A Hybrid Computational Tool to Analyze the Performance of Electric Machines with Reduced Content of Permanent Magnet

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***Abstract*—Electric vehicles (EVs) are equipped with interior permanent magnet (IPM) or permanent magnet assisted synchronous reluctance (PM-SynRel) machines because of their superior performance in field weakening region. The effect of saturation is more pronounced in these machines. This prohibits the use of simple analytical tools utilizing constant machine parameters for d and q axis inductance (L_d and L_q) from accurately predicting control variables like current advance angle, terminal voltage, torque, etc. As a result, design and optimization of IPM and PM-SynRel machines are constrained to time consuming transient finite element analysis (FEA) performed at rated operating point. A hybrid method is introduced in this paper to estimate the complete torque speed characteristics of IPM and PM-SynRel machines. The hybrid method works by employing simple analytical formulation alongside static FEA, thereby, reducing the computational time by more than 10 times. This enables optimization of electric machines considering complete torque speed characteristics with reduced computational burden. In addition, the accuracy of hybrid method utilizing static FEA is found to be on par with complete transient FEA analysis.**

***Index Terms*—IPM, PM-SynRel, reduced PM and Static FEA**

I. Introduction

IC engine based conventional drivelines are substituted with electric machines to meet pollution norms set by various governing agencies. These electric machines should inherit high torque density with reduced content of rare earth PM [1]. Reduction in the volume of rare earth PM is necessitated to prevent automotive manufactures from the volatility in its price [2]. Additionally, this has resulted in an extensive search for electric machine topologies with increased torque per magnet volume. Electric machine used in most of the current EVs have IPM with V or triangular structures [3]. This is due to the superior field weakening capability of IPM machines. However, PM-SynRel machines is also known to demonstrates wide field weakening capability and employ reduced volume of PM. PM-SynRel machine with rare earth PM is incorporated in BMW i3 and i8 [4]. Therefore, the future trend is towards increasing the fraction of reluctance torque or reducing the PM mass in electric machines. Consequently, it becomes essential to develop simplistic methods beforehand for the design and analysis of IPM and PM-SynRel with reduced quantity of rare earth PMs.

Permanent magnet synchronous machine (PMSM) is found to deliver poor constant power range despite its superior torque density [5]. The reluctance offered to stator flux is much less in an IPM and PM-SynRel machine compared to a PMSM. This ensures wide field weakening operation without the need for excess inverter capability in an IPM and PM-SynRel machine. However, the effects of saturation becomes more pronounced in these machines [2]. This results in a considerable variation of d axis inductance (L_d) and q axis inductance (L_q). Reluctance component of output torque is altered due to the change in values of L_d and L_q. Therefore, accommodating this variation is crucial during the performance estimation of both IPM and PM-SynRel machines.

The effect of saturation is significant in the field weakening region of IPM machines [6]. Analytical approximations like fixed permeability method is tested to study the saturation characteristics of PM-SynRel machines [7]. Hybrid method comprising both analytical and 2-D FEA is used to account for skew and cross coupling in [8]. Lumped parameter models to analyze IPM machines for traction application is proposed in [9]. However, none of the available literature provides a tool to segregate IPM machines which are influenced heavily by saturation. Parameter estimation is performed either at constant speed or locked rotor operation [6]. Therefore, inductance variation can be estimated using both static and transient FEA analysis. Computation time required for static FEA analysis is much less compared to transient analysis. In contrast, static analysis does not account for the variation in permeance due to stator slotting.

This paper is organized as follows. The modelling of IPM machines is discussed in Section II. The parameter to segregate machines with extensive influence of magnetic saturation is identified in Section III. This is followed by comparing the estimation of L_d and L_q obtained with static and transient FEA analysis for both IPM and PM-SynRel machines in Section IV. Finally, Section V highlights the significance of hybrid

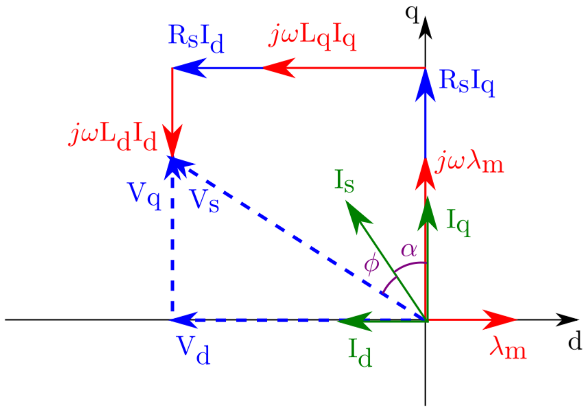


Fig. 1. Phasor diagram for IPM machines

computational tool utilizing static FEA over the conventional methods of analyzing IPM machine.

II. Modelling of IPM machines

Electric machines are generally designed for the rated operating conditions [10]. However, wide field weakening is the major requirement for machines used in EVs. Therefore, analysing the entire torque speed characteristics is crucial to understand the performance. The prediction of torque speed characteristics using commercial software packages is cumbersome and time consuming. Consequently, the mathematical formulation required to estimate the entire torque speed characteristics of electric machines will be discussed in this section.

The phasor diagram in terms of dq components is shown in Fig. 1. The stator of the electric machine has three phase distributed winding. Electromagnetic components of the three phase winding like voltage, current, flux linkage and inductance are used in terms of their dq counterpart for the mathematical formulation [11]. This simplifies control and understanding of the electric machines. d component of the voltage (V_d) is given in (1) while (2) provides the q component (V_q). Here, R_s, I_d, I_q, L_d, L_q, λ_m and ω refers to per phase resistance of stator winding, d-axis current, q-axis current, d-axis inductance, q-axis inductance, PM flux linkage and electrical speed respectively.

$$V_d = R_s I_d + L_d \frac{dI_d}{dt} - \omega L_q I_q \tag{1}$$

$$V_q = R_s I_q + L_q \frac{dI_q}{dt} + \omega L_d I_d + \omega \lambda_m \tag{2}$$

As previously mentioned, obtaining the steady state torque speed envelope is the primary goal. This requires the evaluation of winding parameters in terms of its steady state variables. The dq components of voltage and current are related to steady state phasor of voltage (V_s) and current (I_s) by (3) and (4) respectively. The mechanical shaft torque (T_{em}) and power (P_{out}) are given by (5) and (6), respectively.

$$V_s = \sqrt{V_d^2 + V_q^2} \tag{3}$$

$$I_s = \sqrt{I_d^2 + I_q^2} \tag{4}$$

$$T_{em} = 3\frac{P}{2}\{\lambda_m I_q + \{L_d - L_q\} I_d I_q\} \tag{5}$$

$$P_{out} = T_{em}\omega_{rad} \tag{6}$$

The output torque and terminal voltage are influenced by the value of current components along the d and q axis. Appropriate selection of I_d and I_q is necessary to extract the maximum possible torque and limit the terminal voltage in the field weakening region. The estimation of I_d and I_q is also very essential for controlling the machine. I_d and I_q are related to the stator current as given by (7) and (8) respectively. Conventionally, look up tables are employed due to the heavy saturation present in these machine. Degradation of machine properties with ageing mechanism [12] and continuous change in ambient temperature for EVs demands the estimation of electromagnetic characteristics in real time.

$$I_d = -I_s \cos\alpha \tag{7}$$

$$I_d = I_s \sin\alpha \tag{8}$$

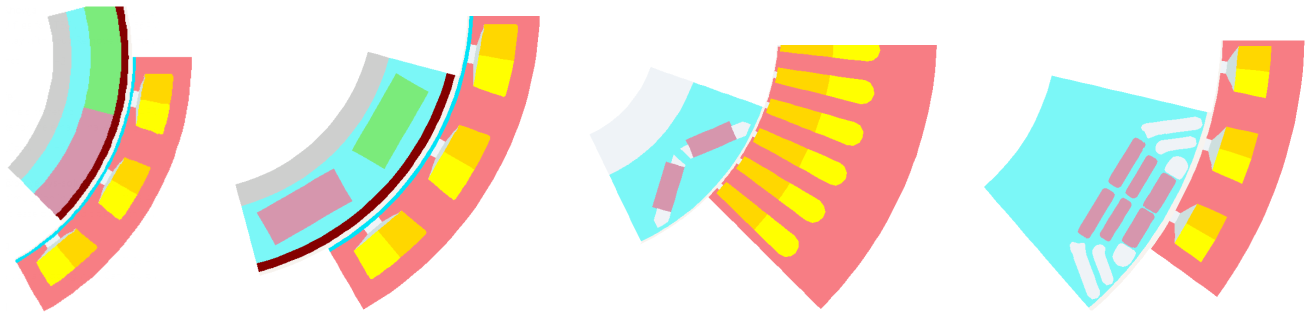
I_q is obtained in terms of I_s and I_d as shown in (9) after rearranging (4). Using binomial expansion, (9) is expanded to obtain the relationship between current component as given by (10) [13]. Steady state terminal voltage is obtained from (3) after eliminating the transient terms present in (1) and (2). Substituting the value of I_q from (10) into the steady state voltage equation results in (11). Finally, a cubic equation in terms of I_d is obtained. Due to low DC link voltage in EVs, the stator current is high unlike conventional electric machines. This results in a high resistance drop irrespective of the low resistance offered by the stator windings. Therefore, (11) is derived by considering the stator resistance.

$$I_q = \sqrt{I_s^2 - I_d^2} = (I_s^2 - I_d^2)^{1/2} \tag{9}$$

$$(I_s^2 - I_d^2)^{1/2} \approx I_s - \frac{1}{2} I_s^{-1} I_d^2 = I_q \tag{10}$$

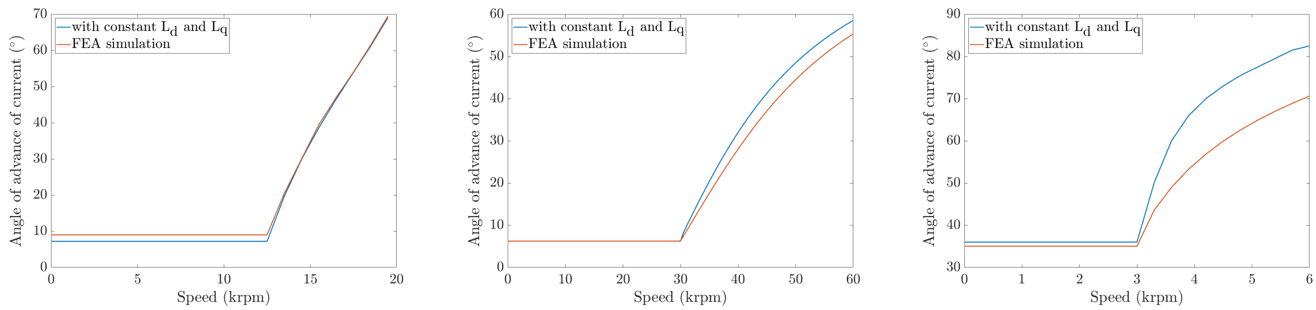
$$\begin{aligned} &\left(-\frac{X_d R_s}{I_s} - \frac{X_q R_s}{I_s} \right) I_d^3 + \left(-\frac{E R_s}{I_s} + X_d^2 - X_q^2 \right) I_d^2 \\ &+ \left(-2EX_d + 2X_d I_s R_s + 2X_q I_s R_s \right) I_d \\ &+ \left(E^2 - V_s^2 + 2ER_s I_s + R_s^2 I_s^2 + X_q^2 I_s^2 \right) = 0 \end{aligned} \tag{11}$$

Above mentioned derivation is based on the assumption that the maximum available stator current is constant. This is extensively valid for EVs where the maximum input current available from the converter is curtailed to a predefined value.



(a) (b) (c) (d)

Fig. 2. Structure of rotor in (a) PMSM (Machine 1) (b) IPM with high PM torque (Machine 2) (c) IPM with high reluctance torque (Machine 3) and (d) PM-SynRel (Machine 4)



(a) (b) (c)

Fig. 3. Current advance angle in (a) PMSM (b) IPM with high PM torque (c) IPM with high reluctance torque

TABLE I
OUTPUT FROM ALL CONFIGURATION

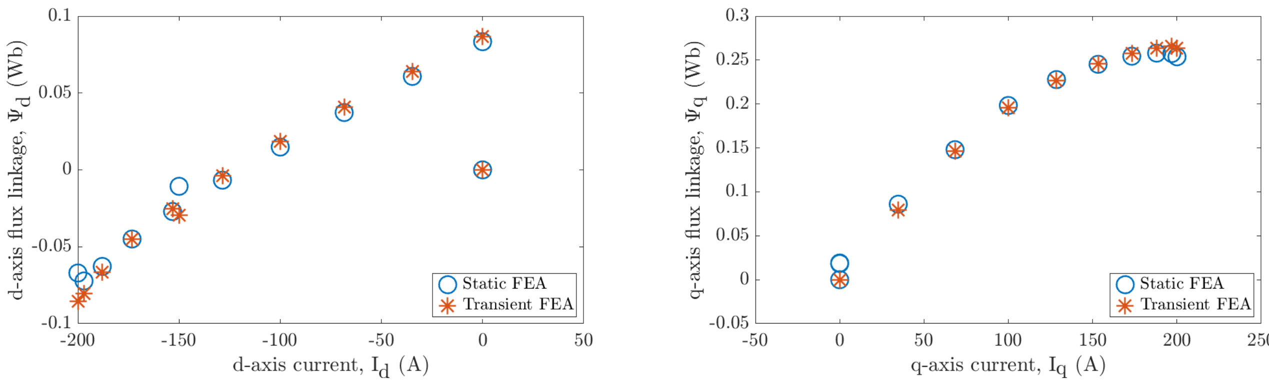
Parameters	Machine 1	Machine 2	Machine 3	Machine 4	Unit
Machine Type	Halbach surface mount PMSM	IPM with interior flat PM	IPM with V shaped barrier	PM-SynRel machine	—
Winding Configuration	Concentrated	Concentrated	Distributed	Distributed	—
DC-Link Voltage	1080	500	600	470	V
Modulation Index	0.944	0.95	0.95	0.95	—
Rated speed	14	30	3	2.8	krpm
Maximum speed	25	60	6	15	krpm
d-axis inductance	0.34	0.32	0.878	0.47	mH
q-axis inductance	0.35	0.36	1.686	0.96	mH
No-load flux linkage	60	12	96	170	mV.s

The hybrid method works by first calculating I_d using (11). All the other values required for solving (11) are computed beforehand. X_d and X_q are highly nonlinear and depend on the operating point of the machine in the torque speed plane. Therefore, these values are computed using static or transient FEA. The significance of static over transient FEA will be highlighted in the subsequent sections. After I_d is obtained, (7) is used to calculate the value of current advance angle.

III. Impact of Increased Reluctance Torque

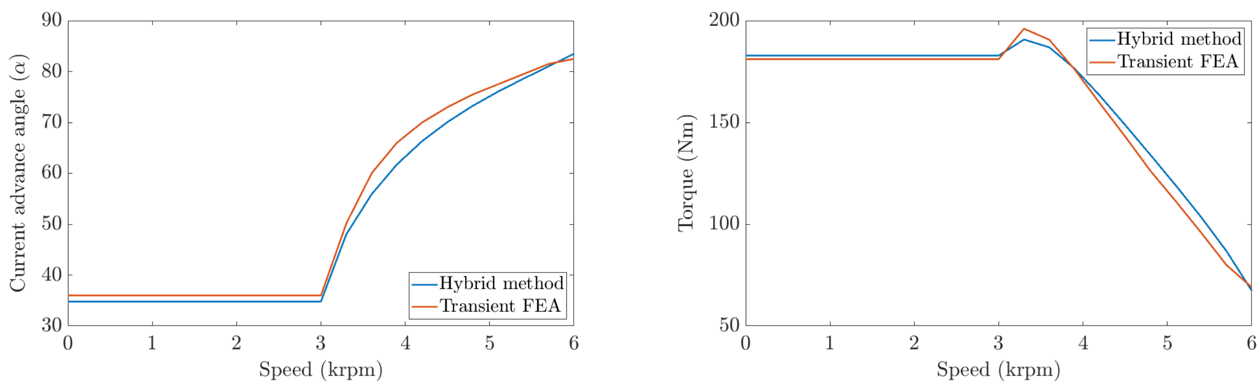
Electric machines with significant fraction of reluctance torque are necessary to reduce PM content. This is enabled by using IPM and PM-SynRel machines. Optimization is the most time consuming and significant phase of electric machine design. Currently, reduction in volume of PM is one of the main constraint imposed during optimization. In addition, wide constant power operation is desired from electric machines installed in EVs. This necessitates all feasible designs of IPM and PM-SynRel machines to satisfy the entire torque speed characteristics. However, heavy computation cost involved in transient FEA analysis makes it undesirable to be used for estimating the complete torque speed characteristics. Consequently, it becomes essential to use simple analytical formulations as described in the previous section. The applicability of these mathematical models on machines with low PM content and need for modifications is presented in this section.

Three different electric machines, namely, machine 1,2 and 3 as shown in Figs. 2(a), 2(b) and 2(c) with parameters listed in Table I and varied PM content are analyzed to understand the significance of saturation. IPM machines with V shaped flux barrier and interior flat magnets are selected to represent electric machine with high reluctance torque and high PM torque, respectively. The current advance angle calculated from analytical and mathematical models using fixed machine



(a) (b)

Fig. 4. Comparison between static and transient FEA for machine 3 based on flux linkage along (a) d-axis and (b) q-axis



(a) (b)

Fig. 5. Comparison between hybrid method and transient FEA for machine 3 using (a) current advance angle and (b) complete torque-speed characteristics

parameters for a PMSM, IPM with high PM content and IPM with low PM content are shown in Figs. 3(a), 3(b) and 3(c) respectively. The fraction of PM torque is 1, 0.7 and 0.4 in these machines. The value of current advance angle calculated using analytical formulation is found to deviate significantly from FEA results with the reduction in volume of PM. The error between FEA and analytical calculation is less than $\pm$ 5% for machines 1 and 2. This is due to the high PM content in these machine compared to machine 3.

The current advance angle calculated with constant L_d and L_q for an IPM with more reluctance torque or machine 3 is considerably away from the actual value as shown in 3(c). The demagnetizing stator flux at rated speed is much less compared to the maximum speed of operation. Therefore, saturation of rotor is high at the rated speed. This results in reduced values for L_d and L_q at rated speed. Based on this, analytical method employing constant inductance is found to use a lesser value for L_d and L_q in the field weakening region. Hence, the results obtained with both methods are found to be very different.

A ratio between no load induced voltage and rated terminal voltage is identified to be used as an indicator for segregating IPM and PM-SynRel machines having high influence of rotor saturation. Based on this, electrical machines with values greater that 0.65 are analyzed with constant machine parameters. In contrast, partial FEA modelling utilizing static or transient FEA in conjunction with simple analytical formulation is necessary to account for the variation of L_d and L_q in electric machines with the ratio less than 0.65.

IV. Comparison between Hybrid Method and Transient FEA

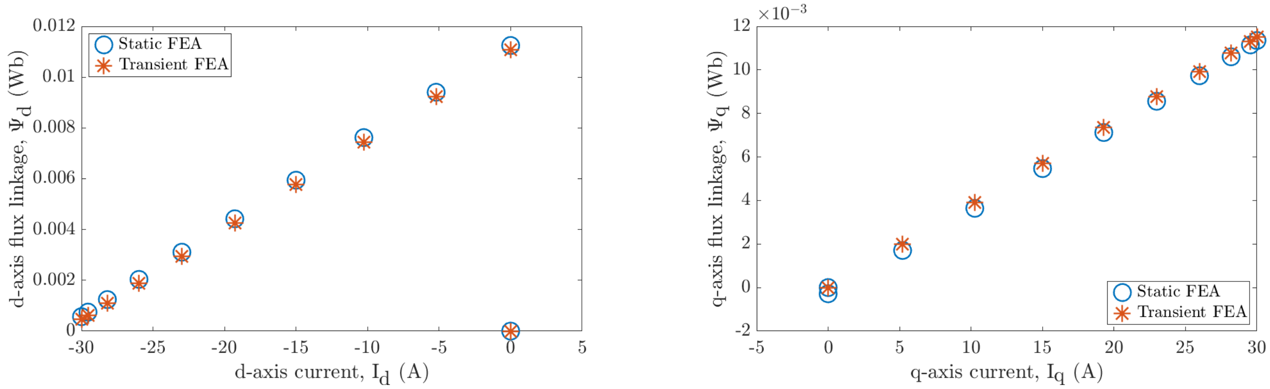
The hybrid method works by utilizing static FEA alongside simple analytical formulation presented in Section II. As previously mentioned, the values of L_d and L_q vary significantly in the field weakening region. L_d and L_q are estimated from their respective flux linkage using (12) and (13). Static FEA is used to calculate the flux linking in stator winding. This is then converted to their dq counterparts by Park's transformation. The application of hybrid method for estimating the performance of IPM and PM-SynRel machines is discussed in this section.

$$L_d = \frac{\lambda_d}{I_d} \tag{12}$$

$$L_q = \frac{\lambda_q - \lambda_m}{I_q} \tag{13}$$

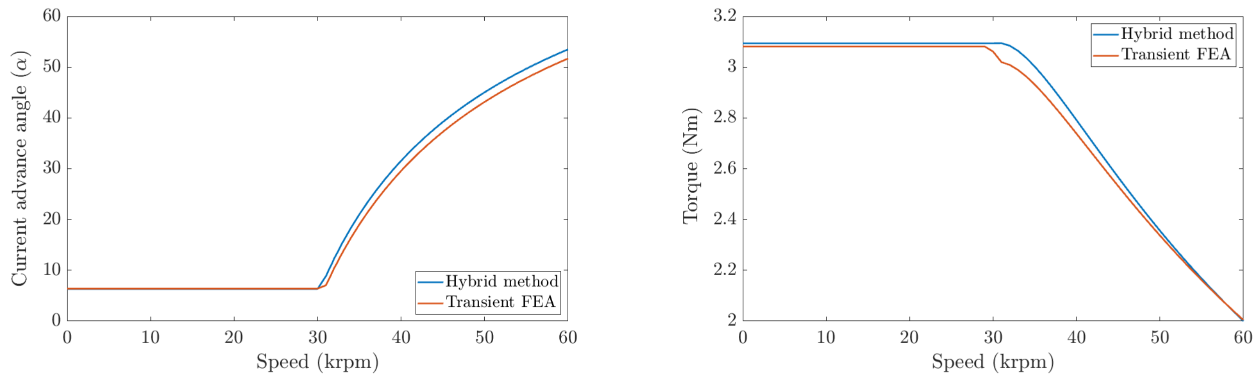
A. IPM machine

d and q axis flux linkage obtained using static and transient FEA analysis for machine 3 are compared in Figs. 4(a) and



(a) (b)

Fig. 6. Comparison between static and transient FEA for machine 2 based on flux linkage along (a) d axis and (b) q axis



(a) (b)

Fig. 7. Comparison between hybrid method and transient FEA for machine 2 using (a) current advance angle and (b) complete torque-speed characteristics

4(b). The results from static and transient FEA are found to match very closely. Therefore, flux linkage calculated from static FEA is sufficient to determine the variation in L_d and L_q using (12) and (13) respectively. The value of current advance angle calculated after incorporating the results of static FEA is shown in Fig. 5(a). Hybrid method is found the calculate current advance angle with higher accuracy as shown in Fig. 5(a) compared to pure analytical formulation given in Fig. 3(c). The torque speed characteristics computed using hybrid method is shown in Fig. 5(b). The error between transient FEA and hybrid calculation is less than $\pm$ 5% in an IPM machine with low PM content. In addition, computation time is reduced by more that 10 times using the hybrid model. This proves the superiority of hybrid model over complete analytical formulation with constant machine parameters and transient FEA for an IPM machine with low PM content.

Hybrid method is also implemented on machine 2 for completeness. The flux linkage obtained using static and transient FEA along d and q axis are found to match very closely as shown in Figs. 6(a) and 6(b). The comparison of current advance angle and torque for IPM with high PM torque are shown in Figs. 7(a) and 7(b) respectively. The error between transient FEA and hybrid calculation is found to be less than $\pm$ 5% even in case of machine 2 with high PM torque.

B. PM-SynRel machine

PM-SynRel machines are the ideal choice to reduce volume of PM and realize wide field weakening operation as desired in EVs. The hybrid method is implemented on a PM-SynRel machine as shown in Fig. 2(d) with rated and maximum speed of 2800 and 15000 rpm respectively. The parameters of the PM-SynRel are given under machine 4 in Table I. The current advance angle and torque speed characteristics computed using the hybrid method with static FEA are compared to the transient FEA analysis in Figs. 8(a) and 8(b) respectively. Unlike the IPM machines, current advance angle and torque are computed at different values of current to encompass the entire torque speed plane for the PM-SynRel machine.

An admirable match is observed between the results of hybrid method and transient FEA as shown in Figs. 8(a) and 8(b) for the entire torque speed plane. However, quantification of these results is essential to understand the impact of variation in stator current on the hybrid formulation. The error between hybrid method and transient FEA is calculated and plotted in Fig. 9. The average error between hybrid method and transient FEA is found to be less than 10 Nm for all values of stator current. In addition, the range over which error in torque varies is found to decrease with the value of stator current. Therefore, it is possible to estimate the electromagnetic performance in all types of electric machine including machines with low PM

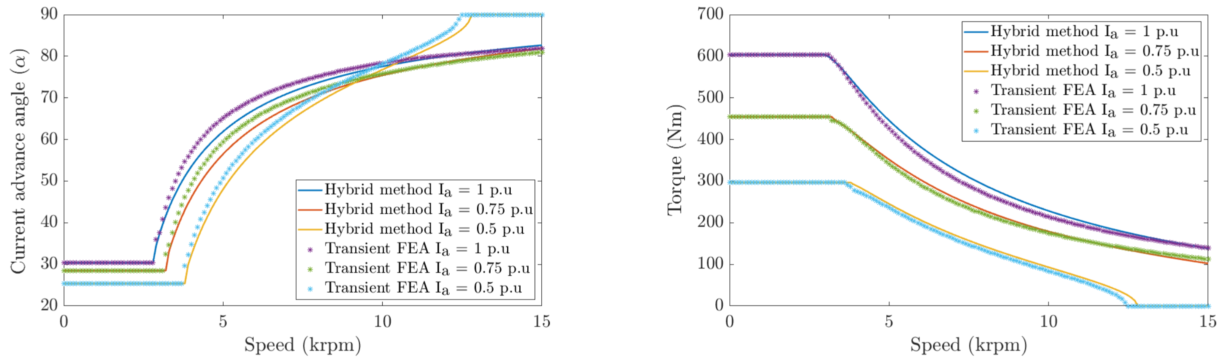


Fig. 8. Comparison between hybrid method and transient FEA for PM-SynRel machine using (a) current advance angle (b) complete torque-speed characteristics

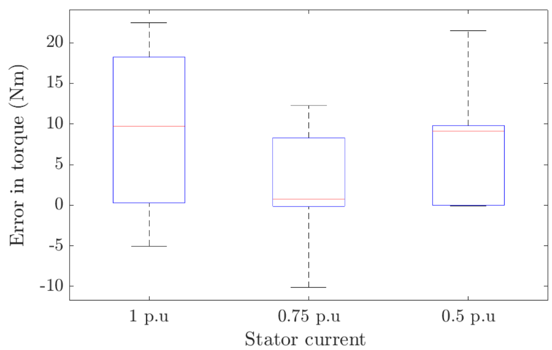


Fig. 9. Error in torque for PM-SynRel machine

torque using the computationally efficient hybrid method.

V. Conclusion

IPM and PM-SynRel machines with reduced content of PM are employed in EVs. The effect of rotor saturation is significant in the field weakening region of these machines. This prohibits simple analytical models from accurately predicting the performance of IPM and PM-SynRel machines. Therefore, a hybrid method utilizing static FEA analysis is cooperation with simple analytical formulation is introduced in this paper. The major outcomes of this are as follows:

- Ratio between no load and rated terminal voltage is identified as a parameter to segregate machines with high influence of rotor saturation like IPM machines with low PM content and PM-SynRel machines.
- The accuracy of hybrid method is on par with time consuming transient FEA.
- Predicting the electromagnetic performance with hybrid method is computationally more efficient and 10 times faster compared to transient FEA.
- It is easier to estimate the entire torque speed characteristics with hybrid method. This extends the scope of optimization which is conventionally based on rated operating conditions.
- Hybrid method is also found to efficiently compute the torque speed characteristics for all value of stator current.

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