A General Proof Method for Modal Predicate Logic without the Barcan Formula.*

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Abstract.

We present a general proof method for normal systems of modal predicate logic with identical inference rules for each such logic. Different systems are obtained by changing the conditions under which two formulas are considered complementary. The paper extends previous work in that we are no longer confined to models in which the Barcan formula and its converse hold. This allows the domain of individuals to vary from world to world. Modifications to the original inference rules are given, and a semantic justification is provided.

1 Introduction

Modal logics are primarily concerned with the dual notions of necessity and possibility, but they can also provide a basis for reasoning about knowledge, belief, time and change, e.g. [Halpern & Moses, 1985]. Automated reasoning in modal logics is made difficult, however, by (i) the absence of a normal form for expressions containing modal operators, and (ii) problems associated with possible individuals when we quantify into modal expressions.

This paper generalizes the proof method for modal predicate logic first described in Jackson [1987] and axiomatized in Jackson & Reichgelt [1987]. As before, the inference rules are identical for each system; different systems differ only with respect to the definition of complementarity between formulas. The conditions under which we allow formulas in sequents to unify depend upon the properties of the accessibility relation in the underlying Kripke semantics.

In the original presentation, the Barcan formula, $(\forall x)L\alpha \supset L(\forall x)\alpha$, and its converse always held, so the domain of individuals was invariant between possible worlds. This is not suitable for all applications because, as we pass from world to world, new individuals may come into existence, while extant individuals may cease to exist. The present work releases the proof method from such a restriction. This is done by indexing terms with the world in which they are introduced, and then imposing additional constraints upon the unification of formulas containing modal terms. The intuition is that if two formulas are complementary then their terms denote the same individuals.

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The outline of the paper is as follows. First, we present the original definition of modal unification, which preserves both the Barcan formula and its converse. Then we give the inference rules of the original proof theory, and illustrate the proof method. Next we generalize to models with varying domains via the definition of modal term unification, and provide a semantic justification for the modified inference rules. Finally, we discuss some related work.

2 M-Unification

Our logical language is defined in the usual way. We use the connectives $\supset$ and $\neg$, the universal quantifier $\forall$ and the necessity operator L.

In the proof theory, a formula has an index associated with it, representing the world in which it is true or false. Indices are defined as arbitrary sequences of *world symbols* separated by colons. The set of world symbols is defined as the union of the set of numerals {0, 1, 2, ...} called *world constants*, the set of *world variables* {u, v, w}, possibly with subscripts, and the set of skolemized world symbols which are formed from new n-ary function symbols and n-tuples of variables.

A world symbol that is not a world variable is called *ground*, as is an index whose world symbols are all ground. If $s_1:...:s_n$ is an index, then we call s_1 the *end symbol* and s_n the *start symbol*, written $end(s_1:...:s_n)$ and $start(s_1:...:s_n)$ respectively. If $s_1:s_2:...:s_n$ is an index, then s_2 is the *parent symbol* of s_1 written $parent(s_1)$. Thus indices are read from right to left.

The original proof theory begins by defining a special form of unification that corresponds to a particular definition of complementarity.

Definition 1. Two formulas are *complementary* iff there exists a world in which they have opposite truth values.

A standard model for a system of modal propositional logic is a structure (W, R, V), where W is a set of worlds, R is a relation on W^2 and V is a valuation function from atomic sentences to 2^W. A system of modal logic can be specified semantically in terms of the properties of the accessibility relation R that hold in all standard models of the system [Chellas, 1980]. We make use of this fact in the definition of modal unification, by making the conditions under which formulas are complemetary relative to R.

Definition 2. Two indexed formulas are *R-complementary* iff they are complementary under R.

To discover whether two *indexed* formulas are complementary prior to resolution, in addition to unifying the formulas we need to unify their indices, in such a way that two indices $s_1:...:s_m$ and $t_1:...:t_n$ unify iff s_1 and t_1 denote the same world.

Definition 3. If s is a world symbol then the *denotation* of s, [s], is defined as follows: (i) if s is ground, then $[s] \in \{\{w\} \mid w \in W\}$, else $[s] \in 2^W$; (ii) if s, t are ground and $s \neq t$, then $[s] \neq [t]$; (iii) if s is not ground, then $[s] = \{w \mid \langle parent(s), w\rangle \in R\}$.

Making the denotation of a ground symbol a singleton set instead of a possible world simplifies the presentation.

Theorem 1. Two world symbols, s and t, denote the same world for some $w \in W$ iff $[s] \cap [t] \neq \{\}$.

Proofs of theorems, suppressed for reasons of space, can be found in Jackson & Reichgelt [1988]. *World unification* can now be defined as follows.

Definition 4. Two world-indices i and j *w-unify* with unification σ iff:
(i) start(i) = start(j), and
(iia) if end(i) and end(j) are ground and end(i) = end(j), then $\sigma = \{\}$, else
(iib) if end(i) is ground and end(j) is a world variable and $\langle parent(end(j)), end(i)\rangle \in R$, then $\sigma = \{end(i)/end(j)\}$, else
(iic) if end(i) and end(j) are world variables and either $\langle parent(end(j)), end(i)\rangle \in R$ or $\langle parent(end(i)), end(j)\rangle \in R$ or parent(end(i)) w-unifies with parent(end(j)), then $\sigma = \{end(i)/end(j)\}$.

By convention, the numeral 0 denotes the real world, and this will be the start symbol for all indices considered below.

The only difficult case is (iic), where neither end symbol is ground, so we are dealing with two arbitrary worlds accessible from their respective parent worlds. If one of the worlds is accessible from the parent of the other, or the parents can be shown to be identical (by a recursive application of w-unification), then the two worlds can be deemed identical. However, the argument applies only if we can assume that world variables always have a non-empty denotation. Thus we insist that R be serial in any application of case (iic). This also applies when a variable occurs in the ground symbol of (iib), e.g. as an argument to a skolem function.

It can now be shown that w-unification is both sound and complete with respect to standard models.

Theorem 2. Two world indices $s_1:...:s_m$ and $t_1:...:t_n$ w-unify iff s_1 and t_1 denote the same world.

Proof is by case analysis on the definition of w-unification.

In the propositional case, we allow the *modal unification* (m-unification) of two formulas iff the formulas are identical and their indices w-unify. In the first order case, the valuation function V assigns to the symbols of the language and we induce an assignment to the complex expressions. Complementarity now requires that there be a unification that renders two formulas of opposite truth value identical in the same world.

The only interesting departure from the treatment given above is where two indexed formulas are not allowed to m-unify because the substitutions derived from the formulas and the indices are not consistent.

Definition 5. Two first order formulas with associated indices p_i and q_j *m-unify* iff (i) formulas p and q unify with unification θ; (ii) indices i and j w-unify with unification σ; (iii) θ and σ are consistent.

Theorem 3. Two indexed formulas m-unify under accessibility relation R iff they are R-complementary.

Proof follows straightforwardly from Theorems 1-2 and Definitions 1-5.

3 Proof Theory with Barcan Formula

The proof theory that we define is sequent based. We define a sequent as $S \leftarrow T$, where S, T are possibly empty sets of formulas with world indices associated with them. If S and T are both empty, then we call the sequent empty. The reading of $S \leftarrow T$ is that if all the formulas in T are true then at least one of the formulas in S is true.

Let S, T, S_1, S_2, T_1, T_2 be sets of indexed formulas and $S\sigma$ be the result of applying substitution σ to S. Let i and i' be arbitrary indices, and p and q be any propositions. Let $\Pi(x)$ be any propositional function of x, and $\Pi(a/x)$ be the result of uniformly substituting a for x in Π. Vertical bars will delimit the scope of indices with respect to compound formulas, e.g. $|p \supset q|_i$. Then we have the following inference rules.

R1. If $S_1, p_i \leftarrow T_1$ and $S_2, p'_j \leftarrow T_2$, and p_i, p'_j m-unify with unification σ, then $S_1\sigma, S_2\sigma \leftarrow T_1\sigma, T_2\sigma$.

R2. If $S, |p \supset q|_i \leftarrow T$ then $S, q_i \leftarrow p_i, T$.

R3. If $S \leftarrow |p \supset q|_i, T$ then $S \leftarrow q_i, T$.

R4. If $S \leftarrow |p \supset q|_i, T$ then $S, p_i \leftarrow T$.

R5. If $S, \neg p_i \leftarrow T$ then $S \leftarrow p_i, T$.

R6. If $S \leftarrow \neg p_i, T$ then $S, p_i \leftarrow T$.

R7. If $S \leftarrow Lp_i, T$ then $S \leftarrow p_{n:i}, T$ where
(i) n is a new ground world symbol if i is a ground index and p does not contain any free variables
(ii) else n is $f(w_j, ..., w_k, x_1, ... x_m)$ where f is a skolem function of world variables $w_j, ..., w_k$ and free individual variables $x_1, ... x_m$ in p.

R8. If $S, Lp_i \leftarrow T$ then $S, p_{w:i} \leftarrow T$ where w is a new world variable.

R9. If $S \leftarrow |(\forall x)\Pi(x)|_i, T$ then $S \leftarrow |\Pi(c/x)|_i, T$ where
(i) if p contains no free variables and i is a ground index, then c is a new constant
(ii) else c is $f(w_j, ..., w_k, x_1, ... x_m)$ where f is a skolem function of world variables $w_j, ..., w_k$ and free individual variables $x_1, ... x_m$ in p.

R10. If $S, |(\forall x)\Pi(x)|_i \leftarrow T$ then $S, |\Pi(y/x)|_i \leftarrow T$ where y is a new individual variable.

A proof of a formula α is defined as a finite sequence of sequents $\Sigma_0, \ldots, \Sigma_n$ where Σ_0 is the sequent $\leftarrow |\alpha|_0$, Σ_n is the empty sequent, and every sequent but Σ_0 has been obtained from one or more previous sequents by applying an inference rule.

Thus every proof consists of attempting to construct a countermodel for the formula in question by showing that its negation has a model. Every successful proof discovers a contradiction in the putative countermodel.

Example 1. Now consider the proof of the Barcan formula (BF) in the weakest normal system, K.

1		$\leftarrow$	$\vert(\forall x)L\Pi(x) \supset L(\forall x)\Pi(x)\vert_0$	
2	$\vert(\forall x)L\Pi(x)\vert_0$	$\leftarrow$		R4, 1
3	$\vert L\Pi(y/x)\vert_0$	$\leftarrow$		R10, 2
4	$\vert\Pi(y/x)\vert_{w:0}$	$\leftarrow$		R8, 3
5		$\leftarrow$	$\vert L(\forall x)\Pi(x)\vert_0$	R3, 1
6		$\leftarrow$	$\vert(\forall x)\Pi(x)\vert_{1:0}$	R7,5
7		$\leftarrow$	$\vert\Pi(c/x)\vert_{1:0}$	R9, 6
8		$\leftarrow$		R1, 4, 7

The proof succeeds with substitution {1/w, c/y} at line 8. But the critical step is line 4, where we allow y to range over individuals in w, an arbitrary world accessible from world 0.

Example 2. The proof of the converse of the Barcan formula (FB) is also straightforward in K:

1		$\leftarrow$	$\vert L(\forall x)\Pi(x) \supset (\forall x)L\Pi(x)\vert_0$	
2	$\vert L(\forall x)\Pi(x)\vert_0$	$\leftarrow$		R4, 1
3	$\vert(\forall x)\Pi(x)\vert_{w:0}$	$\leftarrow$		R8, 2
4	$\vert\Pi(y/x)\vert_{w:0}$	$\leftarrow$		R10, 3
5		$\leftarrow$	$\vert(\forall x)L\Pi(x)\vert_0$	R3, 1
6		$\leftarrow$	$\vert L\Pi(c/x)\vert_0$	R9, 5
7		$\leftarrow$	$\vert\Pi(c/x)\vert_{1:0}$	R7, 6
8		$\leftarrow$		R1, 4, 7

The crucial step is line 7, where we effectively 'export' an individual from world 0 to world 1.

Example 3. Consider the proof of $L(\forall x)\Pi(x) \supset LL(\forall x)\Pi(x)$ in K4. Lines 2-4 are identical with Example 2.

5	$\leftarrow$	$\vert LL(\forall x)\Pi(x)\vert_0$	R3, 1
6	$\leftarrow$	$\vert L(\forall x)\Pi(x)\vert_{1:0}$	R7, 5
7	$\leftarrow$	$\vert(\forall x)\Pi(x)\vert_{2:1:0}$	R7, 6
8	$\leftarrow$	$\vert\Pi(c/x)\vert_{2:1:0}$	R9, 7

We can only apply R1 and resolve lines 4 and 8 with substitution {c/y, 2/w} if R is transitive, since 2 must be accessible from 0, the parent of w.

Example 4. Finally, consider the proof of $L(\forall x)\Pi(x) \supset L(\forall x)L\Pi(x)$ in K4. Lines 2-4 are identical with Example 2.

5	$\leftarrow$	$\vert L(\forall x)L\Pi(x)\vert_0$	R3, 1
6	$\leftarrow$	$\vert(\forall x)L\Pi(x)\vert_{1:0}$	R7, 5
7	$\leftarrow$	$\vert L\Pi(c/x)\vert_{1:0}$	R9, 6
8	$\leftarrow$	$\vert\Pi(c/x)\vert_{2:1:0}$	R7, 7

Note that the sequent at line 8 in Example 4 is identical with the sequent at line 8 in Example 3. Yet individual, c, was introduced in world 2 in Example 3 and world 1 in Example 4. The notation of the original proof method does not permit such distinctions.

4 Proof Theory without Barcan Formula

A first order modal model is a structure (W, R, U V), where R is a relation on W^2 as before, U is a universe of individuals, and V is a set of valuation functions V_w, one for each $w \in W$.

If BF and FB hold, then U is common to all worlds. For all constants c in the language, $V_w(c) \in U$, while for all k-place predicates p_k, $V_w(p_k) \in U^k$. A ground atomic formula $p_k(c_1, \ldots, c_k)$ is then true at world w just in case $(V_w(c_1), \ldots, V_w(c_k)) \in V_w(p_k)$, and $V_w((\forall x)p_k(\ldots x \ldots))$ = true iff, for each V'_w that is just like V_w except that it assigns some member of U to x, $V_w(p_k(\ldots x \ldots))$ = true.

If BF and FB do not hold, then U is a collection of universes U_w, one for each $w \in W$. Now $V_w(p_k) \in (U^+)^k$, where U^+ is the union of all the sets in U. $V_w((\forall x)p_k(\ldots x \ldots))$ = true iff, for each V'_w that is just like V_w except that $V_w(x) \in U_w$, $V_w(p_k(\ldots x \ldots))$ = true.

The latter follows Kripke [1963] in all essentials. It interprets 'everything has p in w' as 'everything in the universe of w has p.'

To generalize the proof theory, we need to complicate our notation. In addition to indices on a formula, we also attach *term indices* to the individual terms in a formula to indicate in which world the terms were introduced. This requires a more restrictive definition of complementarity between formulas, which depends on the notion of an individual in one world being the 'counterpart' of another individual in another world.

Definition 6. Two indexed terms, c^i and d^j, are *R-counterparts* iff they denote the same individual under accessibility relation R.

Definition 7. Two indexed formulas are now *RT-complementary* iff they are R-complementary and their corresponding indexed terms are R-counterparts.

Definition 8. If $c^{s:\ldots:0}$ is an indexed term, then the denotation of c with respect to [s], $[c]_s$, is defined as follows. Let U_s be $\{u \mid u \in U_w$ for some $w \in [s]\}$.
(i) If c is ground, then $[c]_s \in \{\{u\} \mid u \in U_s\}$.
(ii) If c is not ground, then $[c]_s = U_s$.
(iii) If c, d are ground and $c \neq d$, then $[c]_s \neq [d]_s$.

If c is a term, then the relationship between $[c]_s$ where s is some world symbol and $V_w(c)$ where w is some world can be explicated as follows. $[c]_s$ will always be a set of individuals, whereas $V_w(c)$ will always be an individual. If c is ground, then $[c]_s \in \{\{V_w(c)\} \mid w \in [s]\}$, else $[c]_s = \{V'_w(c) \mid w \in [s]$ and V'_w is just like V_w except that it assigns some $u \in U_w$ to c$\}$.

Theorem 4. Two indexed terms, $c^{s:...:0}$ and $d^{t:...:0}$, denote the same individual iff $[c]_{st} \cap [d]_{st} \neq \{\}$, where $[c]_{st}$ is the denotation of c with respect to $[c]_s \cap [c]_t$ and $[d]_{st}$ is the denotation of d with respect to $[d]_s \cap [d]_t$.

Such an individual will be a member of $U_{st} = \{u \mid u \in U_w$ for some $w \in [s] \cap [t]\}$. We can now define *term unification* as follows.

Definition 9. Two indexed terms, c^i and d^j, *t-unify* iff (i) terms c and d unify with unification θ; (ii) indices i and j w-unify with unification σ; (iii) θ and σ are consistent.

Theorem 5. Two indexed terms t-unify iff they are counterparts.

We also need to extend m-unification to *modal term unification*, so that two indexed formulas unify only if their indexed terms t-unify.

Definition 10. Two indexed formulas p_i and q_j *mt-unify* iff (i) p_i and q_j m-unify with unification θ; (ii) corresponding indexed terms in p and q t-unify with unification σ; (iii) θ and σ are consistent.

Theorem 6. Two indexed formulas mt-unify iff they are RT-complementary.

This leads to the following modification to R1.

R1'. If $S_1, p_i \leftarrow T_1$ and $S_2, p'_j \leftarrow T_2$,
and p_i, p'_j mt-unify with unification σ,
then $S_1\sigma, S_2\sigma \leftarrow T_1\sigma, T_2\sigma$.

Now we need to modify R9 and R10, so that term indices are appropriately introduced.

R9'. If $S \leftarrow |(\forall x)\Pi(x)|_i$, T then $S \leftarrow |\Pi(c/x)^i|_i$, T where
(i) if p contains no free variables and i is a ground index, then c is a new constant
(ii) else c is $f(w_j, ..., w_k, x_1, ..., x_m)$ where f is a skolem function of world variables $w_j, ..., w_k$ and free individual variables $x_1, ..., x_m$ in p.

R10'. If S, $|(\forall x)\Pi(x)|_i \leftarrow T$ then S, $|\Pi(y/x)^i|_i \leftarrow T$
where y is a new individual variable.

These modifications are sufficient to frustrate the critical step in Example 1; lines 4 and 7 are now

4	$\lvert\Pi(y/x)^0\rvert_{w:0}$	$\leftarrow$	R8, 3
7		$\leftarrow \lvert\Pi(c/x)^{1:0}\rvert_{1:0}$	R9', 6

The term indices fail to unify, so the proof fails. If domains are allowed to vary between worlds, then there is no reason why c shuld be in the range of y.

The last line of Example 2 requires that we resolve $|\Pi(y/x)^{w:0}|_{w:0}$ and $|\Pi(c/x)^0|_{1:0}$. Here the term indices unify, but the substitution so derived is not consistent with the substitution derived from the unification of the world indices. Given variable domains, there is no reason to suppose that c will exist in an arbitrary world accessible to world 0.

In Example 3, we can happily resolve $|\Pi(y/x)^{w:0}|_{w:0}$ with $|\Pi(c/x)^{2:1:0}|_{2:1:0}$ under transitivity. This is as it should be, since y can range over individuals in any world accessible from 0, and 2 is accessible from 0. In Example 4, however, we cannot resolve $\Pi(y/x)^{w:0}|_{w:0}$ and $|\Pi(c/x)^{1:0}|_{2:1:0}$ under any accessibility relation. c was introduced in world 1, and may not exist in world 2. The new proof method thus enables distinctions that were beyond the scope of the original method.

Lest this seem unduly restrictive, there are still conditions under which we can let variables in one world range over individuals in another, even under the weaker semantics.

Theorem 7. BF is true at world w if U_v is a subset of U_w for all v such that wRv, and FB is true at world w if U_w is a subset of U_v for all v such that wRv.

The reader is invited to consult Hughes & Cresswell [1968, Ch.10] for the background. The obvious corrolaries are that BF and FB are true in a model iff they are true at every world in the model, and valid iff they are true in all models. However, we need not require BF and FB to be valid, or even true in a model, to apply Theorem 7 to individual worlds when attempting to construct a countermodel for some formula.

Theorem 7 suggests the following amendments to R7-R8.

R7'. If $S \leftarrow Lp_i$, T, then $S \leftarrow Lp_{n:i}$, T iff
$U_{parent(n)}$ is a subset of U_w for all w such that $\langle parent(n), w\rangle \in R$ where
(i) if i is a ground index and p contains no free variables, then n is a new ground world symbol
(ii) else n is $f(w_j, ..., w_k, x_1, ..., x_m)$ where f is a skolem function of world variables $w_j, ..., w_k$ and free individual variables $x_1, ..., x_m$ in p.

R8'. If S, $Lp_i \leftarrow T$, then S, $p_{w:i} \leftarrow T$ iff
U_w is a subset of $U_{parent(n)}$ for all w such that $\langle parent(n), w\rangle \in R$ where w is a new world variable.

Alternative modifications to R7-R8, in conjunction with R1', R2-R6 and R9'-R10, will enable the derivation of one of BF and FB, but not the other. Retaining R8, but replacing R7 with the following enables FB.

R7". If $S \leftarrow Lp_i$, T, then $S \leftarrow Lp(n:i/i)_{n:i}$, T where
(i) if i is a ground index and p contains no free variables, then n is a new ground world symbol
(ii) else n is $f(w_j, ..., w_k, x_1, ..., x_m)$ where f is a skolem function of world variables $w_j, ..., w_k$ and free individual variables $x_1, ..., x_m$ in p

(iii) p(n:i/i) is the result of uniformly substituting n:i for i throughout p.

Retaining R7, but replacing R8 with the following enables BF.

R8". If S, $Lp_i \leftarrow$ T, then S, $p(w{:}i/i)_{w:i} \leftarrow$ T where
(i) w is a new world variable and
(ii) p(w:i/i) is the result of uniformly substituting w:i for i throughout p.

Allowing term indices to be updated in step with world indices ensures that the corresponding subset relations between universes always hold. Thus R7" ensures that no individuals disappear as we pass from world to world, while R8" ensures that no new individuals come into existence. These rules are useful in applications where universes shrink or expand monotonically.

5 Related Work

Moore [1985] proposes a modal logic of knowledge which is essentially a first-order axiomatization of the model theory of S4. Variables of quantification in the metalanguage range over rigid designators, i.e. terms that have the same denotation in each possible world (p.335). Thus his semantics preserves both BF and FB.

Abadi & Manna [1986] present a non-clausal resolution proof method for several systems of modal logic. There are different inference rules for different systems, so the method is more complex than ours. In particular, there are complicated rules for extracting quantifiers from within formulas. Inference rules can introduce new operators, unlike our rules which only eliminate operators. The Barcan formula and its converse always hold in their semantics (p.178).

Konolige [1986, Section 3.3] takes *id constants* supplied by a 'naming map' to be rigid designators which always denote the extension of an individual's name in the actual world. He then follows Kripke in extending all valuation functions to cover every individual in a model, so that neither BF nor FB is valid in his semantics. The treatment of quantification in his deduction model of belief is therefore similar in spirit to our treatment in a possible worlds model, if the naming map is partial.

Unlike Konolige, we do not define the value of the denotation function from indexed terms to possible individuals as the term's denotation in the real world. Indeed, the term may not have a denotation in the real world, if BF is not valid. Another difference is that we allow for the case where BF or FB are true in certain worlds without being valid. Here the relevant consideration is not the partial nature of the naming map, but the relations of set inclusion that hold among the universes of worlds that are accessible to each other. Finally, our method is more general, because we take the prevailing accessibility relation into account when computing complementarity.

Wallen's matrix proof method [1987] is most closely related to ours, in that formulas are given *prefixes* which stand for worlds in which they are true. Such prefixes do not contain variables, and therefore do not use skolem functions to encode dependencies, as we do. Dependencies are encoded in the order of symbols in a prefix, and modal substitutions are derived which render prefixes identical. Wallen's inference rules do not appear to be commutative, as ours are; different orders of application result in different dependencies, not all of which may be resolved.

Wallen defines two notions of complementarity, one for constant and one for varying domains. The latter encodes the interaction between modal substitutions and first-order substitutions (which render formulas identical). His presentation is not couched in terms of the validity of BF and FB, although any such encoding must invalidate them both for varying domains.

In summary, this paper presents a proof method for modal predicate logics without the Barcan formula or its converse. The method is suitable for theorem proving in all fifteen normal systems, including applications with varying domains. The key technique is that of mt-unification, in which we insist that corresponding terms have the same denotation. The proof method has been fully implemented, and can be shown to be sound and complete [Jackson & Reichgelt, in preparation]. An adaptation of the method for nonmonotonic reasoning is described in [Jackson, 1988; Jackson & Reichgelt, in press].

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