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A Ginzburg-Landau Type Theory of Quark Confinement*)

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An infrared effective theory of quark confinement is proposed with the aid of a dual vector potential and a complex scalar monopole field. A static potential between charged sources is composed mainly of the linear and the Yukawa interaction. The existence of a light neutral axial-vector boson and a massive scalar boson not composed of quarks is suggested.

Quark confinement is not yet completely solved. In 1981, 'tHooft[1] proposed an interesting idea of how to isolate relevant dynamical variables at the hadron mass scale in QCD. Fixing the "non-abelian part" of the gauge redundancy reduces the $SU(3)$ gauge symmetry to that of the maximal abelian group $U(1)\times U(1)$. (This is called abelian projection.) One gets an effective theory with two different kinds of electric charges and two photons. One also gets point-like singularities which are magnetic monopoles with respect to $U(1)\times U(1)$. An important suggestion by 'tHooft is that the magnetic monopoles must play an essential role in quark cofinement.

The aim of this paper is to develop the 'tHooft idea and to construct an infrared effective theory of quark confinement. We restrict ourselves to the $SU(2)$ case for simplicity. After the abelian projection is made, one has three kinds of dynamical variables. They are (1) monopoles, (2) a diagonal gluon a_μ (called photon hereafter) and (3) off-diagonal gluons $A_\mu{}^\pm$ and quarks ψ which are charged with respect to the photon. The system can be regarded as QED with monopoles.

The Maxwell equations[2] with respect to the photon are

$$\partial_\nu F^{\mu\nu}=j^\mu\,, \tag{1}$$

$$\partial_\nu {}^*F^{\mu\nu}=k^\mu$$

$$=g\sum_{l=1}^{N}n_l\int d\tau_l\frac{d\bar{x}_l{}^\mu(\tau_l)}{d\tau_l}\delta^4(x-\bar{x}_l(\tau_l))\,, \tag{2}$$

where ${}^*F_{\mu\nu}=2^{-1}\,\varepsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}$. Here $g=4\pi/e$ and $n_l g$ is the magnetic charge of the l-th monopole. The electric current j^μ is derived from the original QCD Lagrangian:

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$$j^\mu = ie\{A^{-\nu}D^\mu A_\nu^+ - A^{+\nu}D^{*\mu}A_\nu^- + A^{-\mu}D^\nu A_\nu^+ - A^{+\mu}D^{*\nu}A_\nu^-$$
$$+2A_\nu^+ D^{*\nu}A^{-\mu} - 2A_\nu^- D^\nu A^{+\mu}\} + \frac{e}{2}(\bar{\psi}_1\gamma^\mu\psi_1 - \bar{\psi}_2\gamma^\mu\psi_2) + \tilde{j}^\mu\,, \tag{3}$$

where $D_\mu \equiv \partial_\mu + iea_\mu$ and $\tilde{j}_\mu$ comes from a non-abelian gauge-fixing term not specified here.

When the monopoles exist, the abelian photon part of the QCD Lagrangian must be modified.[2] Although it is possible to rewrite it in terms of a_μ alone,[3] we introduce another vector potential B_μ following Zwanziger.[4] The Lagrangian is given explicity as follows:[2]

$$\mathcal{L}(x;\bar{x}_1,\cdots,\bar{x}_N) = \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4\,, \tag{4}$$

$$\mathcal{L}_1 = -\frac{1}{8}(\partial_\mu a_\nu - \partial_\nu a_\mu)^2 - \frac{1}{8}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2 - \frac{1}{4n^2}[n_\mu(\partial^\mu a^\nu - \partial^\nu a^\mu + \varepsilon^{\mu\nu\alpha\beta}\partial_\alpha B_\beta)]^2$$
$$-\frac{1}{4n^2}[n_\mu(\partial^\mu B^\nu - \partial^\nu B^\mu - \varepsilon^{\mu\nu\alpha\beta}\partial_\alpha a_\beta)]^2\,, \tag{5}$$

$$\mathcal{L}_2 = \partial_\mu A_\nu^+ \partial^\nu A^{-\mu} - \partial_\mu A_\nu^+ \partial^\mu A^{-\nu}$$
$$+\frac{e^2}{4}(A_\mu^+ A_\nu^- - A_\mu^- A_\nu^+)^2 + \bar{\psi}_1(i\gamma_\mu\partial^\mu - M_1)\psi_1$$
$$+\bar{\psi}_2(i\gamma_\mu\partial^\mu - M_2)\psi_2 + \frac{e}{\sqrt{2}}(A_\mu^+\bar{\psi}_1\gamma^\mu\psi_2 + A_\mu^-\bar{\psi}_2\gamma^\mu\psi_1)$$
$$+(\text{a non-abelian gauge-fixing term})\,, \tag{6}$$

$$\mathcal{L}_3 = j_\mu a^\mu + e^2(a_\mu{}^2 A_\nu^+ A^{-\nu} - a_\mu A^{+\mu} a_\nu A^{-\nu})\,, \tag{7}$$

$$\mathcal{L}_4 = k_\mu B^\mu\,, \tag{8}$$

where $F_{\mu\nu}$ is written by

$$F_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu + (n\cdot\partial)^{-1}\varepsilon_{\mu\nu\alpha\beta}n^\alpha k^\beta$$
$$= -\varepsilon_{\mu\nu\alpha\beta}\partial^\alpha B^\beta - (n\cdot\partial)^{-1}(n_\mu j_\nu - n_\nu j_\mu)\,. \tag{9}$$

Here n_μ is an arbitrary fixed four vector. The Lagrangian actually reproduces the Maxwell equations (1) and (2). We shall discuss later abelian gauge-fixing conditions to reduce the degrees of freedom.

The next step is to perform the summation over the monopole trajectories. Various methods have been proposed.[2),5)] Following Bardacki and Samuel[2] and assuming that monopoles with $n = \pm 1$ alone contribute, we get a new Lagrangian:

$$\mathcal{L}(x) = \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + |\partial_\mu\phi + igB_\mu\phi|^2\,, \tag{10}$$

where ϕ is a complex scalar monopole field.

In addition, a mass term of the field ϕ is expected to arise naturally.[5] A case in which monopole trajectories with opposite charges happen to overlap must be excluded in the summation. There must exist a repulsive force of a delta-function type between the monopoles which leads to $\lambda|\phi|^4(\lambda>0)$.[5] To obtain the exact result, one needs more extensive studies of monopole dynamics in QCD. Here we assume that such self-interactions of the ϕ field really arise after the summation is carried out. We propose a Lagrangian

$$\mathcal{L}(x)=\mathcal{L}_1+\mathcal{L}_2+\mathcal{L}_3+|\partial_\mu\phi+igB_\mu\phi|^2-\lambda(|\phi|^2-v^2)^2 \tag{11}$$

as an infrared effective theory of quark confinement of the Ginzburg-Landau type. Note that higher-order ϕ interactions are infrared irrelevant. Without explicit gauge-fixing terms, (11) has an extra magnetic $U(1)$ symmetry in addition to the original electric $U(1)$. Since B_μ is the axial-vector field, the extra $U(1)$ is chiral if the ϕ field can be regarded as a chiral scalar field.

Now we evaluate the potential between static charged sources. We assume that the abelian part is essential and neglect $\mathcal{L}_2$ and $\mathcal{L}_3$ except the static-source interaction. Adding a gauge fixing-term[4]

$$\mathcal{L}_G=\frac{1}{2n^2}[\partial_\mu(n^\nu a_\nu)]^2, \tag{12}$$

one can integrate out a_μ completely. The Lagrangian (11) is reduced to

$$\mathcal{L}(x)=-\frac{1}{4}H_{\mu\nu}^2+|\partial_\mu\phi+igB_\mu\phi|^2-\lambda(|\phi|^2-v^2)^2, \tag{13}$$

where

$$\begin{aligned}H_{\mu\nu}&=\partial_\mu B_\nu-\partial_\nu B_\mu+\varepsilon_{\mu\nu\lambda\sigma}\int d^4z h^\lambda(x-z)j^\sigma(z)\\ &\equiv\partial_\mu B_\nu-\partial_\nu B_\mu+\varepsilon_{\mu\nu\lambda\sigma}(h^\lambda\cdot j^\sigma)\end{aligned} \tag{14}$$

and

$$h^\lambda(x)=-n^\lambda(n\cdot\partial)^{-1}(x). \tag{15}$$

To treat the system (13) analytically, we must make some additional assumptions. Note that (13) is just the dual form of the relativistic Ginzburg-Landau theory with a pair of monopole-antimonopole source.[6] Utilizing the knowledge[6),7] obtained in the analyses of the vortex solution of the latter model, we assume that
(a) the scalar mass $m_\phi=2\sqrt{\lambda}\,v$ is much larger than the vector mass $m_B=\sqrt{2}gv$ and
(b) n_μ is nearly pararell to the direction between the two static charges and the phase of ϕ may be neglected.

The assumption (a) shows that ϕ takes its vacuum expectation value v almost everywhere except inside the vortex region between the static sources.[6,7] It also says that the region inside the vortex (with a radius $\sqrt{2}m_\phi^{-1}$) does not give a sizable energy, so that we neglect the region completely.[8] As for the assumption (b), physical quantities like the static potential are n_μ-independent.[9] Hence n_μ could be exactly pararell if a full treatment would be done. Within the approximation adopted here, however, n_μ cannot be chosen pararell owing to an apparent infrared divergence arising from the center of the vortex. The difference is shown later to be unimportant numerically.

We carry out the functional integration with respect to B_μ, introducing a covariant gauge-fixing term with a gauge parameter α. Neglecting an irrelevant infinity, one gets

$$\int[DB_\mu]\exp i\int d^4x\Big[-\frac{1}{4}(\partial_\mu B_\nu-\partial_\nu B_\mu)^2-\frac{1}{2\alpha}(\partial_\mu B^\mu)^2+\frac{m_B{}^2}{2}B_\mu{}^2$$
$$+\varepsilon^{\mu\mu\lambda\sigma}\partial_\mu B_\nu n_\lambda((n\cdot\partial)^{-1}\cdot j_\sigma)+\frac{n^2}{2}((n\cdot\partial)^{-1}\cdot j^\lambda)\Big(g_{\lambda\sigma}-\frac{n_\lambda n_\sigma}{n^2}\Big)((n\cdot\partial)^{-1}\cdot j^\sigma)\Big]$$
$$=\exp i\int\frac{d^4k}{(2\pi)^4}\Big[\frac{1}{2}j_\mu(-k)\Big\{\frac{g^{\mu\nu}}{k^2-m_B{}^2}+\frac{(-m_B{}^2)}{k^2-m_B{}^2}\frac{n^2}{(n\cdot k)^2}\Big(g^{\mu\nu}-\frac{n^\mu n^\nu}{n^2}\Big)\Big\}j_\nu(k)\Big], \tag{16}$$

where we use an infrared prescription of $(n\cdot k)^{-2}$ such that

$$\frac{1}{2}\Big[\Big(\frac{1}{(n\cdot k)+i\varepsilon}\Big)^2+\Big(\frac{1}{(n\cdot k)-i\varepsilon}\Big)^2\Big]. \tag{17}$$

As the static source, we introduce

$$j^\mu(x)=g^{\mu 0}Q\{\delta(\boldsymbol{r}-\boldsymbol{a})-\delta(\boldsymbol{r}-\boldsymbol{b})\}, \tag{18}$$

$$j^\mu(k)=g^{\mu 0}Q(e^{-i\boldsymbol{k}\cdot\boldsymbol{a}}-e^{-i\boldsymbol{k}\cdot\boldsymbol{b}})2\pi\delta(k^0), \tag{19}$$

$$Q=\begin{cases}\dfrac{e}{2} & \text{for quarks},\\ e & \text{for gluons}.\end{cases} \tag{20}$$

Subtracting the Coulomb self-energy, one gets the static potential

$$V(\boldsymbol{r})=-Q^2\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}\Big\{\frac{1}{\boldsymbol{k}^2+m_B{}^2}+\frac{m_B{}^2}{\boldsymbol{k}^2+m_B{}^2}\frac{1}{(\boldsymbol{n}\cdot\boldsymbol{k})^2}\Big\}, \tag{21}$$

where $\boldsymbol{r}\equiv\boldsymbol{a}-\boldsymbol{b}$ and we have chosen $n_\mu=(0,\boldsymbol{n})$ with $|\boldsymbol{n}|=1$.

We divide the vector $\boldsymbol{r}$ into two parts pararell and perpendicular to $\boldsymbol{n}$:

$$\boldsymbol{r}=\boldsymbol{r}_n+\boldsymbol{r}_t. \tag{22}$$

As previously assumed, we take $|\boldsymbol{r}_t|\sim\sqrt{2}m_\phi{}^{-1}$ which is very small.[8] The length $r_n=|\boldsymbol{r}_n|$ can be approximated by $r=|\boldsymbol{r}|$. Using the Cauchy integral, one gets

$$V(\boldsymbol{r})=-\frac{Q^2}{4\pi}\frac{e^{-m_B r}}{r}+\frac{Q^2 m_B{}^2}{4\pi}K_0\Big(\frac{\sqrt{2}m_B}{m_\phi}\Big)r_n+\frac{Q^2 m_B{}^2}{2}I(\boldsymbol{r}), \tag{23}$$

where

$$I(\boldsymbol{r})=\int\frac{d^2k_t}{(2\pi)^2}\frac{1}{(k_t{}^2+m_B{}^2)^{3/2}}\exp(-\sqrt{k_t{}^2+m_B{}^2}r_n-i\boldsymbol{k}_t\cdot\boldsymbol{r}_t). \tag{24}$$

The function $I(\boldsymbol{r})$ is regular and bounded by

$$|I(\boldsymbol{r})|<(2\pi m_B)^{-1}e^{-m_B r_n}, \tag{25}$$

so that it can be neglected numerically. Then

$$V(\boldsymbol{r})=-\frac{Q^2}{4\pi}\frac{e^{-m_B r}}{r}+\frac{Q^2 m_B{}^2}{4\pi}K_0\left(\frac{\sqrt{2}m_B}{m_\phi}\right)r\,. \tag{26}$$

A few comments are in order.

(1) The potential (26) is composed of the Yukawa and the linear interaction. When $v^2<0$, $m_B=0$, so that (26) is reduced to the usual Coulomb interaction. The vacuum expectation value of ϕ plays the role of the order parameter.

(2) Physically $A_\mu{}^\pm$ as well as a_μ contribute equally in the short-distance region. Around $r\sim 0$, therefore, a new factor $-2(4\pi)^{-1}Q^2$ must be added to the coefficient $-(4\pi)^{-1}Q^2$ of the Yukawa term in (26).

(3) Since the string tension σ is proportional to Q^2, one gets

$$\frac{\sigma_{\text{quark}}}{\sigma_{\text{gluon}}}=\frac{1}{4}\,. \qquad (\text{in } SU(2)) \tag{27}$$

This ratio is 1/3 in $SU(3)$. This may explain the Regge slope difference between Pomeron and ρ trajectories.[10]

(4) The string tension is just the energy per unit length for $r\gg m_B{}^{-1}\gg\sqrt{2}m_\phi{}^{-1}$ in the usual Ginzburg-Landau model with a pair of magnetic monopoles.[8]

Now we try to determine the values of the parameters using the data of Monte-Carlo simulations.[11),12)] The $SU(2)$ static quark potential is measured to be[11]

$$V(r)=-\frac{\alpha}{r}+Kr+\text{constant} \tag{28}$$

with $\alpha\sim 0.244$ and $K\sim(420\ \text{MeV})^2$. On the other hand, the $SU(2)$ Monte-Carlo data of deconfinement transition[12] is

$$T_c\sim 200\ \text{MeV}\,. \tag{29}$$

Since the Lagrangian (13) is an infrared effective theory, we consider only the thermal fluctuation to calculate the transition temperature, neglecting quantum effects. Following Caldi and Nussinov,[13] we get

$$T_c\sim 2v\,. \tag{30}$$

Also we take into account the modification of the coefficient around $r\sim 0$. Then we get roughly

$$m_B\sim 880\ \text{MeV}\,, \qquad m_\phi\sim 18\ \text{GeV}\,, \tag{31}$$

$$v\sim 100\ \text{MeV} \tag{32}$$

and

$$\lambda\sim 8\times 10^3\,. \tag{33}$$

Since the data in $SU(3)$ are not so different from those in $SU(2)$, qualitatively similar results are obtained in $SU(3)$ also.[14] The result (31) is consistent with the assumption $m_\phi\gg\sqrt{2}m_B$. The large value of λ in (33) shows that $|\phi|$ is almost kept to be v as in the case of the non-linear representation.

Finally we make some concluding remarks.

(a) The theory we have proposed in (11) is very attractive in comparison with other models[15),16)] because it does not use any perturbative result and also because it respects chiral flavor symmetry when $M_1=M_2=0$.
(b) There are many problems to be studied numerically and analytically. Among them, it is very important to evaluate $\langle \bar{\psi}\psi \rangle$ within this model and also to calculate λ and v directly from QCD.
(c) More details, numerical calculations and the extension to $SU(3)$ will be published elsewhere.[14)]

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