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A generic dynamical model of gamma-ray burst remnants

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ABSTRACT

The conventional generic model is considered to explain the dynamics of gamma-ray burst remnants very well, no matter whether they are adiabatic or highly radiative. However, we find that, for adiabatic expansion, the model cannot reproduce the Sedov solution in the non-relativistic phase, and thus it needs to be revised. In this paper a new differential equation is derived. The generic model based on this equation is shown to be correct for both radiative and adiabatic fireballs, and in both ultrarelativistic and non-relativistic phases.

Key words: hydrodynamics – relativity – shock waves – gamma-rays: bursts.

1 INTRODUCTION

Since the *BeppoSAX* detection of GRB 970228, X-ray afterglows have been observed from about 15 gamma-ray bursts (GRBs), of which 10 have been detected optically and five also at radio wavelengths (Costa et al. 1997; Kulkarni et al. 1998; Bloom et al. 1998; Piran 1999, and references therein). The cosmological origin of at least some GRBs is thus firmly established. The so-called fireball model (Goodman 1986; Paczyński 1986; Rees & Mészáros 1992, 1994; Mészáros & Rees 1992; Katz 1994; Sari, Narayan & Piran 1996) is strongly favoured, and is successful at explaining the major features of the low-energy light curves (Mészáros & Rees 1997; Vietri 1997; Tavani 1997; Waxman 1997; Wijers, Rees & Mészáros 1997; Sari 1997; Huang et al. 1998b; Dai & Lu 1998a; Dai, Huang & Lu 1999). A variant of this model, in which central engines (e.g. strongly magnetized millisecond pulsars) supply energy to post-burst fireballs through magnetic dipole radiation, has been proposed to account for the special features of the optical afterglows from GRB 970228 and 970508 (Dai & Lu 1998b,c).

Since the expansion of a fireball may be either adiabatic or highly radiative, extensive attempts have been made to find a common model applicable to both cases (Blandford & McKee 1976; Chiang & Dermer 1999; Piran 1999). As a result, a conventional model has been suggested by various authors (e.g. Chiang & Dermer 1999; Piran 1999). A dynamical model should be correct not only in the initial ultrarelativistic phase, which is well described by those simple scaling laws (Mészáros & Rees 1997; Vietri 1997; Waxman 1997), but also in the consequent non-relativistic phase, which is correctly analysed by using the Sedov solution (Sedov 1969; Wijers et al. 1997). Although the conventional model is correct for the ultrarelativistic phase, we find that it cannot match the Sedov solution in the non-relativistic limit. In this paper we will therefore construct a dynamical model that is really capable of describing generic fireballs, no matter whether they are radiative or adiabatic, and no matter whether they are ultrarelativistic or non-relativistic.

2 CONVENTIONAL DYNAMICAL MODEL

A differential equation has been proposed to depict the expansion of GRB remnants (Chiang & Dermer 1999; Piran 1999):

$$\frac{\mathrm{d}\gamma}{\mathrm{d}m} = -\frac{\gamma^2 - 1}{M}, \tag{1}$$

where m is the rest mass of the swept-up medium, γ is the bulk Lorentz factor and M is the total mass in the comoving frame, including the internal energy U. Since the thermal energy produced during the collisions is $\mathrm{d}E = (\gamma - 1)\,\mathrm{d}m\,c^2$, we usually assume that $\mathrm{d}M = (1 - \epsilon)\,\mathrm{d}E/c^2 + \mathrm{d}m = [(1 - \epsilon)\gamma + \epsilon]\,\mathrm{d}m$, where ϵ is defined as the fraction of the shock-generated thermal energy (in the comoving frame) that is radiated (Piran 1999). It is supposed that equation (1) is correct in both the ultrarelativistic and non-relativistic phases, for both radiative and adiabatic fireballs. However, after careful inspection, we find that, during the non-relativistic phase of an adiabatic expansion, equation (1) cannot give a solution consistent with the Sedov results (Sedov 1969).

2.1 Radiative case

In the highly radiative case, $\epsilon = 1$, $\mathrm{d}M = \mathrm{d}m$, equation (1) reduces to

$$\frac{\mathrm{d}\gamma}{\mathrm{d}m} = -\frac{\gamma^2 - 1}{M_{\mathrm{ej}} + m}, \tag{2}$$

where M_{ej} is the mass ejected from the GRB central engine. Then an analytic solution is available (Blandford & McKee 1976;

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Piran 1999):

$$\frac{(\gamma-1)(\gamma_0+1)}{(\gamma+1)(\gamma_0-1)}=\left(\frac{m_0+M_{\rm ej}}{m+M_{\rm ej}}\right)^2, \tag{3}$$

where γ_0 and m_0 are initial values of γ and m respectively. Usually we assume $\gamma_0 \sim \eta/2$, $m_0 \sim M_{\rm ej}/\eta$, where $\eta \equiv E_0/(M_{\rm ej}c^2)$ and E_0 is the total energy in the initial fireball (Waxman 1997; Piran 1999).

During the ultrarelativistic phase, $\gamma \gg 1$, $M_{\rm ej} \gg m$, equation (3) gives $(\gamma+1)m \approx M_{\rm ej}$, or equivalently the familiar power law $\gamma \propto R^{-3}$, where R is the radius of the blast wave. In the later non-relativistic phase, $\gamma \sim 1$, $m \gg M_{\rm ej}$, we have $m^2\beta^2 = 4M_{\rm ej}^2$, or $\beta \propto R^{-3}$, where $\beta = v/c$ and v is the bulk velocity of the material. This is consistent with the late isothermal phase of the expansion of supernova remnants (SNRs) (Spitzer 1968). From these approximations, we believe that equation (2) is really correct for highly radiative fireballs.

2.2 Adiabatic case

In the adiabatic case, $\epsilon = 0$, $dM = \gamma\, dm$, equation (1) also has an analytic solution (Chiang & Dermer 1999):

$$M = (M_{\rm ej}^2 + 2\gamma_0 M_{\rm ej} m + m^2)^{1/2}, \tag{4}$$

$$\gamma = \frac{m+\gamma_0 M_{\rm ej}}{M}. \tag{5}$$

During the ultrarelativistic phase, $\gamma_0 M_{\rm ej} \gg m \gg M_{\rm ej}/\gamma_0$, $\gamma \gg 1$, this solution can produce the familiar power law $\gamma \propto R^{-3/2}$, which is often quoted for an adiabatic blastwave decelerating in a uniform medium. In the non-relativistic limit ($\gamma \sim 1$, $m \gg \gamma_0 M_{\rm ej}$), Chiang & Dermer (1999) have derived $\gamma \approx 1 + \gamma_0 M_{\rm ej}/m$, so that they believe it also agrees with the Sedov solution (Lozinskaya 1992). However, we find that their approximation is not accurate enough, because they have omitted some first-order infinitesimals of $\gamma_0 M_{\rm ej}/m$. The correct approximation can be obtained only by retaining all the first- and second-order infinitesimals, which in fact gives $\gamma \approx 1 + (\gamma_0 M_{\rm ej}/m)^2/2$; then we have $\beta \propto R^{-3}$. This is not consistent with the Sedov solution! We have also evaluated equation (1) numerically: the result is consistent with equations (4) and (5), all pointing to $\beta \propto R^{-3}$, not the relation $\beta \propto R^{-3/2}$ as often quoted in the literature (Chiang & Dermer 1999; Piran 1999).

This discrepancy is serious. First, it means that equation (1) is not a dependable model for non-radiative fireballs, although it can reproduce the major features in the ultrarelativistic phase. Secondly, the expansion of a realistic fireball is widely believed to be highly radiative at first, but after only a few days the expansion will become non-radiative (Sari, Piran & Narayan 1998; Dai et al. 1999). In the non-relativistic phase, therefore, the fireball is likely to be adiabatic rather than highly radiative. However, it is under just this condition that the conventional model fails. So any calculation made according to equation (1) will lead to serious deviations in the light curves in the non-relativistic phase.

3 OUR GENERIC MODEL

Equation (1) is not consistent with the Sedov solution, and we need to revise it. In the fixed frame, since the total kinetic energy of the fireball is $E_{\rm K} = (\gamma-1)(M_{\rm ej}+m)c^2 + (1-\epsilon)\gamma U$ (Panaitescu, Mészáros & Rees 1998), and the radiated thermal energy is $\epsilon\gamma(\gamma-1)\,dmc^2$ (Blandford & McKee 1976), we have

$$d[(\gamma-1)(M_{\rm ej}+m)c^2 + (1-\epsilon)\gamma U] = -\epsilon\gamma(\gamma-1)\,dm\,c^2. \tag{6}$$

For the variable U, it is usually assumed that $dU = (\gamma-1)\,dm\,c^2$ (Panaitescu et al. 1998). Equation (1) has been derived just in this way. However, the jump conditions (Blandford & McKee 1976) at the forward shock imply that $U = (\gamma-1)mc^2$, so we suggest that the correct expression for dU should be $dU = d[(\gamma-1)mc^2] = (\gamma-1)\,dmc^2 + mc^2\,d\gamma$. Here we simply use $U = (\gamma-1)mc^2$ and substitute it into equation (6); then it is easy to obtain

$$\frac{d\gamma}{dm} = -\frac{\gamma^2-1}{M_{\rm ej}+\epsilon m + 2(1-\epsilon)\gamma m}. \tag{7}$$

We expect this equation to describe a generic fireball correctly.

Indeed, in the highly radiative case ($\epsilon = 1$) equation (7) reduces to equation (2) exactly, while in the adiabatic case ($\epsilon = 0$) equation (7) reduces to

$$\frac{d\gamma}{dm} = -\frac{\gamma^2-1}{M_{\rm ej}+2\gamma m}. \tag{8}$$

This equation has an analytic solution:

$$(\gamma-1)M_{\rm ej}c^2 + (\gamma^2-1)mc^2 \equiv E_{\rm K0}, \tag{9}$$

where $E_{\rm K0}$ is the initial value of $E_{\rm K}$. In the ultrarelativistic phase ($\gamma_0 M_{\rm ej} \gg m \gg M_{\rm ej}/\gamma$) we obtain the familiar relation $\gamma \propto R^{-3/2}$, and in the non-relativistic phase ($m \gg M_{\rm ej}$) we obtain $\beta \propto R^{-3/2}$ as required by the Sedov solution.

For any other ϵ-value between 0 and 1, equation (7) describes the evolution of a partially radiative fireball. Unfortunately, we now cannot find an exact analytic solution for equation (7). In the non-relativistic phase, however, by assuming that $m \gg M_{\rm ej}$, we can still get $m(\gamma-1)^{(2-\epsilon)/2} \equiv$ constant, that is,

$$\beta \propto R^{-3/(2-\epsilon)}. \tag{10}$$

4 NUMERICAL RESULTS

We have evaluated equation (7) numerically, bearing in mind that (Huang et al. 1998b)

$$dm = 4\pi R^2 n m_{\rm p}\, dR, \tag{11}$$

$$dR = \beta c\gamma\left(\gamma + \sqrt{\gamma^2-1}\right) dt, \tag{12}$$

where n is the number density of the interstellar medium, $m_{\rm p}$ is the mass of a proton, and t is the time measured by an observer. We take $E_0 = 10^{52}$ erg, $n = 1\ {\rm cm}^{-3}$ and $M_{\rm ej} = 2\times10^{-5}\ {\rm M}_{\odot}$. Figs 1–4 illustrate the evolution of γ, v, R and $E_{\rm K}$ respectively. In these figures, we have set $\epsilon = 0$ (full lines), 0.5 (dotted lines) and 1 (dashed lines). It is clearly shown that our generic model overcomes the shortcomings of equation (1).

For example, for highly radiative expansion, the dashed lines in these figures approximately satisfy $\gamma \propto t^{-3/7}$, $R \propto t^{1/7}$, $\gamma \propto R^{-3}$, $E_{\rm K} \propto t^{-3/7}$ when $\gamma \gg 1$, and $v \propto t^{-3/4}$, $R \propto t^{1/4}$, $v \propto R^{-3}$, $E_{\rm K} \propto t^{-3/4}$ when $\gamma \sim 1$. For adiabatic expansion, the full lines satisfy $\gamma \propto t^{-3/8}$, $R \propto t^{1/4}$, $\gamma \propto R^{-3/2}$ when $\gamma \gg 1$, and $v \propto t^{-3/5}$, $R \propto t^{2/5}$, $v \propto R^{-3/2}$ when $\gamma \sim 1$.

5 DISCUSSION AND CONCLUSION

The conventional dynamical model is successful at describing

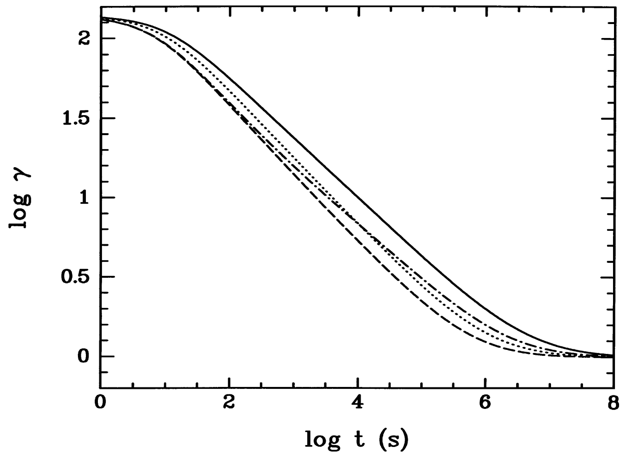

Figure 1. Evolution of the bulk Lorentz factor γ. We take $E_0 = 10^{52}$ erg, $n = 1\,\mathrm{cm}^{-3}$ and $M_{\mathrm{ej}} = 2 \times 10^{-5}\,\mathrm{M}_{\odot}$. The full, dotted and dashed lines correspond to $\epsilon = 0$ (adiabatic), 0.5 (partially radiative) and 1 (highly radiative) respectively. The dash–dotted line is plotted by allowing ϵ to evolve with time (see Section 5).

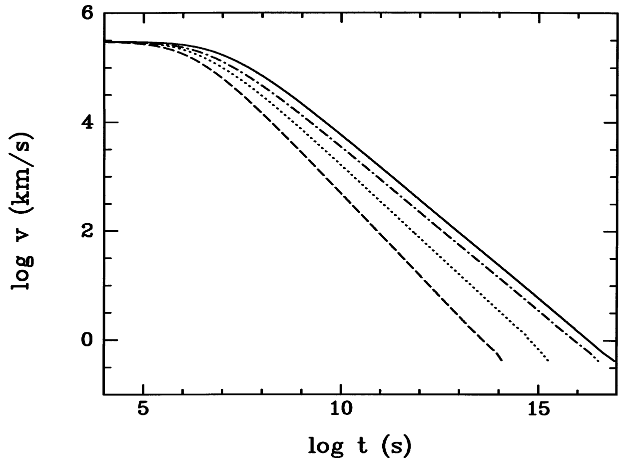

Figure 2. Evolution of the bulk velocity v. Parameters and line styles are the same as in Fig. 1.

highly radiative GRB remnants, but it has difficulty in reproducing the Sedov solution for adiabatic fireballs. This has gone completely unnoticed in the literature. We have constructed a new generic model to overcome this shortcoming. Numerical evaluation has proved that our model is highly credible. We hope that this work will remind researchers of the importance of the transition from the ultrarelativistic to the non-relativistic phase, which might occur as early as 10^6–10^7 s after the initial burst (Huang, Dai & Lu 1998a).

In the above analysis, for simplicity, we have assumed that ϵ is a constant. In realistic fireballs, however, ϵ is expected to evolve from 1 to 0 owing to the changes in the relative importance of synchrotron-induced and expansion-induced loss of energy (Dai et al. 1999). Assuming that electrons in the comoving frame carry a fraction $\xi_{\mathrm{e}} = 1$ of the total thermal energy and that the magnetic energy density is a fraction $\xi_B^2 = 0.01$ of it, we re-evaluate equation (6) numerically. The results are plotted in Figs 1–4 with dash–dotted lines. We see from Fig. 4 that the evolution of ϵ changes $E_{\mathrm{K}}(t)$ dramatically.

It is worth mentioning that SNRs evolve from the non-radiative to the radiative stage, but GRB remnants are just the opposite.

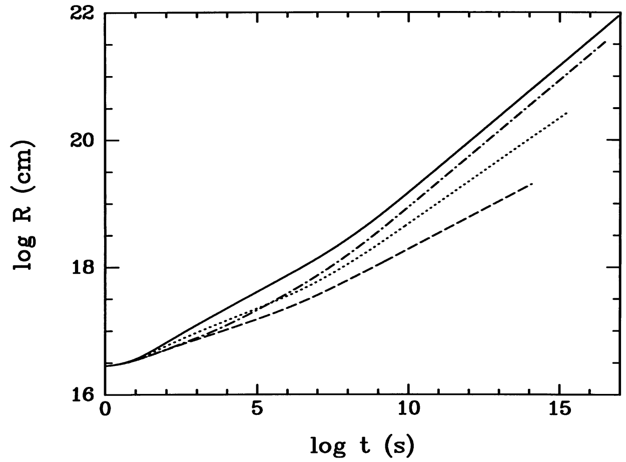

Figure 3. Evolution of the shock radius R. Parameters and line styles are the same as in Fig. 1.

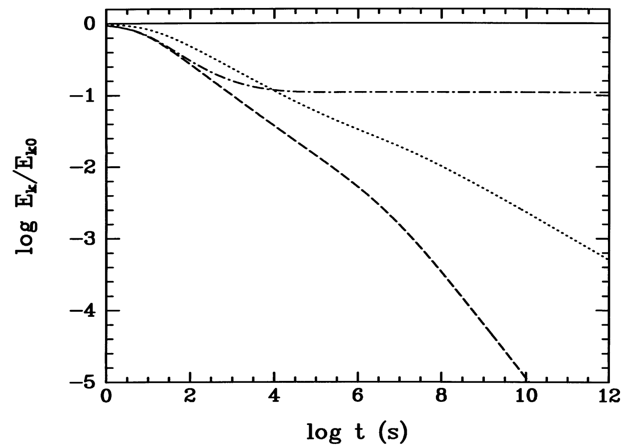

Figure 4. Evolution of the total kinetic energy E_{K}. Parameters and line styles are the same as in Fig. 1.

This is not surprising, because GRB remnants radiate mainly through synchrotron radiation while SNRs lose energy as a result of excited ions. It is reasonable to deduce that at very late stages, when the cooling resulting from ions becomes important, GRB remnants may become highly radiative again, in the same way as SNRs do. The transition may occur when the temperature drops to below $\sim 10^6$ K and the velocity is just several tens of kilometres per second. This needs to be addressed in more detail.

Another interesting problem is the possibility that HI supershells might be highly evolved GRB remnants (Loeb & Perna 1998; Efremov, Elmegreen & Hodge 1998). Our Figs 3 and 4 have shown that typical adiabatic GRB fireballs can evolve to $R \sim 1$ kpc at $t \sim 10^6$–10^7 yr, with $v \sim 10\,\mathrm{km\,s^{-1}}$, but highly radiative fireballs are obviously not powerful enough. To discuss this in detail, we should pay attention to the possible adiabatic-to-radiative transition mentioned above.

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