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A GOAL PROGRAMMING MODEL FOR ACHIEVING RACIAL BALANCE IN PUBLIC SCHOOLS

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Abstract—This paper develops a goal programming model for achieving racial balance in segregated public schools. The model is illustrated and offered as an improvement upon linear programming, a model previously applied in the literature which allows a single objective function only and, in comparison with goal programming, requires more variables and constraints. Goal programming, a member of the general multiple objective linear programming (MOLP) model, improves upon these among other disadvantages thereby emerging as a more efficient tool for student assignment problems.

INTRODUCTION

The 1954 landmark Supreme Court decision in Brown vs Board of Education ([2], p. 686) signaled a reversal in the Court's attitude towards racial desegregation in the nation's school system. Prior to this decision many school districts were administered under the "separate but equal" doctrine expressed in the 1896 Supreme Court decision, Plessy vs Ferguson, which stated that "equality of treatment... is essentially achieved...where the races are provided substantially equal facilities even though these facilities are separate."† The 1954 decision, on the other hand, stated that "separate educational facilities are inherently unequal" and therefore the "separate but equal" doctrine does not hold in public education issues involving segregation of the races ([12], p. 686). The decision further stipulated that students should be admitted to schools "... with all deliberate speed." ‡ In 1968 the Supreme Court reiterated its position regarding racial desegregation in the school systems by ruling that the systems must comply more rapidly with the 1954 decision ([6], p. 1689).

These recent Supreme Court decisions confronted the multitude of local school boards across the country with significant administrative problems. For example, students must be assigned to schools to achieve an as yet unspecified race allocation ratio. This assignment should represent some semblance of racial balance and must be effected by using whatever means necessary. Two of the methods available were realignment of school districts and busing of students to distant schools. Inherent in either of these, or any other method, was the criterion that the solution obtained result in a minimal cost consistent with constraints of the system. This paper presents a goal programming model which can be used for the purpose of assigning students to schools.

†Reference ([11], p. 1138).
‡Reference ([2], p. 686)

THE LINEAR PROGRAMMING APPROACH

One approach to the problem noted above would be to randomly assign students to schools until the desired ratio of the races is obtained. While this procedure would undoubtedly result in a workable solution it is nevertheless time consuming and, consequently, expensive. Additionally, there would be no element of optimality, regarding cost minimization, in the solution unless the allocation process was repeated many times.

The literature is replete with articles dealing with the application of operations research techniques to achieve racial desegregation in school systems [1–5, 7, 10, 14]. While other operations research techniques have been used for the design of bus routes and schedules, the application of LP has generally been aimed at the student assignment problem.

The application of LP techniques normally involves a minimization of some function of cost, i.e. total student busing distance, total number of buses required, etc. In one application in the literature, total student travel time was minimized [4] and in another entire population units were assigned to schools [10].

Constraints for the problems were normally constructed around the following criteria:

(1) Each student must be assigned to a school;
(2) Consideration must be given to school capacities;
(3) Racial balance must be achieved within specified ranges.

These criteria resulted in a large number of variables and constraints. For example, in the simple application presented in a later section of this article, in which two races, three schools and four school districts are considered the number of constraints is eighteen and the number of variables forty-seven. One can readily imagine the inordinate size of the variable-constraint matrix in a problem of any appreciable complexity.

As potentially powerful and useful a tool as LP may be, it is not without shortcomings. The basic model is deterministic and, as such, assumes certainty, i.e. inputs are known and constant. These known and constant inputs include objectives as well as constraints, which ordinarily is not the case in a real-world situation. Normally a practical problem involves trade-offs between several objectives rather than the optimization of a single objective. The typical LP application considers only the maximization (or minimization) of a single objective.

Recent efforts by Charnes, Cooper and Niehaus[3], Lee[9], Steuer and Oliver[13] and others, while not totally negating the above shortcomings, have considerably diminished their effect. The continually developing techniques in Multiple Objective Linear Programming (MOLP) are adding a great deal of diversity and utility to the original LP model.

THE MULTIPLE OBJECTIVE LINEAR PROGRAMMING APPROACH

MOLP techniques allow a decision-maker to give consideration to more than one objective when deriving solutions to allocation type problems.† Of course a solution to an MOLP problem is not to be regarded as a maximum (or minimum as the case may be), but as one which satisfied the constraints and simultaneously optimizes the objective according to the weighting scheme imposed on them. In other words, the solution obtained will represent an "optimal" solution which in real-world situations is undoubtedly more valuable than an artificial maximum.

Presently there are, at least four methods of treating MOLP problems [3, 12]:

(1) Point estimate weighted sum method.
(2) Feasible region reduction method.
(3) Goal programming.
(4) Vector maximum method.

While these methods will not be discussed in depth, it can be noted that with the exception of the "Goal Programming Method," an assumption of non-satiety (i.e. the utility function is monotonically non-decreasing) on the decision maker's part is implicit. This means that the higher the value of the objective function the more satisfied the decision maker. In many real world situations this assumption is invalid, making the application of MOLP techniques utilizing the assumption questionable. Additionally, the "Feasible Region Reduction Method" may result in an empty feasible region and thus no feasible solution.

Because of the above disadvantages and the relative simplicity in applying "Goal Programming" the authors regard it as the most potentially useful of the four models noted. Consequently "Goal Programming" is utilized in this paper.

†In this example shown later in the paper, only one objective function was converted to a goal. Previously, school officials had to specify the objective to be optimized. Typically, this objective was tied to a cost factor: minimization of either miles or number of students bussed. With the use of goal programming, both of these objectives may be incorporated as well as other goals such as minimizing the likelihood of suburban white schools and urban black schools, or maximizing the use of schools with special interest facilities (for example, indoor swimming pools, industrial/technical equipment or academic subjects of infrequent demand).

Primarily because of the simplicity of application and structuring of the problem, the "composite objective function by weighting all deviation variables" method of goal programming[8] is used in this paper. This method requires the statement of the objective functions in the form of constraints. An acceptable level of achievement is specified and the objective becomes an equality constraint. If the objective need not meet this exact level of achievement, then deviation variables, allowing for some movement from the equality position, are inserted into the objective-turned-constraint (which will be called a goal).

The objective functions are all restated as goals. In the LP setting, the new objective function of the problem is the sum of all the deviation variables from the goals and the desire is to minimize the deviations with the objective of achieving the goals. Thus, the form of the problem appears as a normal LP tableau, only some of the constraints are actually the goals to be optimized.

THE GOAL PROGRAMMING MODEL

Formulation of this model, as in the previously mentioned LP methods, centers around the following three general constraints:

(1) Each student must be assigned to a school;
(2) Consideration must be given to school capacities; and
(3) Racial balance must be achieved within specified ranges.

Before the constraints can be made more specific pertinent variables must be defined:

- i identifying number for race or ethnic group;
- j identifying number for a specific school;
- k identifying number for a population subdivision, i.e. tract, block, neighborhood, etc.;
- X_{ijk} number of students of race i assigned to school j from tract k;
- d_{ijk} distance students of race i assigned to school j from tract k must travel;
- P_{ik} percentage of students of race i in tract k;
- P_k total number of students in tract k;
- C_j maximum student capacity of school j; and
- DP_{ij} desired percentage of race i assigned to school j.

The following general information was assumed for the example presented herein:

$i = 1, 2$, i.e. minority $i = 1$, majority $i = 2$.
$j = 1, 2, 3$
$k = 1, 2, 3, 4$

In that only two races or ethnic groups are considered in the example, the following simplications can be made.

- P_{ik} becomes P_k percentage of minority students in track k;
- DP_{ij} becomes DP_j desired percentage of minority students assigned to school j and;
- DP_j becomes DP overall percentage of minority in the general population (the desired percentage in each school is normally the same).

The specific constraints can now be formulated as follows:

1. Each student must be assigned to a school.
   (a) Minority students where $i = 1$

$$\sum_{j=1}^{3} X_{1jk} = p_k P_k \text{ for each } k. \tag{1}$$

   (b) Majority students where $i = 2$

$$\sum_{j=1}^{3} X_{2jk} = (1 - p_k)P_k \text{ for each } k. \tag{2}$$

2. Consideration for school capacities.

$$\sum_{i=1}^{2} \sum_{k=1}^{4} X_{ijk} \leq C_j \text{ for each } j. \tag{3}$$

3. Specified racial balance.

$$\sum_{k=1}^{4} X_{1jk} \leq DP{*}C_j \left[\sum_{k=1}^{4} P_k \Big/ \sum_{j=1}^{3} C_j \right] \text{for each } j \tag{4}$$

The factor

$$\left[\sum_{k=1}^{4} P_k \Big/ \sum_{j=1}^{3} \right] C_j$$

acts as a correction for over or under utilization of school capacities.

4. The objective function in earlier LP models now becomes the last constraint. In this example total student miles from their assigned schools is used.

$$\sum_{i=1}^{2} \sum_{j=1}^{3} \sum_{k=1}^{4} \mathrm{d}_{ijk} X_{ijk} \leq \text{arbitrary constant.} \tag{5}$$

In order to enhance the comprehensibility of the model, some simplifying assumptions are made, with regard to input data for the example presented. These assumptions are:

1. Racial mix is homogeneous within a tract.
2. Distances are measured as a straight line distance from the centroid of the tract population to the different schools.
3. Total student busing miles is directly proportional to total student miles from their assigned schools.

The above assumptions do not constitute limitations of the model itself but rather of the example presented herein, and are made for the sake of simplicity. Indeed the versatility of the model extends well beyond these assumptions.

AN APPLICATION OF THE GOAL PROGRAMMING MODEL

For demonstration purposes hypothetical data were assumed and used as input for the model. These data are shown below; and in Tables 1–3.

Overall Minority Percentage in Population
$D = 13.1\%$

A graphical representation of the hypothetical distances is shown in Fig. 1. While the school locations are pre-determined, the centroid (i.e. population center) of the tract populations must be computed. Of course as the tract sizes are reduced this computation becomes less difficult.

In order to enable the reader to more readily visualize the model and simultaneously highlight the fact that even a simple LP problem requires a relatively large capacity computer, the initial tableau is presented in Fig. 2. This tableau represents the actual input data to the LP program, for the initial run. Note that with the GP model a solution can be obtained by utilizing commonly available LP computer programs, rather than sophisticated MOLP programs since all goals are incorporated as constraints and only one objective function is used. As was previously pointed out, this was one of the reasons for the development of the GP model, as opposed to other modeling possibilities.

Table 1. Tract populations and minority percentages

Tract #	Student Population	Minority Percentage
1	P_1 = 1500	P_1 = 4.0
2	P_2 = 2500	P_2 = 5.0
3	P_3 = 3500	P_3 = 30.0
4	P_4 = 2500	P_4 = 3.0

Table 2. School capacities

School #	Capacity
1	C = 3000
2	C = 4500
3	C = 3500

Table 3. Distance from schools to the centroid of individual tract population

Tract #	School #1	School #2	School #3
1	d_{11} = 1.0	d_{21} = 3.4	d_{31} = 5.2
2	d_{12} = 3.4	d_{22} = 1.0	d_{32} = 2.1
3	d_{13} = 2.7	d_{23} = 2.5	d_{33} = 2.6
4	d_{14} = 5.4	d_{24} = 4.1	d_{34} = 1.4

The objective function, in the initial tableau shown in Fig. 2, contains zeros for all of the original variables, 999s for all of the artificial variables and "positive one" values for all of the deviation variables. This objective function is then minimized. Zeros, in the objective function, for the original variables allow them to enter the solution at no cost to the objective function, hence "free access" in a sense. If for any reason one of the original variables was not desired in the final solution (perhaps due to an infeasible road route), an arbitrarily large number (999), in lieu of zero, could be inserted in the objective function as a coefficient for this variable. Because of its high cost, the variable would then be excluded from the final solution.

In a similar fashion, the insertion of 999s as objective function coefficients for the artificial variables will cause them to be absent in the final solution. If they were to be present, the problem is normally infeasible.

The objective function coefficients for the deviation variables were set at "positive one" values for the initial

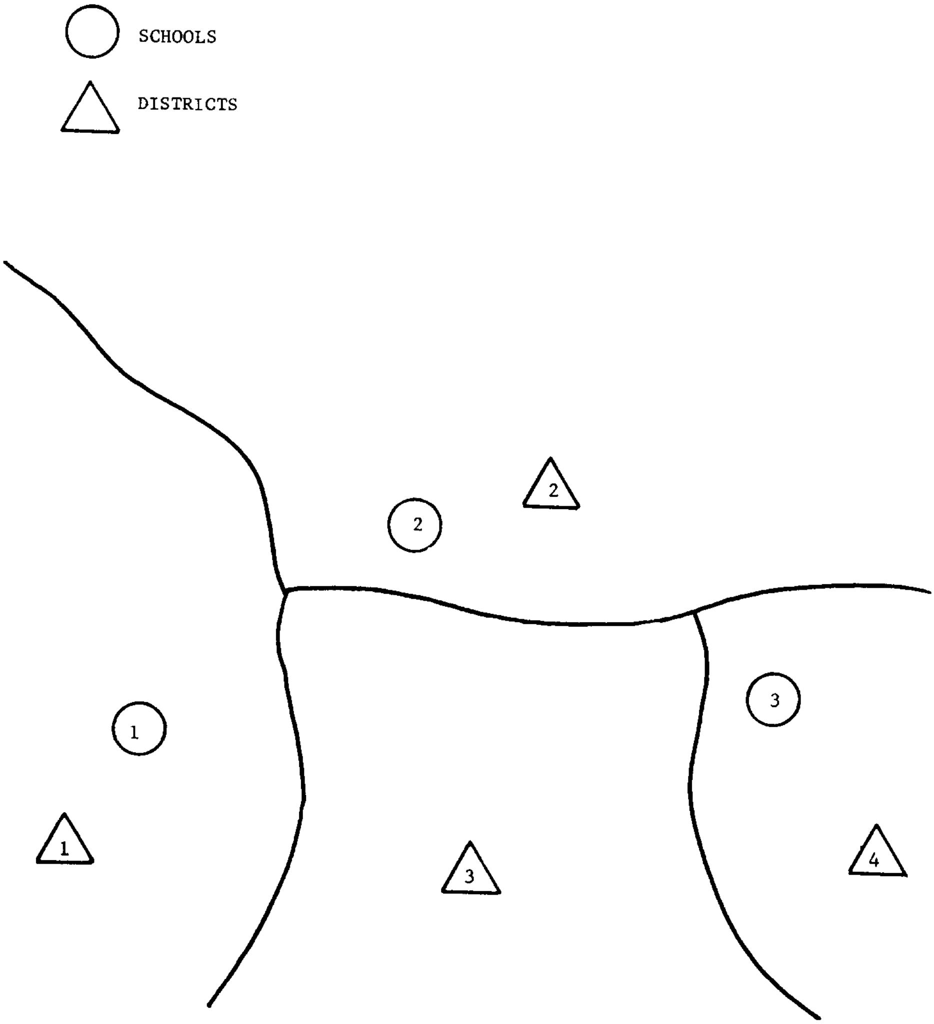

Fig. 1. Map of schools and districts.

trial run. This was done to obtain a "feel" for the problem and its possible solutions.

Any desired weighting of the problem may be accomplished by changing the coefficient in the objective function. For example, where other researchers have squared distances, a simple coefficient change in the objective function for any one of the variables will produce similar results.

Sensitivity analysis is a powerful tool which can be used to determine the magnitude of a coefficient change necessary to effect a change in the make-up of the final solution. However, in the example presented herein, changes in the coefficients were determined intuitively. Consequently, a follow-up run was made utilizing rather large changes in the coefficients which did effect a change in the final solution and satisfied the explanatory function of the example.

Table 4 contains the school make-ups, by tract, for trial solution No. 1, i.e. all deviation variables assigned equal weights in the objective function. The assumed data resulted in a total school overcapacity of 1000 students, all of which were assigned to school No. 1. Additionally the minority percentage for school No. 1 was 17.8%—considerably higher than the desired 13.1%.

In order to alter these two values, in the subsequent run, a cost (i.e. objective function coefficient) of 1000 was assigned to the capacity deviation variable (D_1) and to the positive racial composition variable (D_5). Note that since in this example there was an overabundance of school capacity, it was unnecessary to provide a negative deviation variable for the school capacity constraints. However, had the situation been otherwise, a negative deviation variable would have been included in the school capacity constraints.

In trial solution No. 2 (shown in Table 5), the overcapacity shifted to schools No. 2 and No. 3, while the minority percentage clustered much more closely around the desired percentage of 13.1% than in trial solution No. 1 (11.9–17.8 for solution No. 1, 11.9–14.7 for solution No. 2). This improvement was accomplished while incurring an increase in busing mileage of less than 1%.

Admittedly further "fine tuning" of the objective function is possible in order to further refine the solution. However, the two trial solutions shown should be sufficient to demonstrate the potential of the goal programming model.

CONCLUDING REMARKS

The goal programming model developed in this paper offers several advantages over the LP models applied to

Table 4. School make-up trial solution No. 1

Tract		School #1	School #2	School #3
1	Minority	60	---	---
	Majority	1,440	---	---
2	Minority	---	125	---
	Majority	---	2,375	---
3	Minority	297	411	342
	Majority	203	1,589	658
4	Minority	---	---	75
	Majority	---	---	2,425
Total		2,000	4,500	3,500
Underutilization of School		1,000	---	---
Percent Minority		17.8	11.9	11.9

Total Bussing Miles = 16,450

Table 5. School make-up trial solution No. 2

Tract		School #1	School #2	School #3
1	Minority	60	---	---
	Majority	1,440	---	---
2	Minority	---	125	---
	Majority	---	2.375	---
3	Minority	297	411	342
	Majority	1,203	1,247	---
4	Minority	---	---	75
	Majority	---	---	2,425
Total		3,000	4,158	2,842
Underutilization of School		---	342	658
Percent Minority		11.9	12.9	14.7

Total Bussing Miles = 16,548

the student assignment problem. Fewer variables and constraints are required in the goal programming model than with the LP model because the constraint with positive and negative deviation variables replaces an upper and a lower constraint for racial composition and/or school capacities. No artificial variables are required for the GP constraint hence reducing the number of variables needed. For the example presented fifteen constraints and forty-three variables were required. To use the LP model with the same data, eighteen constraints and forty-seven variables would have been required. Since the number of iterations required to obtain a solution is dependent upon the number of constraints, it is easy to see the double savings resulting from the use of the goal programming model (i.e. a smaller matrix and fewer required iterations). Because the example only had 18 constraints and 47 variables, the reduction in the number of constraints and variables was relatively small. However, this reduction could be very significant in any practical-size problem consisting of many more constraints and variables.

As mentioned earlier, use of the LP model allowed consideration of a single objective function only, whereas the "Goal Programming" approach, being a member of the general MOLP model, allows the consideration of more than one objective simultaneously. This greatly enhances the versatility of the output and provides the decision maker with a catalog of feasible solutions rather than a group of "optimal" solutions based upon a single objective. There should be little disagreement regarding the proposition that the MOLP model more closely approximates real-world situations than does the LP model.

When making changes in the problem formulation (such as squaring of the distance) for purposes of obtaining alternate solutions, considerable effort may be required with the LP model. However, with the goal programming model practically all desired changes can be obtained by modification of the objective function coefficient, resulting in much less effort. The increased effort required in changing the LP model naturally means increased opportunity for the commission of errors.

Activity #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	.22
Constraints	X_{111}	X_{121}	X_{131}	X_{112}	X_{122}	X_{132}	X_{113}	X_{123}	X_{133}	X_{114}	X_{124}	X_{134}	X_{211}	X_{221}	X_{231}	X_{212}	X_{222}	X_{232}	X_{213}	X_{223}	X_{233}	X_{214}
Min. in Tract 1	1	+1	+1																			
Min. in Tract 2				1	+1	+1																
Min. in Tract 3							1	+1	+1													
Min. in Tract 4										1	+1	+1										
Maj. in Tract 1													1	+1	+1							
Maj. in Tract 2																1	+1	+1				
Maj. in Tract 3																			1	+1	+1	
Maj. in Tract 4																						1
School Cap. 1	1			+1			+1			+1			+1			+1			+1			+1
School Cap. 2		1			+1			+1			+1			+1			+1			+1		
School Cap. 3			1			+1			+1			+1			+1			+1			+1	
Racial Comp. 1	1			+1			+1			+1												
Racial Comp. 2		1			+1			+1			+1											
Racial Comp. 3			1			+1			+1			+1										
Total Bus **Dist.**	d_{11}	d_{21}	d_{31}	d_{12}	d_{22}	d_{32}	d_{13}	d_{23}	d_{33}	d_{14}	d_{24}	d_{34}	d_{11}	d_{21}	d_{31}	d_{12}	d_{22}	d_{32}	d_{13}	d_{23}	d_{33}	d_{14}
Obj. Function	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Blanks indicate a zero coefficient for the variable.

$$*CF = \text{Capacity Factor} = \sum_{k=1}^{4} P_K / \sum_{j=1}^{3} C_j$$

Fig. 2. Busing to achieve desegregation in public schools. A goal programming model.

23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	
X_{224}	X_{234}	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9	D_{10}	D_{11}	
		+1																			$= P_{11}\ P_1 = 60$
			+1																		$= P_{12}\ P_2 = 125$
				+1																	$= P_{13}\ P_3 = 1050$
					+1																$= P_{14}\ P_4 = 75$
						+1															$= P_{21}\ P_1 = 1440$
							+1														$= P_{22}\ P_2 = 2375$
								+1													$= P_{23}\ P_3 = 2450$
+1	+1								+1												$= P_{24}\ P_4 = 2425$
										+1											$= c_1 = 3000$
+1											+1										$= c_2 = 4500$
	+1											+1									$= c_3 = 3500$
													+1	-1							$= DPC_1(CF)* = 357$
															+1	-1					$= DPC_2(CF)* = 536$
																	+1	-1			$= DPC_3(CF)* = 417$
d_{24}	d_{34}																		+1	-1	= Arbitary 1600 mi.
0	0	999	999	999	999	999	999	999	999	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	

Fig. 2 (*Contd*).

As discussed earlier, the goal programming model is one of a number of potential models encompassed in the general MOLP model and, as such, possesses the advantage that the MOLP model has over the LP model. However, the goal programming model is capable of being used with commonly existing LP computer programs and does not require the use of the more sophisticated and scarce MOLP computer programs.

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