A generalized frequency-temperature viscoelastic model

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Abstract. The design of passive damping treatments using viscoelastic materials requires both an accurate numerical analysis approach, usually making use of the finite element method, and a realistic means of material description. While the former has been widely studied and several and valuable approaches have become available during the last years, the latter is still an issue requiring additional efforts. The experimental characterization, the data modeling and finally the constitutive models able to be directly used along analytic and numerical analysis, are still important research areas. Several viscoelastic models, able to be directly applied into a finite element analysis either in a time or in a frequency domain analysis, are available and have been widely used during the last years in most of the published work. Despite the general description and straightforward use that such modeling approaches may provide, temperature effect is usually disregarded and isothermal analysis are usually performed. Moreover, this temperature effect is naturally not directly considered as an input parameter for most of the viscoelastic material models and isothermal conditions are also considered in the experimental characterization data analysis. This work presents an extended viscoelastic model, based on well known isothermal models, where the temperature-frequency superposition effect is directly considered. The extended model is applied to the analysis of the experimental data using a data fitting procedure to identify a set of global parameters able to represent the effect of the frequency and the temperature.

1. Introduction

When dealing with the design of damping treatments, a complete and accurate characterization of the viscoelastic damping materials is required. These materials may evidence a high dependency upon the frequency and the temperature, and experimental methodologies are usually applied to identify its complex modulus, either the extensional or the shear complex modulus.

In this work, a frequency-temperature parametric model is presented and numerically implemented. Based on a published parametric isothermal model, the reduced frequency approach is applied onto such model to provide the description for both the frequency and the temperature dependency. The model is implemented and assessed using the experimental data obtained from the application of an in-house testing device designed for the characterization of viscoelastic materials. The final goal for this model is to provide the ability to be directly fitted to the experimental data, without the need for pre-analysis and filtering tasks, and midterm procedures such as the required shift factor fitting process. This extended model, combining both the temperature and the frequency dependency, shall be regarded as an universal descriptor for the viscoelastic materials complex modulus, providing a valuable tool for data sharing and publication, or even as a constitutive model in a numerical analysis procedure.

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1.1. Linear viscoelasticity

A polymeric material, with viscoelastic behavior, presents a material structure formed by reticular cross-linked chains. Such structure evidences a high temperature dependency, resulting from the different chain mobility level that depends upon the temperature. Thus, below its transition temperature, the vitreous range, the material exhibits a high stiffness value, whereas the stiffness value decreases significantly for temperatures above this transition temperature. A similar effect is observed when the material is cyclic loaded with high and low frequency, respectively.

Disregarding the temperature influence, considering isothermal conditions, it can be experimentally observed that a viscoelastic material, when harmonically loaded with a predefined frequency, presents a harmonic response with the same frequency, yet presenting a phase lag between the applied load and the strain response, which is directly related to the damping characteristic of the material. This observation, which is also sustained by the linear viscoelastic theory [1], allows establishing a stress-strain relation as:

$$\sigma(\omega) = E'(\omega)[1 + j\eta(\omega)]\varepsilon(\omega) = [E'(\omega) + jE''(\omega)]\varepsilon(\omega) \tag{1}$$

where $E'(\omega)$ is the storage modulus and is associated to the recovered deformation strain energy, $E''(\omega)$ is the loss modulus and is associated to the dissipated strain energy, and $\eta(\omega)$ stands for the loss factor and corresponds to the ratio between the loss modulus and the storage modulus, i.e. it represents directly the damping characteristic of the viscoelastic material [2]. The storage modulus and the loss modulus can be gathered into a complex frequency and temperature dependent variable, the extensional complex modulus $\bar{E}(\omega, T)$, or the shear complex modulus $\bar{G}(\omega, T)$ that is in fact the design parameter for surface constrained and integrated damping treatments.

1.2. Experimental characterization of viscoelastic materials

Usually, the complex modulus is obtained from experimental tests where damped beams (indirect methods) [3] or viscoelastic materials samples (direct methods) [4–6] are applied. While the former configuration is based on the analysis of the perturbation produced by the application of the damping patch onto the dynamic response of elastic beams, the latter uses simple rod or shear patch samples which are harmonically loaded with a prescribed frequency and amplitude loading. Conventionally, such tests are performed inside a controlled environment chamber, applying isothermal conditions to the damping material sample.

Indirect methods using cantilever beams are historically a reference characterization method for viscoelastic damping materials. In fact, this methodology is still in use by several damping materials manufacturers as a reference for other characterization approaches. Using this approach, the complex modulus data is not identified directly but rather calculated from the perturbation effect that a damping patch introduces onto the modal parameters (natural frequencies and corresponding modal loss ratios) of elastic beams, usually made from aluminum or stainless steel. The calculation can be performed by using the well-known RKU equations, initially proposed by Ross, Kerwin and Ungar [7,8], that provide a simple but imprecise approach. Recently, to reduce the inaccuracy caused by the assumed displacement field and spatial description of the mode shapes of the beam, finite element models [9,10] have been used as a valuable replacement for the RKU equations.

When direct methods are applied, the stress-strain relation is evaluated through the cross analysis between the applied load signal and the induced displacement or elongation produced onto the specimen. This relation, when described as a stress/strain relation, defines directly the complex modulus value for each individual frequency/temperature pair.

Independently of the applied characterization methodology, a large dataset of complex modulus values is collected, which can only be directly applied in a direct frequency analysis [11] to generate a partial frequency domain response model.

1.3. Frequency – temperature superposition principle

If the material is considered as a thermo-rheologically simple material, opposing to those exhibiting a thermo-rheologically complex behavior, it follows the frequency-temperature superposition principle. For such simple materials, only the primary glass-transition is present, whereas the thermo-rheologically complex materials exhibit a

secondary glass-transition effect and, in addition to the speed, also the sequence of the molecular processes changes with the temperature.

The frequency-temperature superposition principle states that an inverse and equivalent relation exists between the frequency and the temperature effects, i.e., the complex modulus modification induced by a frequency change can be identical to the one produced by an inverse modification of temperature. This equivalence principle is usually considered to provide the complex modulus data for frequency/temperature pairs different from those gathered experimentally. This methodology is conventionally applied to generate the nomogram representation [12], which is in fact the unique dissemination method currently used by the damping materials manufacturers or even by most of the research laboratories and other research entities.

Since the frequency and the temperature effects are related, the complex modulus function can be described in terms of a general parameter, called reduced frequency, defined as:

$$\omega_r = \omega.\alpha_T\left(T\right) \tag{2}$$

where α_T represents the shift factor function, which is temperature dependent.

The shift factor definition is based on the observation that storage and loss modulus (or alternatively loss factor) curves obtained for different temperatures may be horizontally shifted in a logarithmic representation to form a single continuous master curve, the basis of a material nomogram. The shift factor distribution can be represented by simple temperature dependent equations, such as the Arrhenius model [13]:

$$\log\left[\alpha_T\left(T\right)\right] = T_A\left(\frac{1}{T} - \frac{1}{T_0}\right) \tag{3}$$

or the Williams-Landel-Ferry (WLF) model [13]:

$$\log\left[\alpha_T\left(T\right)\right] = -C_1\frac{T - T_0}{B_1 + T - T_0} \tag{4}$$

where T is the absolute temperature parameter (kelvin), T_0 is the reference absolute temperature (kelvin), and T_A, C_1 and B_1 are model constant parameters ($T_\infty = T_0 - B_1$).

1.4. Parametric models

The direct use of the complex modulus data can only be feasible if a Direct Frequency Analysis (DFA) is performed. This numerical technique is based on a frequency step analysis along a pre-defined frequency range, where the spatial model, described in the frequency domain, is solved for each individual frequency step [11]. Considering steady harmonic loading conditions, and assuming a synchronous and phase-lagged harmonic response, the frequency response model can be directly evaluated at each frequency step. Since the spatial model is solved for discrete frequency steps within the frequency range under analysis, it is possible to continuously update the stiffness matrix according to the complex modulus data.

Parametric models, like the Anelastic Displacement Fields (ADF) [14,15], the Golla-Hughes-McTavish (GHM) [16] and the fractional derivative [17] models, are commonly applied to provide a direct time domain representation for time domain analysis. Despite the major interest that such parametric modeling approaches may provide for those seeking for a time domain representation of the frequency domain complex modulus data, one interesting feature of these models is related to its inherent ability to represent the isothermal complex modulus data along a relatively wide frequency range through a limited set of constant model parameters. This is an important feature for data archiving and dissemination and therefore such parametric models represent a valuable alternative solution to the large data table containing the storage modulus and loss factor values for each frequency. Nevertheless, these models are conventionally defined as parametric functions in frequency, and temperature relation is lost, requiring the use of the shift factor distribution to recover this temperature dependency.

Both the GHM and ADF models are widely used, especially as constitutive models for time domain finite element analysis. Despite providing a straightforward solution, both models may require the application of several series or sets of parameters to attain a valuable level of representation accuracy; in practice a minimum of three parameter series is usually reported as a minimum value. This requisite is directly responsible for the considerable size increase

of the spatial model, which is directly proportional to the applied number of parameter series, being of utmost importance to define a compromise solution between the model representation accuracy and the corresponding computational cost.

The fractional derivative model, which is based on the work of Bagley and Torvik [17,18] defines the constitutive relation through a fractional derivative operator, such as the 5 parameter model [19] described in the frequency domain as:

$$G(\omega) = \frac{G_0 + G_1(j\omega)^\alpha}{1 + b_1(j\omega)^\beta} \tag{5}$$

where G_0, G_1 and b_1 are constant parameters, α and β are fractional parameters, being all determined through a fitting procedure to the experimental data. These parameters are identified through a fitting procedure of the model to the measured nomogram representing the complex modulus data within a predefined frequency range for a given temperature value.

Contrary to the ADF and GHM models, the fractional derivative model can provide a good representation of the complex modulus function along the frequency using a single series. On the other hand, this model cannot be used in a time domain analysis in a straightforward manner and special approaches must be employed [20–24]. Nevertheless, since a small set of parameters may be required for a precise representation of the complex modulus data along a wide frequency range, this model may play a special role among the parametric models for viscoelastic material properties dissemination.

It is important to note that when the fractional derivative model described in Eq. (5) assumes identical values for parameters α and β, the model represents positive values for both the storage and the loss modulus along the frequency range. In this case, since the primary use of the fractional derivative model is to represent the complex modulus within the analyzed frequency range, the 5 parameters model is used without compromising the result's validity, while maintaining the generalized form of the model.

2. Proposed frequency-temperature parametric model

Considering the frequency-temperature superposition principle, it may be expected the graphical representation of the complex modulus components (storage and loss modulus) along the frequency range for different temperatures to define a unique and continuous surface. The projection of this surface onto the storage/loss modulus plane describes a single curve known as Wicket plot. Figure 1 depicts the graphical representation of the complex modulus function for different temperatures along a frequency range between 0.001 Hz and 10 KHz (complex modulus data is taken from the manufacturer's nomogram plot of the 3M ISD112 damping material [25]).

This observation suggests that a parametric model depending simultaneously upon the frequency and the temperature may be developed. In this work, the reduced frequency concept, ruled by the frequency-temperature superposition principle, is applied directly onto the fractional derivative model. Such approach permits to perform a direct fitting procedure of the model describing such surface onto the complex modulus data measured for different frequency and temperature values, without the need to perform a fitting preprocess to describe the shift factor. This global fitting procedure, which determines the set of parameters of the model that minimizes the error between the experimental data points and the model representation, has also significant value due to the enhanced capability to identify systematic and random erroneous data when performing the three-dimensional fit to the temperature-frequency model.

3. Experimental data

The experimental data was obtained from an in-house developed testing device [6,26], which is based on a direct characterization approach. The testing device, depicted in Fig. 2, reproduces experimentally a single degree of freedom system where a thin viscoelastic material specimen (4) represents part of the system's stiffness and the upper moving bar (2) represents the mass of the system. This upper bar is guided by two lamina springs (3), minimizing
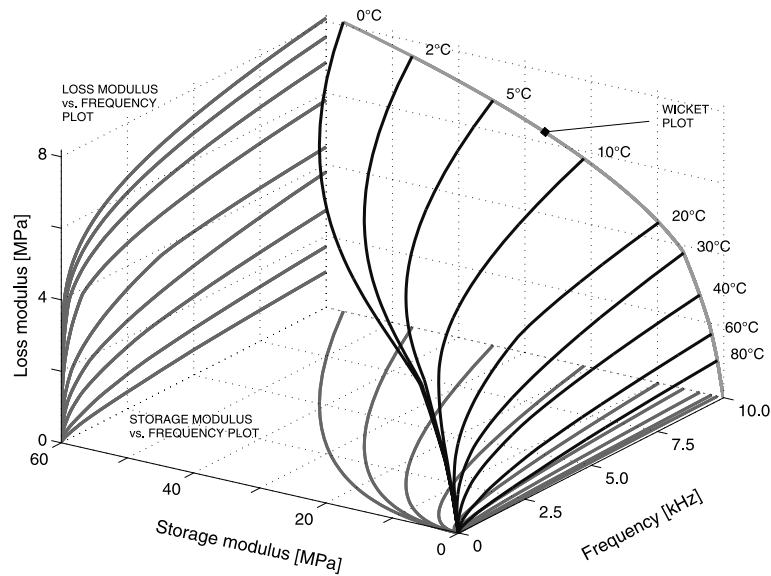
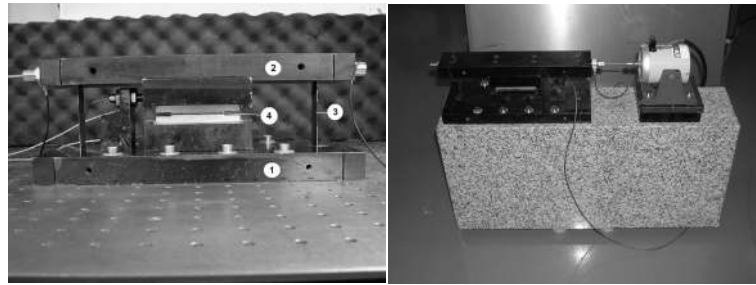Fig. 1. Three-dimensional representation of the complex modulus.

Fig. 2. Viscoelastic characterization assembly.

thus the contribution of the system response along the spurious degrees of freedom. The device is attached onto a heavy and stiff granite block, which also supports the electrodynamic shaker. A load cell is used to measure the load applied to the moving bar and two different transducers are applied to measure the system's response: one proximity displacement transducer and one piezoelectric accelerometer. Both transducers were previously calibrated and the entire system was previously characterized, determining the equivalent stiffness value for the guiding lamina springs and the effective active mass. The entire assembly, which is mounted on soft rubber pads to minimize the external noise effects and reduce the rigid body modes frequency range, is placed inside a climatic chamber to attain different and controlled temperature conditions.

The measured frequency response functions provided the input data for a shear complex modulus calculation procedure, using the geometry parameters of the sample under evaluation to describe such relation. Details on the experimental characterization study and related experimental setup and applied methodology can be found in [6,26].

The shear complex modulus data is described as a large set of complex values depending on the frequency and the temperature. Figure 3 represents the storage and loss modulus values obtained for a 3M ISD112 specimen for several discrete temperatures between $0.9\,°C$ and $34.9\,°C$ within the [0:400] Hz frequency range. Such data usually requires a filtering process to remove the erroneous data due to random errors and measuring related issues. This filtering process is usually performed using the Wicket plot representation, a projection of the complex modulus data onto the real-imaginary plane (Fig. 4) to identify the main curve and make evident which data follows this main tendency.
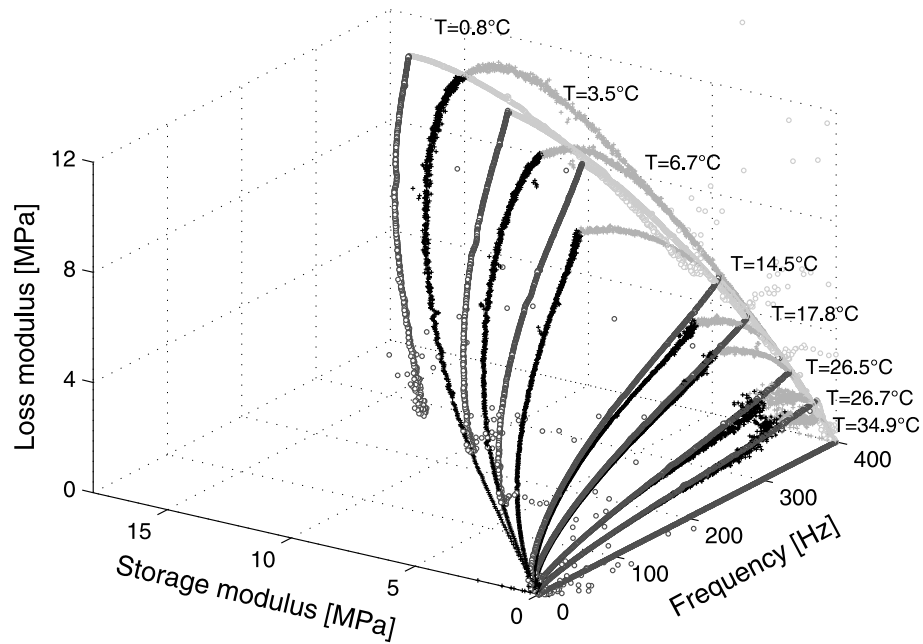
Fig. 3. Representation of the complex modulus data from accelerance (dark grey circle), from receptance (black plus sign) and corresponding projected Wicket plot (light grey).
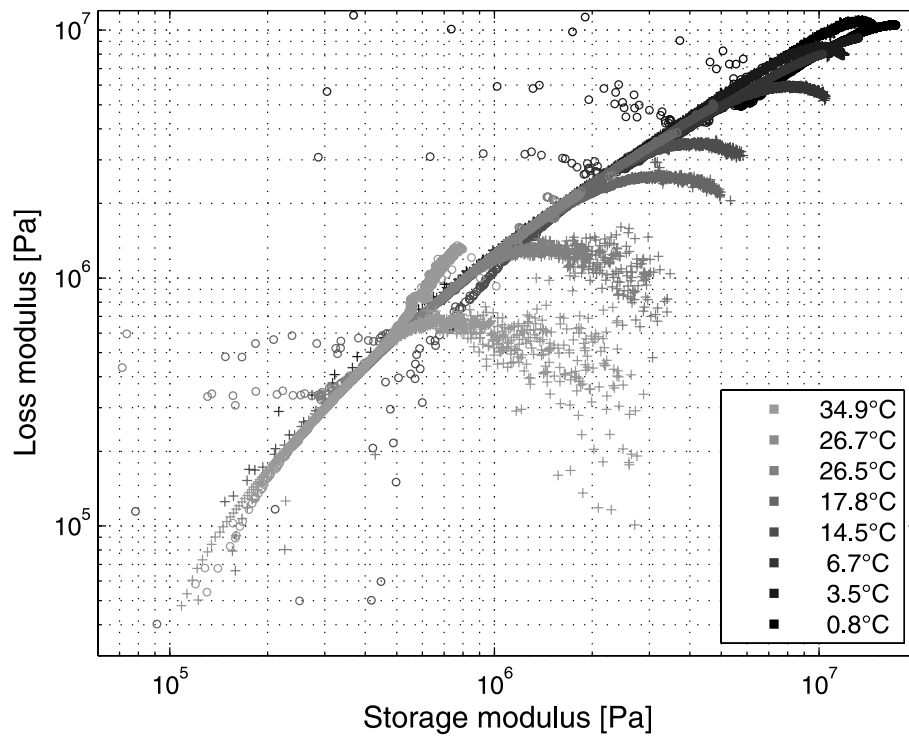

Fig. 4. Wicket plot of complex modulus data calculated from accelerance functions (circle) and receptance functions (plus sign).

Table 1
Fractional derivative model parameters for the 3M ISD112 material

G_0: 1.382E4	G_1: 1.580E5	b_1: 2.200E-3	α: 0.584	β: 0.633

Table 2
Extended fractional derivative model parameters for the 3M ISD112 material

G_0: 1.060E-7	G_1: 5.064E5	b_1: 2.374E-3	α: 0.545	β: 0.725
	C_1: 13.071	T_0: 270.2	T_∞: 117.3	
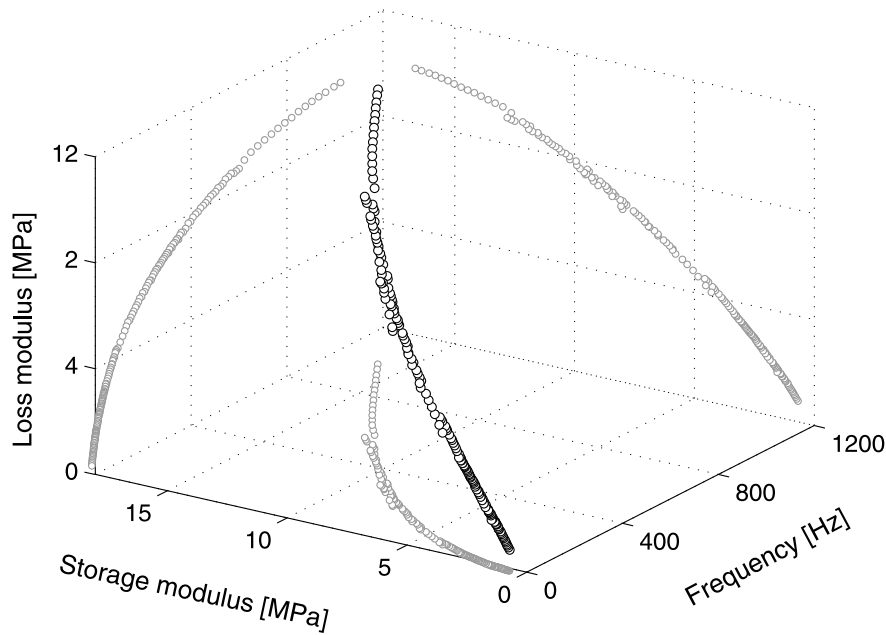
Fig. 5. Master curve of the experimental data (reference temperature 6.8°C).

The scatter of the represented data clearly evidences the rigid body modes effect of the experimental setup at the low frequency range for the accelerance functions and the effect of the high frequency noise populating the high frequency range of the receptance functions. This observation may assist a proper data selection and a spectral filtering procedure in order to remove the data that is unable to be represented by the simplified single degree of freedom model.

4. Frequency parametric models fitting

To apply a data fitting procedure onto the parametric fractional derivative model, a master curve may be constructed (Fig. 5). The master curve construction is based on the frequency-temperature equivalence principle and starts with the determination of the shift factors for the measured temperatures. This is usually performed using a numerical procedure that calculates the graphical horizontal shift of the isothermal curves towards a unique master curve.

Plotting the logarithmic value of the shift factor against the inverse of the corresponding absolute temperature (Fig. 6), the slope of the linear fitting determines the T_A parameter of the Arrhenius model.

The data fitting procedure, using the model equations described in Eq. (5), is performed using an optimization method to determine the set of parameters that best represent the experimental complex modulus data. The identified parameters are listed in Table 1 and were obtained for a single series fractional derivative model. Such fitted parameters are temperature dependent and thus define a fractional derivative model describing a function in frequency that represents the material's complex modulus for the selected reference temperature.
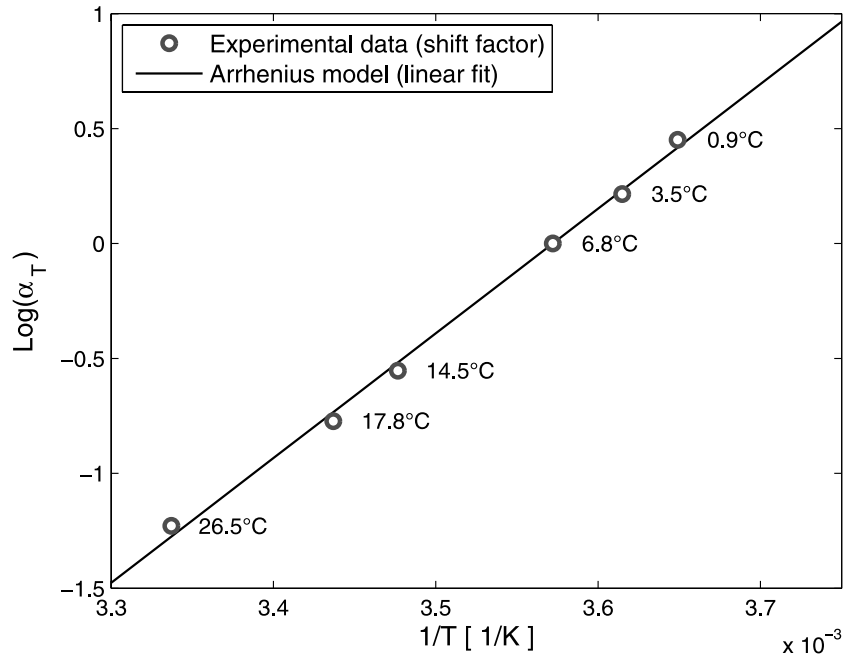
Fig. 6. Arrhenius model fitting of the shift factor values (reference temperature $T_0 = 279.95$ K (6.8°C), $T_A = 5425.6$).
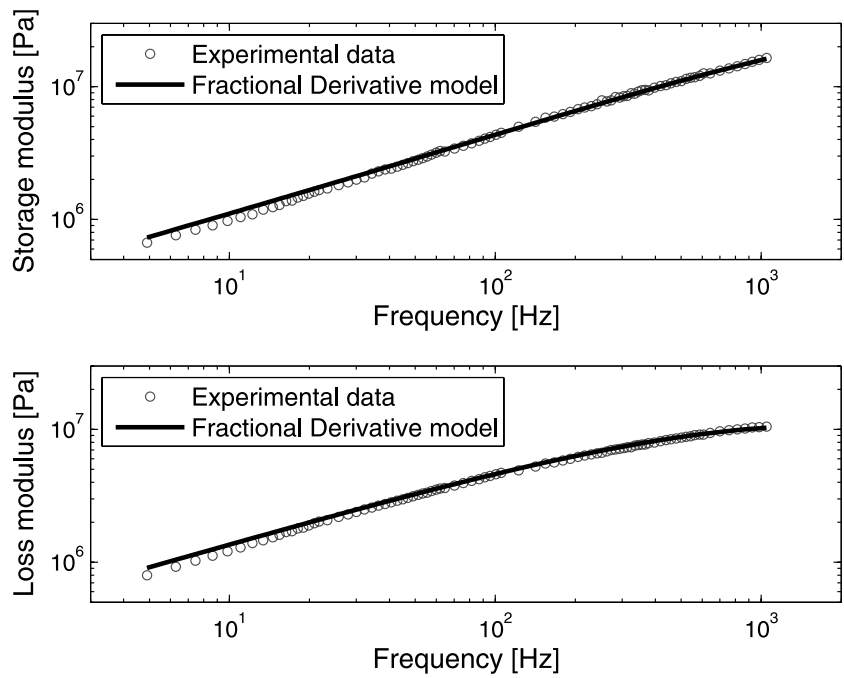

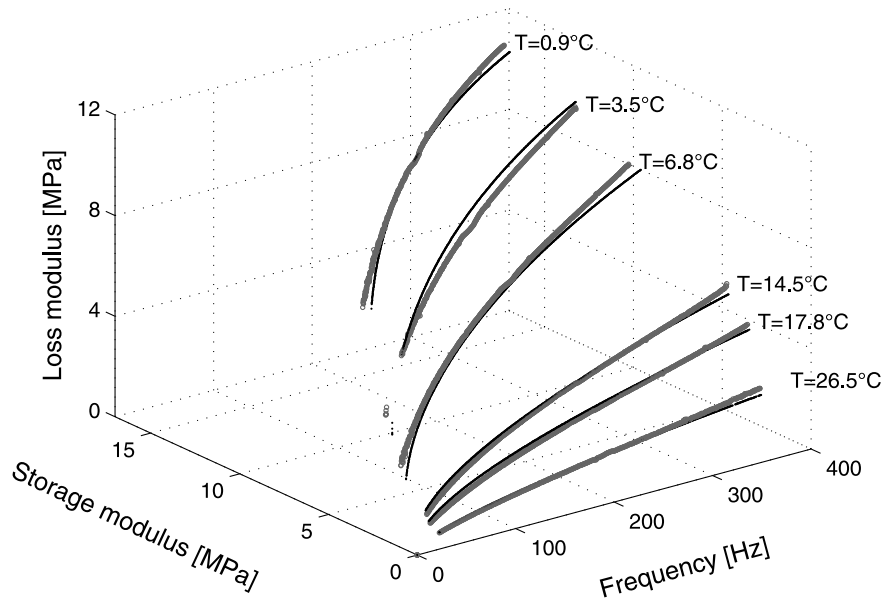Fig. 7. Storage and loss modulus master curves representation for the fractional derivative model.

Fig. 8. Complex modulus representation for the frequency-temperature fractional derivative model (grey circles represent experimental data and black points the corresponding model representation).

Figure 7 represents the experimental complex modulus data master curve along with the fractional derivative model curves (parameters listed in Table 1). A good agreement between the experimental data and the numerical model representation is observed.

5. Proposed frequency-temperature parametric models fitting

As an alternative to the two-dimensional master curve fitting process, the proposed model can be directly applied in a simultaneous frequency and temperature fitting procedure. Using this approach, the shift factor function identification is directly performed along the model fitting, rather than being the initial step for a preprocess task. This new approach avoids the determination of the master curve, which can reduce significantly the level of model fitting error, as the fitting procedure is performed only once. Therefore, instead of a two step fitting procedure, the first one to determine the master curve and consequently the shift factor function, and the second one to identify the model parameters, a single fitting procedure is performed to determine the parameters of the frequency-temperature dependent model describing a surface domain that should contain the experimental data points.

Table 2 presents the fitting procedure results, which were obtained following the same numerical methodology applied for the isothermal or master curve fitting procedure. In this study, the Williams-Landel-Ferry model Eq. (4) was applied to describe the shift factor function describing the frequency-temperature superposition principle. The reduced-frequency variable Eq. (2), defined from the temperature-frequency relation described by the shift factor function Eq. (4), is directly applied in the fractional derivative model Eq. (5), replacing the frequency variable. This combined model is then used in a fitting procedure where the model parameters are determined in order to minimize the sum of the quadratic errors between the experimental data, the storage and the loss modulus data points for different frequencies and temperatures, and the model representation. This is just a simple and expeditious approach, used herein to demonstrate the proposed modeling concept, and other robust numerical strategies can be applied for this optimization/fitting analysis procedure.

To reduce the computational cost and improve the efficiency of the fitting optimization procedure the scatter of the data was reduced by projecting the data onto the real-imaginary plane, representing thus a Wicket plot of the data. This filtering process was assisted by a principal component analysis [27] to identify the main tendency and make evident the non-representative data.
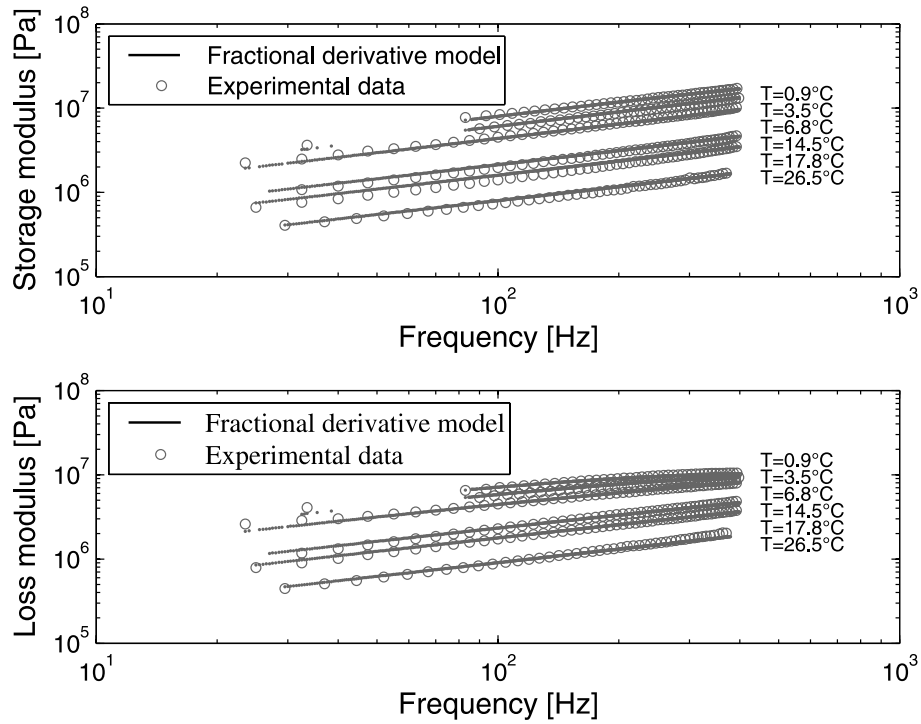
Fig. 9. Storage and loss modulus representation for the frequency-temperature fractional derivative model.
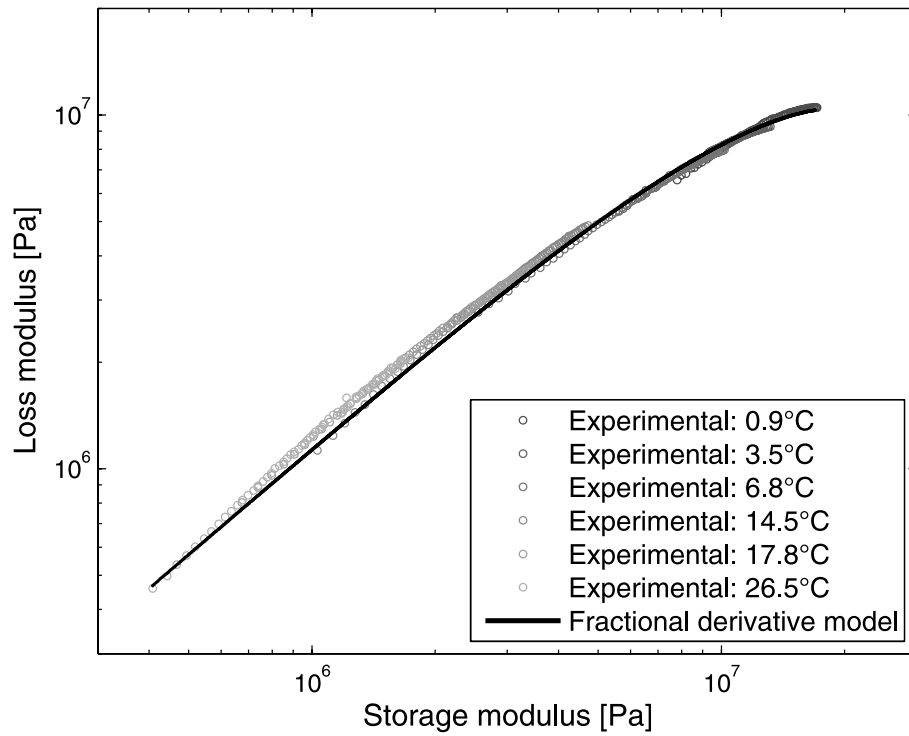

Fig. 10. Wicket plot for the frequency-temperature fractional derivative model.

Figure 8 represents a set of selected experimental complex modulus data curves for different temperatures along with the data described by the generalized fractional derivative model using the parameters listed in Table 2.

Figure 9 presents the same information projected onto the real (Storage modulus) and imaginary (Loss modulus), whereas Fig. 10 depicts the projection of the data, experimental and model representation, onto the real-imaginary plane (Wicket plot).

Both the frequency-temperature representation (Fig. 8) and corresponding projections onto the real and imaginary planes (Fig. 9) and the real/imaginary plane, or Wicket plot (Fig. 10), evidence the excellent representation level provided by the frequency-temperature extended fractional derivative model. It is worthy noting that this model has a single set of parameters, and thus provides a valuable model for data manipulation, filtering and dissemination. Nevertheless, this model presents some inconvenient to be used directly in a time domain analysis, as previously described; but can be an interesting tool to gather into a limited and portable parametric model the complex modulus data measured experimentally or simply recovered from the manufacturer's nomogram.

6. Conclusions

The direct use of the complex modulus data measured experimentally is limited to the direct frequency analysis or iterative modal extraction approaches. Moreover, the archive and dissemination of such data is not straightforward and effective.

Parametric models, which represent the frequency dependency of the complex modulus through a set of fitted parameters, do provide a simple, yet approximate, means of data representation. However, such models are usually applied onto isothermal data sets or onto a master curve which is constructed by considering all the data gathered for different temperatures. This master curve is generated by applying the frequency-temperature equivalence principle and relies on the determination of the shift factor distribution, which must be also retained along with the parametric model. All these fitting procedures are made on two-dimensional domains and a successful result mostly depends on the quality and representative level of the selected reference data, a task that proved to be difficult when projecting and analyzing the data onto two-dimensional domains (storage and loss modulus curves or the Wicket plot representation). These difficulties may be attenuated by performing a highly selective data filtering, which in turn may reduce the data representation and, thus, increase the level of approximation error.

To simplify and minimize the approximation error when defining the usual parametric models, it was proposed an extended fractional derivative model, where the shift factor representation is directly introduced into the parametric model. Therefore, the fitting procedure is performed only once, which minimizes the approximation error and provides a better representation level. Moreover, since a three-dimensional fitting procedure is applied, the systematic and random based erroneous data can be easily highlighted, a difficult feature when analyzing projected Wicket plots.

The identified parameters proved to be representative, both for the original isothermal model and the frequency-temperature extended model. The results demonstrate the feasibility of the proposed approach, which can provide a valuable modeling tool with remarkable interest for data filtering, archive and dissemination purposes.

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