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OPTIMIZATION

A hybrid analytical model for an entire hospital resource optimisation

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Abstract

Given the escalating healthcare costs around the world (more than 10% of the world's GDP) and increasing demand hospitals are under constant scrutiny in terms of managing services with limited resources and tighter budgets. Hospitals endeavour to find sustainable solutions for a variety of challenges ranging from productivity enhancements to resource allocation. For instance, in the UK, evidence suggests that hospitals are struggling due to increased delayed transfers of care, bed-occupancy rates well above the recommended levels of 85% and unmet A&E performance targets. In this paper, we present a hybrid forecasting-simulation-optimisation model for an NHS Foundation Trust in the UK. Using the Hospital Episode Statistics dataset for A&E, outpatient and inpatient services, we estimate the future patient demands for each speciality and model how it behaves with the forecasted activity in the future. Discrete event simulation is used to capture the entire hospital within a simulation environment, where the outputs is used as inputs into a multi-period integer linear programming (MILP) model to predict three vital resource requirements (on a monthly basis over a 1-year period), namely beds, physicians and nurses. We further carry out a sensitivity analysis to establish the robustness of solutions to changes in parameters, such as nurse-to-bed ratio. This type of modelling framework is developed for the first time to better plan the needs of hospitals now and into the future.

Keywords Forecasting · Simulation · Optimisation · Mathematical modelling · Healthcare · Multi-period

1 Introduction and related works

Ageing population combined with increasing demand, ranging from A&E attendance to emergency admissions to referrals, caused unsustainable levels of bed occupancy rates in the United Kingdom (Coggan, 2017). According to the national data published in 2018, 675 patients in need for hospitalisation were admitted to a mental health unit outside of their local area (NHS Digital, 2018), a practice the Department of Health & Social Care has committed to eliminate by 2020 (Department of Health & Social Care 2016). Shortage of beds and staff means that hospitals are potentially at risk of effectively managing patient flows, thus leaving it to be vulnerable to fluctuations in demand. In addition to staff shortages, there are other alarming capacity issues, such as inadequate resourcing of hospital beds. Over the past 30 years, the total number of NHS hospital beds in England has decreased from 299,000 to 142,000 (more than half), despite the increase in the number of patients treated during this period (Ewbank et al. 2017). Evidence suggests that hospitals are struggling due to increased delayed transfers of care, bed-occupancy rates well above the recommended levels of 85%, and unmet A&E performance targets. The UK has fewer hospital beds relative to its population than most comparable health systems (Murray et al. 2017). If staff shortages were to continue at its current rate, we will see a further deterioration in care quality, continued growth in waiting lists, and a risk of undermining sustainability of services into the future. In such a demanding environment, hospital managers tend to better understand their resource needs now and, in the future, to ensure effective and timely delivery of care. In this respect, a range of models and frameworks

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have been developed to assist key decision-makers determine the optimal resource requirements.

Evidence suggests that over the past decade staff shortages and reduction in beds had a serious impact on the healthcare system in the UK. Workforce growth has not kept up with the increasing demand and the pressures around budgeting constraints has made it worse, with lack of investment in most parts of the health service. This applies not just to the NHS in England but for most health systems around the world. To tackle these issues, key decision-makers need a comprehensive plan to improve use of existing resources by minimising waste (e.g. idle capacity) and to re-allocate resources where it is most needed. The NHS analysts usually generate outputs based on "averages", whether if it is related to bed capacity requirements, staffing, or demand for services. Key decisions made based on averages can have severe consequences and even small deviation from the average can potentially be fatal, as most hospitals already operate at full capacity (e.g. bed occupancy rates greater than 95%).

The literature is vast and rich, where different models and techniques are proposed to deal with this problem. However, one problem with these models is that a majority of them focus on a single department, specialty or service with the objective of calculating the optimal solutions for decision variables, such as required beds and staff. Owing to the complexities of a hospital and the convoluted patient pathways, the management requires a hospital-wide model that considers all specialties (and departments) spanning over the entire services made up of inpatient, outpatient and A&E. Given that pressures are apparent in the NHS as a whole (i.e. the entire hospital), modelling a single specialty, a service, or a disease would be inadequate. The management needs to examine every component of their services and see if the required resources are in place now and in the future. Therefore, a comprehensive entire hospital-level model is crucial to bring together all specialties and services within a single framework.

To develop such a comprehensive model in this paper, we first investigated the relevant literature of the last two decades from 1999 to 2021 by searching well-known databases related to healthcare and operational research including HealthSTAR, Medline, INFORMS Online, CINAHL, INSPEC, Science Citation Index, Embase, SIGLE and MathSci databases. We thoroughly determined and listed all the related keywords in healthcare modelling, simulation, scheduling, forecasting, optimisation, mathematical modelling, heuristic, and experimental design. More than 200 papers met our inclusion criteria and articles were categorised as follows.

- Simulation models in the healthcare context (discrete event simulation, system dynamics and agent-based simulation), i.e. Rashwan et al. (2015), Mathews and Long (2015), Lane et al. (2000), Djanatliev and Meier (2016), Oh et al. (2016), Hussein et al. (2017), Babashov et al. (2017), Liu (2011), Demir et al. (2018), Gul et al. (2019) and Cudney et al. (2019).
- Forecasting hospital demand (A&E, outpatient and inpatient specialties), i.e. Aboagye-Sarfo et al. (2015), Gul and Guneri (2015), Zinouri et al. (2018), Cote and Smith (2018), Ordu et al. (2019a), Kaushik et al. (2020) and Piccialli et al. (2021).
- Mathematical modelling in healthcare settings, i.e. Ben Abdelaziz and Masmoudi (2012), Bachouch et al. (2012), Wang et al. (2015), Yahia et al. (2016) and Ordu et al. (2021).
- Hybrid studies in healthcare modelling, i.e. Cappanera et al. (2014), Ghanes et al. (2015), Saadouli et al. (2015), Uriarte et al. (2017), Ordu et al. (2019b), Bahari and Asadi (2020) and Sasanfar et al. (2020).
- Other topics (i.e. scheduling and planning in healthcare) in operational research/operations management in healthcare, i.e. Yeh and Lin (2007), Brailsford and Vissers (2011), Saghaan et al. (2015), Leeftink et al. (2018), Xie and Lawley (2015), Rezaeiahari and Khasawneh (2020), Kluger et al. (2020) and Cinar et al. (2021).

Data mining techniques (i.e. Random Forest, K-Nearest Neighbours) have been also widely used in computer vision and modelling systems, i.e. Garg et al. (2018), Kumar et al. (2018), Gupta et al. (2019a), Chhabra et al. (2020), Bansal et al. (2021a), Kumar et al. (2021), Gupta et al. (2021) and Bansal et al. (2021b). In terms of resource planning in healthcare, a wide range of tools and techniques have been proposed including agent-based simulation (Cabrera et al. 2012), discrete event simulation (Izady and Worthington 2012; Rossetti et al. 1999; Ahmed and Alkhamis 2009), queuing theory (Belciug and Gorunescu 2015; Hou et al. 2019), operating room scheduling (Adan et al. 2009; Akbarzadeh et al. 2019; Vandenberghe et al. 2019) and ambulance deployment (Bertsimas and Ng 2019; Talarico et al. 2015; Majzoubi et al. 2012) among others. Centeno et al. (2003) integrated simulation and integer linear programming to find the optimal staffing requirements and schedules for staff over a planning period. A stochastic integer programming model was proposed in Daldoul et al. (2018) to optimise the resource use to reduce the average patient waiting times in a Tunisian university hospital. Burdett and Kozan (2016) presented a multi-objective optimisation model for hospital capacity analysis for the whole hospital using the epsilon-constraint method. Ben Abdelaziz and Masmoudi (2012) studied the problem of hospital bed planning using a multi-objective stochastic programming model which was tested on a set of Tunisian hospitals. Ma and Demeulemeester (2013) presented a multilevel integrative approach to the capacity planning problem using mathematical programming and simulation analysis.

Multi-period optimisation models are preferred whenever the problem parameters evolve over time and since the optimal decisions in different periods are dependent on each other, only a comprehensive optimisation model can lead to valid results. A variety of multi-period optimisation problems have been introduced in the literature such as the problem of supply chain network design (Pasandideh et al. 2015), portfolio optimisation (Zhang et al. 2013) and price optimisation (Gupta et al. 2019b). Hulshof et al. (2013) developed an integer model for resource allocation and elective patient admission plan over multiple periods and for multiple patient groups with uncertain treatment paths. Nezamoddini and Khasawneh (2016) is one of the few optimisation models in the literature along with Hulshof et al. (2013) dealing with the problem of resource optimisation in the healthcare context and as a multi-period optimisation model. They used a capacitated network design approach to model the patient transfers between hospitals and developed a mathematical model to minimise patient waiting times. Benneyan et al. (2012) is another study dealing with a similar problem to ours in which the optimal location of specialty care services in the United States was investigated as a single-period and a multi-period model.

To the best of our knowledge, no comprehensive model has ever been developed at a scale such that it is able to capture the entire hospital patient pathway at a sufficient level of detail, where series of decision variables are optimised, including staff, inpatient acute beds and outpatient consultation rooms. Hence, we attempt to fill this gap and contribute to the academic literature by putting forward an integrated decision support system for hospital managers. The goal of our study is to develop an innovative approach that combines forecasting, simulation and optimisation within an entire hospital, considering all services and specialties, generating a multi-period resource requirement plan for the hospital manager(s). We believe that the combination of these techniques creates a powerful decision-making framework to provide an optimal configuration of hospital services for the better, at a time when hybrid modelling techniques are becoming even more popular in today's complex decision-making environment. In a nutshell, the hybrid forecasting-simulation-optimisation model will deal with the following three decisions which we call forecasting, simulation and optimisation stages respectively.

- Forecasting demand to generate the activity to be fed into the simulation model to capture variation across all services and specialties.
- Simulating the hospital setting to capture the uncertainties around the core dynamics of the hospital, such as length of stay, waiting times, treatment duration, hospital finances. Key outputs from the simulation model will be used as inputs into the optimisation model.
- Determining the essential resource requirements across the entire hospital with an emphasis on high costing resources, i.e. staff (physicians and nurses), beds, and consultation rooms.

As far as we are concerned, there is no multi-period study in the literature combining forecasting, simulation and optimisation for capacity planning in a tactical level. Hence, this study is an attempt to fill this gap by putting forward an integrated model for capacity planning in a hospital. Our model predicts the demand over the next 12 months first before the average length of stay is computed using a simulation model. Then, an optimisation model optimises the number of beds, nurses and physicians needed each month to maximise a weighted average of number of patients admitted, rescheduled, and lost. We believe that our study contributes to the operational research applied to health services community, by filling a major gap in the literature around capacity modelling of an entire hospital, for the benefit of patients, staff, tax-payers and beyond. In collaboration with a hospital in England, we developed a novel hybrid approach that combines these three stages to capture the monthly needs for resources (beds, nurses and physicians) of the entire hospital to ensure the system is able to cope with demand, so that effective treatment can be provided as and when necessary. The optimisation will consider a multi-period healthcare resource allocation approach, whereby resource requirements will be generated for each month over a period of a year, rather than a relying on a single figure. Given that seasonality plays a role in demand for services (e.g. winter bed crisis), the number of beds or consultation room needs are likely to differ for each month. A multi-period set of resource outputs will empower the management with a wealth of intelligence to ensure their services are effective, safe and sustainable, thus the opportunity of achieving all key performance-related targets. Note that the model is generic such that it can be applied to any health systems around the world, subject to availability of local data and with slight modifications to the constraints.

The remainder of the paper is organised as follows. It will follow by explaining our methodology for forecasting the demand, simulation of the hospital flow and optimising the resources need in Sect. 2. The results of our integrated model for a real-world case study will be presented in Sect. 3 before conclusion are given in Sect. 4 with some avenues for further research proposed.

2 Methodology

2.1 The structure of the proposed hybrid framework

Patients are typically admitted to NHS hospitals through one of the following three routes: emergency patient arrivals (known as Accident & Emergency or A&E), outpatient attendances and inpatient admissions. Once a patient is in A&E, they can be admitted to an inpatient specialty (i.e. a ward) as a non-elective patient, referred to an outpatient specialty or discharged. Some A&E patients might need a surgery and use theatre facilities, which can affect health resources planning. A&E patients can also utilise bed capacity of inpatient departments when referred to an inpatient specialty. Elective patients are mostly referred by general practitioners (GPs) to inpatient specialties in hospitals. This demand is known by the hospital administrations in advance and is relatively more manageable since these patients are non-urgent cases, which are planned or delayed depending on the situation of hospital resources (i.e. availability of beds, theatres, physicians, nurses, diagnostics, etc.). Inpatient specialties are interconnected with A&E and outpatients who require wide range of resources. Outpatient arrivals are regular attendances at hospitals and are usually referred by GPs or A&E. An outpatient attendance is normally complete the same day of attendance and in some cases, patients might require a follow-up attendance. The number of follow-ups depends on the patient's condition, e.g. severity of disease (illness), age group, and co-morbidity. Figure 1 depicts the possible routes between patient groups as non-elective inpatients (I/NE), elective inpatients (IE), outpatient first visits (O/F),

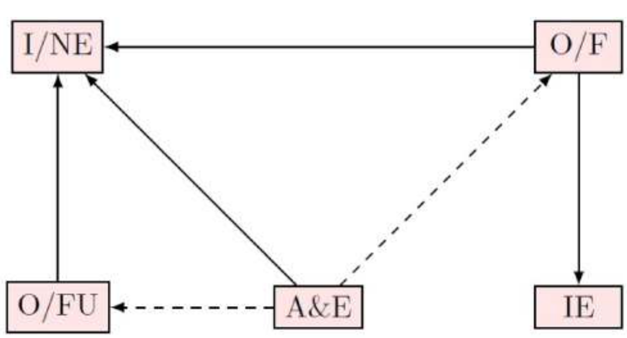


Fig. 1 The flow of patients between types. *O/FU* outpatient follow-up, *O/F* outpatient first, *I/E* inpatient elective, *I/NE* inpatient non-elective and *A&E* accident and emergency

Table 1 Specialties in the hospital

Specialties	A&E	O/F & O/FUP	I/E	I/NE
A&E	X			
General Surgery		X	X	X
Urology		X	X	
Trauma & orthopaedics		X	X	X
ENT		X	X	
Ophthalmology		X	X	
Oral Surgery		X	X	
Anaesthetics		X		
General Medicine		X	X	X
Gastroenterology		X	X	
Clinic Haematology		X	X	
Cardiology		X	X	X
Dermatology		X		
Neurology		X		
Rheumatology		X		
Paediatrics		X	X	X
Geriatric Medicine				X
Obstetrics		X		X
Gynaecology		X	X	X
Clinical Oncology		X	X	
Medical Oncology			X	
Radiology			X	

outpatient follow-ups (O/FU) and A&E patients. One should note that the dashed lines in the figure refer to paths which are feasible, but rarely happen. In order to provide further information on the problem settings, Table 1 shows the list of specialties in the hospital and the type of patients treated in each.

Individual patient pathways across the entire hospital were modelled using the Simul8 software, where 600 observed frequency distributions were established using both the national HES data and local datasets provided by the hospital. A large and extremely complex hospital-level DES was modelled to capture variation, allowing us to explore an array of policies and interventions the hospital had planned to make, including the impact of closure of a nearby hospital. Key outputs from the simulation model provided highly accurate estimates (i.e. average length of stay) were then fed into the optimisation model. Figure 2 demonstrates the hybrid model used in the paper.

2.2 Forecasting & discrete event simulation

Using a comprehensive modelling framework, we first forecast demand for all services and specialties. We conducted a comparative forecasting study by contrasting

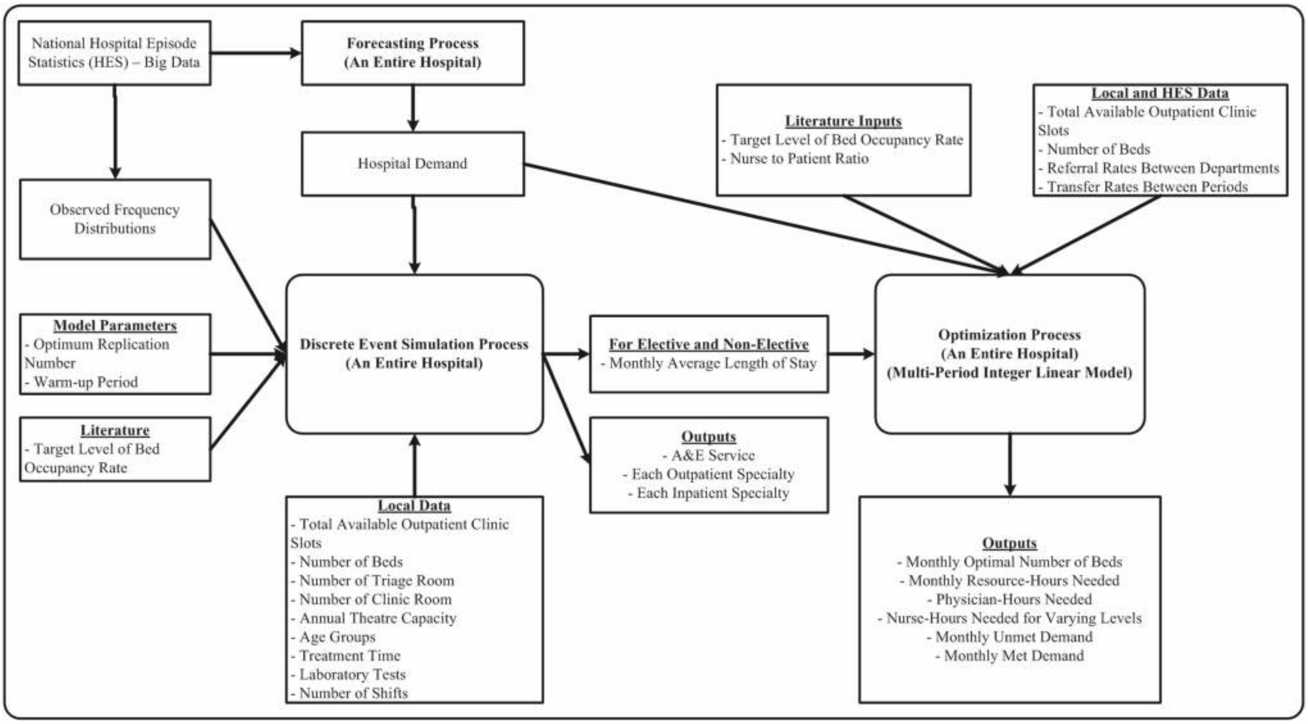


Fig. 2 The structure of the hybrid analytical model

forecasting models and periods to determine the best model-period combination to use for our problem. To this end, we used four forecasting methods (i.e. ARIMA, exponential smoothing, the seasonal and trend decomposition using Loess function and stepwise linear regression) and three forecasting periods (i.e. daily, weekly and monthly). We developed a total number of 768 forecasting models as follows: 12 forecasting models for A&E (four methods $\times$ three periods), 456 forecasting models for outpatient specialties (four methods $\times$ three periods $\times$ 19 specialties $\times$ 2 types of attendance, i.e. first and follow-up appointments) and 300 forecasting models for inpatient specialties (four methods $\times$ three periods $\times$ 25 specialties). We selected the best 64 models based on the forecast accuracy using the Mean Absolute Scaled Error (MASE) (Hyndman and Koehler 2006). These included 38 outpatient specialties (first and follow-ups), 25 inpatient specialties (elective and non-electives) and one for A&E. The forecast demands (i.e. 64 models) were then fed into the simulation model as activity. In addition to activity-related inputs, hundreds of other input parameters were established (approximately 600 statistical distributions for the simulation model using both primary and secondary data). Numerous primary data were collected from the hospital for each specialty, e.g. treatment time, outpatient consultation time for first/follow-up attendances, etc. Secondary data were also obtained from the hospital, such as number of outpatient clinic slots for each specialty, number of beds for each ward to name a few.

Figure 3 shows the predicted monthly demand of the hospital for each of the five demand types, showing that outpatients account for the majority of the demand (65%), while the share of demand for A&E and inpatients are 20% and 15%, respectively. Out of the five types, outpatient/follow-up patients constitute the largest per cent of the demand (48%). Inpatient elective, however, constitutes the lowest among the five types with almost 6% of the demand.

Following the forecasting stage, a discrete event simulation (DES) model was developed to investigate patient pathways of an entire hospital, including all specialties and services, inclusive of A&E. The simulation model was then converted into a decision support system (DSS) to enable the hospital management to assess how possible changes in resources and interventions (e.g. staff, beds, rooms and clinic slots) affect key metrics of interest (e.g. activity, utilisation, financial implications) in the safety of a simulation environment. The model uses a variety of input parameters as shown in Table 2 (in total 362 input parameters were estimated).

Demand for each service was forecasted using forecasting techniques as explained in the previous section. A&E patients are classified and prioritised according to severity of injuries in triage rooms. Further investigations can be carried out in A&E, such as X-rays, urinalysis, and biochemistry. There are A&E beds for urgent cases

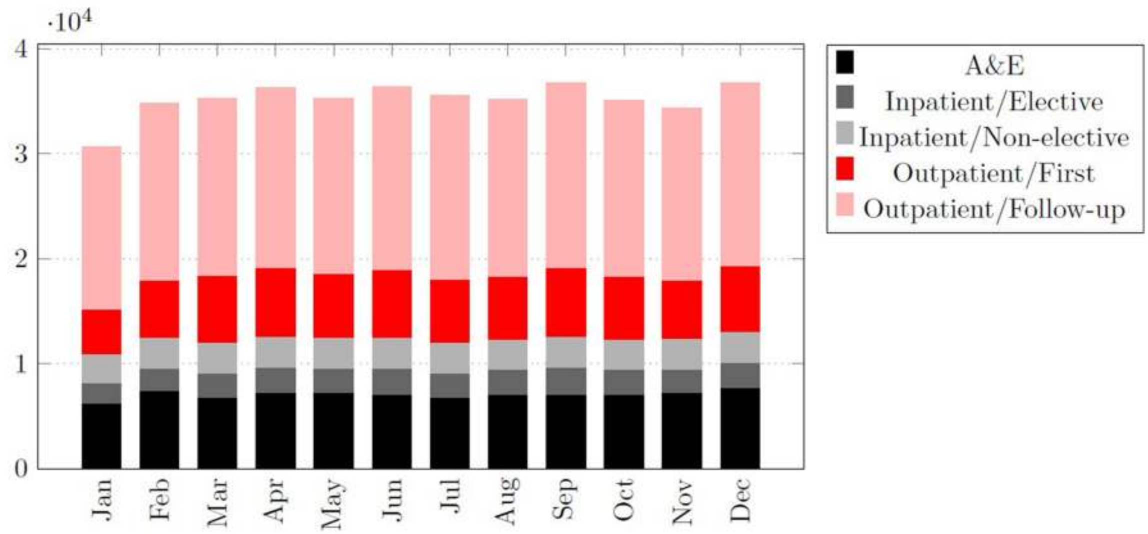


Fig. 3 Monthly demand broken down for each patient group

Table 2 Input parameters used in the simulation modelling

Input parameters	A&E	OS	IS
Forecasted demand	X	X	X
Beds	X		X
Triage room	X		
Clinic room	X	X	
HRG tariff (for financial inputs) (distribution)	X	X	X
Age groups (distribution)	X	X	X
Laboratory processes (distribution)	X		
Shifts	X		
Severity of injuries (distribution)	X		
Pre-assessment (distribution)	X	X	X
Treatment time (distribution)	X	X	X
Discharge time (distribution)	X		
Time for first appointment (distribution)		X	
Follow-up number (distribution)		X	
Length of period (distribution)		X	
Total available outpatient clinic slots		X	
Time for first admission (distribution)			X
Length of stay (distribution)			X
Total number of theatre procedure annual capacity			X
Percentage of inpatient admissions end up having a surgery (distribution)			X

A&E accident and emergency services, *IS* inpatient services, *OS* outpatient services

where patients are either discharged after a few hours of using a bed or admitted to an inpatient ward for a longer stay. In England, if a patient is transferred to an inpatient ward from A&E, this is known as a non-elective admission, which is approximately 51% of all admissions. There are two possible ways of been admitted to inpatient services, either as non-elective from A&E or an elective admission. In the case of an elective admission, an appointment is made at the request of a general practitioner (GP), or a consultant from outpatient services. Treatment may involve a surgical procedure, thus use of operating theatres, or a patient is treated (cared for) until they are deemed to be t for discharge. Patients could wait a few days to 18 weeks for an appointment and possibly more depending on availability of resources (including a bed, as inpatient services involve patients staying in hospital for a period of time). All the processes, including use of human and nun-human resources, diagnostic and treatment procedures, etc. are captured within the simulation model.

Attendance to outpatient services is usually through GPs, where patients are referred to a department or specialty depending on their condition. A typical outpatient attendance may involve a diagnostic and/or a treatment procedure carried out by a physician. Patients are either discharged home, a follow-up appointment could be requested by a physician (for a check-up or further treatment) or referred to another outpatient specialty.

Healthcare Research Groups (HRGs) is an indicator which classifies similar clinical "conditions" or "treatments" in terms of level of resources used in healthcare systems (England 2016). HRG tari was therefore used for financial inputs and the HRG codes were assigned to patients according to their diagnostics. Distributions were established based on age groups (i.e. 0–15, 16–30, 31–50, 51–65, 65+) as requested by the hospital management. We estimated the distributions (number of follow ups) related to how many times in a year a patient attends an outpatient clinic for follow-up treatments due to similar health conditions.

This simulation model generates a variety of outputs, such as activity, bed occupancy rates, financial outputs, staffing hours and theatre utilisation, which could all be used in the optimisation section. However, for the sake of this study, we focused on length of stay (LoS) which is within control of the hospital management, depending on patients' condition, specialty, diagnostics and treatment, whereas others, such as staff numbers and costs, number of beds and theatres are not (as they are in shortage anyway). LoS of a patient is a measure of how long the patient stays in a bed based on diagnostics, specialty and age group and is a critical input parameter to the optimisation model as it regulates the pace at which patients move in the system. The accuracy of this input is therefore critical as it determines the number of patients that can be treated per unit of time and the level of resources utilised in inpatient services.

By modelling the entire hospital, we will have captured the stochastic behaviour of hospital and specialties, where the average LoS is calculated for future financial years based on diagnostics, specialty and age group.

To validate our simulation model, black-box and white-box validations were carried out and the model was checked for its face validity. During the model development, we closely worked with key personnel in the hospital and their feedback was carefully considered in development of the model, which was continually improved accordingly, testing each unit for extreme conditions and logical consequences. Following the validation steps, it was decided that the model had passed white-box validation tests. In the final demonstration of the model, which was for face validation, key staff (service managers, director of the hospital, physicians, and nurses) were convinced that the model is appropriate for further use. This validation process was conducted comparing actual outputs and simulated outputs within the confidence interval range of 95%. Table 3 provides a summary of the validation outputs for trauma & orthopaedics outpatients, as a sample, where the actual figures for each parameter are compared against the simulated outputs. As a result, our simulation model was validated according to the black-box validation technique.

2.3 Integer optimisation

The optimisation part deals with a planning problem at the tactical level which is maximising an aggregate measure of served demand over a period of 12 months given a set of assumptions and in presence of constraints on the hospital

Table 3 Validation results for trauma & orthopaedics outpatients

Output parameters	Simulation	Actual	Differences	Percentage (%)
Total first attendance	10,643 (10,568; 10,717)	10,601	42 (− 33; 116)	0.4 (− 0.31; 1.09)
Total follow-up attendance	21,025 (20,710; 21,340)	20,758	267 (− 48; 582)	1.29 (− 0.23; 2.80)
Total DNAs	2990 (2861; 3118)	3088	− 98 (− 227; 30)	− 3.17 (− 7.35; 0.97)
Total cancellation	9103 (8896; 9310)	8916	187 (− 20; 394)	2.1 (− 0.22; 4.42)
First to follow-up ratio	1.98 (1.95; 2.00)	1.96	0.02 (− 0.01; 0.04)	1.02 (− 0.51; 2.04)
Total number of clinic attendance	31,668 (31,317; 32,018)	31,359	309 (− 42; 659)	0.99 (− 0.13; 2.10)
Clinic utilisation (%)	86.29 (85.33; 87.24)	85.45	0.84 (− 0.12; 1.79)	0.98 (− 0.14; 2.09)
Physician hours (hours)	16,389 (16,169; 16,608)	16,268	121 (− 99; 340)	0.74 (− 0.61; 2.09)
Total revenue ($million)	4.028 (3.991; 4.066)	4.014	0.014 (− 0.023; 0.052)	0.35 (− 0.57; 1.30)

resources. It is based on the notion of Master Production Scheduling (MPS) which is rooted in the literature of production planning. The classical approach for generating MPS assumes infinite capacity, fixed processing times, and a single scenario for demand forecasts (Korpeoglu et al. 2011). However, we relax two of these assumptions by considering finite resource capacities (beds, nurses and physicians) as well as using several scenarios for realisation of the problem parameters. The proposed model is a multi-period integer programming model which is based on a set of key assumptions (based on rounds of discussion with key personnel in the hospital) as follows:

- The demand for each specialty is composed of its direct plus the referral demand from other specialties. As an example, those patients who start their pathway as an Ear, Nose and Throat patient, might be directed to the radiology ward for further examination.
- There is a transfer of patients between periods (months). In other words, a patient can start and finish the stay in two different periods.
- The priority of emergency patients is higher compared to non-emergency ones, and this is reflected in the parameter values.
- There is a hierarchy for admissions in the hospital, so the more urgent cases are always admitted with a higher level of priority.
- In case the hospital does not have the capacity needed, patients will be blocked and lost which can lead to either a rescheduling for elective patients or a diversion to another hospital.
- The model takes only one hospital into consideration and capacity planning for other hospitals is beyond the scope of this paper.
- Non-emergency patients can be either admitted or be put on a waiting list. However, the emergency cases must be admitted and if not, the demand is unmet, and a penalty is incurred. The latter is called "loss/blocking probability" which equals the fraction of refused admissions (Bekker et al. 2017).
- The service times are identical for patients of each specialty-type combination.
- The length of each period is considered to be one month.
- Demand is non-stationary in the model which means that it fluctuates in different periods.
- Although the hospital has other resources such as junior doctors, incorporating them in the model is beyond the scope of this paper. This does not lead to an oversimplification of the hospital as the contribution of these resources and their incurred cost to the hospital is negligible compared to beds, nurses and physicians.
- All the hospital resources (beds, nurses and physicians) are available throughout the year. In other words, we assume that the hospital has contingency plans for unavailability of any of these resources.
- There is no addition or removal of a specialty within the planning horizon and all the specialties are fully operational throughout the planning period.
- Physicians visit inpatients every day during the patient's length of stay and the duration of these visits is almost constant. This time incorporates the time to examine the patients, writing a report and any other extra activity.

2.3.1 Definition of parameters

Before presenting the model, its sets, indices, parameters and variables are introduced as follows (Tables 4, 5 and 6).

2.3.2 Decision variables

The variables used in the optimisation model are defined in Table 7.

Figure 4 demonstrates the relationship between the model variables. The number of patients of a specific type and specialty to be admitted in each period (x_{ijt}) equals the total demand from that period (d_{ijt}) as well as all the patients which have been transferred from previous periods ($w_{ij,t-1}$). Please note that each w_{ijt} may include patients from any of the periods in $\{1, \ldots, t-1\}$. In each period, the demand is either met by admission (x_{ijt}) or the demand is transferred to the next periods (w_{ijt}) or the demand is not

Table 4 Definition of sets

Notation	Definition
$\mathcal{G}_1$	Set of A&E patients
$\mathcal{G}_2$	Set of inpatients
$\mathcal{G}_3$	Set of outpatients
$\mathcal{G}$	Set of patient types ($\mathcal{G} = \mathcal{G}_1 \cup \mathcal{G}_2 \cup \mathcal{G}_3$)
$\mathcal{S}$	Set of specialties
$\mathcal{T}$	Set of periods

Table 5 Definition of indices

Notation	Definition
$i \in \mathcal{G}$	Index of patient groups
$j \in \mathcal{S}$	Index of specialties
$t \in \mathcal{T}$	Index of periods

Table 6 Definition of parameters

Notation	Definition
d_{ijt}	Predicted demand of patient group $i \in \mathcal{G}$ for specialty $j \in \mathcal{S}$ at period $t \in \mathcal{T}$
ξ	The target bed occupancy ratio
ω_t^1	Number of days in period $t \in \mathcal{T}$ a nurse works
ω_t^2	Number of days in period $t \in \mathcal{T}$ a physician works
ω_t^3	Number of days in period $t \in \mathcal{T}$ a bed is available
ψ^1	Number of available nurses
ψ^2	Number of available physicians
ψ^3	Number of available beds
σ_{ij}	Length of treatment for patients in group $i \in \mathcal{G}$ in specialty $j \in \mathcal{S}$
μ_{ij}	Length of bed occupancy for patients $i \in \mathcal{G}$ in specialty $j \in \mathcal{S}$
τ_{ij}^1	Average time a nurse spends for patient type $i \in \mathcal{G}$ in specialty $j \in \mathcal{S}$
τ_{ij}^2	Average consultancy time for patient type $i \in \mathcal{G}$ in specialty $j \in \mathcal{S}$
θ_1	Minimum ratio of demand to be met on-time
θ_2	Maximum ratio of demand to be unmet
θ_3	Maximum ratio of demand to be rescheduled
δ_1	Weight for using a nurse (can be a monetary value or not)
δ_2	Weight for using a physician (can be a monetary value or not)
δ_3	Weight for using a bed (can be a monetary value or not)

Table 7 Definition of variables

Notation	Definition
x_{ijt}	Number of patients of group $i \in \mathcal{G}$ and specialty $j \in \mathcal{S}$ to admit at period $t \in \mathcal{T}$
y_{jt}^1	Number of nurses at specialty $j \in \mathcal{S}$ at period $t \in \mathcal{T}$
y_{jt}^2	Number of physicians at specialty $j \in \mathcal{S}$ at period $t \in \mathcal{T}$
z_{ijt}	Number of beds for group $i \in \mathcal{G}$ at specialty $j \in \mathcal{S}$ at period $t \in \mathcal{T}$
u_{ijt}	Unmet demand of group $i \in \mathcal{G}$ and specialty $j \in \mathcal{S}$ at period $t \in \mathcal{T}$ (zero for $i \in \{1,3\}$)
w_{ijt}	Demand of group i and specialty j rescheduled at period t to a later period (zero for $i \in \{1,3\}$)

met due to the constraints on the number of beds (u_{ijt}). One should note that the term *Length of Treatment* is used to refer to the duration of a patient's visit which is normally a fraction of a day for A&E and outpatients.

2.3.3 Objective function and constraints of optimisation model

Now, the optimisation model can be formulated as an integer optimisation model as follows.

$$\min \sum_{j \in \mathcal{S}} \sum_{t \in T} \left[\delta^1 y_{jt}^1 + \delta^2 y_{jt}^2 + \delta^3 \sum_{i \in \mathcal{G}} z_{ijt} \right] \tag{1}$$

$$x_{ijt} \leq d_{ijt} \quad \forall i \in \mathcal{G}; \forall j \in \mathcal{S}; \forall t \in \mathcal{T} \tag{2}$$

$$\sum_{i \in \mathcal{G}} \sum_{j \in \mathcal{S}} x_{ijt} \geq \theta_1 \sum_{i \in \mathcal{G}} \sum_{j \in \mathcal{S}} d_{ijt} \quad \forall t \in \mathcal{T} \tag{3}$$

$$\sum_{i \in \mathcal{G}} \sum_{j \in \mathcal{S}} u_{ijt} \leq \theta_2 \sum_{i \in \mathcal{G}} \sum_{j \in \mathcal{S}} d_{ijt} \quad \forall t \in \mathcal{T} \tag{4}$$

$$\sum_{i \in \mathcal{G}} \sum_{j \in \mathcal{S}} w_{ijt} \leq \theta_3 \sum_{i \in \mathcal{G}} \sum_{j \in \mathcal{S}} d_{ijt} \quad \forall t \in \mathcal{T} \tag{5}$$

$$w_{ij1} + x_{ij1} + u_{ij1} = d_{ij1} \quad \forall i \in \mathcal{G}; \forall j \in \mathcal{S} \tag{6}$$

$$w_{ijt} + x_{ijt} + u_{ijt} = w_{ij(t-1)} + d_{ijt} \quad \forall i \in \mathcal{G}; \forall j \in \mathcal{S}; \forall t \in \mathcal{T} : t > 1 \tag{7}$$

$$\sum_{j \in \mathcal{S}} y_{jt}^1 \leq \psi^1 \quad \forall t \in \mathcal{T} \tag{8}$$

$$\sum_{j \in \mathcal{S}} y_{jt}^2 \leq \psi^2 \quad \forall t \in \mathcal{T} \tag{9}$$

$$\sum_{i \in \mathcal{G}} \sum_{j \in \mathcal{S}} z_{ijt} \leq \psi^3 \quad \forall t \in \mathcal{T} \tag{10}$$

$$\sum_{i \in \mathcal{G}} \tau_{ij}^1 \sigma_{ij} x_{ijt} \leq w_t^1 y_{jt}^1 \quad \forall j \in \mathcal{S}; \forall t \in \mathcal{T} \tag{11}$$

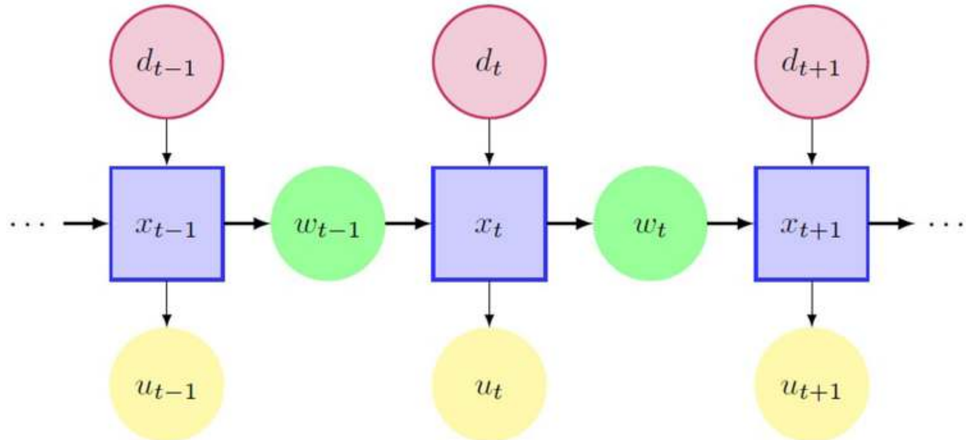


Fig. 4 The relation between the model variables

$$\sum_{i\in\mathcal{G}} \tau_{ij}^2 \sigma_{ij} x_{ijt} \le w_t^2 y_{jt}^2 \quad \forall j \in \mathcal{S}; \forall t \in \mathcal{T} \tag{12}$$

$$\mu_{ij} x_{ijt} \le \xi w_t^3 z_{ijt} \quad \forall i \in \mathcal{G}; \forall j \in \mathcal{S}; \forall t \in \mathcal{T} \tag{13}$$

$$u_{ijt} = 0 \quad \forall i \in \{1,3\}; \forall j \in \mathcal{S}; \forall t \in \mathcal{T} \tag{14}$$

$$w_{ijt} = 0 \quad \forall i \in \{1,3\}; \forall j \in \mathcal{S}; \forall t \in \mathcal{T} \tag{15}$$

$$x_{ijt} \in \mathbb{Z}^+ \quad \forall j \in \mathcal{S}; \forall t \in \mathcal{T} \tag{16}$$

$$y_{jt}^1, y_{jt}^2, y_{jt}^3 \in \mathbb{Z}^+ \quad \forall j \in \mathcal{S}; \forall t \in \mathcal{T} \tag{17}$$

$$u_{ijt} \in \mathbb{Z}^+ \quad \forall i \in \mathcal{G}; \forall j \in \mathcal{S}; \forall t \in \mathcal{T} \tag{18}$$

$$w_{ijt} \in \mathbb{Z}^+ \quad \forall i \in \mathcal{G}; \forall j \in \mathcal{S}; \forall t \in \mathcal{T} \tag{19}$$

Objective function 1 minimises a weighted measure of resource usage including nurses, physicians and beds. The weights associated with these resources are determined by the hospital manager and reflect the significance of each resource, its availability, and the hospital policies. Constraint 2 guarantees that the patients to admit in each (group, specialty, period) triple does not exceed the demand. Constraint 3 guarantees that the total admitted demand is more than a threshold and constraints 4 and 5 ensure that the ratio of unmet and waiting demand are less than certain thresholds. Constraints 6 and 7 deal with the flow conservation constraints (please refer to Figure 4). Constraints 8–10 are the constraints on the number of nurses, physicians and beds, respectively. Constraints 11 and 12 are constraints on the number of nurses and physicians (respectively) required to serve the patients and constraint 13 addresses the bed occupancy ratio. Constraints 14–15 forbid having an unmet demand for A&E and I/NE patients. Finally, constraints 16–19 ensure that all the decision variables take positive integer values.

3 Results and discussion

To evaluate the performance of the mathematical model and its sensitivity to different parameters, we used the data from the hospital. In these experiments, we assumed that a nurse visits a patient four times a day, while a physician visits once during the patient's length of stay (based on our data collection from the hospital). Moreover, physicians work five days a week as opposed to nurses who are available seven days each week. Treatment time by a nurse and a physician in inpatient care is 15 min with the only exception of physicians treatment time for outpatient first visits which takes 45 min. Moreover, a combination of $(\delta^1, \delta^2, \delta^3) = (1, -0.1, -0.2)$ has been used for all the analysis unless mentioned otherwise. All the experiments are coded in Python and solved on a Core i7 laptop with 16 GigaByte memory under Windows 10.

Figure 5 shows the monthly number of clinical slots needed in four of the hospital specialties. One can see that the number of clinical slots needed for some specialties in outpatients (first and follow-up appointments), such as cardiology varies slightly, while the variation for others such as trauma & orthopaedics (T&O) is higher. T&O is related to injuries in accidents and patients can be referred to outpatient services from A&E for further examination. Due to uncertainties in A&E activity, it is not surprising to observe fluctuation in the use of clinical slots for T&O in outpatients and hence, this output is justified.

Figure 6 demonstrates the difference between the average number of beds needed for each speciality and the status quo for a case where 1200 nurse-hours and 600 physician-hours are available with $\theta_1 = 0.95$. The issue with a sub-optimal allocation of beds to specialties is evident in this figure where some specialties such as geriatric medicine are under-capacity, others such as general surgery have far more beds allocated to them than what they actually need. Figure 7 splits the bed requirements into

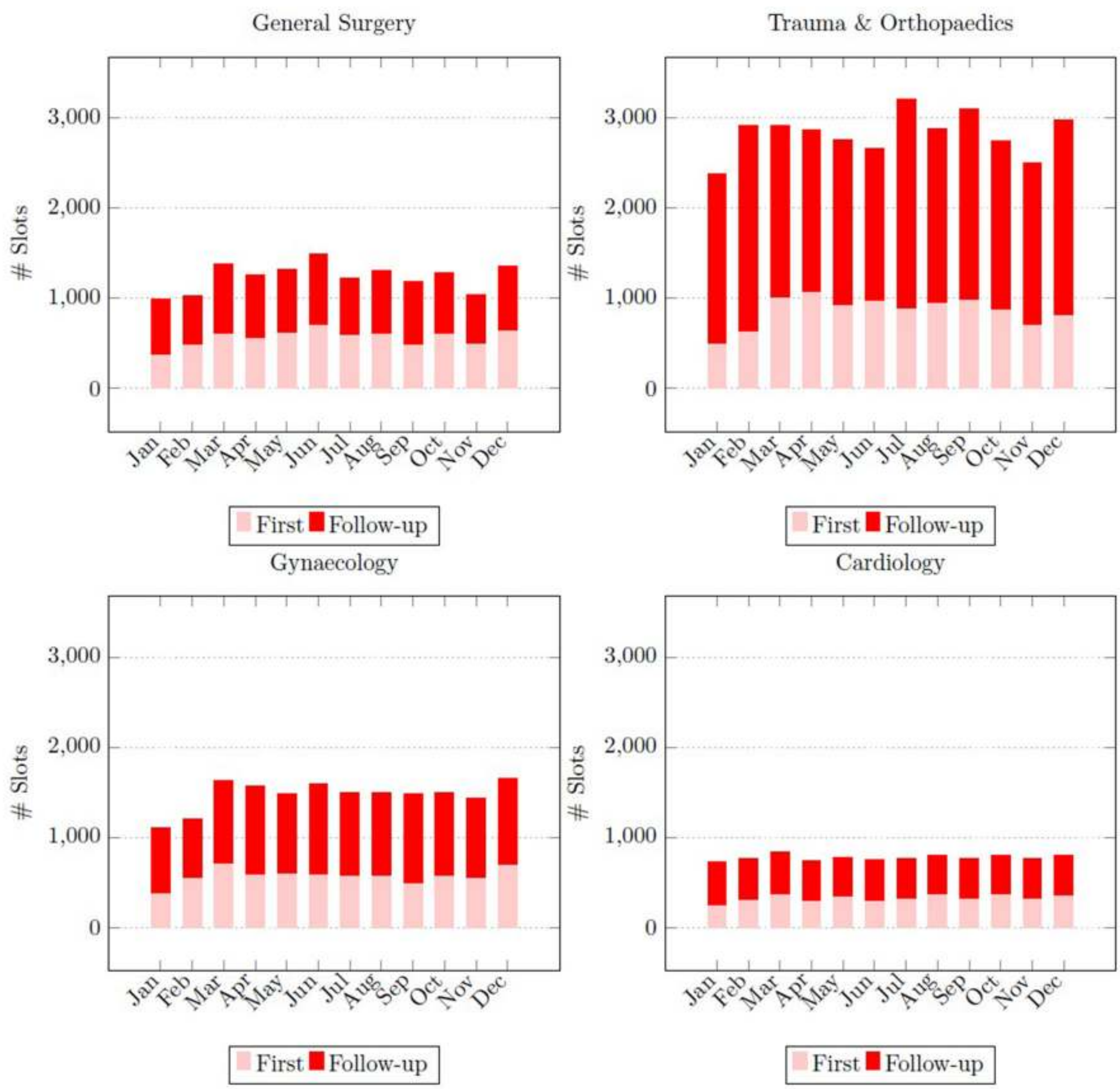
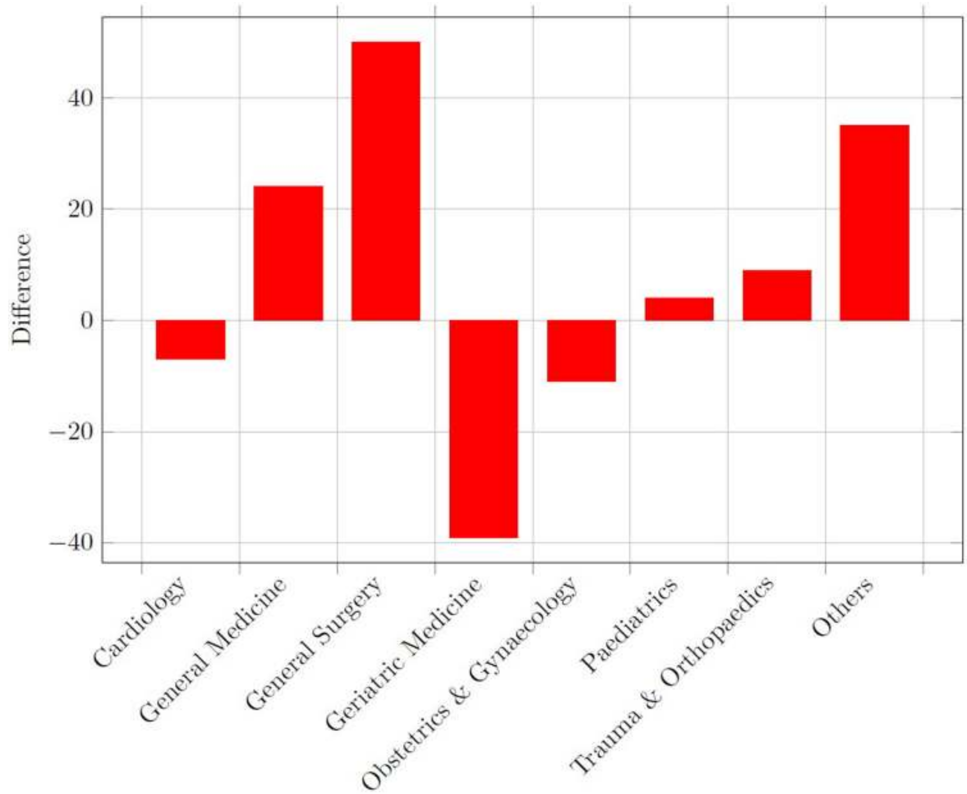


Fig. 5 Clinical slots needed for each period in four specialties

months and depicts the number of beds needed in each specialty when $(\psi^1, \psi^2) = (150, 100)$ with the dashed at line showing the current number of beds in each specialty. Table 8 sets out the total number of beds needed for the two sub-types of inpatients and the total for a selected set of specialties over a 12-month period. This can be a valuable information for the bed management team of a hospital at the individual specialty level. Usually, the bed management teams are only able to calculate the average required number of beds using average length of stay (LoS) and the average number of patients discharged per month. However, both assumptions are invalid as LoS is exponentially distributed and using average numbers is incorrect. Moreover, using average number of discharged patients per month is also misleading as there are variations each month. As a result, the calculated required bed capacity by hospital analysts is inaccurate and unreliable as evidenced in Table 8. This output is of significant value to the hospital manager as this enables them to achieve key performance-related targets, such as referral to treatment, waiting times for elective admissions, and delayed transfers of care.

Figure 8 depicts the number of hours nurses and physicians are needed each month for $\theta_1 = 0.95$ where the seasonality of the resource requirement is clearly demonstrated. Directors of NHS Trusts in England (e.g. Director of Nursing and Director of Workforce) are responsible for planning staff needs according to increasing demand. For example, as part of a Director of Nursing job remit, they are tasked with establishing the number of nurse shifts required each month. In most cases these are calculated using simple averages, whereas using Fig. 8, the optimal number of shifts could easily be derived, a crucial piece of intelligence for efficient running of all hospital services.

Fig. 6 Difference between the current bed allocation and the optimal one for $\theta_1 = 0.95$ with an optimal average of 488 beds needed



Note that a hospital is made up of web of interactions between and within services, and without adequate number of resources (including beds and staff) it is almost impossible to deliver care in an effective manner. In this respect, our model opens flood gates of opportunities for the hospital management to ensure that there are no threats posed to the delivery and quality of care.

Figure 9 demonstrates the sensitivity of the objective function to variations of the weight associated with admissions (δ^1) for three different levels of nurse availability. As expected, there is an almost consistent linear increase in the objective function with an increase in δ^1.

4 Conclusion and future research avenues

Even well beyond the winter pressures, hospitals are forced to deploy extra measures to cope with the increasing demand. Inpatient hospital admissions increased by 20% from 2010/11 to 2019/20 (National Health Services England 2020). Consequently, around 90% of NHS Trusts in England used extra hospital beds to cope with the demand in 2019, and according to British Medical Association, there is little sign that this practice will come to an end, not to mention the devastation caused by the pandemic (BMA 2020). The NHS waiting lists hit a record high of 4.7 million people (O'Down 2021). Similar challenges are faced by most (if not) all hospitals around the world. Coupled with bed shortages, the NHS hospitals has a shortage of nearly 84,000 full time equivalent staff, severely affecting key groups, such as nurses, doctors, health visitors and midwives (The King's Fund 2021).

There is an urgent need by hospital management teams around the world to establish capacity requirements (both beds and staff) for optimal use of scarce resources, which amounts to around 50% of the total NHS budget (£150.4 billion in 2019/20). As a result of this need, we presented a hybrid forecasting-simulation-optimisation model for resource allocation in a hospital setting. We presented the hybrid model and explained the steps taken in developing the model for an NHS Foundation Trust in England. The hybrid model is composed of three fundamental stages, namely forecasting, simulation and optimisation which were used sequentially to propose an annual resource requirement plan for hospital managers. We presented the results of running the hybrid model on the hospital data and examined the sensitivity of the model outputs to its parameters.

Our findings will be immensely useful for day-to-day functioning of a hospital, for example, service managers will be able to schedule staff with confidence (particularly during annual leaves); bed management teams will be able to allocate and re-allocate beds at times of high demand accordingly; the outpatient booking teams will be able to

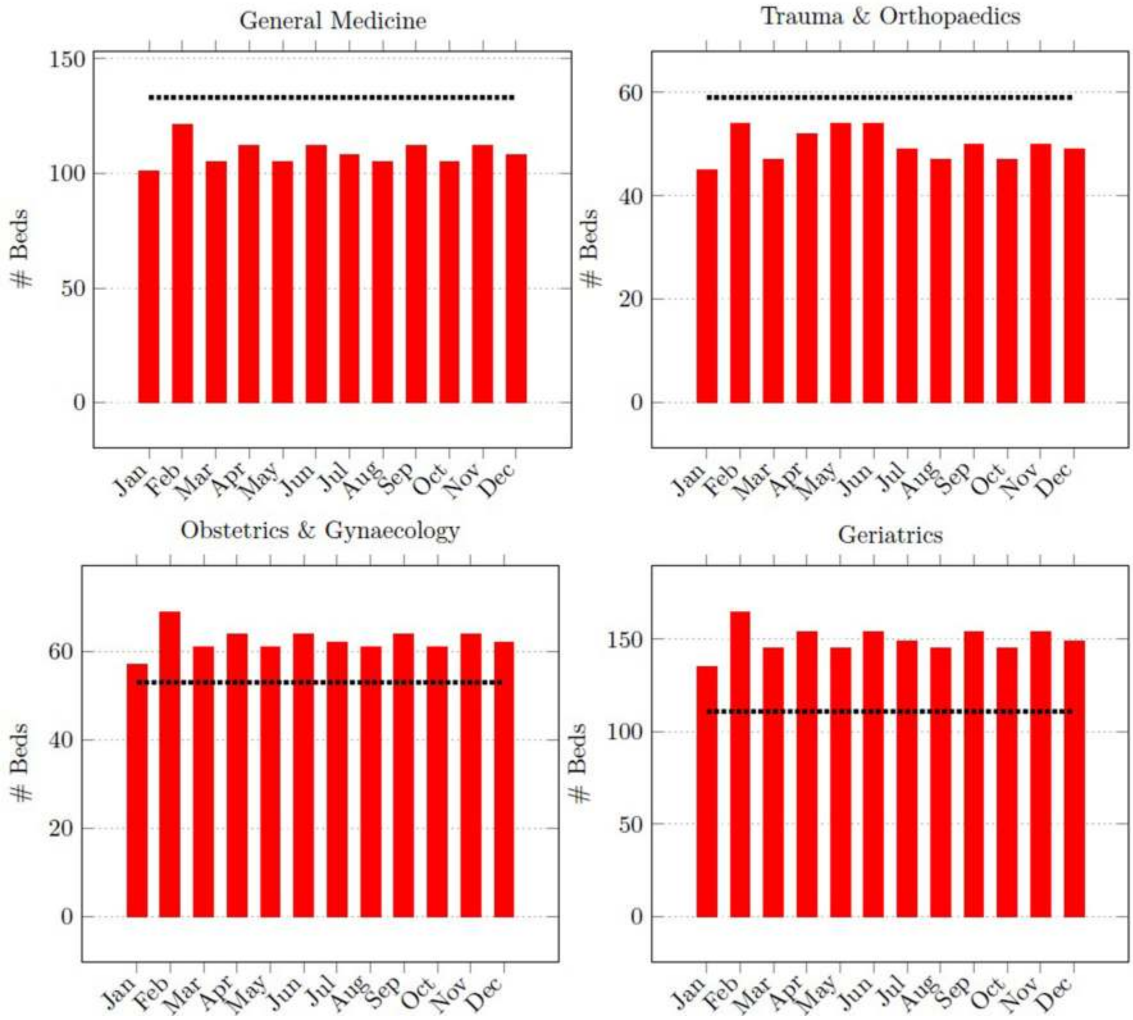


Fig. 7 Number of beds needed each month in four sample specialties for 150 nurses and 100 physicians with the current number of allocated beds in densely dotted at lines

make better use of outpatient consultation rooms (either first or follow-up referrals), particularly for specialties with high referrals at specific months of the year. The top management team will be able to assess whether they have necessary resources in place to ensure key performance indicators are met (as measured by the Department of Health and Social Care in England), such as emergency admissions for acute conditions, and emergency readmissions within 30 days of discharge from hospital. During high demand, high referral periods physicians and nurses feel burnt out due to intense workload. Better planning using the model presented in this study can alleviate these pressures, which will inevitably increase staff morale and have a positive impact on patient satisfaction, thus fewer adverse events. Therefore, the practical benefits of this model spans across the entire health system with the intention of assisting decision-makers for an effective delivery of care at the point of need. The hybrid framework will facilitate the planning of services and speed up the pace of change.

The hybrid modelling framework can be adapted for a wide range of sectors, whether if its retail, manufacturing, tourism or public services. Most systems need to establish optimal resource requirements under scarce resources. The first stage of our modelling framework forecasts demand (e.g. sales). The second stage simulates the system by capturing complexity with all its uncertainties with the aim of experimenting scenarios of interest (e.g. sales under varying economic growths). The final stage then optimises resource requirements possibly with the objective of maximising profit, minimising loss (particularly those in the private sector). Furthermore, the model could easily be extended to examine other parts of the hospital, for instance, theatre utilisation (via the simulation model) and monthly optimal theatre session requirements for each specialty (via the multi-period integer linear

Table 8 Optimal number of beds for each specialty (electives, non-electives and total)

	Month	General Surgery	T&O	General Medicine	Cardiology	Paediatrics	Geriatric Medicine	Obstetrics	Gynaecology	Others	Totals
Total	1	53	45	101	34	11	135	42	15	12	448
	2	64	54	121	41	14	165	51	18	13	541
	3	59	47	105	36	11	145	45	16	12	476
	4	65	52	112	38	12	154	48	16	12	509
	5	61	54	105	30	11	145	45	16	12	479
	6	64	54	112	30	12	154	48	16	12	502
	7	61	49	108	29	12	149	46	16	12	482
	8	62	47	105	28	11	145	45	16	12	471
	9	68	50	112	30	12	154	48	16	12	502
	10	61	47	105	28	11	145	45	16	12	470
	11	62	50	112	30	12	154	48	16	12	496
	12	62	49	108	29	12	149	46	16	12	483
Electives	1	9	19	1	3	1	0	0	4	2	39
	2	10	23	2	3	2	0	0	5	2	47
	3	12	20	2	3	1	0	0	4	2	44
	4	15	21	2	3	1	0	0	4	2	48
	5	14	20	2	3	1	0	0	4	2	46
	6	14	21	2	3	1	0	0	4	2	47
	7	12	21	2	3	1	0	0	4	2	45
	8	15	20	2	3	1	0	0	4	2	47
	9	18	21	2	3	1	0	0	4	2	51
	10	14	20	2	3	1	0	0	4	2	46
	11	12	21	2	3	1	0	0	4	2	45
	12	13	21	2	3	1	0	0	4	2	46
Non-electives	1	44	26	100	31	10	135	42	11	10	409
	2	54	31	119	38	12	165	51	13	11	494
	3	47	27	103	33	10	145	45	12	10	432
	4	50	31	110	35	11	154	48	12	10	461
	5	47	34	103	27	10	145	45	12	10	433
	6	50	33	110	27	11	154	48	12	10	455
	7	49	28	106	26	11	149	46	12	10	437
	8	47	27	103	25	10	145	45	12	10	424
	9	50	29	110	27	11	154	48	12	10	451
	10	47	27	103	25	10	145	45	12	10	424
	11	50	29	110	27	11	154	48	12	10	451
	12	49	28	106	26	11	149	46	12	10	437

programming). Therefore, our modelling framework can be generalised either beyond hospital (healthcare) context or expanded to other parts of the hospital.

No model is ever perfect, and our model is no exception, hence several enhancements can be made on the hybrid forecasting-simulation-optimisation approach. For instance, other forecasting models can be tested, such as singular spectrum analysis, deep learning for time series forecasting and neural networks. The simulation model might include more operational level modelling of some specialties. These specialties are needed to be modelled deeply in terms of improving performance metrics, for example, majority of A&E services in the world have higher patient volume than other specialties. In the hospital studies in the paper, the A&E department is visited by an average of 227 patients per day and this department has a limited number of physicians and nurses. In this situation, additional data might be required to be collected in

Fig. 8 Monthly resource-hours needed

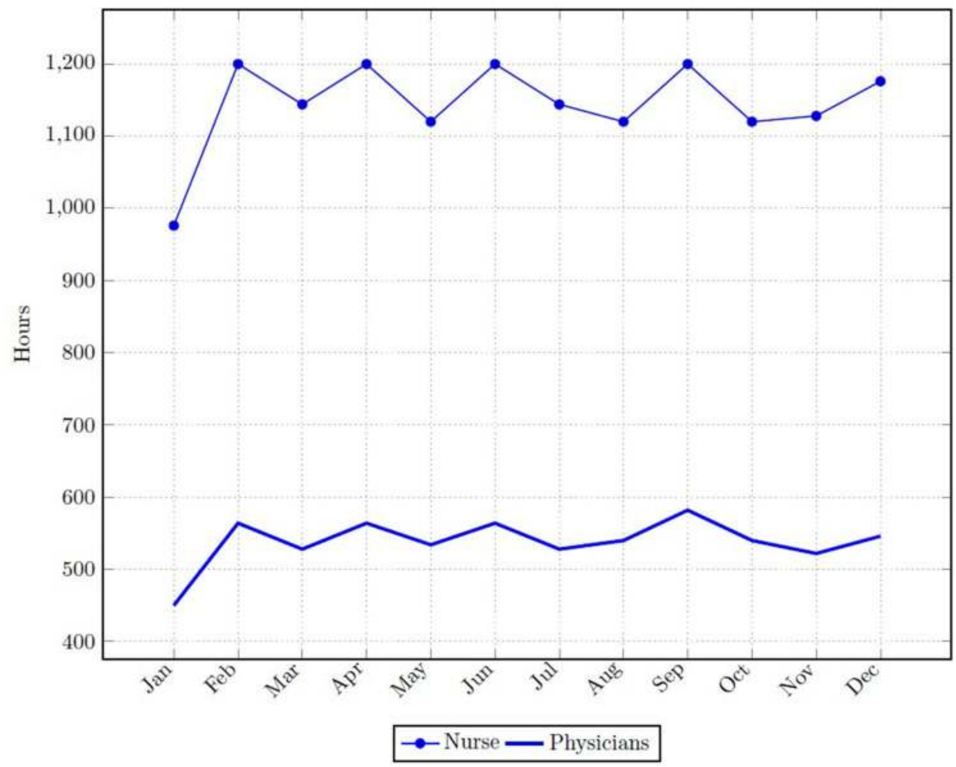
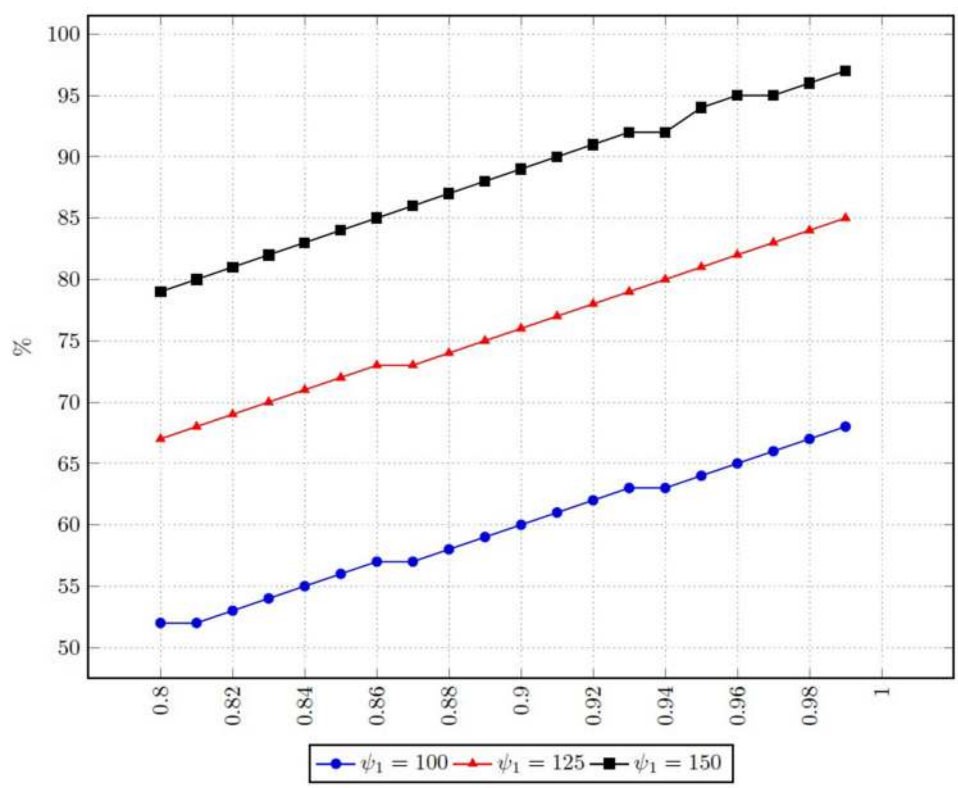


Fig. 9 Sensitivity of the objective function to δ_1 and for three levels of nurse availabilities



addition to available data. Moreover, one can consider a network of hospitals with intra-flow of patients to propose a plan for more than a single hospital. Last but not least, using simulation-based optimisation for our problem can be an interesting avenue for future research. It is indeed a challenge to develop a simulation model when all the phases of simulation modelling are considered. One of the most important needs is data requirements, made up of S input parameters and T statistical distributions, covering the entire hospital, including A&E, inpatient and outpatient services (X specialties, Y departments and Z wards). Secondly, the need for a wider team made up of domain experts. For instance, this research had many professionals involved in the conceptualisation and verification/validation stage of simulation model development from the management team, i.e. Director of Finance, Director of Performance, Director of Transformation, Clinical Director, several consultants and nurses. As part of future research, a decision support tool including a user-friendly interface which consists of control buttons for ease of use with integrated optimisation models will be developed, so that it can be used by key decision-makers (even without a simulation modeller) at the click of a button.

Authorship Contributions M. Ordu designed the structure of the proposed Hybrid Analytical Model. M. Ordu and E. Demir developed the forecasting models and the entire hospital simulation model. All authors developed the mathematical model using multi-period. All authors discussed and wrote the paper together.

Declaration

Conflicts of interest The authors declare that they have no conflict of interest.

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