A generalization of carnap's inductive logic

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A GENERALIZATION OF CARNAP'S INDUCTIVE LOGIC

1. Carnap's continuum of inductive methods for a family of monadic predicates, the so-called λ-continuum or λ-system, is restricted by the assumption that the size of the family is known at the start of the learning process. In the proper inductive situation we do not have this information.

In this article a two-dimensional system will be constructed that is appropriate to deal with the proper inductive situation. This system describes a learning process both for relative frequencies of properties and for the size of the family; it may therefore be called a generalization of Carnap's continuum.

2. The λ-system can be formulated in an interpreted language as follows. The individuals $(t_1, t_2, \ldots)$, called trials, are successive throws with a ball. The ball has at most k colours $(P_1, P_2, \ldots, P_k)$ on its surface. The result of a trial is supposed to be uniquely determinable as the colour of the rest-point.

Let e^s state the result of the first s trials, such that it includes the information, for every P_i, how many trials s_i have resulted in P_i. Let h_i be the hypothesis that the next trial will result in P_i.

According to the λ-system the 'special value' inp $(h_i \mid e^s)$, the inductive probability of h_i on the evidence e^s, is: $(s_i + \lambda/k)/(s + \lambda)$. λ is a parameter for a positive real number.

3. The fundamental assumption of the λ-system is that the size of the family of colours is known to be k. Let us now assume that we do not have this information, but that all other things remain the same.

In order to obtain a suitable terminology we will give each new observed colour a new name in a systematic way. If there have been observed n different colours in the first s trials and if at the $(s+1)$th trial a new colour is instantiated then we will give that colour the new name P_{n+1}.

Let e^s state the result of the first s trials, such that it includes the information how many different colours n are instantiated, and $s_i (1 \leqslant i \leqslant n)$,

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the number of trials that have resulted in P_i. The hypothesis that the next trial will result in $P_i (1 \leqslant i \leqslant n)$ is symbolized by h_i. H is the hypothesis that a new colour will be instantiated at the next trial.

4. Because h_i logically implies $\sim H$ we can prove easily the following basic theorem:

$$\mathrm{inp}(h_i \mid e^s) = \mathrm{inp}(\sim H \mid e^s) \cdot \mathrm{inp}(h_i \mid e^s \,\&\, \sim H); \quad 1 \leqslant i \leqslant n \leqslant s.$$

The second factor at the right side is obviously equal to the corresponding special value of the λ-system with $k = n$, viz. $(s_i + \lambda/n)/(s + \lambda)$. But this function can also be obtained independently along the same lines as Carnap has done for the λ-system.

In order to determine the first factor in the above equation we have to analyse the learning process in detail. There are essentially two sequences of events: the sth trial results in a certain colour resp. that colour is, or is not, already instantiated before the sth trial. The second kind of event is a second order event for it is a relation between the sth event of the first sequence and all the preceding events in that sequence. A consequence of this relational aspect is that H and $\sim H$ start to be interesting after the first trial. The relative frequency of the event 'a new colour' remains the ratio of the number of instantiated colours to the number of trials.

Because there are in the second sequence only two possible events, viz. 'a new colour' or 'a previously instantiated colour', and because these events are mutually exclusive, the second sequence is a proper example to be treated by the λ-system with $k = 2$. There is no reason to identify the parameter for this sequence with that of the first; therefore we introduce the new parameter δ, again for a positive real number.

The two special values of the second sequence that we obtain in this way are:

$$\left.\begin{aligned} \mathrm{inp}(H \mid e^s) &= \frac{n + \delta/2}{s + \delta} \\ \mathrm{inp}(\sim H \mid e^s) &= 1 - \mathrm{inp}(H \mid e^s) = \frac{s - n + \delta/2}{s + \delta} \end{aligned}\right\} \quad 1 \leqslant n \leqslant s.$$

Combining both results we obtain as the special values of the first sequence for instantiated colours:

$$\mathrm{inp}(h_i \mid e^s) = \frac{s - n + \delta/2}{s + \delta} \cdot \frac{s_i + \lambda/n}{s + \lambda}; \quad 1 \leqslant i \leqslant n \leqslant s.$$

Using the laws of the probability calculus we can extrapolate the system to other than special values. The complete system will be called the λ-δ-system. The formulation of the whole system, however, will be much less transparent than the formulation that could be used to present the special values.

It remains to be investigated whether the λ-δ-system has any shortcoming relative to the inductive situation under discussion. But in the last section we will only try to clarify some aspects of the system.

5. If n does not exceed a certain finite N and if s goes to infinity, then the special value for an instantiated colour goes to s_i/s, the same as in the λ-system. Under the mentioned conditions $\mathrm{inp}(H \mid e^s)$ goes to 0; this value is the relative frequency of the event 'a new colour' in an infinite domain if the size of the family of colours is finite.

If s is very large and n is small relative to s then it is very probable but never certain that n is the actual size of the family. In this situation it may be better to introduce, for computational reasons, the new property P_{n+1} defined as the complementary property of the first n properties, and we obtain a Carnapian problem situation with $n + 1$ properties and $s_{n+1} = 0$. This procedure can certainly also be followed at an earlier stage, but then we close our eyes for new and perhaps valuable distinctions.

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