A Hybrid Genetic Algorithm for Selective Harmonic Elimination Control of a Multilevel Inverter with Non-Equal DC Sources

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Abstract-- **The paper presents an optimal solution for eliminating pre-specified order of harmonics from a stepped waveform of a multilevel cascaded inverter topology with non-equal dc sources. The main challenge of solving the associated nonlinear equations, which are transcendental in nature and therefore have multiple solutions, is the convergence of the relevant algorithms and therefore an initial point selected considerably close to the exact solution is required. The paper discusses an efficient hybrid real coded genetic algorithm (HRCGA) that reduces significantly the computational burden resulting in fast convergence. An objective function describing a measure of effectiveness of eliminating selected order of harmonics while controlling the fundamental component is derived. Different cases including seven- and nine-level inverters with different values of modulation indices are reported. The theoretical findings are validated through simulation studies.**

I. INTRODUCTION

Multilevel inverters have drawn tremendous interest in the field of high-voltage and high-power applications such as laminators, mills, conveyors, pumps, fans, blowers, compressors, and so on. The term multilevel starts with the introduction of the three-level inverter [1]. By increasing the number of levels in a given topology, the output voltages have more steps generating a staircase waveform, which approaches closely the desired sinusoidal waveform and also has reduced harmonic distortion [1]-[6]. One promising technology to interface battery packs in electric and hybrid electric vehicles are multilevel inverters because of the possibility of high VA rating and low harmonic distortion without the use of a transformer [4].

Various multilevel inverters structures are reported in the technical literature, such as: diode-clamp multilevel inverters (neutral-clamp), capacitor-clamp multilevel inverters (flying capacitor), cascaded multi-cell with separate dc sources and hybrid inverters that are derived from the above mentioned topologies with the aim to reduce the amount of semiconductor elements. However, the cascaded multi-cell inverter appears to be superior to other multilevel inverters for high power rating applications [1]. It is worth noting that in most of the works reported in the technical literature, the level of the dc sources was assumed to be equal and constant, which is probably not be the case in applications even if the sources are nominally equal [4].

A key issue in designing an effective multilevel inverter is to ensure that the total harmonic distortion (THD) of the output voltage waveform is within acceptable limits. Selective harmonic elimination pulse-width modulation (SHE-PWM) has been intensively studied in order to achieve low THD [4]. The common characteristic of the SHE-PWM method is that the waveform analysis is performed using Fourier theory [7]-[11]. Sets of non-linear transcendental equations are then derived, and the solution is obtained using an iterative procedure, mostly by a Newton-Raphson method [3], [9]-[11]. This method is derivative-dependent and may end in local optima; however, a judicious choice of the initial values alone guarantees convergence [6], [7]. Another approach uses Walsh functions [6] where solving linear equations, instead of non-linear transcendental equations, optimizes the switching angles. In references [4] and [5] these transcendental equations are converted into polynomial equations where the resultant theory is applied to determine the switching angles to eliminate specific harmonics. However, as the number of dc sources increases, so does the degree of the polynomials of these equations which also increase the computation difficulty.

In this paper, a multilevel inverter based on the cascaded converter topology with non-equal dc sources is studied (Fig. 1). The main objective of this paper is to introduce a minimization technique assisted with a hybrid genetic algorithm (GA) in order to reduce the computational burden associated with the solution of the nonlinear transcendental equations of the selective harmonic elimination method. An accurate solution is guaranteed even for a number of switching angles that is higher than other techniques would be able to calculate for a given computational effort. Hence, it seems to be a promising method for applications when a high

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number of dc sources are sought in order to eliminate more low-order harmonics to further reduce the THD.

The paper is organized as follows. Section II presents the formulation of the problem along with analysis for the generalized stepped voltage waveform. Section III discusses the implementation of the hybrid genetic algorithm. Results for a number of selected cases are provided in Section IV and finally conclusions are summarized in Section V.

II. PROBLEM FORMULATION AND ANALYSIS

Taking into consideration the waveform characteristics of odd and half-wave symmetry, the Fourier series expansion of the generalized stepped voltage waveform shown in Fig. 2 is given as:

$$V_{out}(\theta) = \sum_{n=1,3,5,\ldots}^{\infty} B_n \sin(n\theta) \quad (1)$$

where B_n is given by:

$$B_n = \sum_{n=1,3,5,\ldots}^{2N-1} \frac{4V_{dc}}{n\pi}(V_1 \cos(n\alpha_1) + V_2 \cos(n\alpha_2) + \cdots + V_M \cos(n\alpha_N)) \quad (2)$$

where,

$n = 1, 3, \ldots, 2N-1$ (odd harmonics only),

N is the number of switching angles per quarter cycle, and

M is the number of dc sources (i.e. inverter cells) and the product $V_M V_{dc}$ is the value of the M^{th} dc source

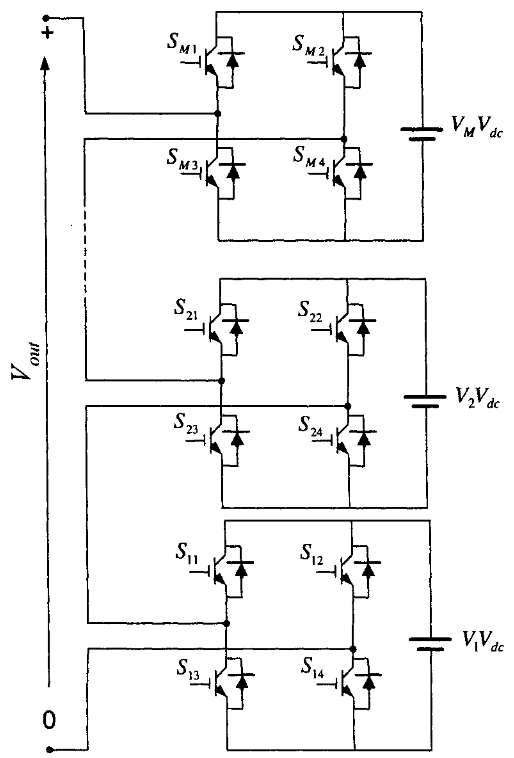

Fig. 1: Single-phase structure of a multilevel cascaded H-bridge based inverter

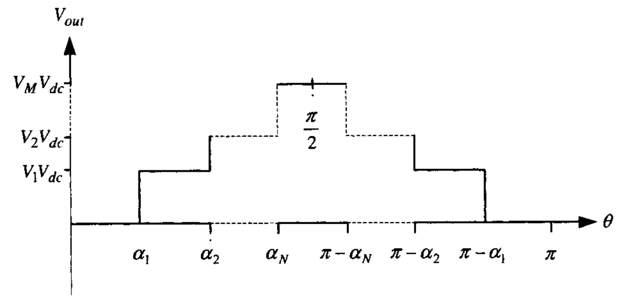

Fig. 2: Generalized stepped voltage waveform

Equation (2) has N variables (i.e. $\alpha_1, \alpha_2, \ldots, \alpha_N$) and a set of solutions is obtainable by equating $N-1$ harmonics to zero and assigning a specific value to the fundamental. Solutions to (2) can be obtained through iterative approaches [3], [9]-[11], resultant theory [4], [5], Walsh function [6] or through the minimization approach [7], [8].

In order to proceed with optimization/minimization an objective function describing a measure of effectiveness of eliminating selected order of harmonics while maintaining the fundamental at pre-specified value must be defined. Hence, the objective function is defined as

$$F(\alpha_1, \alpha_2, \ldots, \alpha_N) = \left(\sum_{k=1}^{N} V_1 \cos(\alpha_k) - A_0\right)^2 + \left(\sum_{k=1}^{N} V_2 \cos(3\alpha_k)\right)^2 + \cdots + \left(\sum_{k=1}^{N} V_M \cos((2N-1)\alpha_k)\right)^2 \quad (3)$$

where,

α_k is the k^{th} switching angle,

$$A_0 = \frac{M\, m_i\, \pi}{4} \quad (4)$$

$$m_i = \frac{H_1}{M\, V_{dc}} \quad (5)$$

noting that m_i is the modulation index, $(0 < m_i < 1)$, and H_1 is the fundamental component.

The optimal switching angles are obtained by minimizing eqn. (3) which is subject to the constraints of eqn. (6) and consequently the selected harmonics are eliminated. These optimal angles are generated for different operating points and then stored in look-up tables to be used to control the inverter for a given operating point.

$$(0 \le \alpha_1 \le \alpha_2 \le \ldots \le \alpha_N \le \frac{\pi}{2}) \quad (6)$$

III. HYBRID GENETIC ALGORITHM IMPLEMENTATION

Genetic algorithms (GAs) are highly suited to search spaces which are not well defined or have a high number of local minima, which plague more traditional calculus-based search methods. By removing the need for auxiliary information regarding the optimization surface the computational requirements are greatly reduced.

The GAs necessitate the need for the optimization variables to be coded as population of strings transformed by three genetic operators: selection, crossover, and mutation.

The presented hybrid genetic algorithm combines a standard real coded GA and the phase-2 of conventional search technique.

A. Phase-1 (Real Coded) Algorithm

The implementation of real coded genetic algorithm is summarized in the following steps:

1- A population of N_P trail solution is initialized. Each solution is taken as a real valued vector with their dimensions corresponding to the number of variables (switching angles). The initial components of α_i are selected in accordance with a uniform distribution ranging between 0 and 1.
2- The fitness score for each solution vector α_i is evaluated, after converting each solution into corresponding switching instants α_a using upper and lower bounds.
3- Roulette wheel based selection method is used to produce N_P offspring from parents.
4- Arithmetic crossover and non-uniform mutation operators are applied to offspring to generate next generation parents.

The algorithm proceeds to step 2, unless the best solution does not change for a pre-specified interval of generations.

B. Phase-2 (Direct Search Optimization Method) Algorithm

After the phase-1 is halted, satisfying the halting condition described in the previous section, optimization by direct search and systematic reduction of the size of search region method is employed in the phase-2. In the light of the solution accuracy, the success rate, and the computation time the best vector obtained from the phase-1 is used as an initial point for the phase-2.

The optimization procedure based on direct search and systematic reduction in search region is found effective in solving various problems in the field of nonlinear programming [12]. This direct search optimization procedure is implemented as follows,

1- The best solution vector obtained from the first phase of the hybrid algorithm is used as an initial point $\alpha(0)$ for phase-2 and an initial range vector is defined as

$$R(0) = RMF \times Range \tag{7}$$

Where, *Range* is defined as the difference between the upper and lower bound, and *RMF* is a range multiplication factor. The value of *RMF* varies between 0.0 and 1.0.

2- N_S trail solution vectors around $\alpha(0)$ are generated using the following relationship:

$$\alpha_i = \alpha(0) + \alpha(0).*rand(1,n) \tag{8}$$

where, α_i is the i^{th} trail solution vector, $(.^*)$ represents element-by-element multiplication operation, and $rand(1,n)$ is a random vector, whose element value varies from -0.5 to 0.5.

3- For each feasible trail solution vector the objective function value and the trail solution set are found, which minimize $F(\alpha)$ and this is equated to $\alpha(0)$ as follows:

$$\alpha(0) = \alpha_{best} \tag{9}$$

where, α_{best} is the trail solution set with minimum $F(\alpha)$.

4- The range is then reduced by an amount given by $R(0) = R(0)*(1-\beta)$, where β is the range reduction factor, with a typical value of 0.05.
5- The algorithm proceeds to step 2, unless the best solution does not change for a pre-specified interval of generations.

IV. RESULTS AND DISCUSSION

A program was developed using the software package MATLAB 6.0 [13] in order to verify the validity of the proposed algorithm. The program is run for a number of independent trials. The proposed technique has been applied to different cases in order to confirm its ruggedness. The simulation results are obtained accordingly using the PSIM software package [14] and a discussion on the results is presented in the following sections.

It should be noted that in the following discussion, the level of the dc sources are non-equal and can be measured, however each dc source has a nominal value of 1 p.u. Furthermore, all the cases presented concern a single-phase system. However, this does not reduce the way the algorithm can be applied in the three-phase system. The location of harmonics to be eliminated vary between the single and three-phase case since the triplen harmonics can be eliminated by the converter structure and there is no need to be included in the elimination process.

A. Case 1: V_1=1p.u., V_2=0.85p.u., V_3=0.75p.u.

The algorithm was used to find the switching angles for the abovementioned case. However, it is worth noting that the solution does exist for a limited range of modulation index, i.e., $0.63 \le m_i \le 0.91$. Fig. 3 illustrates the variation of the switching angles vs. the modulation index. As an example, an operating point when $m_i = 0.87$ is chosen which sets the fundamental voltage to be 2.61 p.u. Fig. 4 shows the spectrum of the output waveform, where it is clear that the targeted low frequency harmonics (3^{rd} and 5^{th}) are eliminated and the fundamental component is equal to 2.61 p.u. as desired.

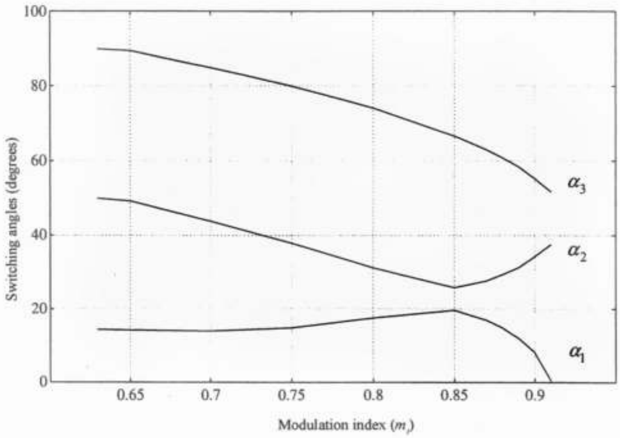

Fig. 3: Switching angles vs. modulation index (*case 1*)

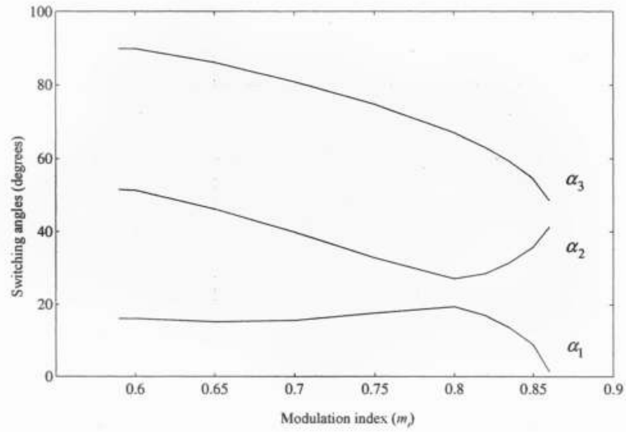

Fig. 5: Switching angles vs. modulation index (*case 2*)

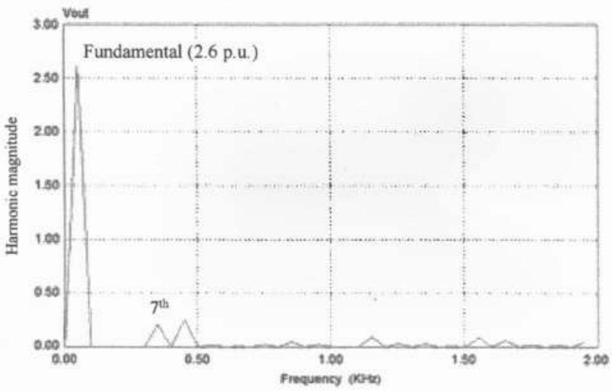

Fig. 4: Spectrum of output voltage (*case 1*)

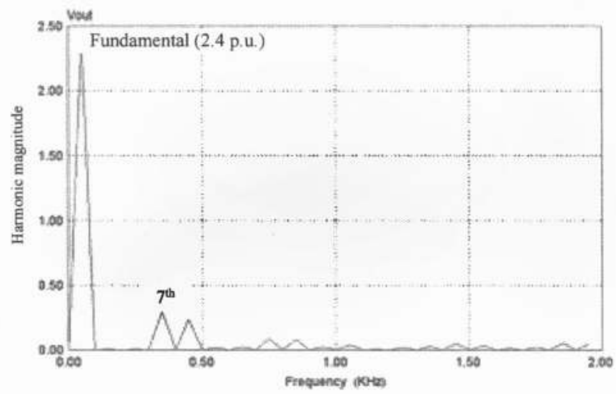

Fig. 6: Spectrum of output voltage (*case 2*)

B. Case 2: V_1=1p.u., V_2=0.75p.u., V_3=0.70p.u.

The proposed technique is applied to minimize the defined cost function for the above stated case. As it can be seen from Fig. 5, the solution exists for the following range of modulation index $0.59 \leq m_i \leq 0.86$. One particular operating point was chosen to demonstrate the effectiveness of the proposed method, i.e., $m_i = 0.80$, which sets the fundamental component to 2.4 p.u.. The spectrum of the output waveform is depicted in Fig. 6 where it is clear that the selected harmonics (3^{rd} and 5^{th}) are totally eliminated and the desired fundamental is achieved.

C. Case 3: V_1=1p.u., V_2=0.96p.u., V_3=0.90p.u., V_4=0.93p.u.

In this case the problem of four dc sources is considered. As such there are four degrees of freedom that offer the elimination of three low order harmonics and maintaining the fundamental at specific value. Fig. 7 shows the switching angles variations against a range of modulation index ($0.71 \leq m_i \leq 0.85$). To validate the effectiveness of the method, the inverter was tested with one operating point that is $m_i = 0.76$ and the spectrum of output voltage is illustrated

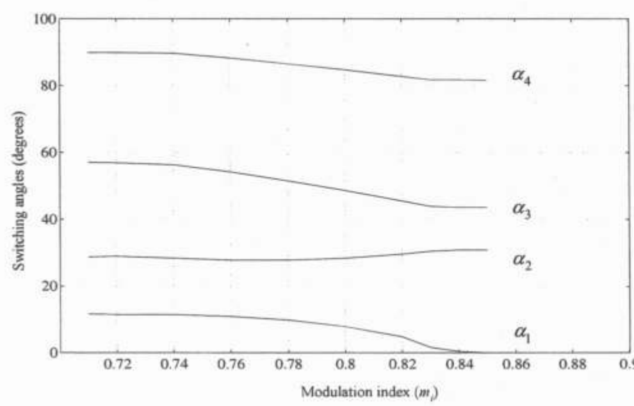

Fig. 7: Switching angles vs. modulation index (*case 3*)

in Fig. 8, where the elimination of targeted harmonics is clearly evident and the fundamental is maintained at 3.04 p.u.. It is worth noting that the next significant harmonics is shifted to the 9^{th} in this case.

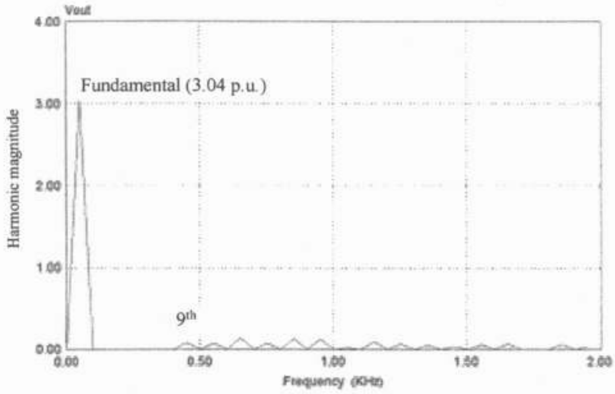

Fig. 8: Spectrum of output voltage (*case 3*)

D. Case 4: V_1=1p.u., V_2=0.95p.u., V_3=0.92p.u., V_4=0.88p.u.

A different combination of dc source level is selected in this case to indicate the feasibility and ruggedness of the proposed technique. The same number of levels as of case 3 was chosen however, the measured dc levels are different. The switching angles are plotted in Fig. 9, where the variation of the angles with a certain range of modulation index ($0.72 \le m_i \le 0.84$) is clearly seen. An operating point of $m_i = 0.80$ was chosen and applied to the inverter to indicate the effectiveness of the proposed algorithm for eliminating selected harmonics (i.e. 3^{rd}, 5^{th} and 7^{th}) while the fundamental is controlled at 3.2 p.u. and the spectrum of the output voltage is shown in Fig. 10.

E. Performance Index

In order to indicate the usefulness and effectiveness of the method, a quality factor is chosen as a performance index. The total harmonic distortion (THD) is very useful parameter to evaluate the performance of the inverter, and therefore the THD is considered in this paper. For practical reasons, the THD is calculated using eqn. (10) and up to the 41^{st} harmonic is taken into account. Typically, the inverter low pass filter eliminates the higher order of harmonics. Furthermore, this factor is calculated for different number of pulses and for all values of the modulation index.

$$THD = \frac{\sqrt{\sum_{n=3,5,\cdots}^{41} V_n^{\ 2}}}{V_1} \tag{10}$$

Fig. 11 depicts the variation of THD with modulation index for various numbers of dc levels.

The conventional optimization technique of Newton-Raphson (NR) is developed and employed to the same operating points/cases and for the same computational efforts. The maximum number of iterations for both techniques is set to 200 iterations and the degree of accuracy or halting conditions is also the same for both methods, i.e., 0.00001. A favorable comparison for the computational times as well as THD was obtained and tabulated in Table 1.

It is clearly evident that the proposed technique is more than two times faster than the conventional Newton-Raphson

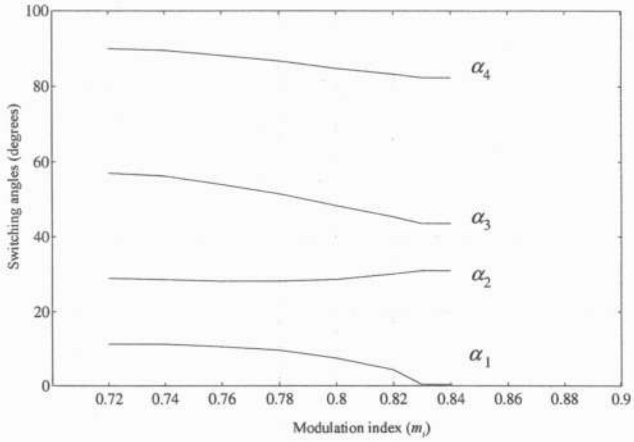

Fig. 9: Switching angles vs. modulation index *(case 4)*

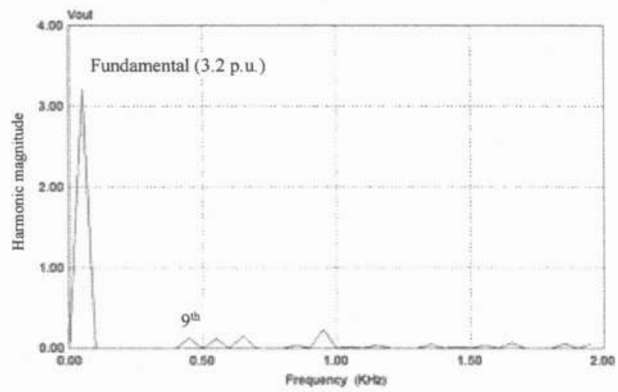

Fig. 10: Spectrum of output voltage (*case4*)

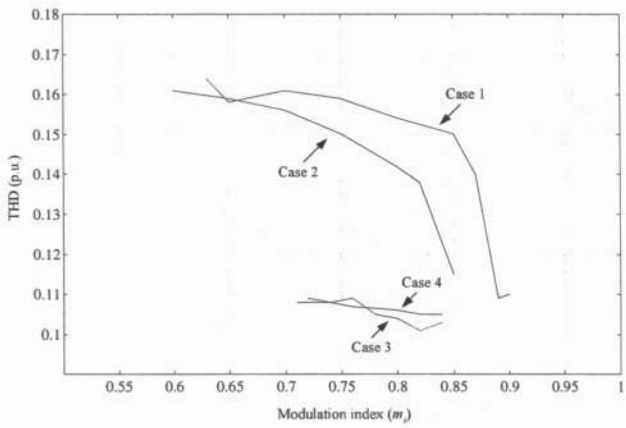

Fig. 11: THD vs. modulation index

method which strongly supports the claim of the superiority of the introduced technique as far as the computational burden is concerned. On the other hand, the THD is same for the first two cases and slightly different for the others which indicate that the proposed technique can provide a different set of solutions as it is clearly observed from the switching angles.

V. CONCLUSION

A method to generate optimal switching angles in order to eliminate a certain order of harmonics is introduced in this paper. A cost function describing the selective harmonic elimination in cascaded multilevel inverter with non-equal dc sources is formulated and addressed. A two phase genetic algorithm, namely the real-coded and direct search is proposed to overcome the computational burden and to ensure the accuracy of the calculated angles. The algorithm was developed using MATLAB software and is run for a number of times independently to ensure the feasibility and the quality of the solution. This method was found to be superior to conventional techniques that may fail to converge if higher pulses per quarter-wave are sought. The algorithm finds the complete set of solutions and confirms that more than one exists as expected. However, only one set of solutions is documented and plotted in this paper. The ruggedness of the method is confirmed with a different cases and operating points including seven- and nine-level inverters and for various modulation indices. As a figure of merit, the THD was used to evaluate the performance of the proposed method and results are plotted for different cases. Furthermore, a computational time-based comparison with conventional optimization technique is provided.

Table 1: Comparison between the proposed technique (HRCGA) and Newton-Raphson (NR)

Operating point/case	Technique/method Proposed technique (HRCGA) Newton-Raphson (NR)	Switching angles per-quarter cycle (degrees)				THD (p.u.)	Computational time (seconds)
		α_1	α_2	α_3	α_4		
Case 1	HRCGA	16.8586	27.5682	62.9737	-	0.147	3.3350
	NR	16.8471	27.6376	63.0600	-	0.147	7.2710
Case 2	HRCGA	19.2150	27.0217	67.0602	-	0.174	3.2150
	NR	19.2270	26.9927	67.0956	-	0.174	7.0400
Case 3	HRCGA	10.7531	28.1350	54.0599	88.1460	0.107	3.4650
	NR	8.9798	27.6707	55.7793	87.3180	0.117	10.8960
Case 4	HRCGA	8.6856	27.8823	49.5169	85.4003	0.113	3.4650
	NR	5.7681	24.8342	53.9078	82.5417	0.136	9.3030

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