Ann. Geophysicae 17, 95–100 (1999) © EGS – Springer-Verlag 1999

Annales Geophysicae

A generation mechanism for chorus emission

V. Y. Trakhtengerts

Institute of Applied Physics, Nizhny Novgorod, Russia

Received: 21 November 1997 / Revised: 30 April 1998 / Accepted: 25 May 1998

Abstract. A chorus generation mechanism is discussed, which is based on interrelation of ELF/VLF noise-like and discrete emissions under the cyclotron wave-particle interactions. A natural ELF/VLF noise radiation is excited by the cyclotron instability mechanism in ducts with enhanced cold plasma density or at the plasmapause. This process is accompanied by a step-like deformation of the energetic electron distribution function in the velocity space, which is situated at the boundary between resonant and nonresonant particles. The step leads to the strong phase correlation of interacting particles and waves and to a new backward wave oscillator (BWO) regime of wave generation, when an absolute cyclotron instability arises at the central cross section of the geomagnetic trap, in the form of a succession of discrete signals with growing frequency inside each element. The dynamical spectrum of a separate element is formed similar to triggered ELF/VLF emission, when the strong wavelet starts from the equatorial plane. The comparison is given of the model developed using some satellite and ground-based data. In particular, the appearance of separate groups of chorus signals with a duration 2–10 s can be connected with the preliminary stage of the step formation. BWO regime gives a succession period smaller than the bounce period of energetic electrons between the magnetic mirrors and can explain the observed intervals between chorus elements.

Key words. Magnetospheric physics (Energetic particles, trapped). Space plasma physics (wave-particle interactions; waves and instabilities).

1 Introduction

Chorus emissions are the most intriguing signals among natural ELF/VLF radiation. They consist of a succession of discrete elements with rising frequency, with a repetition period of $T \sim 0.1$–1 s. According to ground-based observations the typical duration of chorus event is 0.5–1 h or more. Satellite measurements show that chorus is generated in a narrow region near the equatorial cross section of a magnetic flux tube, and appear as whistler waves whose wave vector $\mathbf{k}$ is close to the direction of the magnetic field line $\mathbf{H}$. As a rule, the chorus is accompanied by hiss emission which serves as a lower frequency background for the discrete elements. For more detailed information about chorus emissions one can refer to Helliwell (1965, 1969) and Sazhin and Hayakawa (1992).

It is generally accepted that the generation mechanism of chorus is connected with the cyclotron instability (CI) of radiation belt electrons. The similarity of the spectral forms of chorus elements and of triggered ELF/VLF emissions has stimulated the application of the theory of triggered signals to an explanation of chorus. Helliwell's (1967) paper was important in this connection; there, a phenomenological model of the generation of discrete emissions was developed. The mechanism was based on the cyclotron resonance of energetic electrons with whistler mode waves, which manifest themselves as backward wave oscillators. For ducted whistlers of frequency ω with the wave vector $\mathbf{k}\|\mathbf{H}$, the cyclotron resonance condition is

$$\omega - \omega_H = kv \tag{1}$$

where ω_H is the electron gyrofrequency, v is the electron velocity component along the magnetic field, and $k = |\mathbf{k}|$. In that paper, the idea of the second order cyclotron resonance was first formulated; not only does Eq. (1) hold, but its full derivative along the particle path was made to be zero. This was suggested as the necessary condition for the most effective wave-particle interaction in the inhomogeneous (dipolar) magnetic field, when the interaction length was maximal. By using this condition Helliwell (1967) could explain numerous types of discrete emissions with different frequency versus time characteristics.

Further analytical and computational calculations (Nunn, 1974a; Karpman *et al.*, 1974) confirmed the idea of the second order resonance and permitted a connection to be made through nonlinear currents, between the parameters of a triggered emission and those of the initial quasi-monochromatic whistler wave. During the last 20 y numerous analyses have been undertaken to explain different features of triggered emissions. For more detailed information, see the comprehensive reviews of Omura *et al*. (1991) and Helliwell (1993), and papers cited there. The general conclusion of the developed nonlinear theory of triggered emissions can be formulated as follows: the generation of triggered signals of different spectral forms is connected with the formation of electron beams under the action of the initial wave packet, and the generation of secondary waves by these beams under conditions which obey second order cyclotron resonance; those are fulfilled on a long trajectory of the beam due to the change of emitted wave frequency and to nonlinear effects. Some signals with rising frequency appear, if the beam starts from the equatorial cross section of a magnetic flux tube.

In the case of chorus emission their spectral forms could be explained in the same manner (see, for example, Nunn, 1974b, 1986; Vomvoridis *et al*., 1982), but the cause of the beam formation in the case of chorus is not clear: the initial quasi-monochromatic whistler signal is absent. The computational analysis of a cyclotron instability with the distribution function of energetic electrons as measured in satellite experiments (Nagano *et al*., 1996; Nunn *et al*., 1997) showed that a chorus-like element appeared from weak initial whistler waves, but the CI growth rate had to be one or two orders of magnitude greater than followed from estimates based on the observed smooth distribution function of the energetic electrons.

A second problem arises for chorus generation. That is how to explain the appearance of a succession of discrete elements. Bespalov and Trakhtengerts (1978) suggested the mechanism based on a resonance between the group oscillations of a wave packet reflected from the conjugate ionospheres and bounce oscillations of energetic electrons. However, the experiments have revealed that a repetition period in the succession is often smaller than both the bounce period of electrons between the magnetic mirror points and the wave propagation time between the conjugate ionospheres (see Table 1).

The solution of these two problems seems to be linked to the character of the quasi-linear (QL) relaxation of a smooth distribution function of energetic electrons in the process of CI development. This leads to the formation of a specific step-like deformation of the distribution function, which drastically changes all further development of the wave-particle interactions (Trakhtengerts *et al*., 1996). The instability of a distribution function with a step-like deformation in application to the problem of discrete emissions in the magnetosphere was first considered by Trakhtengerts *et al*. (1986). It was shown that a whistler wave amplification was increased by one to two orders of magnitude in comparison with smooth distribution functions. Nunn and Sazhin (1991) pointed out step-like deformation as a possible source of chorus emissions. Trakhtengerts (1995) revealed that the step-like deformation could be the cause of new generation regimes of the cyclotron instability, which were accompanied by a succession of discrete signals. Here we shall consider the peculiarities of chorus generation coming from these general ideas. Some experimental data are analysed from the viewpoint of this theoretical model.

2 A theoretical model for chorus generation

We suggest that chorus generation begins in regions of a dense cold plasma, which correspond to separate ducts or to the plasmapause. Energetic electrons come into the generation region during their magnetic drift (in longitude, primarily) from an injection place. The initial distribution function of injected electrons is anisotropic but smooth in velocity space with $\alpha = u_0^2/v_0^2 > 1$, where u_0 and v_0 are the electrons' characteristic velocities across and along the magnetic field, respectively, and noise-like ELF/VLF emissions are excited in the process of the CI development. At this stage a step-like deformation of the distribution function is formed, which divides all of velocity space into two regions, corresponding to electrons in resonance with waves and to electrons for which the resonance condition (1) is not fulfilled (Trakhtengerts *et al*., 1996). The characteristic time for this preliminary quasilinear stage τ_{QL} is equal to (Bespalov and Trakhtengerts, 1986; Trakhtengerts *et al*., 1996):

Table 1. Parameters of chorus, taken from satellite and ground-based data

	Geos-1 $\varphi = 7.51°$	Geotail $X_{SM} = +6R_E$	Poker Flat Rocket	Sodankylä PA
L/f_{HL}, kHz	6.6/3.04	$<10(\sim 9)/2$	5.6/5	5/6.4
f, kHz	1–1.5	0.5–1	1–3	1.8–3
hiss	+			+
I_f	600 $(\text{m}\gamma)^2$/Hz	?	?	
Chorus amplitude b_{max}	$\sim$200 mγ	150 mγ		
df/dt, kHz/s	0.7	0.6	1–7	$\sim$1.5
T, s	1	1.1	0.2–0.6	0.3–0.4
Energy W_b, keV	20	20	10	20
τ_B, s	1.4–1.8	2–2.55	1.75–2.2	1.1–1.52
T_{BWO}	>0.2	>0.4	>0.2	>0.15

$$\tau_{QL} = \tau_B(\Delta\theta_L)^2/D, \qquad D \approx \left(\frac{4\pi e}{mcw_0}\right)^2\left(\frac{a\varepsilon}{k}\right), \qquad (2)$$

where θ_L is the electron's pitch-angle in the equatorial plane, τ_B is the bounce period, $w_0 = \sqrt{u_0^2 + v_0^2}$, e and m are charge and mass of electrons, c is the velocity of light, a is the characteristic scale of the magnetic field inhomogeneity, k is the whistler wave vector, ε is the whistler wave energy density, and D is the pitch angle diffusion coefficient.

After the formation of the step on the distribution function, the preliminary stage is finished, and a new very fast hydrodynamic stage of the CI starts, with the growth rate (Trakhtengerts, 1995)

$$\gamma_{HD} \approx \omega_{HL}(\Delta N_h/N_c)^{1/2} \gg \gamma_{sm} \qquad (3)$$

where ω_{HL} is the electron gyrofrequency at the equator, N_h and N_c are hot (energetic electrons) and cold plasma densities, respectively, ΔN_h is the step height, and γ_{sm} is the growth rate for a smooth distribution function with the same density of energetic electrons. This stage begins, when the step width $k\Delta v$ becomes less than the growth rate γ_{HD}

$$|k\Delta v| < \gamma_{HD} \qquad (4)$$

The hydrodynamic stage of the CI manifests itself as the transition to a new backward wave oscillator (BWO) generation regime, when the absolute CI develops in a narrow region which is symmetrical about the equatorial plane. Usually, the absolute instability appears in generators of electromagnetic emissions due to the positive back connection which is organised by mirrors. These mirrors reflect waves into the generation volume, and wave intensity grows in time up to the saturation level which is determined by nonlinear effects only. In the case of BWO generation, a whistler wave packet meets at the entrance of the generation region the phase-bunched electron beam (the hot wave mode) which has been formed by the previous wave packet and amplifies the next packet more strongly. Actually this is the positive back connection through a hot wave mode. Two necessary conditions have to be satisfied in the case of BWO regime: (1) the wave has to interact with the beam moving in the opposite direction, and (2) velocity spread of the beam (step) must be as small as defined in Eq. (4) for the hydrodynamic stage of CI to be realised. In the opposite case to Eq. (4), the thermal motion of the beam suppresses the hot wave mode. Both these conditions are fulfilled for the whistler mode wave if the distribution function F of the electron beam is a delta-function

$$F \sim \delta(v_m - v)\delta(u_m - u) \qquad (5)$$

or as a function with a step-like deformation

$$F \sim \Phi(u, v)\Xi(v_m - v) \qquad (6)$$

where $\Phi(u, v)$ is a smooth function of u and v, $\Xi(x)$ is equal to unity, if $x > 0$, and to zero, if $x < 0$. Here v_m is the velocity corresponding to the step.

The BWO regime is well known in electronic devices, and among these the gyrotrons (Gaponov-Grekhov and Petelin, 1980) are closest to the magnetosphere cyclotron masers. Gyrotrons are based on the CI as well, but in a homogeneous magnetic field and with the beam distribution function as a delta-function as in Eq. (5). Analytical estimations, computational simulation and laboratory modelling have led to a rather full picture of BWO regimes in gyrotrons (Ginzburg and Kuznetsov, 1981). The BWO regime is excited when the beam density exceeds some threshold value. With an increase of the beam density the BWO regime changes from a continuous regime (when the wave amplitude is constant) to a periodic regime, which is accompanied by a periodic modulation of wave intensity, and then to stochastic modulation. These bifurcations are characterised by a single dimensionless parameter, an effective length L_{eff}, which is equal to

$$L_{\text{eff}} = \gamma_{HD} l (v_m^2 v_g)^{-1/3} \qquad (7)$$

where γ_{HD} is the hydrodynamic growth rate in the case of the distribution function as a delta-function (Ginzburg and Kuznetsov, 1981), l is the working length (the geometrical length of a device along the magnetic field, if it is homogeneous), and v_g is the group velocity. In this case the threshold value of L_{eff} for BWO generation (the continuous regime) is equal to two. The first bifurcation, the transition to the periodic regime, takes place, when $L_{\text{eff}} \approx 3$, and the stochastic behaviour begins when $L_{\text{eff}} \geq 6$. The periodic regime is characterised by the modulation period

$$T_M \simeq 1.5 l(1/v_g + 1/|v_m|) \qquad (8)$$

Calculations (Trakhtengerts, 1995) with the distribution function (6), which is important for chorus generation, showed the possibility of the BWO generation regime too; moreover, the effective length in this case can be written as

$$L_{\text{eff}}^{\text{st}} = \gamma_{HD} l (v_g |v_m|)^{-1/2} \qquad (9)$$

where γ_{HD} is determined by the formula (3). This length plays the same role as L_{eff} (7) in the case of a delta-function. In the case of a step, the BWO threshold value $L_{\text{thr}}^{\text{st}} = \pi/2$ (Trakhtengerts, 1995). Taking into account the similarity of the physical processes and of the equations describing the BWO regimes in both distribution functions (5) and (6), it is possible to suppose that an increase of $L_{\text{eff}}^{\text{st}}$, $L_{\text{eff}}^{\text{st}} \approx p L_{\text{thr}}^{\text{st}}$, and $p \sim 2$, results in the transition to the BWO periodic regime with the modulation period T_M, see Eq. (8). We connect this latter value with the repetition period of chorus elements.

Now we discuss the length l of the magnetosphere generator, which determines L_{eff} and T_M. This length depends strongly on the inhomogeneity of the geomagnetic field. At first we estimate the minimal value of l, l_0, neglecting the second order cyclotron resonance effects. In this case the wave frequency is fixed, and l_0 can be found from the condition that the phase mismatching

$$\int_{-l/2}^{l/2} \mathrm{d}z(\Delta/v_m) = \pi/2 \qquad (10)$$

where $\Delta = \omega - \omega_H - kv$, and the point $z = 0$ corresponds to the equatorial cross section of the magnetic flux tube. According to Helliwell (1967) and Trakhtengerts (1995), this condition gives, for the dipole magnetic field,

$$l_0 \simeq (R_0^2 L^2/k)^{1/3} \tag{11}$$

where R_0 is the Earth's radius, and L is the McIlwain parameter. For real conditions, the second order cyclotron resonance is important; then the cyclotron resonance is supported along the beam trajectory due to a suitable change of the wave frequency $\omega(z)$ and to a nonlinear change of the electron velocity component v.

Indeed, after the transition to the periodic BWO regime, we deal with the particular case of triggered emissions, when a succession of quasi-monochromatic wavelets starts from the equatorial region of the magnetic flux tube and triggers signals with rising frequency. The parameters of these signals can be estimated from existing theoretical models of triggered emissions and from earlier considerations. According to computer simulations (Nunn, 1974a, 1984; Vomvoridis and Denavit, 1980; Omura *et al.*, 1991) the optimal condition for the generation of discrete emissions, corresponding to the maximum value of the nonlinear growth rate, can be written as

$$0.2 \leq |S| \leq 0.8 \tag{12}$$

where $S = \Omega_{tr}^{-2}\left[-\frac{3|v_m|}{2}\frac{\partial\omega_H}{\partial z} - \left(1 + \frac{\omega_H}{2\omega}\right)\frac{\mathrm{d}\omega}{\mathrm{d}t}\right]$

and the trapping frequency Ω_{tr} is determined by the expression

$$\Omega_{tr} = (ku\omega_H b)^{1/2} \tag{13}$$

In Eqs. (12) and (13), $\partial\omega_H/\partial z = 2\omega_{HL} z a^{-2}$, in the parabolic approximation for dipole magnetic field, $a = (\sqrt{2}/3)R_0 L$, and $b = H_\sim/H$, where $H_\sim$ is the whistler magnetic field amplitude. Actually, the relation (12) corresponds to the fulfillment of the second order cyclotron resonance condition.

We estimate the wave amplitude $H_\sim$ from the relation, which is valid in the case of an absolute instability (Trakhtengerts, 1984),

$$(\Omega_{tr}/\gamma_0) \simeq 32/3\pi \tag{14}$$

where γ_0 is the initial growth rate. In our case it is the growth rate of the BWO generation after the transition to the periodic regime (subscript per). According to Trakhtengerts (1995), γ_0 can be presented in the form

$$\gamma_0 \equiv (\gamma_{BWO})_{\mathrm{per}} \simeq \frac{\pi}{4}\left(\frac{\sqrt{v_g|v_m|}}{v_g + |v_m|}\right)\gamma_{HD} \tag{15}$$

where γ_{HD} is determined by the expression (3). Taking into account that the generated wave propagates from the equator into an increasing magnetic field, we have $S < 0$, which corresponds to the frequency of separate chorus elements rising as time increases. Making $S = -0.5$, which corresponds to the middle value of S from the optimal range of Eq. (12) and substituting Ω_{tr} from Eq. (14), we find from Eq. (12)

$$\frac{\mathrm{d}f}{\mathrm{d}t} = 1.5\frac{\omega\gamma_0^2}{\omega_H + 2\omega}(1 + S_0) \tag{16}$$

where $f = \omega/2\pi$, $S_0 = (|v_m|/3\gamma_0^2)(\partial\omega_H/\partial z)$.

It is seen from Eq. (16) that, when $S_0 \ll 1$, the nonlinear effects mainly determine the fulfillment of the second order cyclotron resonance and the corresponding frequency change. For that case, the interaction length l is close to the value l_0 in Eq. (11). For the other limiting case, when $S_0 \gg 1$, the interaction length is maximum and can be determined from the cyclotron resonance condition in Eq. (1). For that case we take the motion of an electron in the adiabatic approximation ($b \to 0$) and substitute the minimum (ω_1) and the maximum (ω_2) values of the chorus frequency to find the two points on the field line, at $z = 0$ and $z = l_m/2$, respectively. For rough estimations the resulting formula for l_m can be written as

$$l_m(\gg l_0) \sim 2a\left(\sqrt{1 + 4A\frac{\Delta\omega}{\omega_{HL}}} - 1\right)^{1/2} \tag{17}$$

where $\Delta\omega = \omega_2 - \omega_1$, $A \sim 1$. The real interaction length is

$$l_0 < l < l_m. \tag{18}$$

3 Discussion

We can summarise our theoretical model as follows. Energetic electrons come into the generation region (a detached cold plasma cloud, or the plasmapause) in process of their magnetic drift. After that the CI switches on and QL relaxation begins, which prepares the transition to the BWO generation regime. We can estimate the duration of this stage, taking into account Eq. (2) and using GEOS-1 data (Hattori *et al.*, 1991; see also Table 1) to estimate the wave energy ε, τ_B and the characteristic velocity of electrons $w_0 = (u_0^2 + v_0^2)^{1/2}$. Substituting the corresponding values into Eq. (2), we obtain (for $L = 6.6$)

$$\tau_{QL} \sim 3\tau_B \sim 5\,\mathrm{s}$$

This is in accordance with the supermodulation period of the chorus intensity, which is seen in experiments (Sazhin and Hayakawa, 1992). With our point of view these modulations are interpreted as spatial-temporal features of chorus generation, which are due to local different stages of QL relaxation before the BWO regime starts. The spatial scale of such localisations across the magnetic field near the equatorial plane is

$$r \sim v_D\tau_{QL} \sim (1 - 3) \times 10^2\,\mathrm{km}$$

where v_D is the electron's magnetic drift velocity at the equator.

After the formation of a step-like deformation on the distribution function the fast stage of chorus generation occurs, with a total duration $\Delta t \geq \tau_B$. The succession period of chorus elements, T_M, is determined by Eq. (8) and, for the particular case of GEOS-1 data

(Table 1) with Eq. (18) taken into account, can occupy the interval

$$0.2 < T_M < \tau_B \sim 1.4 - 1.8\,\mathrm{s}$$

The experimental data are in this interval too.

The pulsating aurora can be utilised for verification of the above general picture of chorus generation. Actually, the particular type of pulsating aurora, pulsating patches, are apparently connected with CI development in a separate duct of enhanced cold plasma density (Trakhtengerts *et al.*, 1986; Demekhov and Trakhtengerts, 1994). This phenomenon is accompanied by ELF hiss and chorus and demonstrates the behaviour predicted by this model: the period between neighbouring patches is equal to 2–20 s and corresponds to the preliminary phase; the burst of several chorus elements appears in the final stage of an optical flash and has a duration about 1–2 s.

The important check point for a theory of chorus generation is a comparison of the observed repetition period T of chorus elements with the predictions of the BWO regime (T_M) and with the bounce period τ_B of the energetic electrons. Such a comparison is given in Table 1, where some satellite and ground-based experimental data are collected. The satellite data are taken from Hattori *et al.* (1991) and Nagano *et al.* (1996). The ground-based Poker Flat data (Skoug *et al.*, 1996) were accompanied by the launch of a rocket, measuring precipitated electron fluxes. Sodankylä observations (Tagirov *et al.*, submitted 1997) were made simultaneously with the optical measurements of a pulsating aurora, which permitted the energy of the energetic electrons and the L value of chorus generation to be determined. From the comparison of T with τ_B and T_M^{min}, it is seen that T is considerably less than τ_B and is comparable with T_M, if we take into account the formula (8) and the inequality (18).

We can estimate other parameters of chorus, using the expressions (13)–(16). In particular, its fast temporal growth scale is determined by γ_0^{-1} in Eq. (15). Using as the growth rate for the smooth distribution function $\gamma_{\rm sm} \sim 4\,\mathrm{s}^{-1}$, the step height $b \sim 0.3$, $\omega_{HL} \sim 2 \times 10^4\,\mathrm{s}^{-1}$ and $\omega \sim \omega_{HL}/2$, we find from Eq. (15): $\gamma_0 \sim 80\,\mathrm{s}^{-1}$. This is in accordance with typical experimental values. The frequency growth rate can be estimated from Eq. (16). For $\omega \sim \omega_{HL}/2$ and $\gamma_0 \sim 80\,\mathrm{s}^{-1}$ we find ($S_0 \ll 1$): $\mathrm{d}f/\mathrm{d}t \sim 2.4\,\mathrm{kHz/s}$. The chorus amplitude can be found from the relations (13)–(15). For $ku \sim \omega_{HL}/2 \sim 10^4\,\mathrm{s}^{-1}$ and for the same of γ_0, we obtain the wave magnetic field amplitude $B_\sim \sim 30\,\mathrm{m}\gamma$. This value is smaller than the satellite data show (Table 1). Apparently, more accurate calculations of the nonlinear stage of the BWO regime are needed. In any case, it is possible to conclude that there is a good qualitative agreement of the theory with experimental data.

4 Conclusion

The model developed gives a rather complete picture of chorus generation. On the one hand this model connects directly with the theory of triggered ELF-VLF emissions, and on the other it opens new possibilities for an explanation of the most intriguing features of chorus generation, such as its connection with hiss, the large growth rates, and the appearance of a succession of discrete elements with small repetition period. These possibilities are based on a specific step-like deformation of the electron distribution function, which appears at the slow preliminary stage of quasi-linear relaxation. It is possible to connect with this stage the supermodulation of the chorus intensity, which is characterised by the period $T_S \sim 5$–$10\,\mathrm{s}^{-1}$. It would be interesting to examine some experimental, especially satellite, data concerning classical chorus (the connection with hiss, supermodulation, repetition period, and dependence of chorus parameters on its intensity) from the viewpoint of this model. Computer simulations of the BWO generation regime in an inhomogeneous magnetic field are also very important, including the preliminary stage of QL relaxation.

Acknowledgements. The author is grateful to M. Rycroft for reading this manuscript and useful comments. This work was supported by the Russian Foundation for Basic Research, grant 96-02-16473a, and by the INTAS grant 94-2753.

Topical Editor K.-H. Glassmeier thanks M. Parrot and S. Sazhin for their help in evaluating this paper.

References

Bespalov, P. A., and V. Y. Trakhtengerts, On the theory of the ELF/VLF emissions of chorus type, *Geomagn. Aeron.*, **18,** 428, 1978, (English translation).

Bespalov, P. A., and V. Y. Trakhtengerts, The cyclotron instability in the Earth radiation belts, in *Reviews of Plasma Physics*, Ed. M. A. Leontovich, vol 10, 155–192, Plenum, New York, 1986.

Demekhov, A. G., and V. Y. Trakhtengerts, A mechanism of formation of pulsating aurorae, *J. Geophys. Res.*, **99,** 5831–5841, 1994.

Gaponov-Grekhov, A. V., and M. I. Petelin, Masers on cyclotron resonance, in *Science and Mankind*, 283, Znanie, Moscow, 1981, (in Russian).

Ginzburg, N. S., and S. P. Kuznetsov, Periodic and stochastic regimes in electron generators with distributed interaction, in *Relativistic HF Electronics*, 101–144, Institute of Applied Physics, Gorky, USSR, 1981, (in Russian).

Hattori, K., M. Hayakawa, D. Lagoutte, M. Parrot, and F. Lefeuvre, Further evidence of triggering chorus emissions from wavelets in the hiss band, *Planet. Space Sci.*, **39,** 1465–1473, 1991.

Helliwell, R. A., *Whistlers and Related Ionospheric Phenomena*, Standford University Press, Palo Alto, Calif., 1965.

Helliwell, R. A., A theory of discrete emissions from the magnetosphere, *J. Geophys. Res.*, **72,** 4773–4790, 1967.

Helliwell, R. A., Low-frequency waves in the magnetosphere, *Rev. Geophys.*, **7,** 281–303, 1969.

Helliwell, R. A., 40 years of whistlers, *Mod. Radio Sci.*, 189–212, 1993.

Karpman, V. I., Y. N. Istomin, and D. R. Shklyar, Nonlinear theory of a quasimonochromatic whistler mode packet in inhomogeneous plasma, *Phys. Plasmas*, **16,** 685–703, 1974.

Nagano, I., S. Yagitani, H. Kojima, and H. Matsumoto, Analysis of wave normal and Poynting vectors of the chorus emissions, *J. Geomagn. Geoelectr.*, **48,** 299–307, 1996.

Nunn, D., A self-consistent theory of triggered VLF emissions, *Planet. Space Sci.*, **22,** 349–378, 1974a.

Nunn, D., A theoretical investigation of banded chorus, *J. Plasma Phys.*, **11,** 189, 1974b.

Nunn, D., A quasi-static theory of triggered VLF emissions, *Planet. Space Sci.*, **32,** 325–350, 1984.

Nunn, D., A nonlinear theory of sideband stability in ducted whistler mode waves, *Planet. Space Sci.*, **34,** 429–451, 1986.

Nunn, D., and S. S. Sazhin, On the generation mechanism of hiss-triggered chorus, *Ann. Geophysical*, **9,** 603–613, 1991.

Nunn, D., Y. Omura, H. Matsumoto, I. Nagano, and S. Yagitani, The numerical simulation of VLF chorus and discrete emissions observed on the Geotail satellite using a Vlasov code, *J. Geophys. Res.*, **102,** 1997, (in Press).

Omura, Y., D. Nunn, H. Matsumoto, and M. J. Rycroft, A review of observational, theoretical and numerical studies of VLF triggered emissions, *J. Atmos. Terr. Phys.*, **53,** 351–368, 1991.

Sazhin, S. S. and M. Hayakawa, Magnetospheric chorus emissions: a review, *Planet. Space. Sci.*, **40,** 681–697, 1991.

Skoug, R. M., S. Datta, M. P. McCarthy, and G. K. Parks, A cyclotron resonance model of VLF chorus emissions detected during electron microburst precipitation, *J. Geophys. Res.*, **101,** 21,481–21,491, 1996.

Trakhtengerts, V. Y., Relaxation of plasmas with anisotropic velocity distribution, in *Handbook of Plama Physics, vol. 2, Basic Physics II*, Eds. A. A. Galeev and R. N. Sudan, 519–552, Elsevier, N. Y., 1984.

Trakhtengerts, V. Y., Magnetosphere cyclotron maser: backward wave oscillator generation regime, *J. Geophys. Res.*, **100,** 17,205–17,210, 1995.

Trakhtengerts, V. Y., M. J. Rycroft, and A. G. Demekhov, Interrelation of noise-like and discrete ELF/VLF emissions generated by cyclotron interactions, *J. Geophys. Res.*, **101,** 13,293–13,303, 1996.

Trakhtengerts, V. Y., V. R. Tagirov, and S. A. Chernous, Flow cyclotron maser and impulsive VLF emissions, *Geomagn. Aeron.*, **26,** 99–106, 1986.

Vomvoridis, J. L., and J. Denavit, Nonlinear evolution of monochromatic whistler wave packet in a nonuniform magnetic field, *Phys. Fluids*, **23,** 174–183, 1980.

Vomvoridis, J. L., T. L. Crystal, and J. Denavit, Theory and computer simulations of magnetospheric triggered VLF emissions, *J. Geophys. Res.*, **87,** 1473–1489, 1982.