

# A Hierarchy of Topological Tensor Network States

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Garching, Germany

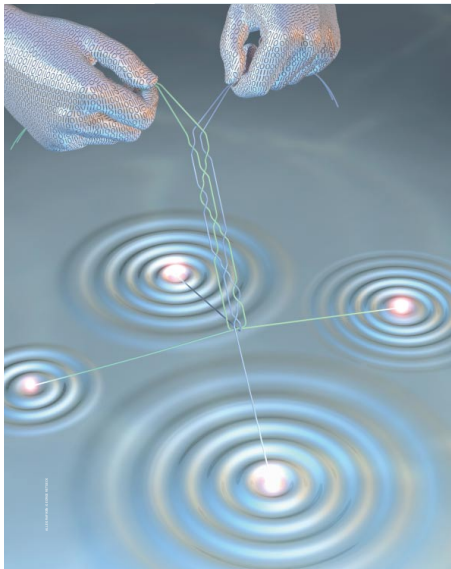
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Córdoba, Argentina

<sup>3</sup>Ludwig-Maximilians-Universität  
Munich, Germany

<sup>4</sup>ETH Zürich  
Zurich, Switzerland

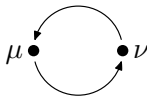
Quantum Engineering of States and Devices, Obergurgl 2010

# Topological Order and Quantum Computation



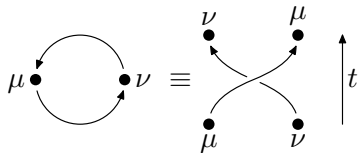
# Anyons

## ► Braiding in 2D



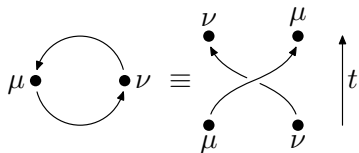
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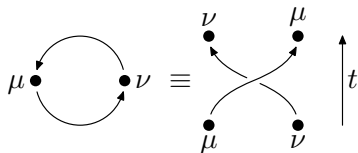
## ► Braiding in 2D



$$|\psi_{\mu\nu}\rangle \mapsto U |\psi_{\mu\nu}\rangle$$

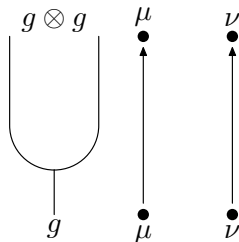
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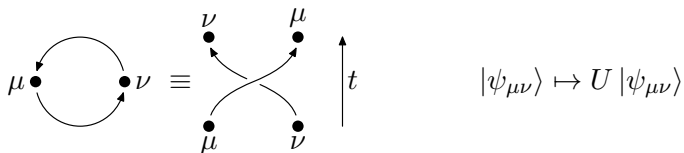
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## ► Symmetry and Hopf algebras

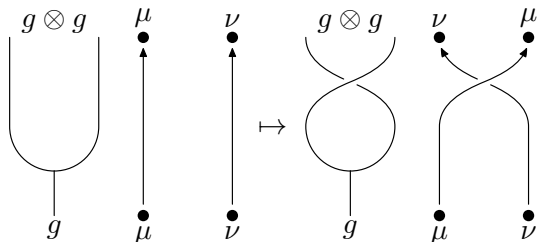


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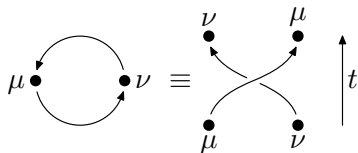


## ▶ Symmetry and Hopf algebras



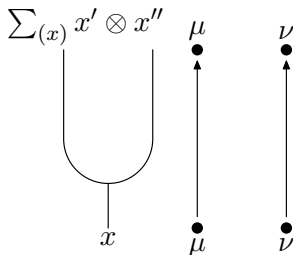
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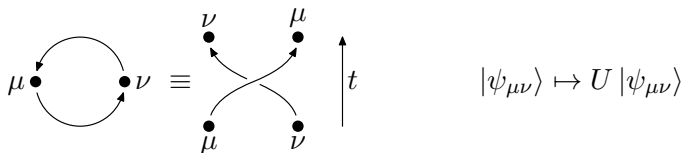
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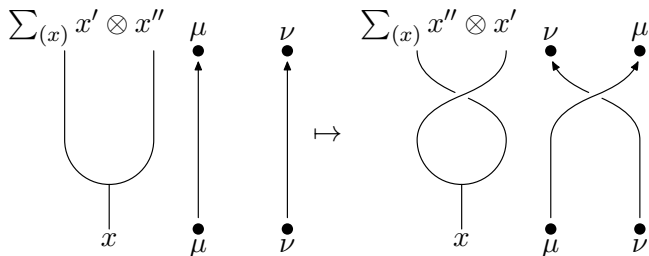


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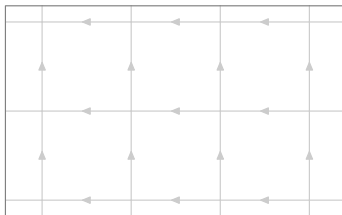


## ▶ Symmetry and Hopf algebras



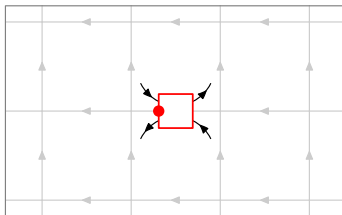
# Tensor networks

- ▶ Traditional approach (AKLT, MPS, PEPS)



# Tensor networks

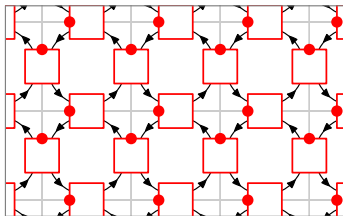
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$$\langle g_1, \dots, g_N | \psi \rangle =$$

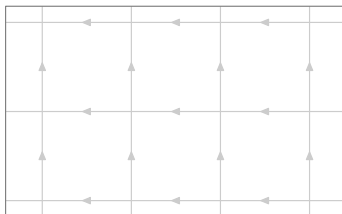


# Tensor networks

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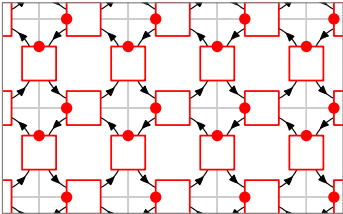
$$\langle g_1, \dots, g_N | \psi \rangle =$$

- ▶ Our approach

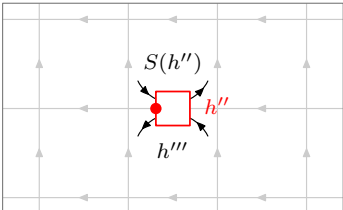


# Tensor networks

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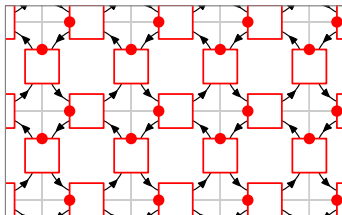
- ▶ Our approach

$$\sum_{(x)} \left[ \begin{array}{c} \leftarrow \leftarrow \leftarrow \leftarrow \\ \uparrow \uparrow \uparrow \uparrow \\ \leftarrow \leftarrow \leftarrow \leftarrow \\ \uparrow \uparrow \uparrow \uparrow \\ \leftarrow \leftarrow \leftarrow \leftarrow \end{array} \right] S(h'') \left[ \begin{array}{c} \leftarrow \leftarrow \leftarrow \leftarrow \\ \uparrow \uparrow \uparrow \uparrow \\ \leftarrow \leftarrow \leftarrow \leftarrow \\ \uparrow \uparrow \uparrow \uparrow \\ \leftarrow \leftarrow \leftarrow \leftarrow \end{array} \right] |x'\rangle$$


# Tensor networks

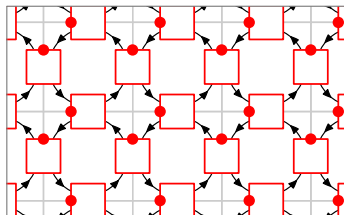
- ▶ Traditional approach (AKLT, MPS, PEPS)

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- ▶ Our approach

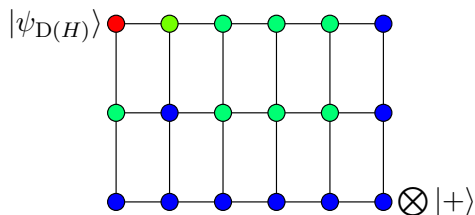
$$|\psi\rangle = \sum_{(x_i)}$$



$$\bigotimes_{i=1}^N |x'_i\rangle$$

# Hierarchy of Topological TN States

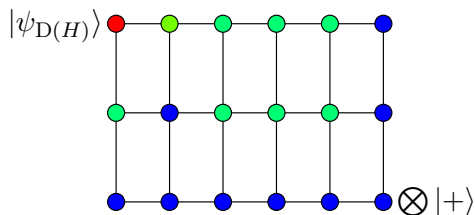
- ▶ Quantum states from Hopf algebras



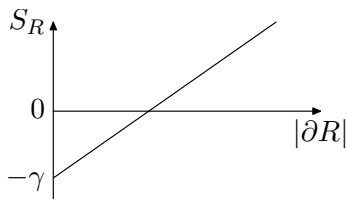


# Hierarchy of Topological TN States

- ▶ Quantum states from Hopf algebras



- ▶ Topological entanglement entropy



# Summary

- ▶ Hopf algebras as natural tool to describe systems with topological order
- ▶ General framework for tensor network states based on Hopf algebras
- ▶ Hierarchy of topological states related by charge condensation