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A Global Adaptive Learning Control for Robotic Manipulators

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Abstract— **This paper addresses the problem of designing a global adaptive learning control for robotic manipulators with revolute joints and unknown dynamics. The reference signals to be tracked are assumed to be smooth and periodic with known period. By developing in Fourier series expansion the input reference signals of every joint, an adaptive learning PD control is designed which 'learns' the input reference signals by identifying their Fourier coefficients: global asymptotic tracking and local exponential tracking of both the input and the output reference signals is obtained when the Fourier series expansion of each input reference signal is finite, while arbitrary small tracking errors are achieved otherwise. The resulting control is not model based and depends only on the period of the reference signals and on some constant bounds on the robot dynamics.**

I. INTRODUCTION

In this paper we refer to the tracking control of robot manipulators with revolute joints. As known [1], control laws based on feedback from the position and velocities of the joints have been shown to be globally asymptotically stable, provided that the gravity terms are compensated. It has been also shown that PD controllers may be used for trajectory tracking, with accuracy related to the velocity feedback gains [2]. Moreover, such control algorithms are robust with respect to uncertainties on the inertia parameters; namely, even though the inertia parameters are not known, the global asymptotic stability is ensured. Conversely, uncertainties on the gravity parameters may lead to undesired steady-state errors [3].

When the robot dynamics are highly uncertain, adaptive and learning control laws have been developed in order to cope with the model uncertainties. Adaptive controls require the assumption that the robot dynamics can be expressed as the product of known functions and unknown parameters [4]. On the other hand, learning controls require that the reference trajectory is periodic with known period. The key idea is to use the information obtained in the preceding trial to improve the performance in the current one. Under the assumption that the accelerations are measured and a resetting precedure is performed at the beginning of each trial, learning control laws were initially proposed in [1], [2]. In [5] three adaptive iterative learning controllers are proposed that guarantee L_2 convergence to zero of the position and velocity tracking errors, only requiring a position and velocity errors resetting at the begin of each trial: if the exact reset is not guaranteed, then the position error can be made arbitrarily small by increasing the feedback gains. In [6] two control laws with velocity estimation are proposed, which assure local uniform asymptotic convergence of the position error to zero. The first one is an adaptive control law which provides an estimation of the inverse dynamics (assuming that the reference input signal is linearly parametrized by unknown parameters). The second one is a learning control law and assumes that the reference input signal can be represented by an integral of the product of a known differentiable kernel and an unknown influence function: no robustness analysis is provided for reference input signals which do not belong to such a class. In [7] an adaptive control law and a learning control law are combined in order to achieve an L_2 convergence of the position and velocity errors, provided that an exact reset of the joint angles and velocities can be assured at the beginning of each trial. In [8] four adaptive PID control laws are applied to a robot arm, with revolute joints, which has been linearized along the desired trajectory. The control law consists of a PID feedback part and a learning part which learns the input reference. Asymptotic tracking is achieved in the first three control schemes, provided that the feedback gains satisfy some inequalities, while an adaptation on the feedback gains is used in the fourth one. An adaptive-learning control law is proposed in [9] in which the L_2 convergence is achieved when the target of the adaptive control, which requires a linear parametrization of the robot dynamics, is to track a periodic reference signal. In [10] a hybrid adaptive/learning control is presented, which, combining the iterative learning and the adaptive control approaches, achieves global asymptotic convergence to zero of the joint errors: the proposed controller requires, as usual in iterative learning algorithms, infinite memory and do not guarantee exponential convergence.

This paper addresses the problem of designing a global adaptive learning PD control for robotic manipulators with revolute joints and unknown dynamics. The reference signals to be tracked are assumed to be smooth and periodic with known period. By developing in Fourier series expansion the input reference signals of every joint of the manipulator, an adaptive learning PD control is designed which 'learns' the input reference signals by identifying their Fourier coefficients: global asymptotic tracking and local exponential tracking of both the input and the output reference signals is obtained when the Fourier series expansion of each input reference signal is finite, while arbitrary small tracking errors are achieved otherwise. The resulting control is not model based and depends only on the period of the reference signals and on some constant bounds on the robot dynamics. The control structure consists of a linear part (proportional

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and derivative) plus a learning part which reconstructs the unknown reference input signal. The structure of the learning part is obtained by adapting to the multi-input multi-output robot model the method already used in [11] for local state feedback control of single-input single-output feedback linearizable systems and in [12] for local output feedback control of single-input single-output systems in output feedback form. Preliminary local results for robot control were obtained in [13]. The results here presented are global and are based on the choice of a different Lyapunov function proposed in [3], [14].

II. SYSTEM DEFINITION AND ASSUMPTIONS

Consider the dynamics of an n-link rigid robot with rotational joints as described by

$$H(q)\ddot{q} + C(q,\dot{q})\dot{q} + E(q) + F(\dot{q}) = u \quad (1)$$

where: q is the $n \times 1$ vector of the joint coordinates; $H(q)$ is the inertia matrix, which is symmetric positive definite and bounded for any q; $C(q,\dot{q})$ takes into account the Coriolis and centrifugal forces and is linear with respect to $\dot{q}$ and bounded with respect to q; $F(\dot{q})$ is the friction vector; u is the vector of the applied torques; E(q) is the vector of the gravity forces given by $E(q) = \partial U(q)/\partial q$ where U(q) is the gravitational energy which is bounded for any q. The vector E(q) and its partial derivative with respect to q are also bounded. We list in the following the properties owned by the robot model (1) and the assumptions under which the control algorithm is designed.

Assumption 2.1: The reference signal $q_r(t) \in C^N$(with $N > 5$) is periodic with known period T and such that $||q_r(t)|| \leq B_0$, $||\dot{q}_r(t)|| \leq B_1$, $||\ddot{q}_r(t)|| \leq B_2$ with B_0, B_1, B_2 known positive constant reals.

Property 2.1: Given a proper definition of C that is not unequivocally defined by the form $C(q,\dot{q})\dot{q}$ the matrix $\dot{H} - 2C$ is skew-symmetric. One possible definition for the elements of C which leads to the skew-symmetry of $\dot{H} - 2C$ is

$$C_{i,j}(q,\dot{q}) = \frac{1}{2}\left[\dot{q}^T \frac{\partial H_{i,j}}{\partial q} + \sum_{k=1}^{n}\left(\frac{\partial H_{i,k}}{\partial q_j} - \frac{\partial H_{j,k}}{\partial q_i}\right)\dot{q}_k\right]$$

which implies that

$$\begin{aligned} \dot{H}(q) &= C(q,\dot{q}) + C^T(q,\dot{q}) \\ C(q,x_1)x_2 &= C(q,x_2)x_1. \end{aligned} \quad (2)$$

Property 2.2: The inertia matrix $H(q)$ is such that

$$\begin{aligned} H_m \leq \|H(q)\| &\leq H_M,\ \forall q \in \Re^n \\ \left\|\dot{H}(q)\right\| &\leq H_{DM}\,\|\dot{q}\|,\ \forall q,\dot{q} \in \Re^n \\ \|H(q) - H(q_r)\| &\leq k_H\,\|q - q_r\|,\ \forall q,q_r \in \Re^n. \end{aligned}$$

Property 2.3: The matrix $C(q,\dot{q})$ is such that

$$\begin{aligned} \|C(q,\dot{q}_r)\| &\leq C_M\,\|\dot{q}_r\|,\ \forall q,\dot{q}_r \in \Re^n \\ \|C(q,\dot{q}_r) - C(q_r,\dot{q}_r)\| &\leq k_C\,\|q - q_r\|,\ \forall q,q_r,\dot{q}_r \in \Re^n. \end{aligned}$$

Property 2.4: The vector of the gravity forces $E(q)$ is such that

$$\begin{aligned} \|E(q)\| &\leq E_M,\ \forall q \in \Re^n \\ \|E(q) - E(q_r)\| &\leq k_E\,\|q - q_r\|,\ \forall q,q_r \in \Re^n. \end{aligned}$$

Assumption 2.2: The friction vector $F(\dot{q})$ is such that $F(0) = 0$ and

$$\|F(\dot{q}) - F(\dot{q}_r)\| \leq F_M\,\|\dot{q} - \dot{q}_r\| \qquad \forall \dot{q},\dot{q}_r \in \Re^n.$$

Assumption 2.3: The bounds H_m, H_M, H_{DM}, k_H, C_M, k_C, E_M, k_E, F_M defined in Properties 2.2-2.4 and Assumption 2.2 are known positive reals.

The bounded periodic reference input $u_r(t) \in \Re^n$ of period T, corresponding to the reference $q_r(t)$, can be computed as

$$u_r = H(q_r)\ddot{q_r} + C(q_r,\dot{q}_r)\dot{q}_r + E(q_r) + F(\dot{q}_r). \quad (3)$$

From (3), from Properties 2.1-2.4 and from Assumption 2.2, the reference input $u_r(t)$ satisfies the inequality

$$\|u_r(t)\| \leq F_M B_1 + E_M + C_M B_1 + H_M B_2 \triangleq B^{(0)} \quad (4)$$

$\forall t \in [0,T]$, with $B^{(0)} \geq 0$ a known constant real by virtue of Assumption 2.1. Subtracting (3) from (1) and taking (2) into account, we obtain the error dynamics

$$\begin{aligned} u - u_r = {} & H(q)\ddot{\tilde{q}} + [H(q) - H(q_r)]\,\ddot{q}_r + C(q,\dot{q})\dot{\tilde{q}} \\ & + C(q,\dot{q}_r)\dot{\tilde{q}} + [C(q,\dot{q}_r) - C(q_r,\dot{q}_r)]\,\dot{q}_r \\ & + [E(q) - E(q_r)] + F(\dot{q}) - F(\dot{q}_r) \end{aligned} \quad (5)$$

where $\tilde{q} = q - q_r$ and $\dot{\tilde{q}} = \dot{q} - \dot{q}_r$. Let $\theta_i = [\theta_{i,1},\theta_{i,2},\cdots,\theta_{i,p_i}]^T$ be the vector of the first p_i Fourier coefficients of the Fourier series expansion of the i-th component of $u_r(t) = [u_{r,1}(t)\cdots,u_{r,n}(t)]^T$, where $1 \leq i \leq n$ and p_i is an odd number. There exist n positive reals ϵ_{Mi} such that (see [15]) $u_{r,i}(t) = \sum_{k=1}^{p_i}\theta_{i,k}\phi_{i,k}(t) + \epsilon_i(t) = \phi_i^T(t)\theta_i + \epsilon_i(t)$ where $|\epsilon_i(t)| \leq \epsilon_{Mi}$ and with $\phi_i(t) = [\phi_{i,1}(t),\cdots,\phi_{i,p_i}(t)]^T$ and

$$\begin{aligned} \phi_{i,1}(t) &= 1, \\ \phi_{i,2j}(t) &= \sqrt{2}\sin(2\pi jt/T), \\ \phi_{i,2j+1}(t) &= \sqrt{2}\cos(2\pi jt/T), \\ j &= 1,\ldots,(p_i - 1)/2\ . \end{aligned} \quad (6)$$

Consequently, we can write

$$u_r(t) = \Phi^T(t)\Theta + \epsilon(t) \quad (7)$$

where $\epsilon(t) = [\epsilon_1(t),\cdots,\epsilon_n(t)]^T \in \Re^n$, $||\epsilon(t)|| \leq \epsilon_M = (\sum_{i=1}^n \epsilon_{Mi}^2)^{1/2}$, $\Theta = [\theta_1^T,\cdots,\theta_n^T]^T \in \Re^{\sum_{i=1}^n p_i}$ and

$$\Phi^T(t) = \begin{bmatrix} \phi_1^T(t) & 0 & \cdots & 0 \\ 0 & \phi_2^T(t) & \cdots & 0 \\ \vdots & 0 & \ddots & 0 \\ 0 & 0 & \cdots & \phi_n^T(t) \end{bmatrix} \in \Re^{n\times\sum_{i=1}^n p_i}.$$

Since by Assumption 2.1 $q_r(t) \in C^N$, then $u_r(t) \in C^{N-2}$ and ϵ_{Mi} is such that (see [15])

$$\epsilon_{Mi} = \begin{cases} 4B_i^{(N-2)} \left(\frac{T}{2\pi}\right)^{N-2} \frac{N-2}{N-3}, & p_i = 1 \\ 4B_i^{(N-2)} \left(\frac{T}{2\pi}\right)^{N-2} \frac{2^{N-3}}{N-3} \frac{1}{(p-1)^{N-3}}, & p_i > 1 \end{cases}$$

where $B_i^{(N-2)} = \sup_{0 \le t \le T}(|d^{N-2}(u_{r,i}(t))/dt^{N-2}|)$. By virtue of the Bessel inequality we have $||\Theta||^2 \le \frac{1}{T}\int_{-T/2}^{T/2} ||u_r(\tau)||^2 d\tau$ which, in view of (4), implies

$$||\Theta|| \le B^{(0)} \ . \tag{8}$$

Since the reference signal $u_r(t)$ defined by (3) and (7) is unknown, we introduce the estimate $\hat{u}_r(t) = \Phi^T(t)\hat{\Theta}(t)$ with $\hat{\Theta}^T(t) = [\hat{\theta}_1^T(t), \cdots, \hat{\theta}_n^T(t)]$. Since Θ is bounded by a known bound, we use the projection algorithm proj$(\chi, \hat{\Theta})$ so that the estimate $\hat{\Theta}(t)$ is constrained to belong to a suitable region. We define $\dot{\hat{\Theta}} = a$ proj$(\chi, \hat{\Theta})$, in which a is a positive adaptation gain, χ is a suitable function and proj$(\chi, \hat{\Theta})$ is given by

$$\text{proj}\left(\chi, \hat{\Theta}\right) = \begin{cases} \chi, & \text{if} \quad \beta(\hat{\Theta}) \le 0 \\ \chi, & \text{if} \quad \beta(\hat{\Theta}) > 0 \quad \text{and} \\ & \quad \chi^T \text{grad}\left(\beta(\hat{\Theta})\right) \le 0 \\ \chi_p, & \text{if} \quad \beta(\hat{\Theta}) > 0 \quad \text{and} \\ & \quad \chi^T \text{grad}\left(\beta(\hat{\Theta})\right) > 0 \end{cases}$$

where $\beta(\hat{\Theta}) = (||\hat{\Theta}||^2 - r^2)/(\alpha^2 + 2\alpha r)$, grad$[\beta(\hat{\Theta})] = (2\hat{\Theta})/(\alpha^2 + 2\alpha r)$, $\chi_p = \chi - \beta(\hat{\Theta})(\text{grad}[\beta(\hat{\Theta})]\text{grad}[\beta(\hat{\Theta})]^T)/(\text{grad}[\beta(\hat{\Theta})]^T\text{grad}[\beta(\hat{\Theta})])\chi$ in which α is an arbitrary positive constant and r is the radius of the ball $S \subset \Re^n$, centered at the origin, containing Θ. According to (8), $r = B^{(0)}$ in our case. By definition, proj$(\chi, \hat{\Theta})$ is Lipschitz continuous and if $\hat{\Theta}(0) \in S$ then the following properties hold ([16]), $\forall t \ge 0$,

$$\begin{aligned} \tilde{\Theta}^T(t)\text{proj}(\chi, \hat{\Theta}(t)) &\ge \tilde{\Theta}^T(t)\chi, \\ \left\|\hat{\Theta}(t)\right\| &\le \alpha + B^{(0)}, \quad \forall t \ge 0, \\ \left\|\text{proj}(\chi, \hat{\Theta})\right\| &\le \|\chi\| \end{aligned} \tag{9}$$

with $\tilde{\Theta} = \Theta - \hat{\Theta}$. From (6), (8), (9) and since $r = B^{(0)}$ we obtain

$$||\Phi^T(t)\hat{\Theta}|| \le \sqrt{p}\left\|\hat{\Theta}\right\| \le \sqrt{p}\left(B^{(0)} + \alpha\right) \tag{10}$$

where $p \ge \max_{1 \le i \le n}\{p_i\}$. Finally we define $\bar{\gamma}_1 = H_M/(k)^{1/2}$, $\bar{\gamma}_2 = 2H_M/(kH_m)^{1/2}$, $\bar{\gamma}_3 = [(2\gamma_1^*)/k + 2[(\gamma_1^*)^2 + k\gamma_2^*]^{1/2}/k]^2$, $\bar{\gamma}_4 = [(2\gamma_3^*)/(3k) + 2[(\gamma_3^*)^2 + 3k\gamma_4^*]^{1/2}/(3k)]^2$, $\bar{\gamma}_5 = 32p(B^{(0)} + \alpha)^2/k^2$, $\bar{\gamma}_6 = H_M/k$, $\bar{\gamma}_7 = 32p(B^{(0)} + \alpha)^2(2 + A_p^2)/(k^2A_p^2)$, $\bar{\gamma}_8 = 64H_mA_v^2p(B^{(0)} + \alpha)^2/(k^2H_mA_V^2)$, $\bar{\gamma}_9 = H_M/k$, $\bar{\gamma}_{10} = 1/k$, $\bar{\gamma}_{11} = 1/k^2$, $\bar{\gamma}_{12} = 1$, $\bar{\gamma}_{13} = 1/[8Gp^2(1+8A_P^2)]$, $\bar{\gamma}_{14} = [H_M/[8Gp^2k(1 + 8A_P^2)]]^{2/3}$, $\bar{\gamma}_{15} = [H_M^2/[2GH_mp^2k(1 + 8A_P^2)]]^{2/3}$, $\bar{k} = 128p(B^{(0)} + \alpha)^2/(H_mA_V^2)$ where $\gamma_1^* = F_M + K_1 + K_2 + K_3 + C_MB_1 + k$, $\gamma_2^* = 2H_M + C_M/(2\sqrt{2}) + 2C_MB_1 + F_M$, $\gamma_3^* = K_1 + K_2 + K_3 + k$, $\gamma_4^* = 2C_MB_1 + B_2K_H + K_CB_1 + K_E + F_M + \frac{k}{2}$, $G = H_M/4 + 6 + 2[(T/2 + 1)a + K_HB_2 + K_CB_1 + K_E]^2 + 4[\omega H_M/2 + H_{DM}B_1 + 2C_MB_1 + F_M + H_M]^2 + 4a^2(T/2 + 1)^2 + 16A_V^2[C_M + H_{DM}]^2$, $K_1 = \max[B_2\sqrt{k_H^2 + 8H_M^2}, 2\sqrt{2}B_2H_M + B_2k_H]$, $K_2 = \max[B_1\sqrt{k_C^2 + 8C_M^2B_1^2}, 2\sqrt{2}B_1^2C_M + B_1k_C]$, $K_3 = \max[\sqrt{k_E^2 + 8E_M^2}, 2\sqrt{2}E_M + k_E]$, α, A_P, A_V are arbitrary positive constants and k, a are positive reals to be defined in the control design. We are now ready to state and prove the main result: the proof is constructive and contains the control design.

Theorem 2.1: Consider system (1) satisfying Assumptions 2.2, 2.3 and a reference output signal $y_r(t)$ satisfying Assumption 2.1. Consider the dynamic control algorithm

$$\begin{aligned} u(t) &= -\Phi^T(t)\hat{\Theta}(t) - \gamma k\tilde{q}(t) - \sqrt{\gamma}k\dot{\tilde{q}}(t) \\ \dot{\hat{\Theta}}(t) &= a \text{ proj}\left[\sqrt{\gamma}\Phi(t)\dot{\tilde{q}}(t) + \Phi(t)\frac{\tilde{q}(t)}{1 + 2\left\|\tilde{q}(t)\right\|^2}, \hat{\Theta}(t)\right] \\ \hat{\Theta}(0) &= \hat{\Theta}_0 \end{aligned} \tag{11}$$

where $k, \gamma \in \Re$ are positive reals, $||\hat{\Theta}_0|| \le B^{(0)}$ and $\hat{\Theta}$ is an estimation of the vector Θ defined in (7). Assume that $k \ge \bar{k}$ and $\gamma > \max_{1 \le i \le 15}\{\bar{\gamma}_i\}$. Then:

(i) All closed loop signals are bounded and, in particular, $||\hat{\Theta}(t)|| \le B^{(0)} + \alpha$, $||\tilde{q}(t)|| \le A_P + 2r_0$, $||\dot{\tilde{q}}(t)|| \le A_V + 2r_0(k\gamma/H_m)^{1/2}$ with $r_0 = (||\tilde{q}(0)||^2 + ||\dot{\tilde{q}}(0)||^2)^{1/2}$.

(ii) The tracking errors $||\tilde{q}(t)||$ and $||\dot{\tilde{q}}(t)||$ converge globally uniformly asimptotically into the region $||\tilde{q}(t)||^2/A_P^2 + ||\dot{\tilde{q}}(t)||^2/A_V^2 \le 1$.

(iii) The errors $||\tilde{q}(t)||$, $||\dot{\tilde{q}}(t)||$ and $||\tilde{\Theta}(t)||$ converge globally uniformly asymptotically and locally exponentially into the region $||\tilde{q}(t)||^2/E_P^2 + ||\dot{\tilde{q}}(t)||^2/E_V^2 + ||\tilde{\Theta}(t)||^2/E_S^2 \le 1$ where $E_P = O(1/p_m^{N-4})$, $E_V = O(1/p_m^{N-9/2})$, $E_S = O(1/p_m^{N-9/2})$ as $p_m \to \infty$ with $p_m = \min_{1 \le i \le n}\{p_i\}$. Moreover $\limsup_{t\to\infty}|\Phi^T\hat{\Theta} - u_r(t)| \le E_U = O(1/p_m^{N-5})$.

(iv) If $\epsilon(t) = 0$, $\forall t \ge 0$, the equilibrium point $(\tilde{q}^T, \dot{\tilde{q}}^T, \tilde{\Theta}^T) = 0$, of the closed loop system (5), (11) is globally uniformly asymptotically and locally exponentially stable.

Remark 2.1: The control law is not model based and consists of the sum of two terms: a PD linear term and a learning term which reconstructs the reference torque corresponding to the desired output trajectory. The order of the controller is equal to $\sum_{i=1}^n p_i$, with p_i being the number of estimated Fourier coefficients of the i-th joint torque.

Remark 2.2: As shown by Property (iii), the accuracy obtained by the proposed controller can be improved by increasing the number of the estimated Fourier coefficients for each joint reference torque. If the joint reference inputs have a finite Fourier series expansion, the joint tracking errors converge globally (and locally exponentially) to zero, while the estimates $\hat{\Theta}(t)$ converge towards the true values Θ.

Remark 2.3: The bounds E_P, E_V, E_S, E_U can be arbitrarily reduced by increasing the number p_i of the estimated Fourier coefficients of each input reference torque $u_{r,i}(t)$

with $1 \leq i \leq n$.

Proof. Consider the function

$$V = \sqrt{\gamma}\left[\frac{1}{2}\gamma\tilde{q}^T K_P \tilde{q} + \frac{1}{2}\dot{\tilde{q}}^T H(q)\dot{\tilde{q}}\right] + \frac{\dot{\tilde{q}}^T H(q)\tilde{q}}{1+2\|\tilde{q}\|^2},$$

which is such that $V_m = 0.25(\gamma)^{3/2}k||\tilde{q}||^2 + 0.25H_m(\gamma)^{1/2}||\dot{\tilde{q}}||^2 \leq V \leq (\gamma)^{3/2}k||\tilde{q}||^2 + H_M(\gamma)^{1/2}||\dot{\tilde{q}}||^2 = V_M$ provided that $\gamma \geq \max\{\bar{\gamma}_1, \bar{\gamma}_2\}$. From Assumption 2.1,2.2 and Properties 2.2-2.4, differentiating V, we obtain

$$\begin{aligned}\dot{V} \leq & -\left[\sqrt{\gamma}\left(\sqrt{\gamma}\frac{k}{2} - \gamma_1^*\right) - \gamma_2^*\right]\|\dot{\tilde{q}}\|^2 \\ & - [\sqrt{\gamma}\,(\sqrt{\gamma}k - \gamma_3^*) - \gamma_4^*]\frac{\|\tilde{q}\|^2}{1+2\|\tilde{q}\|^2} \\ & + \sqrt{\gamma}\,\|\dot{\tilde{q}}\|\left\|-u_r(t) - \Phi^T(t)\hat{\Theta}(t)\right\| - \gamma\frac{k}{2}\|\dot{\tilde{q}}\|^2 \\ & + \left\|-u_r(t) - \Phi^T(t)\hat{\Theta}(t)\right\|\frac{\|\tilde{q}\|}{1+2\|\tilde{q}\|^2} \\ & - \frac{k}{2}\frac{\|\tilde{q}\|^2}{1+2\|\tilde{q}\|^2} \end{aligned} \quad (12)$$

where γ_1^*, γ_2^*, γ_3^*, γ_4^* have already been defined. From (12), completing the squares, choosing $\gamma \geq \max\{\bar{\gamma}_3, \bar{\gamma}_4\}$ and recalling (4) and (10) we obtain

$$\dot{V} \leq -\gamma\frac{k}{4}\varphi\left(\|\tilde{q}\|, \|\dot{\tilde{q}}\|\right) + \frac{4p(B^{(0)}+\alpha)^2}{k} \quad (13)$$

in which $\varphi(||\tilde{q}||, ||\dot{\tilde{q}}||) = ||\dot{\tilde{q}}||^2 + ||\tilde{q}||^2/(1 + 2||\tilde{q}||^2)$. From (13), it follows that $\dot{V} \leq 0$ if $\varphi(||\tilde{q}||, ||\dot{\tilde{q}}||) \geq 16p(B^{(0)}+\alpha)^2/(\gamma k^2)$. Since the level curves of the function $\varphi(||\tilde{q}||, ||\dot{\tilde{q}}||)$ are closed only if $\varphi(||\tilde{q}||, ||\dot{\tilde{q}}||) < 0.5$, the closed loop trajectories are bounded $\forall t \geq 0$ provided that $\gamma > \max\{\bar{\gamma}_3, \bar{\gamma}_4, \bar{\gamma}_5\}$. From the expressions of the functions V_m and V_M we obtain that $||\tilde{q}||$ and $||\dot{\tilde{q}}||$ converge uniformly asimptotically into the region

$$\frac{||\tilde{q}||}{A_P^2} + \frac{||\dot{\tilde{q}}||}{A_V^2} \leq 1, \quad A_P, A_V \in \Re^+ \quad (14)$$

provided that $k > \bar{k}$ and choosing $\gamma > \max_{1\leq i\leq 8}\{\bar{\gamma}_i\}$ (Property (ii) of Theorem 2.1). If $||\tilde{q}(0)||^2/A_P^2 + ||\dot{\tilde{q}}(0)||^2/A_V^2 \geq 1$ and $||\tilde{q}(t)||^2/A_P^2 + ||\dot{\tilde{q}}(t)||^2/A_V^2 \geq 1$, $V_m(t) \leq V(t) \leq V_M(0)$ so that, since $\gamma \geq \bar{\gamma}_9$, it follows that $||\tilde{q}(t)|| \leq A_P + 2r_0$ and $||\dot{\tilde{q}}(t)|| \leq A_V + 2r_0(\gamma k/H_m)^{1/2}$, $\forall t \geq 0$ (Property (i) of Theorem 2.1). Moreover, there exists a finite time instant $t^* \geq 0$ such that

$$\frac{||\tilde{q}(t)||}{4A_P^2} + \frac{||\dot{\tilde{q}}(t)||}{4A_V^2} \leq 1, \ \forall t \geq t^*. \quad (15)$$

Consider the function $W = V + ||\tilde{\Theta}||^2/(2a)$ which is such that $W_m = V_m + ||\tilde{\Theta}||^2/(2a) \leq W \leq V_M + ||\tilde{\Theta}||^2/(2a) = W_M$ provided that $\gamma \geq \max\{\bar{\gamma}_1, \bar{\gamma}_2\}$. Differentiating W and recalling (11) we obtain $\dot{W} \leq -\gamma k\varphi(||\tilde{q}||, ||\dot{\tilde{q}}||)/4 + \epsilon_S^2/k$ where $\epsilon_S = \sup_{t\in[0,T]}||\epsilon(t)||$. If $\gamma \geq \max_{1\leq i\leq 9}\{\bar{\gamma}_i\}$ the closed loop trajectories are such that $||\tilde{q}(t)|| \leq 2A_P \ \forall t \geq t^*$, so that

$$\dot{W} \leq -\frac{\gamma k}{4(1+8A_P^2)}\left(\|\dot{\tilde{q}}\|^2 + \|\tilde{q}\|^2\right) + \frac{\epsilon_S^2}{k}. \quad (16)$$

Consider the function

$$U = W + \frac{1}{2}a^*\left\|Q\tilde{\Theta} - \Phi H(q)\dot{\tilde{q}}\right\|^2 \quad (17)$$

in which $a^* > 0$ is yet to be defined and $Q(t)$ is the matrix solution of $\dot{Q} = -Q + \Phi(t)\Phi^T(t)$, $Q(0) = (T/2)I$ from which, since $\int_t^{t+T}\Phi(\tau)\Phi^T(\tau)d\tau \geq (T/2)I > 0$ ($\forall t \geq 0$) we have $(T/2)e^{-T}I < Q(t) \leq (T/2)I + pI$ with $p \geq ||\Phi||^2 = \max_{1\leq i\leq n}\{p_i\}$. From (5), (11) and (17), we obtain $\dot{U} \leq \dot{W} + a(Q\tilde{\Theta} - \Phi H(q)\dot{\tilde{q}})^T(Q\dot{\tilde{\Theta}} - Q\tilde{\Theta} + \Phi\Phi^T\tilde{\Theta} - \dot{\Phi}H\dot{\tilde{q}} - \Phi\dot{H}\dot{\tilde{q}} + \Phi\left(H(q) - H(q_r)\right)\ddot{q}_r + \Phi C(q,\dot{q})\dot{\tilde{q}} + \Phi C(q,\dot{q}_r)\dot{\tilde{q}} + \Phi(C(q, \dot{q}_r) - C(q_r,\dot{q}_r)) + \Phi\left(E(q) - E(q_r)\right) + \Phi F\dot{\tilde{q}} - \Phi E + \gamma k\Phi\tilde{q} + \sqrt{\gamma}k\Phi\dot{\tilde{q}} + \Phi H\dot{\tilde{q}} - \Phi H\dot{\tilde{q}})$ from which, since $||Q||||\dot{\tilde{\Theta}}|| \leq (T/2+p)\,a(\gamma p)^{1/2}||\dot{\tilde{q}}|| + (T/2+p)\,a\sqrt{p}||\tilde{q}||/(1 + 2||\tilde{q}||^2)$, $||\dot{H}(q_r + \tilde{q})|| \leq H_{DM}||\dot{q}_r|| + H_{DM}||\dot{\tilde{q}}||$, $||C(q,\dot{q})|| \leq C_M||\dot{\tilde{q}}|| + C_M B_1$, by virtue of Properties 2.1-2.4 and Assumption 2.2, we have

$$\begin{aligned}\dot{U} \leq & \ \dot{W} - a^*\left\|Q\tilde{\Theta} - \Phi H(q)\dot{\tilde{q}}\right\|^2 \\ & + a^*\left\|Q\tilde{\Theta} - \Phi H(q)\dot{\tilde{q}}\right\|\delta_1\|\tilde{q}\| \\ & + a^*\left\|Q\tilde{\Theta} - \Phi H(q)\dot{\tilde{q}}\right\|\delta_2\|\dot{\tilde{q}}\| \\ & + a^*\left\|Q\tilde{\Theta} - \Phi H(q)\dot{\tilde{q}}\right\|\delta_3\|\dot{\tilde{q}}\|^2 \\ & + a^*\left\|Q\tilde{\Theta} - \Phi H(q)\dot{\tilde{q}}\right\|\sqrt{p}\epsilon_S \end{aligned} \quad (18)$$

where $\delta_1 = p\sqrt{p}(\gamma k + (T/2+1)a + K_H B_2 + K_C B_1 + K_E)$, $\delta_2 = p\sqrt{p}(k\sqrt{\gamma} + a(T/2+1)\sqrt{\gamma} + \omega H_M/2 + H_{DM}B_1 + 2C_M B_1 + F_M + H_M)$, $\delta_3 = p\sqrt{p}(C_M + H_{DM})$. From (18) and since $||\dot{\tilde{q}}|| \leq 2A_V$, $\forall t \geq t^*$, we have $\dot{U} \leq \dot{W} - a^*||Q\tilde{\Theta} - \Phi H(q)\dot{\tilde{q}}||^2 + a^*||Q\tilde{\Theta} - \Phi H(q)\dot{\tilde{q}}||\delta_1||\tilde{q}|| + a^*||Q\tilde{\Theta} - \Phi H(q)\dot{\tilde{q}}||\delta_4||\dot{\tilde{q}}|| + a^*||Q\tilde{\Theta} - \Phi H(q)\dot{\tilde{q}}||\sqrt{p}\epsilon_S$ with $\delta_4 = \delta_2 + 2\delta_3 A_V$. From (16) and since $||Q\tilde{\Theta} - \Phi H(q)\dot{\tilde{q}}||\delta_1||\tilde{q}|| \leq ||Q\tilde{\Theta} - \Phi H(q)\dot{\tilde{q}}||^2/4 + \delta_1^2||\tilde{q}||^2$, $||Q\tilde{\Theta} - \Phi H(q)\dot{\tilde{q}}||\delta_4||\dot{\tilde{q}}|| \leq ||Q\tilde{\Theta} - \Phi H(q)\dot{\tilde{q}}||^2/4 + \delta_4^2||\dot{\tilde{q}}||^2$, $||Q\tilde{\Theta} - \Phi H(q)\dot{\tilde{q}}||\sqrt{p}\epsilon_S \leq ||Q\tilde{\Theta} - \Phi H(q)\dot{\tilde{q}}||^2/4 + p\epsilon_S^2$ it follows that (recalling that $||c-b||^2 \leq 2(||c||^2 + ||b||^2)$ and $||c-b||^2 \geq ||c||^2/2 - ||b||^2$)

$$\begin{aligned}\dot{U} \leq & -\frac{\gamma k}{4(1+8A_P^2)}||X||^2 + a^*M||X||^2 - \\ & \frac{T^2}{32}e^{-2T}a^*||\tilde{\Theta}||^2 + \left(pa^* + \frac{1}{k}\right)\epsilon_S^2 \end{aligned} \quad (19)$$

where, if $\gamma \geq \max_{1\leq i\leq 12}\{\bar{\gamma}_i\}$, $M = \gamma^2k^2p^3G$, $G = H_M/4 + 6 + 2[(T/2+1)a + K_H B_2 + K_C B_1 + K_E]^2 + 4[\omega H_M/2 + H_{DM}B_1 + 2C_M B_1 + F_M + H_M]^2 + 4a^2(T/2+1)^2 + 16A_V^2[C_M + H_{DM}]^2$ and $X = [\tilde{q}^T, \dot{\tilde{q}}^T]^T$. From (19) we obtain

$$\dot{U} \leq -\gamma\frac{k}{8(1+8A_P^2)}\|X\|^2 - \frac{T^2}{32}e^{-2T}a^*\left\|\tilde{\Theta}\right\|^2 + \frac{2}{k}\epsilon_S^2 \quad (20)$$

where, since $\gamma \geq \bar{\gamma}_{13}$, $a^* = \min\left\{1/(pk), 1/(8G\gamma kp^3(1+8A_P^2))\right\} = 1/(8G\gamma kp^3(1+8A_P^2))$. From (20) it follows that $\dot{U} \leq 0$ provided that $||\tilde{q}|| \geq 16\epsilon_S^2(1+8A_P^2)/(\gamma k^2) \triangleq R_m^2$ and $||\tilde{\Theta}|| \geq 512\epsilon_S^2 G\gamma p^3(1+8A_P^2)e^{2T}/T^2 \triangleq R_M^2$. From (17) and from the expressions of the functions W_m and W_M, if $\gamma \geq \max\{\bar{\gamma}_{14}, \bar{\gamma}_{15}\}$, we have $U_m \leq \gamma\sqrt{\gamma}k||\tilde{q}||^2/4 + \sqrt{\gamma}H_m||\dot{\tilde{q}}||^2/8 + S_m||\tilde{\Theta}||^2 \leq U \leq \gamma\sqrt{\gamma}k||\tilde{q}||^2 + 2\sqrt{\gamma}H_M||\dot{\tilde{q}}||^2 + S_M||\tilde{\Theta}||^2 = U_M$ where $S_m = 1/(2a) + T^2e^{-2T}/(128G\gamma kp^3(1+8A_P^2))$ and $S_M = 1/(2a) + (T+2p)^2/(32G\gamma kp^3(1+8A_P^2))$. From (20) and from the expressions of the functions U_m and U_M, it follows that $||\tilde{q}||$, $||\dot{\tilde{q}}||$ and $||\tilde{\Theta}||$ converge locally exponentially into the region

$$\frac{||\tilde{q}||^2}{E_P^2} + \frac{||\dot{\tilde{q}}||^2}{E_V^2} + \frac{||\tilde{\Theta}||^2}{E_S^2} \leq 1 \tag{21}$$

where $E_P^2 = 4R_m^2 + 8H_M R_m^2/(\gamma k) + 4S_M R_M^2/(\gamma\sqrt{\gamma}k)$, $E_V^2 = 8\gamma kR_m^2/H_m + 16H_M R_m^2/H_m + 8S_M R_M^2/(H_m\sqrt{\gamma})$ and $E_S^2 = \gamma\sqrt{\gamma}kR_m^2/S_m + 2H_M\sqrt{\gamma}R_m^2/S_m + S_M R_M^2/S_m$. Since $k = O(p_m)$, $\gamma = O(1)$, $R_M = O(1/p_m^{N-9/2})$, $R_m = O(1/p_m^{N-2})$ as $p_m \to \infty$ ($p_m = \min_{1\leq i\leq n}\{p_i\}$) then $E_P = O(1/p_m^{N-4})$, $E_V = O(1/p_m^{N-9/2})$, $E_S = O(1/p_m^{N-9/2})$, $\limsup_{t\to\infty}|\Phi^T\hat{\Theta} - u_r(t)| \leq E_U = O(1/p_m^{N-5})$ which implies property (iii) of Theorem 2.1. Moreover, if p_m is sufficiently large , it follows that $E_P < 2A_P$, $E_V < 2A_V$ so that $||\tilde{q}||$ and $||\dot{\tilde{q}}||$ converge in a region smaller than (15): the convergence is exponential in the region obtained by the difference between (15) and the projection of (21) on the plane $||\tilde{\Theta}|| = 0$. From (13), and (20) it follows that, if $\epsilon(t) = 0$ $\forall t \geq 0$, the system is globally uniformly asymptotically and locally exponentially stable (property (iv) of Theorem 2.1).

III. Simulations

The proposed control algorithm has been applied to a two link robot arm with two revolute joints whose dynamic behavior is described by (1) with $u = [u_1, u_2]^T$, $q = [q_1, q_2]^T$, $\dot{q} = [\dot{q}_1, \dot{q}_2]^T$,

$$\begin{aligned}
H(q) &= \begin{bmatrix} \alpha_1 + 2\alpha_3\cos(q_2) & \alpha_2 + \alpha_3\cos(q_2) \\ \alpha_2 + \alpha_3\cos(q_2) & \alpha_2 \end{bmatrix} \\
C(q,\dot{q})\dot{q} &= \begin{bmatrix} -2\alpha_3\sin(q_2)\dot{q}_1\dot{q}_2 - \alpha_3\sin(q_2)\dot{q}_2^2 \\ \alpha_3\sin(q_2)\dot{q}_1^2 \end{bmatrix} \\
E(q) &= \begin{bmatrix} \alpha_4\cos(q_1) + \alpha_5\cos(q_1+q_2) \\ \alpha_5\cos(q_1+q_2) \end{bmatrix} \\
F(\dot{q}_1, \dot{q}_2) &= \begin{bmatrix} F_1\dot{q}_1 \\ F_2\dot{q}_2 \end{bmatrix}.
\end{aligned} \tag{22}$$

In (22) $\alpha_1 = I_1 + m_1L_1^2/4 + m_2(L_1^2 + L_2^2/4) + I_2$, $\alpha_2 = I_2 + m_2L_2^2/4$, $\alpha_3 = m_2L_1L_2/2$, $\alpha_4 = g(m_1L_1/2 + m_2L_1)$, $\alpha_5 = m_2gL_2/2$ and the parameters are such that $2 \leq m_1 \leq 8$ Kg, $1 \leq m_2 \leq 5$ Kg, $1 \leq L_1 \leq 2$ m, $0.5 \leq L_2 \leq 1.5$ m, $0.1 \leq I_1 \leq 0.4$ Kg m^2, $0.05 \leq I_2 \leq 0.2$ Kg m^2, $10 \leq F_1 \leq 20$ Kg m^2/s, $10 \leq F_2 \leq 20$ Kg m^2/s, $g = 9.8$ m/s^2. The cartesian position of the end effector $[x, y]^T$ is described by the kinematic equations

$$\begin{aligned}
x &= L_1\cos(q_1) + L_2\cos(q_1+q_2) \\
y &= L_1\sin(q_1) + L_2\sin(q_1+q_2)\ .
\end{aligned} \tag{23}$$

We assume that the robot arm is at rest at $t = 0$ and its initial configuration is such that $0 < q_2(0) < \pi$ with $x(0) = 0.75$ m and $y(0) = 0.3$ m. Moreover, in the simulations $m_1 = 5$ Kg, $m_2 = 3$ Kg, $L_1 = 1.5$ m, $L_2 = 1$ m, $I_1 = 0.2$ Kg m^2, $I_2 = 0.2$ Kg m^2, $F_1 = 10$ Kg m^2/s and $F_1 = 15$ Kg m^2/s. The reference output velocity profiles $\dot{x}_r(t)$ and $\dot{y}_r(t)$, which are periodic with known period $T = 18$ sec, are shown in Figure 1 and correspond to the planar trajectory depicted in Figure 2 ($x_r(0) = 0.7$ and $y_r(0) = 0.1$) which is such that $q_r(t) \in C^6$, as required by Assumption 2.1, and $B_0 = 3.5$, $B_1 = 2$, $B_2 = 5$. The desired trajectory in joint coordinates is obtained either by assuming the knowledge of the robot kinematics (23) or by a suitable teaching procedure.

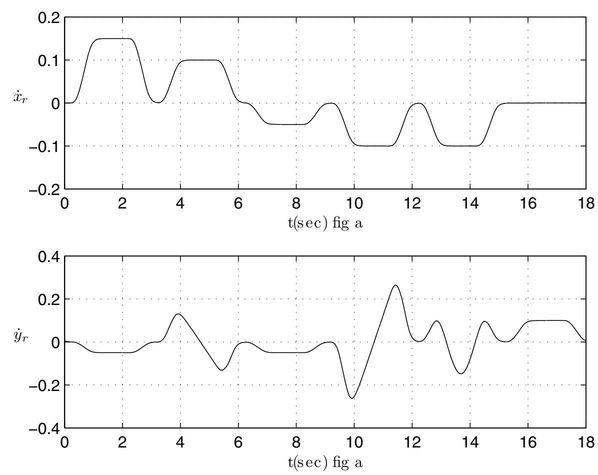

Fig. 1. (a):$dx(t)/dt$ (b): $dy(t)/dt$.

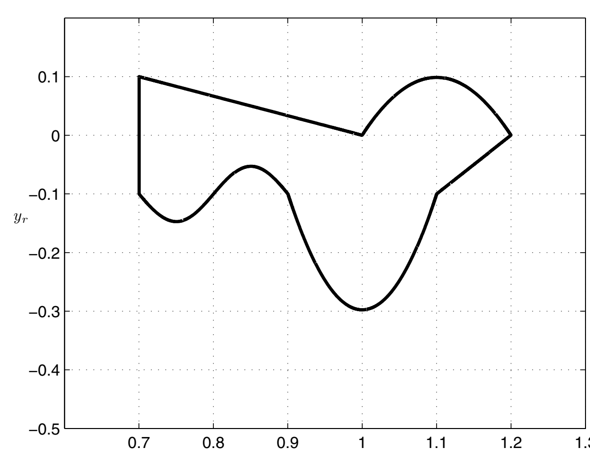

Fig. 2. Planar reference trajectory.

A. Design steps

1) The bounds defined in Properties 2.2-2.4 are computed taking into account the uncertainties on the robot's dynamics: $H_m = 0.6$, $H_M = 48.82$, $H_{DM} = 30$, $k_H = 30$, $C_M = 30$, $k_C = 30$, $E_M = 896.7$, $k_E = 896.7$, $F_M = 164$.
2) According to (4) the bound $B^{(0)} = 1528.8$ is computed. Set the parameters $r = B^{(0)} = 1528.8$, $\alpha = 0.1$ and $a = 50$ and the maximum number of Fourier coefficients to be estimated: $p = 31 \geq \max\{p_1, p_2\}$.
3) The values $A_p = 0.02$ and $A_v = 0.005$ of the maximum steady state tracking errors $||\tilde{q}(t)||$ and $||\dot{\tilde{q}}(t)||$ are set.
4) The minimum values of the control parameters k and γ are computed: $k \geq \bar{k} = 6.1 \cdot 10^{14}$ and $\gamma > \max_{1\leq i \leq 15}\{\bar{\gamma}_i\} = 16$ which are in general highly conservative values.

We consider $k = 500$, $\gamma = 1$ and apply the control law (11)

$$\begin{aligned} u &= -\Phi^T\hat{\Theta} - \begin{bmatrix} 500 & 0 \\ 0 & 500 \end{bmatrix}\tilde{q} - \begin{bmatrix} 500 & 0 \\ 0 & 500 \end{bmatrix}\dot{\tilde{q}} \\ \dot{\hat{\Theta}} &= 50 \text{ proj}\left[\Phi\dot{\tilde{q}} + \Phi\frac{\tilde{q}}{1+2\|\tilde{q}\|^2}, \hat{\Theta}\right] \end{aligned} \qquad (24)$$

with $\hat{\Theta}(0) = 0$, $p_1 = 21$, $p_2 = 21$ so that $\dim(\hat{\Theta}) = 42$. Applying the proposed control law (24) to the robot arm, we obtain the output planar trajectories of Figure 3. Figure 3 shows also the input signals applied to each joint of the robot arm and the position errors $x - x_r$ and $y - y_r$: after four periods the position errors become smaller than 0.5 mm and the steady state design specifications are satisfied.

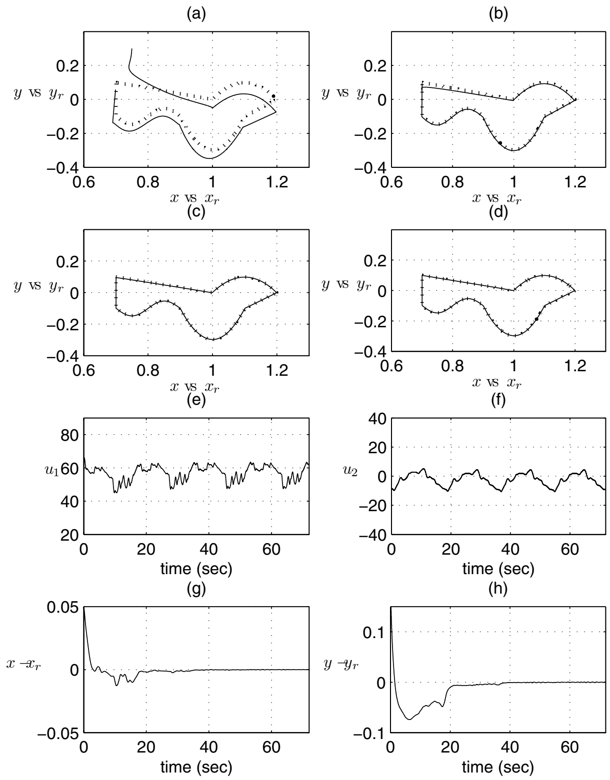

Fig. 3. Actual trajectory (solid line) vs reference trajectory (dotted line) of the end effector: (a) 1st period, (b) 2nd period, (c) 3rd period, (d) 4th period; (e): input to the first joint; (f): input to the second joint; (g): position error $x(t) - x_r(t)$; (h): position error $y(t) - y_r(t)$.

IV. CONCLUSIONS

For robot arms with all revolute joints the problem of tracking a smooth periodic output reference, with known period, has been addressed and solved assuming that some constant bounds on the robot parameters are known. The control structure is independent of the system's nonlinearities: global asymptotic and local exponential convergence to zero or to an arbitrarily small residual set is guaranteed.

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