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A GENERALIZATION OF DAVENPORT'S CONSTANT AND ITS ARITHMETICAL APPLICATIONS

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1. For an additively written finite abelian group G, Davenport's constant $D(G)$ is defined as the maximal length d of a sequence $(g_1, \ldots, g_d)$ in G such that $\sum_{j=1}^{d} g_j = 0$, and $\sum_{j\in J} g_j \neq 0$ for all $\emptyset \neq J \subsetneq \{1, \ldots, d\}$. It has the following arithmetical meaning:

Let K be an algebraic number field, R its ring of integers and G the ideal class group of R. Then $D(G)$ is the maximal number of prime ideals (counted with multiplicity) which can divide an irreducible element of R. This fact was first observed by H. Davenport (1966) and worked out by W. Narkiewicz [8] and A. Geroldinger [4].

For a subset $Z \subset R$ and $x > 1$ we denote by $Z(x)$ the number of principal ideals (α) of R with $\alpha \in Z$ and $(R : (\alpha)) \leq x$. If M denotes the set of irreducible integers of R, then it was proved by P. Rémond [12] that, as $x \to \infty$,

$$M(x) \sim Cx(\log x)^{-1}(\log\log x)^{D(G)-1},$$

where $C > 0$ depends on K; the error term in this asymptotic formula was investigated by J. Kaczorowski [7].

If an element $\alpha \in R \setminus (R^\times \cup \{0\})$ has a factorization $\alpha = u_1 \cdot \ldots \cdot u_r$ into irreducible elements $u_j \in R$, we call r the *length* of that factorization and denote by $L(\alpha)$ the set of all lengths of factorizations of α. For $k \geq 1$, we define sets M_k and M'_k (depending on K) as follows:

M_k consists of all $\alpha \in R \setminus (R^\times \cup \{0\})$ for which $\max L(\alpha) \leq k$ (i.e., α has no factorization of length $r > k$);

M'_k consists of all $\alpha \in R \setminus (R^\times \cup \{0\})$ for which $\min L(\alpha) \leq k$ (i.e., α has a factorization of length $r \leq k$).

If $G = \{0\}$, then $M_k = M'_k$ for all k; in the general case, we have $M_1 = M'_1 = M$ and $M_k \subset M'_k$ for all k.

In this paper, we generalize the results of Rémond and Kaczorowski and obtain asymptotic formulas for $M_k(x)$ and $M'_k(x)$. To do this, we shall define a sequence of combinatorial constants $D_k(G)$ $(k \geq 1)$ generalizing $D(G) = D_1(G)$, and we shall obtain the following result.

THEOREM. *For* $x \ge e^e$ *and* $q \in \mathbb{Z}$, $0 \le q \le c_0 \frac{\sqrt{\log x}}{\log\log x}$, *we have*

$$M_k(x) = \frac{x}{\log x}\left[\sum_{\mu=0}^{q} \frac{W_\mu(\log\log x)}{(\log x)^\mu} + O\left((c_1 q)^q \frac{(\log\log x)^{D_k(G)}}{(\log x)^{q+1}}\right)\right]$$

and

$$M_k'(x) = \frac{x}{\log x}\left[\sum_{\mu=0}^{q} \frac{W_\mu'(\log\log x)}{(\log x)^\mu} + O\left((c_1 q)^q \frac{(\log\log x)^{kD(G)}}{(\log x)^{q+1}}\right)\right],$$

where c_0, c_1 *are positive constants, and* $W_\mu, W_\mu' \in \mathbb{C}[X]$ *are polynomials such that* $\deg W_\mu \le D_k(G)$, $\deg W_\mu' \le kD(G)$, $\deg W_0 = D_k(G) - 1$, $\deg W_0' = kD(G) - 1$, *and* W_0, W_0' *have positive leading coefficients.*

Remarks. 1) For $k = 1$, this is [7, Theorem 1].

2) For $G = \{0\}$, we shall see that $D_k(G) = k$, and we rediscover [9, Ch. IX, § 1, Corollary 1].

3) In another context, the number $M_k'(x)$ was studied in [6].

The main part of this paper is devoted to the definition and the investigation of the invariants $D_k(G)$ and is of purely combinatorial nature. Only in the final section shall we present a proof of the above Theorem using the work of Kaczorowski.

2. Let G be an additively written finite abelian group. We denote by $\mathcal{F}(G)$ the (multiplicatively written) free abelian semigroup with basis G. In $\mathcal{F}(G)$, we use the concept of divisibility in the usual way: $S' \mid S$ if $S = S'S''$ for some $S'' \in \mathcal{F}(G)$. Every $S \in \mathcal{F}(G)$ has a unique representation

$$S = \prod_{g \in G} g^{v_g(S)}$$

with $v_g(S) \in \mathbb{N}_0$; we call

$$\sigma(S) = \sum_{g \in G} v_g(S) \in \mathbb{N}_0$$

the *size* and

$$\iota(S) = \sum_{g \in G} v_g(S) \cdot g \in G$$

the *content* of S. The semigroup

$$\mathcal{B}(G) = \{B \in \mathcal{F}(G) \mid \iota(B) = 0\} \subset \mathcal{F}(G)$$

is called the *block semigroup* of G; we set $\mathcal{B}(G)' = \mathcal{B}(G) \setminus \{1\}$ where $1 \in \mathcal{F}(G)$ denotes the unit element. Every $B \in \mathcal{B}(G)'$ has a factorization $B = B_1 \cdot \ldots \cdot B_r$ into irreducible blocks $B_i \in \mathcal{B}(G)'$; again, we call r the *length* of the factorization and denote by $L(B)$ the set of all lengths of factorizations of B in $\mathcal{B}(G)$. Obviously, B is irreducible if and only if $L(B) = \{1\}$, and $D(G) = \max\{\sigma(B) \mid B \in \mathcal{B}(G)' \text{ is irreducible}\}$.

Now we define, for $k \ge 1$,

$$D_k(G) = \sup\{\sigma(B) \mid B \in \mathcal{B}(G)',\ \max L(B) \le k\}\,.$$

Obviously, $D_1(G) = D(G)$, and we shall see in a moment that $D_k(G) < \infty$ for all $k \ge 1$.

PROPOSITION 1. *Let G be a finite abelian group and $k \in \mathbb{N}$.*

(i) $kD(G) = \max\{\sigma(B) \mid B \in \mathcal{B}(G)',\ \min L(B) \le k\}$
$\qquad = \max\{\sigma(B) \mid B \in \mathcal{B}(G)',\ k \in L(B)\}$.

(ii) $D_k(G) \le kD(G) < \infty$.

(iii) $D_k(G) = \max\{\sigma(B) \mid B \in \mathcal{B}(G)',\ \max L(B) = k\}$.

(iv) *$D_k(G)$ is the smallest number $d \in \mathbb{N}$ with the property that, for every $S \in \mathcal{F}(G)$ with $\sigma(S) \ge d$, there exist blocks $B_1, \ldots, B_k \in \mathcal{B}(G)'$ such that $B_1 \cdot \ldots \cdot B_k \mid S$.*

(v) *If $B \in \mathcal{B}(G)$ is a block satisfying $\sigma(B) > kD(G)$, then there exist blocks $B_1, \ldots, B_{k+1} \in \mathcal{B}(G)'$ such that $B = B_1 \cdot \ldots \cdot B_{k+1}$.*

(vi) *If $G_1 \subsetneq G$ is a proper subgroup, then $D_k(G_1) < D_k(G)$.*

Proof. (i) If $B \in \mathcal{B}(G)'$ is a block such that $\min L(B) \le k$, then there exists a factorization $B = B_1 \cdot \ldots \cdot B_l$ into irreducible blocks $B_j \in \mathcal{B}(G)'$ of length $l \le k$, and therefore

$$\sigma(B) = \sum_{j=1}^{l} \sigma(B_j) \le D(G) \le kD(G)\,.$$

Hence it is sufficient to prove that there exists a block $B \in \mathcal{B}(G)$ such that $\sigma(B) = kD(G)$ and $k \in L(B)$. But if $B_0 \in \mathcal{B}(G)'$ is an irreducible block with $\sigma(B_0) = D(G)$, then $B = B_0^k$ has the required property.

(ii) follows immediately from (i) and the definition of $D_k(G)$.

(iii) Let l be the maximal length of a factorization of a block $B \in \mathcal{B}(G)'$ with $\max L(B) \le k$ and $\sigma(B) = D_k(G)$. If $l < k$, then the block $\overline{B} = B \cdot 0$ satisfies $\sigma(\overline{B}) = D_k(G) + 1$ and $\max L(\overline{B}) = l + 1 \le k$, which contradicts the definition of $D_k(G)$.

(iv) In order to prove that $D_k(G)$ has the indicated property, let $S \in \mathcal{F}(G)$ be such that $\sigma(S) \ge D_k(G)$, set $g = -\iota(S) \in G$ and consider the block $S_g \in \mathcal{B}(G)'$. Since $\sigma(Sg) > D_k(G)$, the block Sg has a factorization of length $\nu > k$, say $Sg = B_1 \cdot \ldots \cdot B_\nu$ with irreducible $B_j \in \mathcal{B}(G)'$ and $v_g(B_\nu) > 0$. This implies $B_1 \cdot \ldots \cdot B_k \mid S$, as asserted.

In order to prove that $D_k(G)$ is minimal with this property, let $B \in \mathcal{B}(G)$ be a block satisfying $\sigma(B) = D_k(G)$ and $\max L(B) = k$, according to (iii). If $B = \prod_{j=1}^{D_k(G)} g_j$ and $d < D_k(G)$, then the element $S_d = \prod_{j=1}^{d} g_j \in$

$\mathcal{F}(G)$ cannot be divisible by a product of k blocks, for this would imply $\max L(B) \geq k+1$.

(v) If $B = g_1 \cdot \ldots \cdot g_\nu$ with $\nu > kD(G)$ then, by (iv), there exist blocks $B_1, \ldots, B_k \in \mathcal{B}(G)'$ such that $B_1 \cdot \ldots \cdot B_k \mid g_1 \cdot \ldots \cdot g_{\nu-1}$, and therefore the assertion follows.

(vi) By (iii), there exists a block $B = g_1 \cdot \ldots \cdot g_N \in \mathcal{B}(G_1)$ such that $N = \sigma(B) = D_k(G_1)$ and $\max L(B) = k$. We pick an element $g \in G \setminus G_1$ and assume that $D_k(G_1) \geq D_k(G)$. By (iv), there exist blocks $B_1, \ldots, B_k \in \mathcal{B}(G)'$ such that $B_1 \cdot \ldots \cdot B_k \mid g_1 \cdot \ldots \cdot g_{N-1} g$; this implies $B_1, \ldots, B_k \in \mathcal{B}(G_1)'$, and therefore there exists a block $B_{k+1} \in \mathcal{B}(G_1)'$ such that $B = B_1 \cdot \ldots \cdot B_k B_{k+1}$, a contradiction. ∎

3. The precise value of $D(G)$ is known only for some special types of abelian groups [2], [3]; see [5] for a survey. In the following proposition we collate those results which we shall either use or generalize in the sequel.

For $n \geq 1$, let C_n be the cyclic group of order n.

PROPOSITION 2. *Let $G = \bigoplus_{i=1}^d C_{n_i}$ be a finite abelian group with $1 < n_d \mid n_{d-1} \mid \ldots \mid n_1$, and set*

$$M(G) = n_1 + \sum_{i=2}^{d} (n_i - 1)\,.$$

(i) *$M(G) \leq D(G) \leq \#G$.*

(ii) *If either $d \leq 2$ or G is a p-group, then $M(G) = D(G)$.*

Proof. [10], [11]; see also [1].

PROPOSITION 3. *Let G be a finite abelian group and $k \in \mathbb{N}$.*

(i) *If $G = G' \oplus G''$, then $D_k(G) \geq D_k(G') + D(G'') - 1$.*

(ii) *If $G = \bigoplus_{i=1}^d C_{n_i}$ with $1 < n_d \,|\, n_{d-1} \,|\, \ldots \,|\, n_1$, then $D_k(G) \geq kn_1 + \sum_{i=2}^d (n_i - 1)$.*

(iii) *$D_k(C_n) = kn$.*

Proof. (i) By Proposition 1(iv), there exist elements $S' \in \mathcal{F}(G')$ and $S'' \in \mathcal{F}(G'')$ such that $\sigma(S') = D_k(G') - 1$, S' is not divisible by a product of k blocks from $\mathcal{B}(G')'$ and $\sigma(S'') = D(G'') - 1$, S'' is not divisible by a block of $\mathcal{B}(G'')'$. If $S' = \prod_{j=1}^{D_k(G')-1} g'_j$ and $S'' = \prod_{j=1}^{D(G'')-1} g''_j$, then the element

$$S = \prod_{j=1}^{D_k(G')-1} (g'_j, 0) \cdot \prod_{j=1}^{D(G'')-1} (0, g''_j) \in \mathcal{F}(G)$$

is not divisible by a product of k blocks of $\mathcal{B}(G)'$, whence

$$D_k(G) > \sigma(S) = D_k(G') + D(G'') - 2\,,$$

by Proposition 1(iv), as asserted.

(ii) If $G = \langle g_1, \ldots, g_d \rangle$ and $\mathrm{ord}(g_i) = n_i$, then the block

$$B = g_1^{kn_1-1} \cdot (g_1 + \ldots + g_d) \cdot \prod_{j=2}^{d} g_j^{n_j-1} \in \mathcal{B}(G)$$

has a unique factorization into irreducible blocks of length k, given by $B = B_1^{k-1} B_0$, where $B_1 = g_1^{n_1}$ and $B_0 = (g_1 + \ldots + g_d) \cdot \prod_{j=1}^{d} g_j^{n_j-1}$. This implies $D_k(G) \geq \sigma(B) = kn_1 + \sum_{j=2}^{d}(n_j - 1)$.

(iii) By Propositions 1 and 2, we have $D_k(C_n) \leq kD(C_n) = kn$, whereas, by (ii), $D_k(C_n) \geq kn$.

4. In this section we generalize the result on groups of rank 2.

PROPOSITION 4. *Let $G = G_1 \oplus G_2$ be a finite abelian group, $\#G_i = n_i$, $n_2 \mid n_1$ and $k \in \mathbb{N}$. Then*

$$D_k(C_n) \leq kn_1 + n_2 - 1\,.$$

For the proof of Proposition 4 we need two technical lemmas.

LEMMA 1. *Let G be a finite abelian group, $m \in \mathbb{N}$, $D(G) < 2m$ and $D(G \oplus C_m) < 3m$. Let $t \in \mathbb{N}$ and $S \in \mathcal{F}(G)$ be such that $\sigma(S) \geq D(G \oplus C_m) + (t-1)m$. Then there exist blocks $B_1, \ldots, B_t \in \mathcal{B}(G)'$ such that $B_1 \cdot \ldots \cdot B_t \mid S$ and $\sigma(B_i) \leq m$ for all $i \in \{1, \ldots, t\}$.*

Proof. It suffices to consider the case $t = 1$, for then the general case follows by a trivial induction argument.

Set $N = D(G \oplus C_m) < 3m$, and let $S = g_1 \cdot \ldots \cdot g_\nu \in \mathcal{F}(G)$ be an element with $\nu = \sigma(S) \geq N$. Let e_m be a generator of C_m, and consider the element

$$S' = \prod_{j=1}^{N} (g_j, e_m) \in \mathcal{F}(G \oplus C_m)\,;$$

by Proposition 1(iv) there exists an irreducible block $S_0' \in \mathcal{B}(G \oplus C_m)'$ such that $S_0' \mid S'$, and we may assume that $S_0' = \prod_{j=1}^{N_0} (g_j, e_m)$ for some $N_0 \leq N$. Since

$$\iota(S_0') = \Big(\sum_{j=1}^{N_0} g_j, N_0 e_m \Big) = (0,0) \in G \oplus C_m\,,$$

we obtain $S_0 = \prod_{j=1}^{N_0} g_j \in \mathcal{B}(G)$ and $m \mid N_0$, whence $m = N_0$ or $2m = N_0$. If $m = N_0$, the assertion follows with $B = S_0$; if $2m = N_0 > D(G)$, then S_0 has a decomposition $S_0 = BB'$ with B, $B' \in \mathcal{B}(G)$ and $\sigma(B) \leq m$, which again implies the assertion. ∎

LEMMA 2. *Let p be a prime, $t \in \mathbb{N}$ and $B \in \mathcal{B}(C_p \oplus C_p)$ a block satisfying $\sigma(B) \geq tp$. Then there exist blocks $B_1, \ldots, B_t \in \mathcal{B}(C_p \oplus C_p)'$ such that $B = B_1 \cdot \ldots \cdot B_t$.*

Proof. The assertion is true for $t = 1$ and also for $t = 2$, as $D(C_p \oplus C_p) = 2p - 1 < 2p$. Therefore we assume that $t \geq 3$ and $B = g_1 \cdot \ldots \cdot g_\nu$ for some $\nu \geq tp$. We apply Lemma 1 with $G = C_p \oplus C_p$, $m = p$ and $S = g_1 \cdot \ldots \cdot g_{tp-1}$. Since $\sigma(S) = tp-1 > (3p-2)+(t-3)p = D(C_p \oplus C_p \oplus C_p)+(t-3)p$, there exist blocks $B_1, \ldots, B_{t-2}, B' \in \mathcal{B}(G)'$ such that $B = B_1 \cdot \ldots \cdot B_{t-2}B'$ and $\sigma(B_j) \leq p$ for all $j \in \{1, \ldots, t-2\}$. This implies

$$\sigma(B') = \sigma(B) - \sum_{j=1}^{t-2} \sigma(B_j) \geq tp - (t-2)p = 2p > D(G)\,,$$

whence $B' = B_{t-1}B_t$ with blocks B_{t-1}, $B_t \in \mathcal{B}(G)'$. ∎

Proof of Proposition 4. By induction on n_2; if $n_2 = 1$, then $D_k(G) = D_k(G_1) \leq kD(G_1) \leq kn_1$ by Proposition 1(ii) and Proposition 2(i).

If $n_2 > 1$, let p be a prime with $p \mid n_2$ and choose subgroups $G_i' \subset G_i$ $(i = 1, 2)$ with $(G_i : G_i') = p$. Set

$$t = kn_1/p + n_2/p\,,$$

and assume that the assertion is true for the subgroup $G' = G_1' \oplus G_2' \subset G$, i.e., $D_k(G') \leq t - 1$. We must prove that every block $B \in \mathcal{B}(G)$ with $\sigma(B) = N \geq kn_1 + n_2$ has a factorization of length $l \geq k + 1$. We set $B = g_1 \cdot \ldots \cdot g_N$ and consider the canonical epimorphism $\pi : G \to C_p \oplus C_p$ with $\ker(\pi) = G'$. The block $B^* = \pi(g_1) \cdot \ldots \cdot \pi(g_N) \in \mathcal{B}(C_p \oplus C_p)$ satisfies $\sigma(B^*) = N \geq tp$ and therefore, by Lemma 2, B^* is a product of t blocks from $\mathcal{B}(C_p \oplus C_p)'$. Taking preimages in G, we obtain a decomposition $B = S_1 \cdot \ldots \cdot S_t$ with $S_i \in \mathcal{F}(G)'$ and $\iota(S_i) = g_i' \in G'$. Since $t > D_k(G')$ and $g_1' \cdot \ldots \cdot g_t' \in \mathcal{B}(G')$, there exist blocks $B_1', \ldots, B_{k+1}' \in \mathcal{B}(G')'$ with $B_1', \ldots, B_{k+1}' \mid g_1' \cdot \ldots \cdot g_t'$ by Proposition 1(v). Hence there exists a decomposition

$$\{1, \ldots, t\} = \bigcup_{\nu=1}^{k+1} J_n \quad \text{(disjoint union)}$$

such that $B_\nu' = \prod_{j \in J_\nu} g_j'$ for all $\nu \in \{1, \ldots, k+1\}$. Putting $B_\nu = \prod_{j \in J_\nu} S_j \in \mathcal{B}(G)$, we obtain $B_1 \cdot \ldots \cdot B_{k+1} \mid B$, and therefore B has a factorization of length $l \geq k + 1$. ∎

PROPOSITION 5. *If $G = C_{n_1} \oplus C_{n_2}$ with $n_2 \mid n_1$, then $D_k(G) = kn_1 + n_2 - 1$.*

Proof. Obvious by Propositions 3 and 4.

5. Proof of the Theorem. Let K be an algebraic number field, R its ring of integers, G the ideal class group, $\mathcal{I}$ the semigroup of non-zero ideals and $\mathcal{H}$ the subsemigroup of non-zero principal ideals of R. We write G additively, and for $J \in \mathcal{I}$ we denote by $[J] \in G$ the ideal class of J. Let $\theta : \mathcal{I} \to \mathcal{F}(G)$ be the unique semigroup homomorphism satisfying $\theta(P) = [P]$ for every maximal P of R. For $J \in \mathcal{I}$, we have $\theta(J) \in \mathcal{B}(G)$ if and only if $J \in \mathcal{H}$. If $\alpha \in R \setminus (R^\times \cup \{0\})$, then $L(\alpha) = L(\theta((\alpha)))$.

Let $\mathcal{M}_k$ be the set of all blocks $B \in \mathcal{B}(G)$ such that $\max L(B) \le k$, and let $\mathcal{M}'_k$ be the set of all blocks $B \in \mathcal{B}(G)$ such that $\min L(B) \le k$. Then

$$M'_k = \{\alpha \in R \setminus (R^\times \cup \{0\}) \mid \theta((\alpha)) \in \mathcal{M}'_k\}$$

and, by Proposition 1,

$$kD(G) = \max\{\sigma(B) \mid B \in \mathcal{M}'_k\}\,, \qquad D_k(G) = \max\{\sigma(B) \mid B \in \mathcal{M}_k\}\,.$$

In particular, the sets $\mathcal{M}_k$ and $\mathcal{M}'_k$ are finite.

After these observations, the Theorem is an immediate consequence of the following Lemma, due to Kaczorowski [7, Lemma 1].

LEMMA 3. *For* $1 \ne S \in \mathcal{F}(G)$, $x \ge e^e$ *and* $q \in \mathbb{Z}$, $0 \le q \le c_0 \dfrac{\sqrt{\log x}}{\log\log x}$, *we have*

$$\#\{J \in \mathcal{I} \mid (R : J) \le x,\ \theta(J) = S\} = \frac{x}{\log x}\left[\sum_{\mu=0}^{q} \frac{W_\mu(\log\log x)}{(\log x)^\mu} + O\left((c_1 q)^q \frac{(\log\log x)^{\sigma(S)}}{(\log x)^{q+1}}\right)\right]$$

with constants $c_0, c_1 \in \mathbb{R}_+$ *and polynomials* $W_\mu \in \mathbb{C}[X]$ *such that* $\deg W_\mu \le \sigma(S)$, $\deg W_0 = \sigma(S) - 1$, *and* W_0 *has a positive leading coefficient.*

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