymbol{z}}) = \frac{1}{2}\boldsymbol{e}^T\boldsymbol{e} \tag{9}$$

which, by definition, is always positive for $\boldsymbol{e} \neq \boldsymbol{0}$ and $V_z(\boldsymbol{0}) = 0$. To impose that V_z is a Lyapunov's function, it suffices that $\dot{V}_z \leq 0$ (see the conditions given in Subsection I-A). With this aim, observe that:

$$\dot{V}_z = \boldsymbol{e}^T\dot{\boldsymbol{e}} = \boldsymbol{e}^T\frac{\partial \boldsymbol{e}}{\partial \tilde{\boldsymbol{z}}}\dot{\tilde{\boldsymbol{z}}} \tag{10}$$

It is important to note that V_z is differentiated only with respect to $\tilde{\boldsymbol{z}}$ since $\tilde{\boldsymbol{u}}$ are input variables for $\boldsymbol{e}$. If we impose that $\dot{\tilde{\boldsymbol{z}}}$ varies according to the gradient of V:

$$\dot{\tilde{\boldsymbol{z}}} = -\boldsymbol{K}_z\left[\frac{\partial V_z}{\partial \tilde{\boldsymbol{z}}}\right]^T = -\boldsymbol{K}_z\left[\frac{\partial \boldsymbol{e}}{\partial \tilde{\boldsymbol{z}}}\right]^T\boldsymbol{e} \tag{11}$$

where $\boldsymbol{K}_z \equiv \text{diag}\{k_{z,1}, k_{z,2}, \ldots, k_{z,n+m}\}$ is a diagonal matrix of arbitrary positive coefficients. Observe that, according to (8), (11) can be also written as:

$$\dot{\tilde{\boldsymbol{z}}} = -\boldsymbol{K}_z\left[\frac{\partial \boldsymbol{\phi}}{\partial \tilde{\boldsymbol{z}}}\right]^T\boldsymbol{\phi} \tag{12}$$

Merging (11) into (10) leads to:

$$\dot{V}_z = -\boldsymbol{e}^T\frac{\partial \boldsymbol{e}}{\partial \tilde{\boldsymbol{z}}}\boldsymbol{K}_z\left[\frac{\partial \boldsymbol{e}}{\partial \tilde{\boldsymbol{z}}}\right]^T\boldsymbol{e} \tag{13}$$

that is a quadratic negative semi-definite function. In our experience, this choice for (11) provides generally good results. However, any other definition for $\dot{\tilde{\boldsymbol{z}}}$ that leads to satisfy the condition of the Lyapunov's function for $\dot{V}_z$ would be valid.

So far, we have defined the differential equations that defines "optimal" trajectory $\tilde{\boldsymbol{z}}(t)$ that the physical state variables $\boldsymbol{z}$ must follow. Now we have to define a set of differential equations that define the "optimal" trajectories $\tilde{\boldsymbol{u}}(t)$. With this aim, let impose the following error functions:

$$\boldsymbol{e}_z = \tilde{\boldsymbol{z}} - \boldsymbol{z}_0 \tag{14}$$

$$\boldsymbol{e}_u = \tilde{\boldsymbol{u}} - \boldsymbol{u}_0 \tag{15}$$

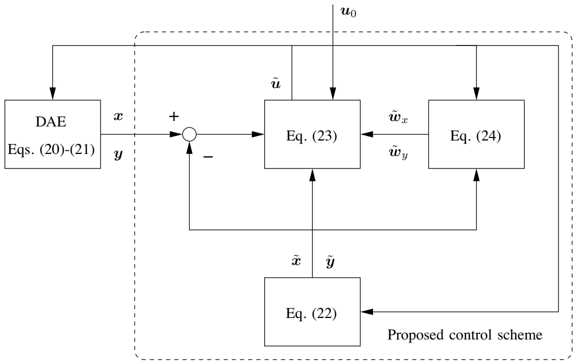

Fig. 1. Synoptic diagram of the proposed control scheme.

that impose that the final point reached by the control system is actually the equilibrium point of the original DAE described by (2). We apply again the technique described above to (14):

$$V_u(\tilde{\boldsymbol{u}}) = \frac{1}{2}(\boldsymbol{e}_z^T \boldsymbol{e}_z + \boldsymbol{e}_u^T \boldsymbol{e}_u) \tag{16}$$

where V_u is function only of $\tilde{\boldsymbol{u}}$ and $\tilde{\boldsymbol{z}}(\tilde{\boldsymbol{u}})$ is an implicit function of $\tilde{\boldsymbol{u}}$. Then, in order to impose the condition of the Lyapunov's stability criterion, one has:

$$\dot{\tilde{\boldsymbol{u}}} = -\boldsymbol{K}_w \frac{\partial \tilde{\boldsymbol{z}}}{\partial \tilde{\boldsymbol{u}}} \boldsymbol{e}_z - \boldsymbol{K}_u \boldsymbol{e}_u \tag{17}$$

where $\boldsymbol{K}_u \equiv \text{diag}\{k_{u,1}, k_{u,2}, \ldots, k_{u,p}\}$ and $\boldsymbol{K}_w \equiv \text{diag}\{k_{w,1}, k_{w,2}, \ldots, k_{w,n+m}\}$, and $\partial \tilde{\boldsymbol{z}} / \partial \tilde{\boldsymbol{u}}$ can be determined by differentiating (4) with respect to $\tilde{\boldsymbol{u}}$, as follows:

$$\begin{bmatrix} \dot{\tilde{\boldsymbol{w}}}_x \\ \boldsymbol{0} \end{bmatrix} = \begin{bmatrix} \partial \tilde{\boldsymbol{f}}/\partial \tilde{\boldsymbol{x}} & \partial \tilde{\boldsymbol{f}}/\partial \tilde{\boldsymbol{y}} \\ \partial \tilde{\boldsymbol{g}}/\partial \tilde{\boldsymbol{x}} & \partial \tilde{\boldsymbol{g}}/\partial \tilde{\boldsymbol{y}} \end{bmatrix} \begin{bmatrix} \tilde{\boldsymbol{w}}_x \\ \tilde{\boldsymbol{w}}_y \end{bmatrix} + \begin{bmatrix} \partial \tilde{\boldsymbol{f}}/\partial \tilde{\boldsymbol{u}} \\ \partial \tilde{\boldsymbol{g}}/\partial \tilde{\boldsymbol{u}} \end{bmatrix} \tag{18}$$

where we have defined:

$$\begin{bmatrix} \tilde{\boldsymbol{w}}_x \\ \tilde{\boldsymbol{w}}_y \end{bmatrix} = \begin{bmatrix} \partial \tilde{\boldsymbol{x}}/\partial \tilde{\boldsymbol{u}} \\ \partial \tilde{\boldsymbol{y}}/\partial \tilde{\boldsymbol{u}} \end{bmatrix} = \frac{\partial \tilde{\boldsymbol{z}}}{\partial \tilde{\boldsymbol{u}}} \tag{19}$$

The set of equations (12), (17) and (18) define the "optimal" trajectories $\tilde{\boldsymbol{z}}(t)$, $\tilde{\boldsymbol{u}}(t)$, $\tilde{\boldsymbol{w}}_x(t)$, and $\tilde{\boldsymbol{w}}_y(t)$. However, such system is fully decoupled from the physical DAE described by (2). So, in this form, (12), (17) and (18) cannot be used as control system. To link the physical DAE and the control system, we substitute $\boldsymbol{u}$ for $\tilde{\boldsymbol{u}}$ in (2) and $\boldsymbol{z}$ for $\boldsymbol{z}_0$ in the first of (14).

The resulting complete system is reported below:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{y}, \tilde{\boldsymbol{u}}) \tag{20}$$

$$\boldsymbol{0} = \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{y}, \tilde{\boldsymbol{u}}) \tag{21}$$

$$\begin{bmatrix} \dot{\tilde{\boldsymbol{x}}} \\ \dot{\tilde{\boldsymbol{y}}} \end{bmatrix} = -\boldsymbol{K}_z \begin{bmatrix} \partial \tilde{\boldsymbol{f}}/\partial \tilde{\boldsymbol{x}} & \partial \tilde{\boldsymbol{f}}/\partial \tilde{\boldsymbol{y}} \\ \partial \tilde{\boldsymbol{g}}/\partial \tilde{\boldsymbol{x}} & \partial \tilde{\boldsymbol{g}}/\partial \tilde{\boldsymbol{y}} \end{bmatrix}^T \begin{bmatrix} \tilde{\boldsymbol{f}} \\ \tilde{\boldsymbol{g}} \end{bmatrix} \tag{22}$$

$$\dot{\tilde{\boldsymbol{u}}} = -\boldsymbol{K}_w \begin{bmatrix} \tilde{\boldsymbol{w}}_x \\ \tilde{\boldsymbol{w}}_y \end{bmatrix} \begin{bmatrix} \boldsymbol{x} - \tilde{\boldsymbol{x}} \\ \boldsymbol{y} - \tilde{\boldsymbol{y}} \end{bmatrix} - \boldsymbol{K}_u(\tilde{\boldsymbol{u}} - \boldsymbol{u}_0) \tag{23}$$

$$\begin{bmatrix} \dot{\tilde{\boldsymbol{w}}}_x \\ \boldsymbol{0} \end{bmatrix} = \begin{bmatrix} \partial \tilde{\boldsymbol{f}}/\partial \tilde{\boldsymbol{x}} & \partial \tilde{\boldsymbol{f}}/\partial \tilde{\boldsymbol{y}} \\ \partial \tilde{\boldsymbol{g}}/\partial \tilde{\boldsymbol{x}} & \partial \tilde{\boldsymbol{g}}/\partial \tilde{\boldsymbol{y}} \end{bmatrix} \begin{bmatrix} \tilde{\boldsymbol{w}}_x \\ \tilde{\boldsymbol{w}}_y \end{bmatrix} + \begin{bmatrix} \partial \tilde{\boldsymbol{f}}/\partial \tilde{\boldsymbol{u}} \\ \partial \tilde{\boldsymbol{g}}/\partial \tilde{\boldsymbol{u}} \end{bmatrix} \tag{24}$$

Figure 1 depicts the proposed approach in a synoptic control diagram.

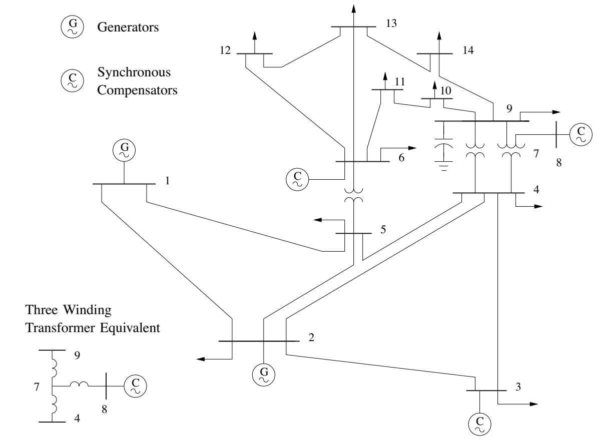

Fig. 2. IEEE 14-bus test system.

A. Practical Aspects of the Proposed Control Technique

The system (20)-(24) is based on three strong hypotheses:

1) All state and algebraic variables $\boldsymbol{x}$ and $\boldsymbol{y}$ are measurable.
2) The control system is centralized and, no matter how large is the power system, the optimal $\tilde{\boldsymbol{u}}$ signals can be sent to the correspondent devices and all measured system variables can be collected in a unique control center.
3) The structure and the parameters of (2) are perfectly known, including system topology and any possible change due to operation or perturbances.

These hypotheses cannot be realistically satisfied, especially in modern decentralized and deregulated power systems.

However, it is not necessary to regulate *all* system variables. The proposed scheme can work and be effective (as we show in the next section) by measuring and regulating only a subset of $\boldsymbol{x}$ and/or $\boldsymbol{y}$ and $\tilde{\boldsymbol{u}}$, respectively. The key point that leads to a feasible implementation of the proposed technique is thus to define a proper subset of $\boldsymbol{x}$, $\boldsymbol{y}$, $\tilde{\boldsymbol{u}}$. Clearly, this choice relies on practitioner experience and know-how.

III. Case Study

The case study considered in this paper is the well-known IEEE 14-bus system. This benchmark network consists of two generators, three synchronous compensators, two two-winding and one three-winding transformers, fifteen transmission lines, eleven loads and one shunt capacitor (see Fig. 2). Not depicted in Fig. 2, but included in the system model, are generator controllers, such as primary voltage regulators (AVRs). All dynamic data of this system as well as a detailed discussion of its transient behavior can be found in [10]. In order to force undamped oscillations the base loading level is increased by 20% and loads are modeled as constant power consumption.

All simulations and plots are obtained using a novel Python-based version of PSAT [11], called Dome.[1] Dome has been compiled based on Python 2.7.1, Numpy 1.5.1, CVXOPT 1.1.4, SuiteSparse 3.7.0, and Matplotlib 1.0.0 and has been executed on a 64 bit Linux Fedora Core 15 platform running on 8 six-core 2.4 GHz AMD Opteron with 512 GB of RAM.

[1] Available at: `www3.uclm.es/profesorado/federico.milano/dome.htm`

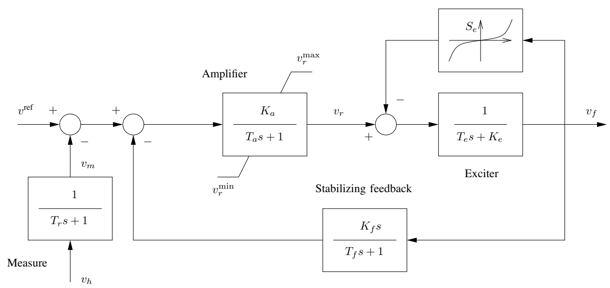

Fig. 3. Automatic voltage regulator Type I control diagram.

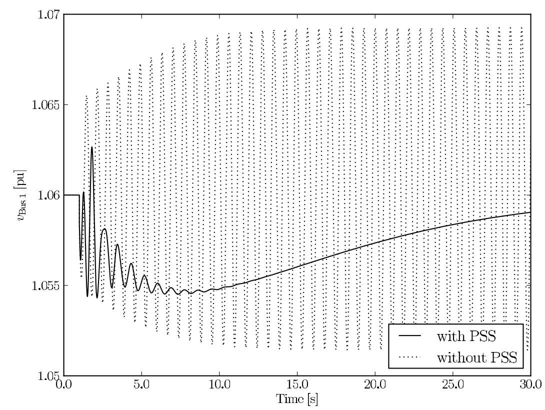

Fig. 4. IEEE 14 bus system: trajectories of the voltage magnitude at bus 1 following line 2-4 outage. The figure compares the results with and without the PSS.

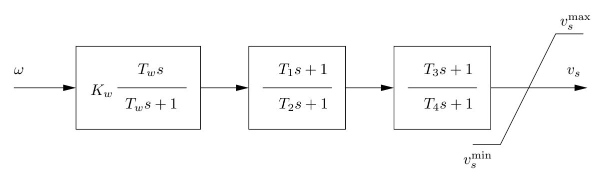

Fig. 5. Power system stabilizer control diagram.

A. AVR and PSS Models

The AVR used in the simulations is depicted in Fig. 3. The equations that describes the AVR are:

$$\begin{aligned}\dot{v}_{r1} &= (K_a(v^{\text{ref}} - v_m - v_{r2} - \frac{K_f}{T_f}v_f) - v_{r1})/T_a \quad (25)\\ \dot{v}_{r2} &= -(\frac{K_f}{T_f}v_f + v_{r2})/T_f\\ \dot{v}_f &= -(v_f(K_e + S_e(v_f)) - v_{r1})/T_e\end{aligned}$$

where v_{r1} and v_{r2} are regulator state variables, v_f is the generator field voltage and other parameters are depicted in Fig. 3. The amplifier state variable v_{r1} is subjected to an anti-windup limit. The bus voltage measure is passed through a lag block:

$$\dot{v}_m = (v_h - v_m)/T_r \tag{26}$$

where v_h is the generator bus voltage or any bus voltage regulated by the AVR and v_m is the state variable used as voltage signal within the AVR model.

In (25), S_e is the ceiling function:

$$S_e(v_f) = A_e e^{B_e|v_f|} \tag{27}$$

where the coefficients A_e and B_e can be determined by experimentally determine two points of S_e.

Equations (25)-(27) are a simplified version of the classic IEEE type DC1 [12]. The IEEE DC1 system includes an additional lead-lag block before the amplifier block. However, this lead-lag block is often neglected and is not considered in this paper.

If no PSS or other additional controller is considered (e.g., over-excitation limiters, under-excitation limiter, etc.), the AVR reference voltage is constant, say:

$$v^{\text{ref}} = v_0^{\text{ref}} \tag{28}$$

Figure 4 shows the effect of the automatic voltage regulation on the bus terminal voltage of synchronous machine 1 of the IEEE 14-bus system. The transient refers to line 2-4 outage at $t = 1$ s. For the considered loading level, the line outage leads to an unstable equilibrium point, as fully discussed in [10]. In fact, a Hopf bifurcation occurs and the system trajectory enters into a stable limit cycle.

Figure 4 also compares the transient response of the IEEE 14-bus system with and without a PSS device connected at generator 1. The PSS considered in this example is a standard scheme (see Fig. 5) that measures the synchronous machine rotor speed ω and provides an additional signal v_s to the AVR reference voltage v^{ref}, so that (28) becomes:

$$v^{\text{ref}} = v_0^{\text{ref}} + v_s \tag{29}$$

A detailed description of the PSS controller and of all data can be found in [10]. As expected, the system with PSS is stable and well damped. Furthermore, the PSS allows recovering the generator bus voltage at the desired value.

B. Proposed Control Scheme

We apply the proposed control strategy in order to define an adaptive control of the AVR reference voltage. With this aim, let rewrite (28) as follows:

$$0 = g_{v^{\text{ref}}}(y, \tilde{u}) = v^{\text{ref}} - \tilde{v}^{\text{ref}} \tag{30}$$

where $y \equiv v^{\text{ref}}$, and $\tilde{u} \equiv \tilde{v}^{\text{ref}}$. Observe also that $g_{v^{\text{ref}}}$ does not depend on state variables. According to Section II, the resulting control equations are:

$$\dot{\tilde{u}} = -K_w\tilde{w}_y(y - \tilde{y}) - K_u(\tilde{u} - v_0^{\text{ref}}) \tag{31}$$

$$0 = \frac{\partial \tilde{g}_{v^{\text{ref}}}}{\partial \tilde{y}}\tilde{w}_y + \frac{\partial \tilde{g}_{v^{\text{ref}}}}{\partial \tilde{u}} \tag{32}$$

$$\dot{\tilde{y}} = -K_y\left(\frac{\partial \tilde{\boldsymbol{f}}}{\partial \tilde{y}}^T\tilde{\boldsymbol{f}} + \frac{\partial \tilde{\boldsymbol{g}}}{\partial \tilde{y}}^T\tilde{\boldsymbol{g}}\right) \tag{33}$$

where, according to (30) and (25)-(27):

$$\frac{\partial \tilde{g}_{v^{\text{ref}}}}{\partial \tilde{y}} = 1, \qquad \frac{\partial \tilde{g}_{v^{\text{ref}}}}{\partial \tilde{u}} = -1 \tag{34}$$

and, hence, $\tilde{w}_y = 1$, whereas $\partial\tilde{\boldsymbol{f}}/\partial\tilde{y}$ and $\partial\tilde{\boldsymbol{g}}/\partial\tilde{y}$ are constant vectors of all zeros but for one element, as follows:

$$\frac{\partial\tilde{f}_{v_{r1}}}{\partial\tilde{v}^{\mathrm{ref}}} = \frac{\partial\tilde{f}_{v_{r1}}}{\partial\tilde{y}} = K_a/T_a \tag{35}$$

$$\frac{\partial\tilde{g}_{v^{\mathrm{ref}}}}{\partial\tilde{v}^{\mathrm{ref}}} = \frac{\partial\tilde{g}_{v^{\mathrm{ref}}}}{\partial\tilde{y}} = 1 \tag{36}$$

Hence (31)-(33) can be rewritten as:

$$\dot{\tilde{u}} = -K_w(y-\tilde{y}) - K_u(\tilde{u} - v_0^{\mathrm{ref}}) \tag{37}$$

$$\dot{\tilde{y}} = -K_y\Big(\frac{K_a}{T_a^2}\Big(K_a\Big(\tilde{y} - \tilde{v}_m - \tilde{v}_{r2} - \frac{K_f}{T_f}\tilde{v}_f\Big) - \tilde{v}_{r1}\Big) + \tilde{y} - \tilde{u}\Big) \tag{38}$$

Observe that the resulting controller is linear. Equation (38) requires the knowledge of estimated AVR variables $\tilde{v}_m$, $\tilde{v}_f$, $\tilde{v}_{r1}$, and $\tilde{v}_{r2}$. To know such quantities would be possible only by defining and computing the whole estimated system (22), which is clearly impractical. To overcome this difficulty, we substitute (38) for the following approximated equation:

$$\dot{\tilde{y}} = -K_y\Big(\frac{K_a}{T_a^2}\Big(K_a\Big(y - v_m - v_{r2} - \frac{K_f}{T_f}v_f\Big) - v_{r1}\Big) + y - \tilde{u}\Big) \tag{39}$$

Figure 6 shows the transient performance of the proposed control scheme and compares it with the PSS response. The gains used in the simulation are: $K_w = 0.25$, $K_u = 4500$, and $K_y = 0.02$. Observe that the propose controller is able to damp oscillations much better than the PSS device.

C. Remarks on the Proposed Control Scheme

Remarkable properties of the proposed control scheme, apart from its dynamic performance, are threefold, as follows.

1) Only local measurements and parameters are required: The proposed scheme can be implemented using a simple electronic board (i.e., the same cheap technology used to implement PSS devices). Hence, the proposed control scheme is cost-less.

2) No information on the controlled system is required: The proposed controller has been designed without any particular hypothesis on the controlled system. The specific issues of the physical system composed of the AVR and the synchronous machine have not to be taken into account. Nevertheless, since the proposed control schemes is intrinsically stable, it is able to remove the Hopf bifurcation and the following limit cycle that occur in the original system.

3) No frequency or active power measurements are required: PSS devices work by emulating a virtual (i.e., lossless) damping in the synchronous machine mechanical equations (see [10] for details). Typical PSS measured quantities are the synchronous machine rotor speed and/or active power production. As a result, the effect of PSS devices is to damp the natural machine oscillations (see Fig. 4). On the contrary, the proposed control scheme does not introduce virtual rotor dampers and, thus, can damp oscillations in a very effective way.

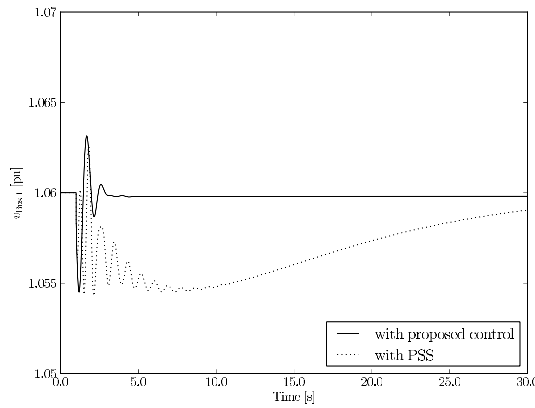

Fig. 6. IEEE 14 bus system: trajectories of the voltage magnitude at bus 1 following line 2-4 outage. The figure compares the effects of the proposed controller and of the standard PSS.

IV. Conclusions

This paper proposes a general control scheme for regulating power system devices based on Lyapunov's second stability criterion. The resulting controller scheme is nonlinear and intrinsically stable because built starting from a Lyapunov's function. The proposed control scheme can be applied to any device. In this paper, we test the proposed procedure with the aim of stabilizing power system oscillations following a Hopf bifurcation. Obtained results are promising as the proposed controller outperforms standard PSS devices.

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