N72-19878

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A GUIDE TO USING METEOROID-ENVIRONMENT MODELS FOR EXPERIMENT AND SPACECRAFT DESIGN APPLICATIONS

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • MARCH 1972

1. Report No. NASA TN D-6596	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle A GUIDE TO USING METEOROID-ENVIRONMENT MODELS FOR EXPERIMENT AND SPACECRAFT DESIGN APPLICATIONS		5. Report Date March 1972
		6. Performing Organization Code
7. Author(s) Donald J. Kessler, MSC		8. Performing Organization Report No. MSC S-273
		10. Work Unit No. 124-12-10-00-72
9. Performing Organization Name and Address Manned Spacecraft Center Houston, Texas 77058		11. Contract or Grant No.
		13. Type of Report and Period Covered Technical Note
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546		14. Sponsoring Agency Code
15. Supplementary Notes		

16. Abstract

In this report, a method is derived for transforming a meteoroid flux (defined relative to the Earth and expressed as a function of meteoroid mass, velocity, and mass density) into a meteoroid flux expressed as a function of penetration thickness (or some other parameter) on a moving spacecraft at some particular distance from the Earth. The procedure for performing the transformation is summarized in this report, and an example is discussed.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
· Meteoroid(s) · Meteoroid Penetration · Meteoroid Velocity · Meteoroid Flux · Prediction of Meteoroid Penetration	

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
None	None	35	$3.00

*For sale by the National Technical Information Service, Springfield, Virginia 22151

A GUIDE TO USING METEOROID-ENVIRONMENT MODELS FOR EXPERIMENT AND SPACECRAFT DESIGN APPLICATIONS

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SUMMARY

In this report, a method is derived for transforming a meteoroid flux (defined relative to the Earth and expressed as a function of meteoroid mass, velocity, and mass density) into a meteoroid flux expressed as a function of penetration thickness (or some other parameter, such as energy or momentum) on a moving spacecraft at some particular distance from the Earth. The meteoroid flux on a moving spacecraft is increased over the meteoroid flux relative to the Earth. Use of specific weighted-average velocities and mass densities is necessary for transforming the meteoroid flux-mass distribution relative to the Earth into a flux-penetration distribution on a moving spacecraft at a specified distance from the Earth. The procedure for performing the transformation is summarized in this report, and an example is discussed.

INTRODUCTION

The conversion of a meteoroid distribution (expressed as a function of mass, velocity, and mass density) into a distribution expressed as a function of some other parameter can be accomplished by several methods. One method, which gives an approximate solution, is to average all meteoroid velocities and densities and convert the mass distribution into a distribution of the new parameter. In the use of such a method, the variables involved are not adequately considered. For example, if a small high-velocity meteoroid penetrates a material to the same thickness as a large low-velocity meteoroid, then flux as a function of penetration thickness could change as the velocity distribution changed, even though the average velocity might remain constant. As is demonstrated in this report, the velocity distribution can be replaced by a weighted-average velocity, where the proper weighting is a function of the equation describing the desired parameter, as well as a function of the slope of a log-flux-versus-log-mass curve.

The approximation is also usually made that the meteoroid environment relative to the Earth is the same as that for an orbiting spacecraft. Actually, both the relative meteoroid impact velocities and the meteoroid flux increase as the spacecraft velocity relative to the Earth increases.

Methods are developed in this report to convert a meteoroid flux expressed as a function of mass, velocity, and mass density (relative to the Earth) to a flux expressed as a function of some other parameter relative to an orbiting spacecraft. It is assumed that the spacecraft has no preferred orientation with respect to the Sun and that the spacecraft orbit about the Earth has no preferred plane. It is shown in this report that this assumption is equivalent to assuming that the geocentric flux is not directional.

The author wishes to thank Kenneth Baker, who first suggested an approach similar to the one taken in this paper; Herbert A. Zook, who assisted the author in numerous instances; and Burton G. Cour-Palais, who suggested that this work be organized and published. The author is also indebted to the other members of the Meteoroid Sciences Branch (dissolved October 1970) at the NASA Manned Spacecraft Center.

SYMBOLS

A normalization constant in the impact-angle distribution equation

B constant coefficient in the penetration equation (or in the equation defining another parameter)

C constant coefficient in the cumulative-flux-versus-mass equation

D normalization constant in the equation for the meteoroid velocity distribution relative to a moving spacecraft

d_G radius of an imaginary sphere concentric to and outside the Earth

F cumulative meteoroid flux, number of impacts per unit area per unit time

F_e cumulative meteoroid flux on a stationary spacecraft with respect to the Earth, number of impacts per unit area per unit time

F_s cumulative meteoroid flux on an orbiting spacecraft, number of impacts per unit area per unit time

f ratio of the mass flux of meteoroids with velocity V_m on an orbiting spacecraft to the mass flux of meteoroids with velocity V_m on a hypothetical spacecraft at the same position and stationary relative to the Earth

f_t ratio of the mass flux of meteoroids with a velocity distribution of V_m on an orbiting spacecraft to the mass flux of meteoroids with a velocity distribution of V_m on a hypothetical spacecraft at the same position and stationary with respect to the Earth

G ratio of the meteoroid mass flux at a distance r from the Earth to the meteoroid mass flux near the surface of the Earth (if Earth shielding is neglected and only meteoroid velocity V_∞ is considered)

G_t ratio of the meteoroid mass flux at a distance r from the Earth to the meteoroid mass flux near the surface of the Earth (if Earth shielding is neglected and only a meteoroid velocity distribution of V_∞ is considered)

i direction from which a meteoroid stream comes

m meteoroid mass

n power to which velocities are averaged

$n(\ )$ number of meteoroids having the property ( )

r distance from the center of the Earth

r_e radius of the Earth

S number density of meteoroids near the Earth

t thickness a meteoroid will penetrate (or exhibit some other behavior), depending on the values of β, γ, δ, and ϵ

V impact velocity of a meteoroid on the spacecraft

V_a representative velocity of meteoroids when converting a flux expressed as a function of mass to a flux expressed as a function of t

V_e escape velocity from the Earth surface

V_G velocity of a meteoroid relative to the Earth before interaction with the Earth gravitational field

V_m relative velocity between the Earth and a meteoroid at some distance from the Earth

V_s velocity of an orbiting spacecraft relative to the Earth

V_∞ relative velocity between the Earth and a meteoroid at the edge of the Earth atmosphere

α exponent of m in the flux-versus-mass equation

β exponent of m in the penetration equation

γ exponent of V in the penetration equation

δ exponent of ρ in the penetration equation

ϵ exponent of $\cos\theta$ in the penetration equation

η Earth shielding factor resulting from "line of sight" Earth shielding

η_e effective Earth shielding factor resulting from the curved paths of meteoroids under the influence of Earth gravity

θ angle between the normal to a surface and the negative velocity vector of an impacting meteoroid

θ_a representative impact angle of a meteoroid when converting a flux expressed as a function of mass to a flux expressed as a function of t

λ angle between the velocity vectors $\vec{V}_m$ and $\vec{V}_s$

ρ meteoroid mass density

ρ_a representative meteoroid mass density when converting a flux expressed as a function of mass to a flux expressed as a function of t

ϕ angle between the normal to the surface of the Earth and the meteoroid negative velocity vector $\vec{V}_m$

ψ half angle of the Earth when viewed from a distance r

ψ_e effective half angle of the Earth when viewed from a distance r, when the curved trajectory of the meteoroid between r and the Earth is considered

DIRECTIONALITY RELATIVE TO THE EARTH

The meteoroid flux relative to the Earth is probably directional, although the degree of directionality is not yet well defined. However, if, from all points on the Earth, meteoroids were observed entering the Earth atmosphere, the total distribution of meteoroid entry angles would be random, as is shown by the following analysis.

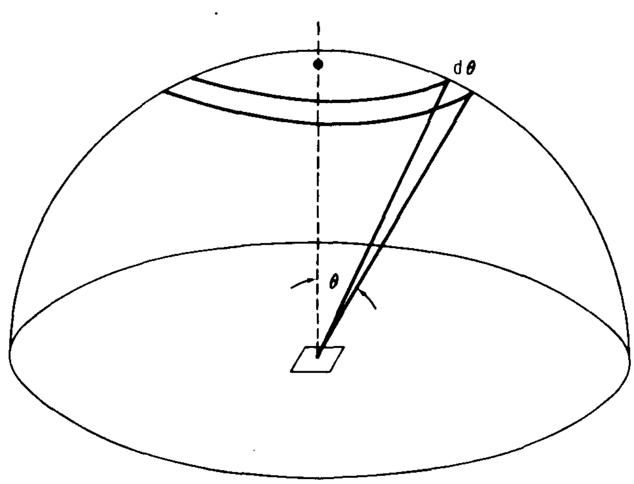

Figure 1. - Random flux on a flat plate.

A random distribution can be described mathematically as follows. Consider a unit-area flat plate sitting under a large hemispherical dome (fig. 1). Each element of surface area of the dome emits an equal number of meteoroids toward the flat plate. (That is, all meteoroid directions are equally probable relative to the plate.) Let θ be the angle between the normal to the flat plate and the direction from which the meteoroids are arriving. The number of meteoroids hitting the plate from angle θ ($\pm d\theta/2$) will be proportional to the element of dome surface area having angle θ (i.e., $\sin\theta$) and the component of plate surface area exposed to direction θ (i.e., $\cos\theta$), or

$$n_\theta(\theta) = A \sin\theta \cos\theta \tag{1}$$

where $n_\theta(\theta)$ is the number of meteoroids hitting the plate from angle θ, and A is a normalization constant. If the total number of meteoroids hitting the plate is normalized to 1, then

$$\int_0^{\pi/2} n_\theta(\theta) d\theta = 1 \tag{2}$$

and the value of A is 2. Thus, the normalized random distribution of meteoroids is

$$n_\theta(\theta) = 2 \sin\theta \cos\theta \tag{3}$$

Zero-Gravity Earth

Consider a group of meteoroids, of uniform flux throughout, which comes from direction i and hits the Earth. (This situation is similar to the Earth passing through a meteoroid stream, as in figure 2.) Neglect, for a moment, Earth gravity. If meteor-observation stations were located over the entire hemisphere exposed to direction i, the entry angles observed would be proportional to the area of the Earth where observation of an entry angle θ was possible (i.e., $\sin\theta$), times the component of that area facing direction i (i.e., $\cos\theta$). Thus, the angular distribution observed by the total of all observers would be

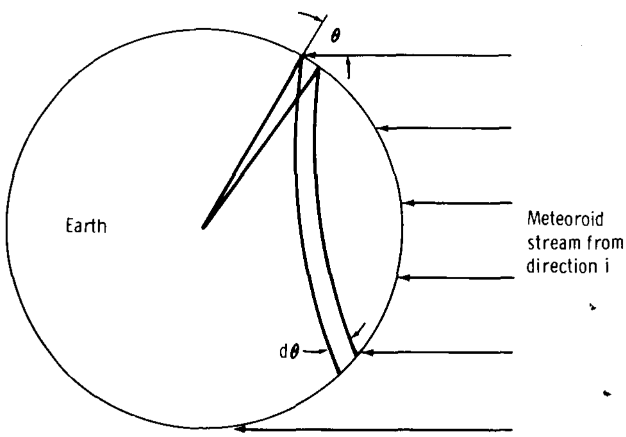

Figure 2.- Meteoroid stream hitting the Earth (gravity neglected).

$$n_{\theta,i}(\theta) = A_i \sin\theta \cos\theta \tag{4}$$

If j directions (streams) existed, the total angular distribution observed over the entire Earth surface would be

$$n_\theta(\theta) = \sum_{i=1}^{i=j} n_{\theta,i}(\theta) \tag{5}$$

or

$$n_\theta(\theta) = \sin\theta \cos\theta \sum_{i=1}^{i=j} A_i \tag{6}$$

If equation (6) is normalized so that the total number of meteoroids striking the Earth is 1, then

$$\sum_{i=1}^{i=j} A_i = 2 \tag{7}$$

and

$$n_\theta(\theta) = 2 \sin\theta \cos\theta \tag{8}$$

Gravitating Earth

Consider a meteoroid stream from direction i having a relative velocity V_G with respect to the Earth before interaction with the Earth gravitational field and a velocity V_∞ upon impact with the Earth. Define an imaginary sphere of radius d_G concentric to and outside of the Earth, such that, if the trajectory of a meteoroid in the stream were to pass the Earth at a distance greater than d_G (when gravity is neglected), then the gravitational force of the Earth would be insufficient to cause the meteoroid to hit the Earth. If the meteoroid trajectory (when gravity is neglected) were to carry the meteoroid a distance of exactly d_G from the Earth, the gravity-affected meteoroid would just graze the Earth, and the meteoroid perigee would be the radius of the Earth r_e (fig. 3).

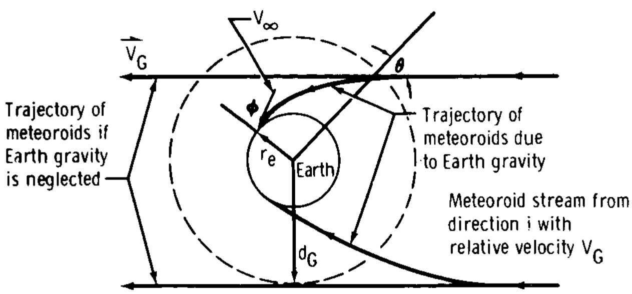

Figure 3.- Meteoroid stream hitting the Earth (gravity included).

Thus, from conservation of angular momentum (ref. 1)

$$r_e V_\infty = d_G V_G \tag{9}$$

Conservation of energy requires that all meteoroids with velocity V_G will have velocity V_∞ when they strike the Earth.

A trajectory (when gravity is neglected) that passes the Earth within a distance d, will pass through the imaginary sphere of radius d_G and make an angle θ between the normal to the surface of the imaginary sphere at the point of intersection and the meteoroid trajectory. The meteoroid will hit the Earth at angle ϕ, which is measured between the normal to the Earth surface at the point of impact and the negative velocity vector $\vec{V}_\infty$. Conservation of angular momentum requires that

$$r_e V_\infty \sin \phi = d_G V_G \sin \theta \tag{10}$$

If equations (9) and (10) are combined

$$\sin \phi = \sin \theta \tag{11}$$

From equation (4), the total distribution of the intersection angles between the meteoroid trajectories and the normals to the surface of the imaginary sphere at the respective points of intersection is

$$n_{\theta,i}(\theta)d\theta = A_i \sin \theta \cos \theta \, d\theta \tag{12}$$

Thus, from equations (11) and (12), it becomes obvious that the total angular distribution of the intersection angles between the meteoroid trajectories and the normals to the surface of the gravitating Earth at the respective points of impact is

$$n_{\phi,i}(\phi)d\phi = A_i \sin \phi \cos \phi \, d\phi \tag{13}$$

If equation (13) is normalized

$$n_{\phi}(\phi)d\phi = 2 \sin \phi \cos \phi \, d\phi \tag{14}$$

Thus, the entry-angle distribution for meteoroids, averaged over the Earth surface, is random, regardless of the initial meteoroid directionality or the effects of Earth gravity. It does not follow, however, that the entry-angle distribution is random at some particular point on the Earth surface.

TRANSFORMATION OF THE FLUX-VERSUS-MASS EQUATION

If a meteoroid flux expressed as a function of meteoroid mass, velocity, impact angle, and mass density is given, the problem arises of converting these distributions to the meteoroid flux expressed as a function of some other parameter. Since the parameter usually required is penetration thickness, the remainder of the analysis will refer to penetration thickness as the parameter to which the transformation is to be made. However, the development will be general enough to include any other parameter, such as energy or momentum.

Assume that the thickness t that a meteoroid will penetrate (over the range of possible values of m, V, ρ, and θ) can be expressed by

$$t = Bm^{\beta}V^{\gamma}\rho^{\delta}\cos^{\epsilon}\theta \tag{15}$$

where m is the meteoroid mass, V is meteoroid velocity, ρ is meteoroid mass density, and θ is the impact angle between the meteoroid trajectory and the normal to the spacecraft surface at the point of impact; B, β, γ, δ, and ϵ are constants that depend on the structure of the spacecraft, the spacecraft penetration mode, and the particular parameter desired (such as penetration thickness). Within the limits of possible values of meteoroid velocity, mass density, and entrance angle, a range of meteoroid masses that will penetrate a thickness t exists. Assume that over this range of mass values, the mass distribution can be expressed as

$$n_m(m)dm = \alpha C m^{-\alpha-1} dm \tag{16}$$

where $n_m(m)$ is the flux of meteoroids of mass m, and α and C are constants. The flux of meteoroids of mass m and larger is then given by integration of equation (16) from $m = m$ to $m = \infty$, or

$$F = Cm^{-\alpha} \tag{17}$$

It would be convenient to use a single representative meteoroid velocity V_a, a single representative meteoroid mass density ρ_a, and a single representative impact angle θ_a in equation (15) and to combine equation (15) with equation (17) to obtain

$$F = CB^{\alpha/\beta}t^{-\alpha/\beta}V_a^{\gamma\alpha/\beta}\rho_a^{\delta\alpha/\beta}\cos^{\epsilon\alpha/\beta}\theta_a \tag{18}$$

The problem is thus reduced to defining V_a, ρ_a, and θ_a so that equation (18) is correct.

Assume that the flux of meteoroids with mass m, velocity V, mass density ρ, and impact angle θ can be expressed as

$$n_t(m, V, \rho, \theta) = n_m(m)n_V(V)n_\rho(\rho)n_\theta(\theta) \tag{19}$$

where $n_V(V)$, $n_\rho(\rho)$, and $n_\theta(\theta)$ are the meteoroid velocity distribution, mass-density distribution, and impact-angle distribution, respectively. Each distribution, except $n_m(m)$, is found by comparing meteoroids of the same mass and is normalized so that

$$\int_{V=0}^{\infty} n_V(V)dV = 1 \tag{20}$$

$$\int_{\rho=0}^{\infty} n_\rho(\rho)d\rho = 1 \tag{21}$$

$$\int_{\theta=0}^{\pi/2} n_\theta(\theta)d\theta = 1 \tag{22}$$

The flux that will penetrate to thickness t and greater is then (ref. 2)

$$F = \int_{\theta=0}^{\pi/2} n_\theta(\theta) \int_{\rho=0}^{\infty} n_\rho(\rho) \int_{V=0}^{\infty} n_V(V) \int_{m=m_t}^{\infty} n_m(m)dm\ dV\ d\rho\ d\theta \tag{23}$$

where m_t is the mass of the meteoroid that will penetrate thickness t at velocity V, mass density ρ, and impact angle θ. The value of m_t is found from equation (15) and is expressed as

$$m_t = t^{1/\beta} B^{-1/\beta} V^{-\gamma/\beta} \rho^{-\delta/\beta} \cos^{-\epsilon/\beta} \theta \tag{24}$$

Thus, by using the expression for $n_m(m)$ in equation (16) and integrating equation (23) over mass

$$F = CB^{\alpha/\beta} t^{-\alpha/\beta} \int_{\theta=0}^{\pi/2} n_\theta(\theta) \int_{\rho=0}^{\infty} n_\rho(\rho) \int_{V=0}^{\infty} n_V(V) V^{\gamma\alpha/\beta} \rho^{\delta\alpha/\beta} \cos^{\epsilon\alpha/\beta} \theta \; dV \, d\rho \, d\theta \tag{25}$$

Since the distributions of V, ρ, and θ are normalized (eqs. (20), (21), and (22)), then by definition

$$\int_{V=0}^{\infty} n_V(V) V^{\gamma\alpha/\beta} dV \equiv \overline{V^{\gamma\alpha/\beta}} \tag{26}$$

$$\int_{\rho=0}^{\infty} n_\rho(\rho) \rho^{\delta\alpha/\beta} d\rho \equiv \overline{\rho^{\delta\alpha/\beta}} \tag{27}$$

$$\int_{\theta=0}^{\pi/2} n_\theta(\theta) \cos^{\epsilon\alpha/\beta} \theta \; d\theta \equiv \overline{\cos^{\epsilon\alpha/\beta} \theta} \tag{28}$$

Thus, equation (25) becomes

$$F = CB^{\alpha/\beta} t^{-\alpha/\beta} \, \overline{V^{\gamma\alpha/\beta}} \, \overline{\rho^{\delta\alpha/\beta}} \, \overline{\cos^{\epsilon\alpha/\beta} \theta} \tag{29}$$

If equations (18) and (29) are compared

$$V_a = \left(\overline{V^{\gamma\alpha/\beta}}\right)^{\beta/\gamma\alpha} \tag{30}$$

$$\rho_a = \left(\overline{\rho^{\delta\alpha/\beta}}\right)^{\beta/\delta\alpha} \tag{31}$$

$$\cos\theta_a = \left(\overline{\cos^{\epsilon\alpha/\beta}\theta}\right)^{\beta/\epsilon\alpha} \tag{32}$$

Thus, the representative values to be used in equation (18) are defined in equations (30), (31), and (32).

The environments defined in references 3 and 4 can be used as examples for using equations (30), (31), and (32). As developed in the section entitled Gravitating Earth, the impact-angle distribution (relative to the entire Earth surface) is random. Thus, for a random directional flux (eq. (8)), equation (32) becomes

$$\cos^{\epsilon\alpha/\beta}\theta_a = \int_{\theta=0}^{\pi/2} 2\sin\theta\cos\theta\cos^{\epsilon\alpha/\beta}\theta\, d\theta \tag{33}$$

or

$$\cos\theta_a = \left(\frac{2}{2+\frac{\epsilon\alpha}{\beta}}\right)^{\beta/\epsilon\alpha} \tag{34}$$

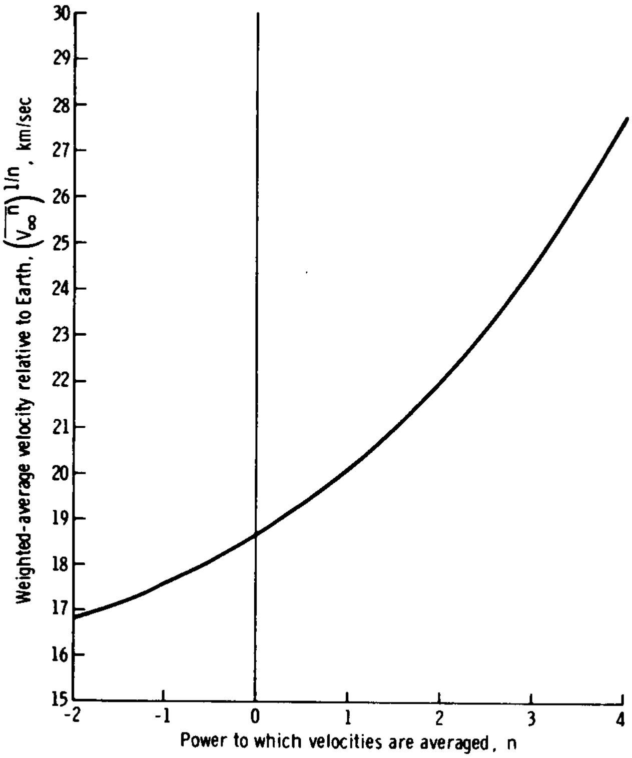

Figure 4. - Weighted-average meteoroid velocity relative to the Earth as a function of weighting (velocity distribution from ref. 3).

The velocity distribution given in reference 3 was used to compute the weighted-average velocity $\left(\overline{V^n}\right)^{1/n}$ for various values of n. The results are shown in figure 4. If spacecraft shielding were being designed relative to this velocity distribution, then

$$n = \frac{\gamma\alpha}{\beta} \tag{35}$$

However, as is developed in the section entitled Meteoroid Flux and Impact-Velocity Distribution for an Orbiting Spacecraft, such a velocity distribution is modified relative to an orbiting spacecraft.

Since a meteoroid mass-density distribution is not given in reference 3, then the average mass density given in reference 3 is also the representative average, or

$$\rho_a = 0.5 \text{ g/cm}^3 \tag{36}$$

However, in the mass-density distribution given in reference 4, 51 percent of the meteoroids have a mass density of 0.37 g/cm^3, 45 percent have a mass density of 2.8 g/cm^3, and 4 percent have a mass density of 8 g/cm^3. Thus, from this mass-density distribution

$$\rho_a = \left(0.51 \times 0.37^{\delta\alpha/\beta} + 0.45 \times 2.8^{\delta\alpha/\beta} + 0.04 \times 8^{\delta\alpha/\beta}\right)^{\beta/\delta\alpha} \text{ g/cm}^3 \tag{37}$$

METEOROID FLUX AND IMPACT-VELOCITY DISTRIBUTION FOR AN ORBITING SPACECRAFT

Thus far in spacecraft design, the assumption has been made that either the meteoroid hazard for an orbiting spacecraft is the same as the meteoroid hazard relative to the Earth, or the meteoroid environment is already defined relative to an orbiting spacecraft. However, this is rarely the case. Meteoroid fluxes and velocity distributions obtained from meteor observations are, by the nature of the observations, relative to the Earth. Although meteoroid fluxes measured by Earth satellites (ref. 4) are relative to an orbiting spacecraft, the velocity distribution used to reduce the penetration-thickness data is obtained from photographic meteors. Thus, mass determinations from the penetration thicknesses are made from a velocity distribution relative to the Earth. Such approximations do not introduce large errors, but the nature of these errors should be known. In this section, the problem of transforming the meteoroid flux and velocity distribution relative to the Earth to a flux and velocity distribution relative to an orbiting spacecraft is discussed.

Velocity Distribution From Uniform-Velocity Meteoroids

Consider a spacecraft in orbit about the Earth with a velocity vector $\vec{V}_s$ relative to the Earth. Consider also a group of meteoroids, each with a velocity vector $\vec{V}_m$ relative to the Earth. The velocity of any one of these meteoroids relative to the spacecraft is given by the vector relationship

$$\vec{V} = \vec{V}_m - \vec{V}_s \tag{38}$$

which can also be expressed as

$$V^2 = V_m^2 + V_s^2 - 2V_m V_s \cos \lambda \tag{39}$$

where λ is the angle between the velocity vectors $\vec{V}_m$ and $\vec{V}_s$, as shown in figure 5.

Thus, a meteoroid velocity distribution develops relative to the spacecraft (depending on the angle λ) from a single meteoroid velocity interval relative to the Earth. As is shown in the section entitled Gravitating Earth, if the spacecraft spends equal time over all parts of the Earth, all directions for $\vec{V}_m$ are equally probable, the exception being the direction shielded by the Earth. However, for the purpose of this analysis, Earth shielding will be ignored.

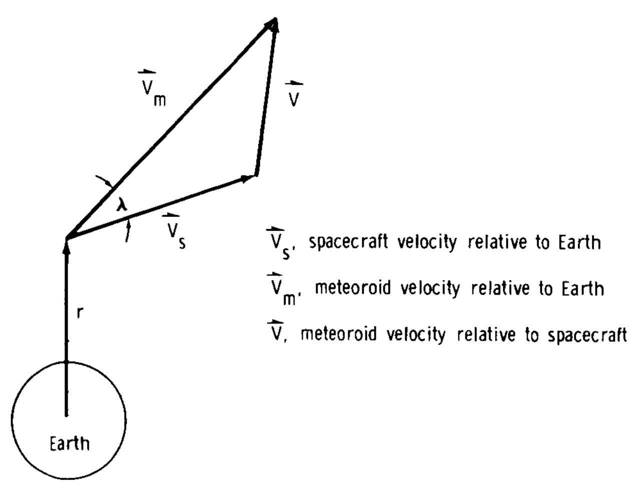

Figure 5. - Vector relationship between the spacecraft velocity relative to the Earth and the meteoroid velocity relative to the Earth.

Thus, the number of meteoroids coming from direction λ will be proportional to the number of directions $\vec{V}_m$ will point as it rotates around the $\vec{V}_s$-axis, or

$$n_\lambda(\lambda) \propto \sin \lambda \tag{40}$$

Since the probability that the spacecraft will encounter meteoroids increases with V, the meteoroid velocity distribution relative to the spacecraft is also proportional to V. (For example, if $V = 0$, the spacecraft never encounters a meteoroid.) Therefore

$$n_s(V)dV \propto V \sin \lambda \, d\lambda \tag{41}$$

If equation (39) is differentiated

$$\sin \lambda \, d\lambda = \frac{V \, dV}{V_m V_s} \tag{42}$$

Thus, if equations (41) and (42) are combined

$$n_s(V)dV = DV^2 dV \tag{43}$$

where V is between $V_m - V_s$ and $V_m + V_s$, and D is the normalization constant.

If the assumption is made that $V_s \leq V_m$, the value of D is evaluated by normalizing the velocity distribution so that

$$\int_{V_m - V_s}^{V_m + V_s} n_s(V)dV = 1 \tag{44}$$

Thus

$$D = \frac{3}{2V_s\left(3V_m^2 + V_s^2\right)} \tag{45}$$

and

$$n_s(V)dV = \frac{3V^2 dV}{2V_s\left(3V_m^{\ 2} + V_s^{\ 2}\right)} \tag{46}$$

where $V_m - V_s \leq V \leq V_m + V_s$, and $V_s \leq V_m$. The function $n_s(V)dV$ is then the fraction of the meteoroids which impacts the spacecraft with a velocity between V and $V + dV$ and which results from meteoroids with velocity V_m relative to the Earth.

If $V_m \leq V_s$, then by exchanging signs on the lower limit of integration in equation (44)

$$n_s(V)dV = \frac{3V^2 dV}{2V_m\left(3V_s^{\ 2} + V_m^{\ 2}\right)} \tag{47}$$

where $V_s - V_m \leq V \leq V_m + V_s$, and $V_m \leq V_s$. The weighted-average velocity $\left(\overline{V^n}\right)^{1/n}$ is found from equation (46) (assuming that in most cases $V_s \leq V_m$).

$$\overline{V^n} = \frac{\displaystyle\int_{V_m - V_s}^{V_m + V_s} V^n n_s(V)dV}{\displaystyle\int_{V_m - V_s}^{V_m + V_s} n_s(V)dV} \tag{48}$$

or

$$\overline{V^n}\left(V_m\right) = \frac{3V_m^{\ n}}{n + 3}\left[\frac{\left(1 + \frac{V_s}{V_m}\right)^{n+3} - \left(1 - \frac{V_s}{V_m}\right)^{n+3}}{\left(1 + \frac{V_s}{V_m}\right)^{3} - \left(1 - \frac{V_s}{V_m}\right)^{3}}\right] \tag{49}$$

where $V_s < V_m$, and the functional dependence is introduced to remind the reader of the V_m dependence.

An example of a direct application of equation (49) follows from considering a spacecraft in orbit about one of the large planets (i.e., Jupiter, Saturn, Uranus, or Neptune). The escape velocities of these planets are so large (compared with their orbital velocities around the Sun) that all meteoroid velocities near the planet are essentially the escape velocity of that planet. Thus, if the escape velocity of one of the planets is unity (i.e., $V_m = 1$), and if a spacecraft is in circular orbit around the planet, $V_s = 1/\sqrt{2}$. If $n = 1$, equation (49) gives an average impact velocity of 1.29, which is 29 percent higher than the meteoroid velocity relative to the planet. If $n = 2$, equation (49) gives a root-mean-square velocity of 1.33.

Before equation (49) can be used with a distribution of V_m, a flux increase factor, which is also a function of V_s and V_m, must be determined. This factor will be derived in the following section.

Increase in Meteoroid Flux on an Earth-Orbiting Spacecraft

The increase in meteoroid velocity relative to an orbiting spacecraft (discussed in the previous section) also leads to an increase in the meteoroid flux on the orbiting spacecraft, as compared with the meteoroid flux and velocity detection on a hypothetical spacecraft located at the position of the orbiting spacecraft, but which is stationary with respect to the planet. If the number density of meteoroids near the Earth is S, the meteoroid flux detected on a spacecraft near the Earth and which is stationary with respect to the Earth is given in reference 5 as

$$F_e = \frac{1}{4} S\left(\overline{V_m^{-1}}\right)^{-1} \tag{50}$$

where V_m is the meteoroid velocity relative to the Earth. Similarly, the meteoroid flux on an orbiting spacecraft will be

$$F_s = \frac{1}{4} S\left(\overline{V^{-1}}\right)^{-1} \tag{51}$$

where V is the meteoroid velocity relative to the orbiting spacecraft.

If only one group of meteoroids, those with velocity V_m, is considered, then the ratio of the meteoroid flux on the orbiting spacecraft to the flux on the stationary spacecraft becomes

$$f(V_m) = \frac{F_s}{F_e} = \frac{\left(\overline{V^{-1}}\right)^{-1}}{\left(\overline{V_m^{-1}}\right)^{-1}} = \frac{1}{V_m \int \frac{n_s(V)dV}{V}} \tag{52}$$

where $n_s(V)$ is given by equation (46) or (47), and the limits of integration are $V_m - V_s$ and $V_m + V_s$ for equation (46) or $V_s - V_m$ and $V_m + V_s$ for equation (47). The flux increase factor then becomes

$$f(V_m) = \frac{3V_m^2 + V_s^2}{3V_m^2} \quad \text{for} \quad V_s \leq V_m \tag{53}$$

and

$$f(V_m) = \frac{3V_s^2 + V_m^2}{3V_s V_m} \quad \text{for} \quad V_s \geq V_m \tag{54}$$

The total flux increase, resulting from all values of V_m, is then given by

$$f_t = \int f n_{V,m}(V_m) dV_m \tag{55}$$

where $n_{V,m}(V_m)$ is a fraction of meteoroids having velocity V_m.

If $V_s \leq V_m$, which is the case for a spacecraft in orbit around the Earth, then

$$f_t = 1 + \frac{V_s^2 \overline{V_m^{-2}}}{3} \tag{56}$$

For example, figure 4 shows that $\left(\overline{V_m^{-2}}\right)^{-1/2}$ has a value of 16.8 km/sec for the velocity distribution given in reference 3. A spacecraft in circular orbit just above the Earth will have a V_s value of 7.6 km/sec. For this case, the value of f_t is 1.07. That is, for a particular meteoroid mass, there will be a 7 percent greater meteoroid flux on the orbiting spacecraft as compared with the meteoroid flux detected on a stationary spacecraft near the Earth.

Total Meteoroid Velocity Distribution Relative to an Orbiting Spacecraft

In the section entitled Velocity Distribution From Uniform-Velocity Meteoroids, it was shown that a velocity distribution develops relative to an orbiting spacecraft from meteoroids having uniform velocity V_m relative to the Earth. This velocity distribution is expressed in equation (46).

In the section entitled Increase in Meteoroid Flux on an Earth-Orbiting Spacecraft, the flux of meteoroids on the orbiting spacecraft was found to increase by the factor of $f(V_m)$ (eq. (53)) over the meteoroid flux detected on a stationary spacecraft at the same position. Thus, the total velocity distribution relative to the spacecraft can be obtained by multiplying the fraction of the meteoroids with velocity V_m by $n_s(V)$ and f for all values of V_m and then summing the numbers having equal values of V.

However, for most applications, all that is required is to obtain a weighted-average velocity. The weighted-average velocity $\overline{V^n}(V_m)$ from the distribution of $n_s(V)$ is given in equation (49). Thus, given a distribution of V_m, the resulting weighted-average velocity relative to the spacecraft can be found from

$$\overline{V^n} = \frac{\int n_{V,m}(V_m) f(V_m) \overline{V^n}(V_m) dV_m}{\int n_{V,m}(V_m) f(V_m) dV_m} \tag{57}$$

The integral in the denominator is f_t, as defined in equation (55) and evaluated in equation (56). Thus, if equations (55), (53), and (49) are combined with equation (57)

$$\overline{V^n} = \frac{1}{2(n+3)V_s f_t} \int n_{V,m}(V_m) V_m^{-2} \left[(V_m + V_s)^{n+3} - (V_m - V_s)^{n+3}\right] dV_m \tag{58}$$

Of course, equation (58) can be numerically integrated for any value of n, but it is simpler to assume integer values and produce a plot of $\overline{V^n}^{\,1/n}$ as a function of n for various values of V_s. Thus, if n = -2, -1, 1, 2, 3, and 4, the following equations are obtained.

$$\left(\overline{V^{-2}}\right)^{-1/2} = \sqrt{f_t}\left(\overline{V_m^{-2}}\right)^{-1/2} \tag{59}$$

$$\left(\overline{V^{-1}}\right)^{-1} = f_t\left(\overline{V_m^{-1}}\right)^{-1} \tag{60}$$

$$\overline{V} = \frac{\overline{V_m} + V_s^2\overline{V_m^{-1}}}{f_t} \tag{61}$$

$$\left(\overline{V^2}\right)^{1/2} = \left(\frac{\overline{V_m^2} + 2V_s^2 + \frac{1}{5}\overline{V_m^{-2}}V_s^4}{f_t}\right)^{1/2} \tag{62}$$

$$\left(\overline{V^3}\right)^{1/3} = \left(\frac{\overline{V_m^3} + \frac{10}{3}V_s^2\overline{V_m} + V_s^4\overline{V_m^{-1}}}{f_t}\right)^{1/3} \tag{63}$$

$$\left(\overline{V^4}\right)^{1/4} = \left(\frac{\overline{V_m^4} + 5V_s^2\overline{V_m^2} + 3V_s^4 + \frac{1}{7}V_s^6\overline{V_m^{-2}}}{f_t}\right)^{1/4} \tag{64}$$

where f_t is given by equation (56).

If the velocity distribution in reference 3 is used, values for $\overline{V_m^n}$ are obtained from figure 4. If values for V_s of 0, 5, 8, and 10 km/sec are assumed, the family of curves shown in figure 6 results. Note that the curve for $V_s = 0$ in figure 6 is the same as the curve in figure 4. The value of n is determined from equation (30).

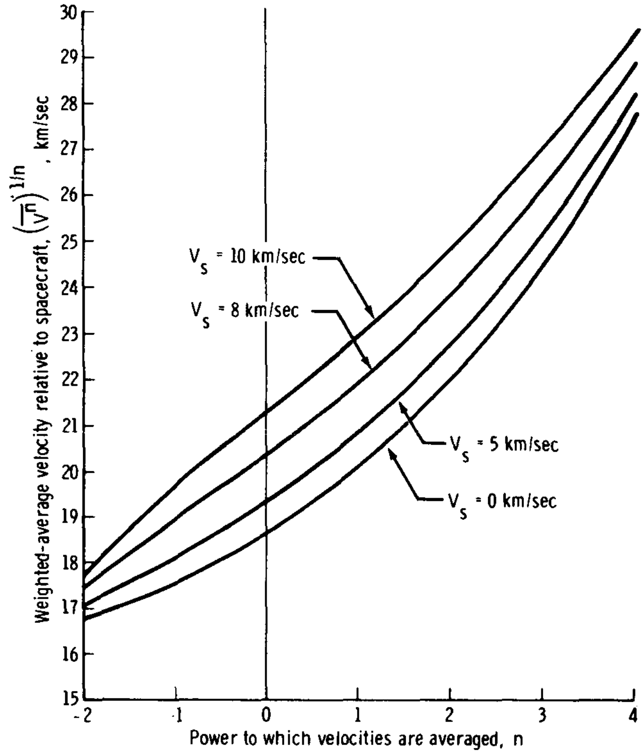

Figure 6. - Weighted-average meteoroid velocity relative to the spacecraft as a function of weighting for various spacecraft velocities (velocity distribution from ref. 3).

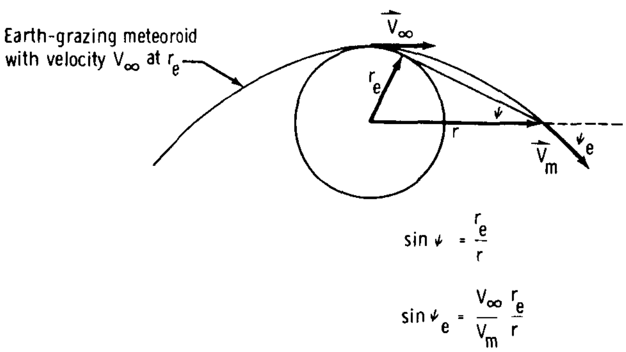

Figure 7. - Earth shielding factor.

METEOROID FLUX AND VELOCITY AS FUNCTIONS OF DISTANCE FROM THE EARTH

As a result of Earth gravity and shielding, both meteoroid flux and meteoroid velocity distribution vary as functions of distance from the Earth. In this section, these variations are discussed relative to the Earth. Velocities relative to a moving spacecraft are found in the section entitled Meteoroid Flux and Impact Velocity Distribution for an Orbiting Spacecraft, after the velocity distribution relative to the Earth is known.

Flux Variation

Earth shielding factor. - The Earth shielding factor is usually calculated by taking the fraction of solid angle that the Earth subtends and subtracting the fraction from 1, or

$$\eta = \frac{1 + \cos\psi}{2} \tag{65}$$

where

$$\sin\psi = \frac{r_e}{r} \tag{66}$$

and r_e and r are the Earth radius and the distance from the center of the Earth, respectively (fig. 7). However, for equation (66), the assumption is made that meteoroids are traveling in a straight line between the Earth and r; this, of course, is the limiting case as a meteoroid velocity increases.

The effective shielding angle ψ_e is found (through conservation of angular momentum (ref. 6)) to be

$$\sin\psi_e = \frac{V_\infty}{V_m}\frac{r_e}{r} \tag{67}$$

where V_∞ is the velocity of the meteoroid at r_e, and V_m is the velocity of the meteoroid at r. Conservation of energy gives V_m as a function of V_∞ and r, or

$$V_m^2 = V_\infty^2 - V_e^2\left(1 - \frac{r_e}{r}\right) \tag{68}$$

Thus, the effective shielding factor becomes

$$\eta_e = \frac{1 + \cos \psi_e}{2} \tag{69}$$

Since η_e is a function of meteoroid velocity, η_e must be integrated over the meteoroid velocity distribution in order to obtain the total shielding factor. However, by examining the ratio η_e/η for various meteoroid velocities (as is done in fig. 8), it can be seen that the maximum error introduced by using η is only 10 percent, at most, and that for most velocity distributions and distances from the Earth, the error is less than 5 percent. Thus, depending on the accuracy required, equations (65) and (66) will usually describe the Earth shielding factor accurately.

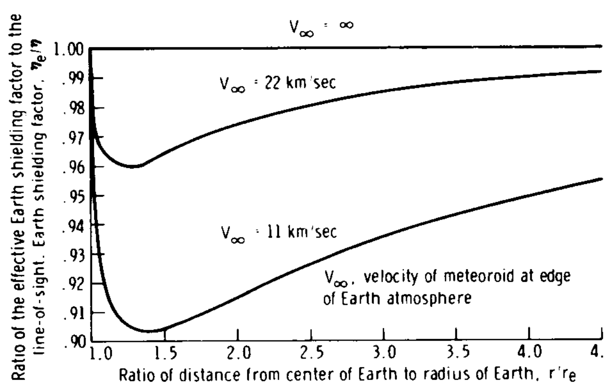

Figure 8. - Comparison of Earth shielding factors.

Gravitation decrease factor. - Meteoroid flux will decrease as distance from the Earth increases, as a result of the inverse variation of the Earth gravitational field. In reference 1, this factor is determined from conservation of angular momentum, and the factor can be expressed as

$$G = \frac{V_m^2}{V_\infty^2} \tag{70}$$

If equation (70) is combined with equation (68)

$$G = 1 - \frac{V_e^{\,2}}{V_\infty^{\,2}}\left(1 - \frac{r_e}{r}\right) \tag{71}$$

To determine the total decrease in flux resulting from all values of V_∞, equation (71) must be integrated over a distribution of V_∞, or

$$G_t = \int G n_\infty\left(V_\infty\right) dV_\infty \tag{72}$$

which gives

$$G_t = 1 - V_e^{\,2}\overline{V_\infty^{\,-2}}\left(1 - \frac{r_e}{r}\right) \tag{73}$$

As an example, when $\left(\overline{V_\infty^{\,-2}}\right)^{-1/2} = 16.8$ km/sec (from fig. 4),

$$G_t = 1 - 0.44\left(1 - \frac{r_e}{r}\right) \tag{74}$$

Meteoroid Variation Relative to the Earth

Because of Earth gravity, the velocity distribution relative to the velocity distribution detected on the Earth changes with distance from the Earth. With increasing distance from the Earth, meteoroid velocities are slower, but the relative number of slower meteoroids also decreases. The net effect is to change some weighted-average velocities only slightly, to increase the weighted averages that weight the higher velocities, and to decrease the weighted averages that weight the lower velocities.

If the velocity distribution at the Earth surface, relative to the Earth, is given by $n_\infty(V_\infty)$, then the velocity distribution at a distance r from the Earth is given by

$$n_{V,m}\left(V_m\right)dV_m = G n_\infty\left(V_\infty\right)dV_\infty \tag{75}$$

where G is given in equation (71) and V_∞ is related to V_m in equation (68). By substituting equations (68) and (71) into equation (75), $n_{V,m}(V_m)$ can be determined. However, since all that is usually required is determination of $\left(\overline{V_m^{\,n}}\right)^{1/n}$, a determination of the new distribution function is not necessary.

Since

$$\overline{V_m^{\,n}} = \frac{\int n_{V,m}(V_m) V_m^{\,n} dV_m}{\int n_{V,m}(V_m) dV_m} \tag{76}$$

equation (75) is combined with equation (76) to give

$$\overline{V_m^{\,n}} = \frac{\int G V_m^{\,n} n_\infty(V_\infty) dV_\infty}{\int G n_\infty(V_\infty) dV_\infty} \tag{77}$$

Substitution of equations (68), (71), and (72) into equation (77) gives

$$\overline{V_m^{\,n}} = \frac{1}{G_t} \int V_\infty^{-2} \left[V_\infty^{\,2} - V_e^{\,2}\left(1 - \frac{r_e}{r}\right)\right]^{(n+2)/2} n_\infty(V_\infty) dV_\infty \tag{78}$$

where G_t is given by equation (73).

Thus, it is necessary to integrate only over the meteoroid velocity distribution at the Earth surface. Equation (78) can be integrated directly for values of $n = -2$, 2, and 4. Thus

$$\left(\overline{V_m^{\,-2}}\right)^{-1/2} = \sqrt{G_t}\left(\overline{V_\infty^{\,-2}}\right)^{-1/2} \tag{79}$$

$$\left(\overline{V_m^{\,2}}\right)^{1/2} = \left[\frac{\overline{V_\infty^{\,2}} - 2V_e^{\,2}\left(1 - \frac{r_e}{r}\right) + \overline{V_\infty^{\,-2}} V_e^{\,4}\left(1 - \frac{r_e}{r}\right)^2}{G_t}\right]^{1/2} \tag{80}$$

$$\left(\overline{V_m^{\,4}}\right)^{1/4} = \left[\frac{\overline{V_\infty^{\,4}} - 3\overline{V_\infty^{\,2}}V_e^{\,2}\left(1 - \frac{r_e}{r}\right) + 3V_e^{\,4}\left(1 - \frac{r_e}{r}\right)^2 - \overline{V_\infty^{\,-2}}V_e^{\,6}\left(1 - \frac{r_e}{r}\right)^3}{G_t}\right]^{1/4} \qquad (81)$$

As an example of the application of equations (79), (80), and (81), by using values for $\left(\overline{V_\infty^{\,n}}\right)^{1/n}$ given in figure 4, the three average velocities in equations (79), (80), and (81) are found for various distances from the Earth. These velocities are then plotted as a function of n, and a smooth curve is fitted to each set of three points. The results are shown in figure 9. Three points may seem insufficient to determine a curve accurately, but when intermediate values of $\left(\overline{V_m^{\,n}}\right)^{1/n}$ are determined by numerically integrating equation (78), the resulting curve proves sufficiently well behaved so that three points may be considered to be adequate.

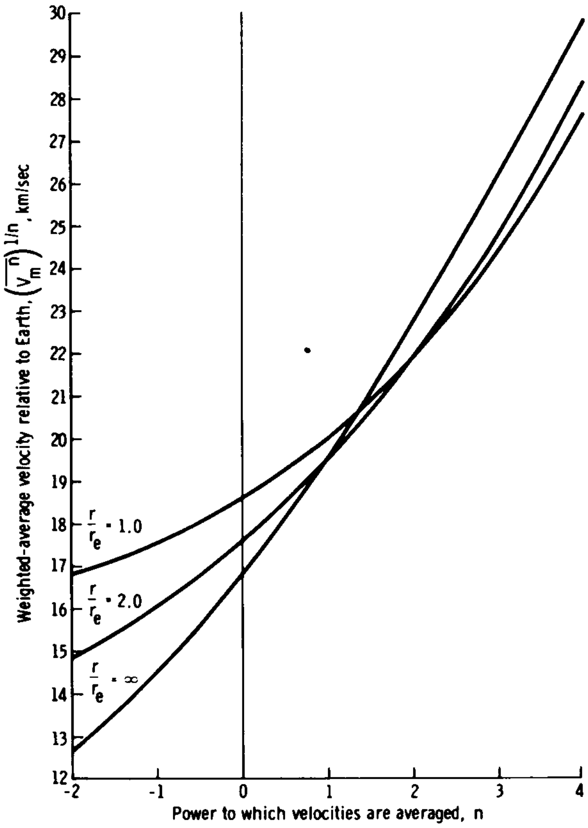

Figure 9. - Weighted-average meteoroid velocity relative to the Earth as a function of weighting for various distances from the Earth (velocity distribution from ref. 3).

To determine the weighted-average velocity of a meteoroid relative to an orbiting spacecraft at some distance from the Earth, equations (59), (62), and (64) can be used, where the values of $\overline{V_m^{\,n}}$ are found at a distance r by using equations (79), (80), and (81). The value of $\left(\overline{V^n}\right)^{1/n}$ for other values of n can be found by plotting $\left(\overline{V^n}\right)^{1/n}$ as a function of n.

CRITERIA

For the calculations made in this report, the assumption was made that the meteoroid environment at the edge of the Earth atmosphere and relative to the Earth is known. The meteoroid flux at the edge of the Earth atmosphere can be expressed as

$$F = Cm^{-\alpha} \tag{17}$$

where F is the number of meteoroids of mass m and larger impacting the atmosphere per unit area per unit time; C and α are constants. The velocity distribution relative to the Earth at the edge of the Earth atmosphere is expressed as $n_\infty(V_\infty)$, and the meteoroid mass-density distribution is given by $n_\rho(\rho)$. The distribution of impact angles into the Earth atmosphere, averaged over the entire surface of the Earth, is random, or

$$n_\theta(\theta) = 2 \sin\theta \cos\theta \tag{8}$$

where θ is the angle between the meteoroid negative velocity vector and the normal vector to the point of impact on the Earth atmosphere. Each of these three distributions is obtained by comparing meteoroids of the same mass.

By using the following method, these distributions may be transformed to a meteoroid flux as a function of some other parameter (e.g., penetration thickness, energy, or momentum) on an orbiting spacecraft at some distance r from the Earth. The assumption is made that this other parameter t can be expressed as

$$t = Bm^\beta V^\gamma \rho^\delta \cos^\epsilon\theta \tag{15}$$

where m is the impacting mass, V is the relative velocity between the meteoroid and impacted spacecraft, ρ is the mass density of the impacting mass, θ is the angle between the negative velocity vector and the normal to the spacecraft surface at the point of impact, and B, β, γ, δ, and ϵ are constants that are characteristic of the particular parameter.

The flux of meteoroids having parameter t and greater is given by

$$F = G_t \eta f_t C B^{\alpha/\beta} t^{-\alpha/\beta} V_a^{\gamma\alpha/\beta} \rho_a^{\delta\alpha/\beta} \cos^{\epsilon\alpha/\beta}\theta_a \tag{82}$$

where G_t is the gravitational decrease factor, η is the Earth shielding factor, f_t is the flux increase factor on Earth-orbiting spacecraft, V_a is the weighted-average velocity of meteoroids relative to the spacecraft, ρ_a is the weighted-average meteoroid mass density, and θ_a is the weighted-average meteoroid impact angle.

Weighted-Average Impact Angle

The weighted-average impact angle on a randomly tumbling surface is given by

$$\cos \theta_a = \left(\frac{2}{2 + \frac{\epsilon\alpha}{\beta}} \right)^{\beta/\epsilon\alpha} \tag{34}$$

Weighted-Average Meteoroid Mass Density

The weighted-average mass density is given by

$$\rho_a = \left(\overline{\rho^{\delta\alpha/\beta}} \right)^{\beta/\delta\alpha} \tag{31}$$

Weighted-Average Impact Velocity

The weighted-average velocity of meteoroids relative to an orbiting spacecraft is

$$V_a = \left(\overline{V^n} \right)^{1/n} \tag{83}$$

where

$$n = \frac{\gamma\alpha}{\beta} \tag{35}$$

The process of determining $\left(\overline{V^n} \right)^{1/n}$ from a distribution of velocity relative to the Earth is described in the criteria section entitled Determination of the Weighted-Average Meteoroid Velocity Relative to an Orbiting Spacecraft at Some Distance From the Earth.

Gravitation and Earth Shielding Decrease Factors

The meteoroid mass flux at a distance r from the Earth is decreased by the factors

$$G_t = 1 - \overline{V_e^{\,2} V_\infty^{-2}} \left(1 - \frac{r_e}{r}\right) \tag{73}$$

and

$$\eta = \frac{1 + \cos \psi}{2} \tag{65}$$

where

$$\sin \psi = \frac{r_e}{r} \tag{66}$$

and where V_e is the escape velocity from the surface of the Earth, r_e is the radius of the Earth, r is the distance from the center of the Earth, and ψ is the half angle subtended by the Earth from a distance r (fig. 7).

Flux Increase Factor for an Orbiting Spacecraft

The meteoroid mass flux on an orbiting spacecraft is increased over the flux on a hypothetical stationary spacecraft at the same position by the factor

$$f_t = 1 + \frac{\overline{V_s^{\,2} V_m^{-2}}}{3} \tag{56}$$

where V_s is the velocity of the spacecraft relative to the Earth, and V_m is the velocity relative to the Earth of a meteoroid at a distance r.

The weighted average $\overline{V_m^{-2}}$ is related to $\overline{V_\infty^{-2}}$ by

$$\overline{V_m^{-2}} = \frac{\overline{V_\infty^{-2}}}{G_t} \tag{84}$$

Determination of Weighted-Average Meteoroid Velocity Relative to an Orbiting Spacecraft at Some Distance From the Earth

Weighted-average meteoroid velocity relative to an orbiting spacecraft. - If the meteoroid velocity distribution at a distance r from the Earth and relative to the Earth is known to be $n_{V,m}(V_m)$, then the representative velocity of the meteoroids relative to the spacecraft is

$$\overline{V^n} = \frac{1}{2(n+3)V_s f_t} \int n_{V,m}(V_m) V_m^{-2} \left[(V_m + V_s)^{n+3} - (V_m - V_s)^{n+3} \right] dV_m \tag{58}$$

or, for particular values of n

$$\left(\overline{V^{-2}}\right)^{-1/2} = \sqrt{f_t} \left(\overline{V_m^{-2}}\right)^{-1/2} \tag{59}$$

$$\left(\overline{V^{-1}}\right)^{-1} = f_t \left(\overline{V_m^{-1}}\right)^{-1} \tag{60}$$

$$\overline{V} = \frac{\overline{V_m} + V_s^2 \overline{V_m^{-1}}}{f_t} \tag{61}$$

$$\left(\overline{V^2}\right)^{1/2} = \left(\frac{\overline{V_m^2} + 2V_s^2 + \frac{1}{5} \overline{V_m^{-2}} V_s^4}{f_t} \right)^{1/2} \tag{62}$$

$$\left(\overline{V^3}\right)^{1/3} = \left(\frac{\overline{V_m{}^3} + \frac{10}{3} V_s{}^2 \overline{V_m} + V_s{}^4 \overline{V_m{}^{-1}}}{f_t}\right)^{1/3} \tag{63}$$

$$\left(\overline{V^4}\right)^{1/4} = \left(\frac{\overline{V_m{}^4} + 5V_s{}^2 \overline{V_m{}^2} + 3V_s{}^4 + \frac{1}{7} V_s{}^6 \overline{V_m{}^{-2}}}{f_t}\right)^{1/4} \tag{64}$$

where V_m is the meteoroid velocity at distance r from the Earth and relative to the Earth, V_s is the velocity of the spacecraft relative to the Earth, and f_t is the flux increase factor on an orbiting spacecraft (eq. (56)).

Usually, it is not necessary to use equation (58) to obtain $\left(\overline{V^n}\right)^{1/n}$ for intermediate values of n. The plot of $\left(\overline{V^n}\right)^{1/n}$ as a function of n is sufficiently well behaved that the curve can be accurately interpolated by using equations (59) to (64). An example is shown in figure 6, where it is assumed that the spacecraft is just above the Earth surface so that $n_{V,m}(V_m) = n_\infty(V_\infty)$. For this example, the velocity distribution at the Earth surface $n_\infty(V_\infty)$ is taken from reference 3. If $n_{V,m}(V_m)$ is not known, but $n_\infty(V_\infty)$ is known, the following section will describe the process to determine $\overline{V_m{}^n}$ from $n_\infty(V_\infty)$.

Weighted-average meteoroid velocity at some distance from the Earth. - The weighted-average meteoroid velocity at a distance r from the Earth and relative to the Earth is given by

$$\overline{V_m{}^n} = \frac{1}{G_t} \int V_\infty{}^{-2} \left[V_\infty{}^2 - V_e{}^2\left(1 - \frac{r_e}{r}\right)\right]^{(n+2)/2} n_\infty\left(V_\infty\right) dV_\infty \tag{78}$$

or, for particular values of n

$$\left(\overline{V_m{}^{-2}}\right)^{-1/2} = \sqrt{G_t}\left(\overline{V_\infty{}^{-2}}\right)^{-1/2} \tag{79}$$

$$\left(\overline{V_m^{2}}\right)^{1/2} = \left[\frac{\overline{V_\infty^{2}} - 2V_e^{2}\left(1 - \frac{r_e}{r}\right) + \overline{V_\infty^{-2}}V_e^{4}\left(1 - \frac{r_e}{r}\right)^{2}}{G_t}\right]^{1/2} \tag{80}$$

$$\left(\overline{V_m^{4}}\right)^{1/4} = \left[\frac{\overline{V_\infty^{4}} - 3\overline{V_\infty^{2}}V_e^{2}\left(1 - \frac{r_e}{r}\right) + 3V_e^{4}\left(1 - \frac{r_e}{r}\right)^{2} - \overline{V_\infty^{-2}}V_e^{6}\left(1 - \frac{r_e}{r}\right)^{3}}{G_t}\right]^{1/4} \tag{81}$$

where V_∞ is the meteoroid velocity relative to the Earth at a distance $r = r_e$ from the Earth. (Values of $\overline{V_\infty^{n}}$ are found from $n_\infty(V_\infty)$, V_e is the escape velocity from the surface of the Earth, r is the distance of the spacecraft from the center of the Earth, r_e is the radius of the Earth, and G_t is the gravitational decrease factor (eq. (73).)

Equations (79) to (81) are often sufficient to plot $\left(\overline{V_m^{n}}\right)^{1/n}$ as a function of n. Thus, intermediate values of the weighted-average meteoroid velocity can be obtained without using equation (78). As an example, figure 9 is a plot in which the meteoroid velocity distribution from reference 3 was used.

A summary of the procedure to find $\left(\overline{V^{n}}\right)^{1/n}$ from $n_\infty(V_\infty)$ is as follows.

1. Numerically obtain the averages $\overline{V_\infty^{-2}}$, $\overline{V_\infty^{2}}$, and $\overline{V_\infty^{4}}$ from $n_\infty(V_\infty)$.

2. Use equations (79) to (81) (and, if necessary, eq. (78)) to plot $\left(\overline{V_m^{n}}\right)^{1/n}$ as a function of n for the particular spacecraft distance from the Earth.

3. Substitute the values obtained for $\left(\overline{V_m^{n}}\right)^{1/n}$ into equations (59) to (64), and plot $\left(\overline{V^{n}}\right)^{1/n}$ as a function of n for the particular spacecraft velocity.

4. Read from this plot the weighted-average velocity that corresponds to the appropriate value of n (given in eq. (35)).

CONCLUSION

Errors are generated by using average meteoroid velocities and mass densities in transforming a meteoroid flux as a function of mass into a flux as a function of penetration thickness at some distance from the Earth. While in some cases the error may be tolerable, in other cases it would be critical. In any event, the use of a properly weighted-average velocity is no more difficult than using the average velocity, once the representative or weighted-average velocities have been determined for the particular spacecraft distance from the Earth and spacecraft velocity relative to the Earth.

Manned Spacecraft Center
National Aeronautics and Space Administration
Houston, Texas, May 27, 1971
124-12-10-00-72

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APPENDIX

ERROR IN METEOROID VELOCITY AND FLUX DUE TO A DIRECTIONAL ENVIRONMENT

All meteoroid-velocity calculations in this report assume that the spacecraft is moving through a meteoroid flux that is random in direction. It is shown that even a directional flux becomes random when averaged over the entire Earth surface. However, a spacecraft will rarely spend equal time over all areas of the Earth. Instead, a spacecraft will circle the Earth in a plane, and its path over the Earth will be a circle around the Earth. Although any directional effects will tend to average out along the spacecraft trajectory, it may be useful to know what errors are introduced when a random directional flux is adopted.

The effects of a directional meteoroid stream during one particular spacecraft orbit can be quickly calculated. Assume that all meteoroids impact the Earth from a single direction in space and that a spacecraft is orbiting the Earth such that the plane of its orbit is perpendicular to the direction of the meteoroid stream. Meteoroids would impact the spacecraft with velocity V given by

$$V = \left(V_m^2 + V_s^2\right)^{1/2} \tag{A1}$$

where V_m is the meteoroid velocity relative to the Earth, and V_s is the spacecraft velocity relative to the Earth.

The ratio of the flux on the moving spacecraft to the flux on a spacecraft stationary relative to the Earth is then given by

$$f = \left(1 + \frac{V_s^2}{V_m^2}\right)^{1/2} \tag{A2}$$

(See the section entitled Velocity Distribution From Uniform-Velocity Meteoroids.) If equation (A2) is integrated over the distribution of V_m (eq. (55) from the text)

$$f_t = \int f n\left(V_m\right) dV_m \tag{A3}$$

and the weighted-average meteoroid impact velocity on the spacecraft is (by using eq. (57) from the text)

$$\overline{V^n} = \frac{1}{f_t} \int n_{V,m}(V_m) V_m^{-1} \left(V_m^2 + V_s^2\right)^{(n+1)/2} dV_m \tag{A4}$$

If equation (A3) is integrated for $n = -1$, 1, and 3

$$\left(\overline{V^{-1}}\right)^{-1} = f_t \left(\overline{V_m^{-1}}\right)^{-1} \tag{A5}$$

$$\overline{V} = \frac{1}{f_t} \left(\overline{V_m} + \overline{V_m^{-1}}\, V_s^2\right) \tag{A6}$$

$$\overline{V^3}^{\,1/3} = \left(\frac{\overline{V_m^3} + 2V_s^2 \overline{V} + V_s^4 \overline{V_m^{-1}}}{f_t}\right)^{1/3} \tag{A7}$$

Equations (A5) and (A6) are identical to equations (60) and (61), respectively, from the text, while equation (A7) is almost identical to equation (63) from the text. Thus, for these particular cases, the value of f_t represents the principal differences in these weighted-average meteoroid velocities.

If equation (A3) is numerically integrated over the meteoroid velocity distribution given by reference 3, and if a value for V_s is assumed to be 7.6 km/sec, then a value of 1.096 for f_t is found (compared with a value of 1.068 for f_t found in the section entitled Increase in Meteoroid Flux Relative to an Earth-Orbiting Spacecraft). Thus, the error in meteoroid velocity and flux is only 2.6 percent, even in the unlikely circumstance that such a highly directional environment is encountered. The error becomes less as V_s becomes smaller and as the high degree of directionality is reduced.

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