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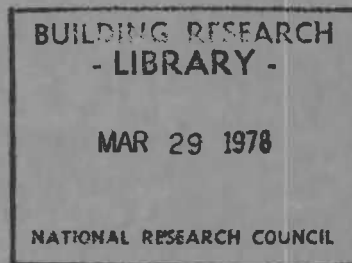
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A LIMIT EQUILIBRIUM ANALYSIS OF PROGRESSIVE FAILURE IN THE STABILITY OF SLOPES

by K. T. Law and P. Lumb

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A limit equilibrium analysis of progressive failure in the stability of slopes¹

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A limit equilibrium method of analysis is proposed for the study of progressive failure in slope stability under a long-term condition. Based on effective stresses, the formulation of the method is derived from consideration of force and moment equilibrium within the soil mass above a prospective slip surface. By dividing the soil mass into a number of vertical slices, recognition of local failure can be made. Once local failure takes place, post-peak strength is assumed to be operative. This then initiates a redistribution of interslice forces and leads to some further local failure. Thus realistic available strengths along the slip surface can be evaluated. This permits the definition of a final safety factor, which is expressed in terms of the actual available reserve of strength. The proposed method has been applied to three well documented case records and encouraging results have been obtained. Based on the assumption that post-peak strengths are given by a friction angle equal to the peak value and a zero cohesion, stability charts have been prepared for design purposes.

Une méthode d'analyse d'équilibre limite est proposée pour l'étude de la rupture progressive dans les problèmes de stabilité à long terme des talus. Basée sur les contraintes effectives, la formulation de la méthode est déduite de considérations d'équilibre des forces et des moments à l'intérieur de la masse de sol au-dessus d'une surface potentielle de rupture. En divisant la masse de sol en un certain nombre de tranches verticales, une rupture locale peut être identifiée. Lorsque cette rupture locale s'est produite, l'on suppose que la résistance post-pic devient opérationnelle. Ce phénomène amorce une redistribution des forces entre les tranches et résulte en une rupture locale additionnelle. Ainsi des valeurs réalistes de résistance le long de la surface de rupture peuvent être évaluées, ce qui permet de définir un facteur de sécurité final en fonction de la résistance réellement disponible en réserve. La méthode proposée a été appliquée à trois cas bien documentés et des résultats encourageants ont été obtenus. En partant de l'hypothèse que la résistance post-pic est donnée par un angle de frottement correspondant au pic et une cohésion nulle, des chartes de stabilité ont été préparées pour fins de calcul.

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Introduction

Limit equilibrium concept is extensively used in conventional stability analyses (e.g. Bishop 1955; Morgenstern and Price 1965; Spencer 1967). This conventional approach assumes a continuous rupture or slip surface along which soil behaves as a rigid plastic material satisfying the Mohr–Coulomb failure criterion. Coupled with other assumptions involving stresses within the soil mass, a solution can be reached. The solution is generally expressed in terms of a safety factor, which is defined as the ratio of the available shear strength of the soil to that required for maintaining equilibrium, and which is assumed to be constant throughout the slip surface.

This conventional method has proved to be of great practical significance in soils exhibiting perfectly plastic behaviour. This is because such soils follow closely the assumptions involved in the method of analysis. In particular, when a slope

failure is imminent, a constant safety factor approximately equal to one is reached throughout the slip surface; this is true even though non-uniform strain may prevail within the soil mass. A number of case histories (e.g. Bishop and Bjerrum 1960) have shown that such soils are amenable to the conventional method of analysis.

The same method, however, has produced misleading results when dealing with soils of a brittle nature (Skempton 1964; Bjerrum 1967). It is now understood that the main reason for the discrepancy can be ascribed to the process of progressive failure. A realistic appraisal of slope stability should, therefore, include the effect of such a process.

A rigorous treatment of progressive failure requires not only a thorough understanding of almost all basic soil behaviour but also a precise knowledge of the initial and long-term conditions in the field. In most instances, however, it is not possible to acquire all the information necessary for a rigorous analysis. In order to obtain a solution, numerical methods, such as the finite element tech-

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nique, have to be used. Although such a complex technique is now available (Lo and Lee 1973), there is still a need for the development of a simple method of solution useful in an ordinary design office. It is for this purpose that the work described in this paper has been carried out.

Method of Analysis

A method involving the division of a slope into slices is used in the present analysis with an assumption of a circular slip surface. Symbols used in this paper are defined in the Notation appendix.

Figure 1a shows a section through a slope of height H and slope β , together with a circular slip surface and a typical slice of mean height h and width b . Figure 1b depicts the various forces acting on a typical slice. These forces are: (i) the weight of the slice W ; (ii) the total normal force at the base; this force is composed of the pore pressure ($ub \sec \alpha$) and the effective pressure P' ; (iii) the interslice forces acting on both sides of the slice; these forces are expressed in terms of vertical and horizontal components; and (iv) the shear force given by the resistance of the soil S .

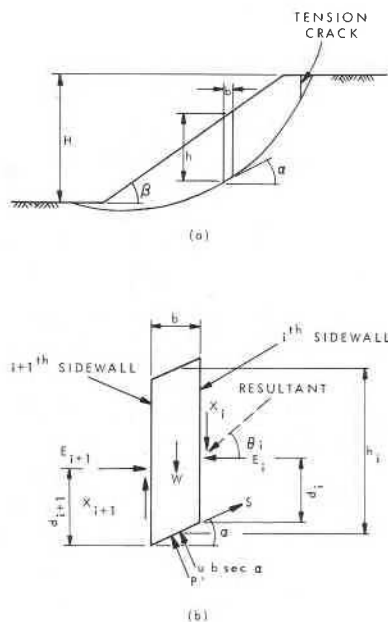


FIG. 1. Dimension of a slip surface and forces on a slice: b = width of the slice; d = vertical distance between the line of application of the interslice force and the base of the slice; E , X = horizontal and vertical components of the interslice force; h = height of the slice; P' , u = effective normal force and pore pressure acting at the base of the slice; S = mobilized shear stress at the base of the slice; W = weight of the slice; α = inclination of the base of the slice; β = slope angle; θ = inclination of the resultant of an interslice force.

Taking moments about the mid-point of the base,

$$[1] \quad E_{i+1} = E_i \frac{d_i + (\tan \alpha - \tan \theta_i)b/2}{d_{i+1} + (\tan \theta_{i+1} - \tan \alpha)b/2}$$

Resolving forces parallel to the base,

$$[2] \quad S = E_i(\cos \alpha + \sin \alpha \tan \theta_i) - E_{i+1}(\cos \alpha + \sin \alpha \tan \theta_{i+1}) + W \sin \alpha$$

The available maximum shear resistance S_p , based on the Coulomb equation, can be obtained by resolving forces vertically. Hence

$$[3] \quad S_p = \{c_p' b + [W(1 - r_u) + (X_i - X_{i+1})] \tan \phi_p'\} / m_{\alpha'}$$

where c_p' , ϕ_p' are the peak values of the cohesion and angle of internal friction of the soil in terms of effective stress; $r_u = ub/W$ (Bishop and Morgenstern 1960) and

$$[4] \quad m_{\alpha'} = \cos \alpha + \tan \phi_p' \sin \alpha$$

By comparing S and S_p , the existence of local failure can be recognized.

Local Failure

Two processes of local failure are considered in this study.

Initiation of Local Failure

This process occurs when at some location, the shear stress due to the effect of gravity and pore pressure has exceeded the maximum available soil strength. This is based on the assumption that, prior to any local failure, the resultant of all the interslice forces acting on a slice is zero. The condition of this type of failure can be expressed as

$$[5] \quad W \sin \alpha > \{c_p' b + W(1 - r_u) \tan \phi_p'\} / m_{\alpha'}$$

Once local failure takes place, the slice is in limiting equilibrium and the stress mobilized at the base is equal to the post-peak resistance of the soil. In this study, it was assumed that immediately after reaching the peak value, the soil resistance would drop abruptly (Fig. 2) to the final post-peak value, S_r . Hence,

$$[6] \quad S_r = \{c_r' b + [W(1 - r_u) + (X_i - X_{i+1})] \tan \phi_r'\} / m_{\alpha'}$$

where

$$[7] \quad m_{\alpha_r'} = \cos \alpha + \tan \phi_r' \sin \alpha$$

Owing to a decrease in resistance in the failed

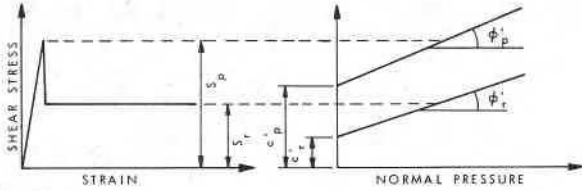


FIG. 2. Shear strength characteristics used in this study.

slice, additional forces are transmitted to neighbouring slices. This leads to the second process of local failure.

Propagation of Local Failure

This process occurs when the maximum available strength of an originally safe slice has been exceeded by the shear stress resulting from the joint effect of gravity, pore pressure and the additional forces produced by failure of nearby slices. In this case, because some primary local failure has taken place, the assumption of zero resultant interslice force is no longer valid. The condition for this type of local failure should, therefore, be written in a more general manner. Hence

$$[8] \quad S > S_p$$

where S and S_p are given by [2] and [3], respectively.

Calculation of Interslice Forces

The procedure to determine the magnitudes of interslice forces comprises four steps.

(1) Slices that fail initially are detected.

(2) The uppermost failed slice is identified and the forces acting on it are calculated. Assuming the soil possesses no tensile strength, a tension crack would be formed along its upslope sidewall. Since the crack is filled with water to a head consistent with the r_u value, the magnitude of the interslice force E_i acting on the upslope sidewall is given by

$$[9] \quad E_i = (r_u \gamma h_i)^2 / 2\gamma_w$$

where γ_w is the density of water.

The line of application of this force is horizontal ($\theta_i = 0$) and is located at d_i above the base given by

$$[10] \quad d_i = \frac{1}{3} r_u h_i \gamma / \gamma_w$$

Bearing in mind that the residual strength is completely mobilized at the base of this slice, one obtains an expression for E_{i+1} by simultaneously solving [2], [6] and [7]. Hence,

$$[11] \quad E_{i+1} = \frac{D_i + E_i \{1 + \tan(\alpha - \phi_r') \tan \theta_i\}}{1 + \tan(\alpha - \phi_r') \tan \theta_{i+1}}$$

where

$$[12] \quad D_i = W(1 - r_u) \tan(\alpha - \phi_r') + W r_u \tan \alpha - \frac{c_r' b \sec^2 \alpha}{1 + \tan \phi_r' \tan \alpha}$$

With two independent equations, [1] and [11], and three unknowns (E_{i+1} , d_{i+1} and θ_{i+1}), the problem is statically indeterminate. To solve it, θ_{i+1} has to be so assumed that the computed E_{i+1} and d_{i+1} should satisfy a set of conditions (Law 1971) for maintaining a physically admissible stress state. The computed quantities are then used to evaluate their counterparts of the next slice.

(3) By working in a downslope direction, the interslice forces on each of the upslope sidewalls can be evaluated one after another. If the slice under consideration has already undergone local failure as determined in step (1), the evaluation of the pertinent quantities is similar to that described in step (2). If the slice is originally safe, the calculation of E_{i+1} is based on [1] with an additional assumption that

$$[13] \quad d_{i+1} = \frac{1}{3} h_{i+1}$$

Again E_{i+1} has to be physically admissible or else θ_{i+1} has to be reassumed. With E_{i+1} and θ_{i+1} known, examination of the possibility of propagation of local failure based on [8] can be carried out. If local failure does occur at this slice, E_{i+1} should be recalculated by solving simultaneously [1] and [11] with an appropriate assumption of θ_{i+1} ; if not, the original E_{i+1} would be retained for estimating the next E value.

(4) Step (3) is terminated when the last slice is reached. If this slice remains safe, progressive failure must have been stopped at some interior point. A final safety factor (to be discussed later) can be computed. If, however, the last slice fails locally, it would produce an unbalanced force. To maintain equilibrium this unbalanced force would be redistributed into the soil mass by recalculating the interslice forces along the slip surface in the upslope direction. Further propagation of local failure would also be examined under the new set of interslice forces. This procedure would be stopped at a slice that remained safe even after the redistribution process. The final safety factor could then be calculated.

It should be noted that step (3) is merely one of the steps in the total computational scheme. It does not mean that local failure must initiate from the top. In many of the cases analyzed, initial local failure detected in step (1) was found to exist both at the top and near the toe. This is in agreement with the work of Bishop (1967) who ex-

amined the distribution of the ratio of the maximum shear stress to the effective normal stress within a slope. The higher this ratio, the greater will be the likelihood of local failure in terms of effective stresses; Bishop also showed that the highest ratio existed at the top and near the toe. The fact that local failure at the top is not observed in the recent finite element analyses (e.g. Dunlop and Duncan 1970; Lo and Lee 1973) is due to one or more of the following reasons: (1) the analysis is based on total analysis, thus disregarding the effective stress state which controls the available strength; (2) the soils near the top are generally more brittle than the soils below. This characteristic, which leads to formation of tension cracks, is seldom included in the analysis; (3) there is no consideration of tension cracks whose existence changes the state of stress.

Final Safety Factor

Because a constant safety factor applied to all slices (as in the conventional limit equilibrium analysis) is not realistic, especially in a slope susceptible to progressive failure, a new definition of safety factor is required. The final safety factor, F_f , as defined in this study, is the ratio of the overall available strength to the actual shear stress required for equilibrium. Hence

$$[14] \quad F_f = \left(\sum_{i=1}^n S_p + \sum_{j=1}^m S_r \right) / \sum_{k=1}^{m+n} S$$

where $\sum_{i=1}^n$ denotes summation over all safe slices of total number n ;

$\sum_{j=1}^m$ summation over all failed slices of total number m ;

$\sum_{k=1}^{m+n}$ summation over the entire slip surface.

By considering equilibrium of the soil mass above the slip surface [14] can be rearranged to give

$$[15] \quad F_f = 1 + \sum_{i=1}^n (S_p - S) / \sum_{i=1}^{m+n} W \sin \alpha$$

The second term of [15] represents the proportion of the actual reserve of strength, which is contributed only by the safe slices. When the shear stress in these slices is equal to the peak shear strength or when $n = 0$, the final safety factor would be equal to 1. When this condition is reached, the soil mass would slide away, breaking all equilibrium conditions derived from statics. It appears inappropriate, therefore, both from a mathematical viewpoint and from practical experience, that the safety factor would fall below 1.

Comparison of Proposed and Conventional Methods

Results of analyses on slopes with varying angle β obtained with the proposed method and that by Bishop and Morgenstern (1960) are presented in Fig. 3. The following data have been used: $r_u = 0.25$; $\phi_p' = \phi_r' = 30^\circ$; $c_p'/\gamma H = 0.05$; $c_r' = 0$.

The method of Bishop and Morgenstern (1960) using peak and post-peak strengths gives two series of safety factors considerably different from each other. The two series of values understandably define the upper and lower limits within which the true safety factor lies. The actual difference between the true value and the limits depends on the magnitude of the effect of progressive failure. With given strength data and r_u value, steeper slopes would result in more extensive zones of local failure (Lo and Lee 1973). Hence the value of the true safety factor would diverge, with increasing slope angles, from the upper limits evaluated using peak strengths. This is precisely the trend predicted by the proposed method. Furthermore, it gives a safety factor significantly higher than that estimated by the conventional method using post-peak strengths. This observation clearly supports the view that calculations based on the post-peak strength generally result in an unnecessarily conservative solution.

Case Records Study

The proposed method was used in the study of

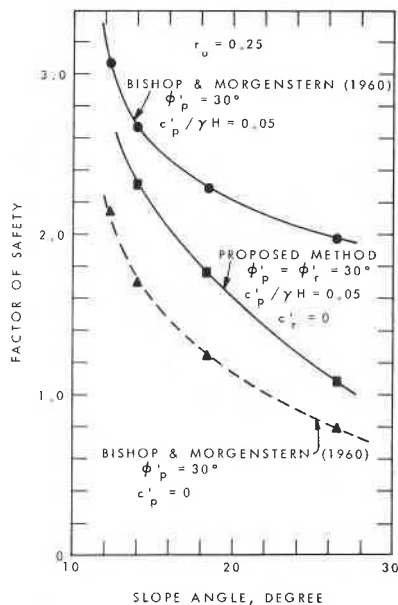


FIG. 3. Comparison of proposed and conventional methods.

three first-time slides, one in a boulder clay, the other two in fissured London clay. The relevant post-peak strength in these cases can be given by the softened strength (Skempton 1970) expressed as $\phi_r' = \phi_p'$, $c_r' = 0$.

Other information pertaining to these records is summarized in Table 1.

Selset (Skempton and Brown 1961)

This slide (Fig. 4a) occurred in a valley slope of the River Lune, near Middleton-in-Teesdale, England. The entire slope was composed of a remarkably uniform intact boulder clay. Sufficient evidence indicated that the failure took place under a long-term condition. With an r_u value of 0.35, for groundwater flow parallel to the slope, the factors of safety from the conventional limit equilibrium method using peak and post-peak strengths are 1.14 and 0.63, respectively.

The application of the proposed method is illustrated in Fig. 4 b and c. The initial assumption of $\tan \theta$ along the slip surface is shown in Fig. 4b. The resulting d/h ratio (Fig. 4c) shows that the line of application of the interslice forces is reasonably close to one-third height. The magnitudes of these forces have also been examined and found to be admissible. With these conditions the proposed method predicts the slope would fail (Table 1).

Northolt (Henkel 1957)

The cutting in London clay at Northolt was first excavated in 1903 and was widened in 1936 with a slope at 2.5:1. The slope, rising from a small toe wall approximately 1 m high (Fig. 5), has a maximum height of 10 m, but the height of the actual slip, which took place in 1955, was only about 6.5 m. With this actual slip height the conventional stability analyses using peak and post-peak strengths give factors of safety of 1.67 and 0.64 respectively.

TABLE 1. Summary of the study of slide case records

	Selset	Northolt	Sudbury
ϕ_p' (deg)	32	20	20
c' (kN/m ²)	8.6	12.0*	12.0*
γ (kN/m ³)	21.8	18.8	18.8
H (m)	12.8	6.5†	7.0
r_u	0.35	0.25	0.30
F_p^\ddagger	1.14	1.67	1.75
F_r^\S	0.63	0.64	0.73
F_t	1.0	1.0	1.0

*From Henkel (1957) and Chandler and Skempton (1974).

†Actual height of the slip.

‡Safety factor from the Bishop simplified method using peak strength.

§Safety factor from Bishop and Morgenstern (1960) using post-peak strength.

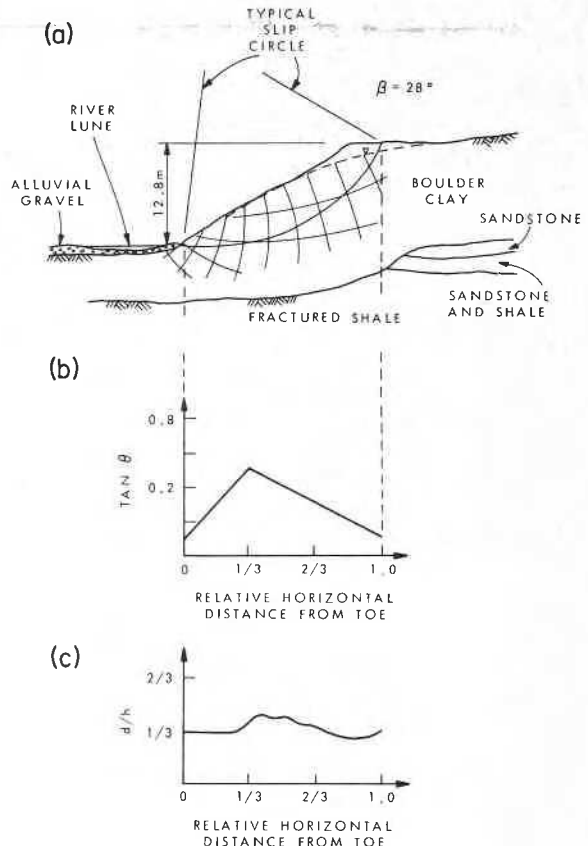


FIG. 4. Analysis of the Selset slide. (a) Slope profile before slide. (b) Assumed $\tan \theta$. (c) Location of line of application of interslice forces.

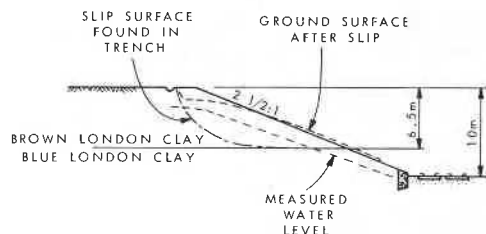


FIG. 5. Section through Northolt cutting (after Henkel 1957).

Neither one of these values is satisfactory in explaining the actual slide. With the same data, however, the slide can be accounted for by progressive failure incorporated in the proposed method (Table 1).

Sudbury Hill (DeLory 1957)

A slide in brown London clay took place in 1949, in a cutting excavated at Sudbury Hill in 1900 (Fig. 6). The post-slide piezometric observation indicated an r_u value in the range of 0.25–0.35. Using the peak strength and the r_u values, the safety

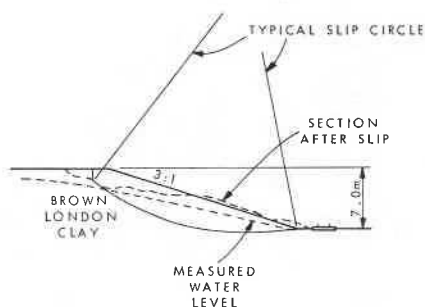


FIG. 6. Sudbury Hill slip in cutting (after DeLory 1957).

factors based on the conventional stability analyses vary from 1.82 to 1.66. These results indicate that the slope should not have failed. The application of the proposed method (Table 1) reveals that a slide would occur at an r_u value exceeding 0.29, a condition consistent with field observations.

Stability Charts

The results of a large number of analyses of slopes under various situations are summarized in the form of stability charts (Figs. 7–11). These charts are so presented that they provide equal convenience either in finding the final safety factor of an existing slope, or in designing a safe slope of given height in a soil of given properties.

The final safety factors given in these charts are obtained using the following data:

- (i) $\phi_r' = \phi_p'$ and $c_r' = 0$.
- (ii) To account for soil variability, $\tan \phi_p'$ and c_p' for each slice are assumed to be two independent statistical variates with coefficients of variation equal to 0.1 and 0.3, respectively. Both variates are described by a Gaussian distribution (Hooper and Butler 1966; Lumb 1966). For each slope, three sets of random Gaussian variates have been employed in the calculation. The average F_t , which is of essentially the same magnitude as that of a deterministic case (Law 1971), has been used in establishing the stability charts.

It has been further assumed that the critical circle for obtaining F_t is identical to that deduced from the Bishop simplified method.

There appear to be some minor irregularities in some of the curves shown in the stability charts. This is caused by a transition of a totally safe slope into one that suffers from local failure. At small inclinations, a slope is free of any local failure, thus giving a curve of a certain curvature. As the slope angle increases, local failure takes place, resulting in a curve of a different curvature. At the transition zone, a minor bend will therefore be noticed.

Example

The Rockcliffe landslide (Eden and Mitchell 1970) is used herein to illustrate the use of stability charts. This slide (Fig. 12) occurred along the south bank of the Ottawa River near Ottawa during the unusually wet spring of 1967. The appropriate strength parameters are $\phi_p' = 45^\circ$, $c_p' = 7.85 \text{ kN/m}^2$ (Jarrett 1970); and $\phi_r' = 43^\circ$, $c_r' = 0$ (Lo and Lee 1974).

Owing to the slight difference in ϕ_p' and ϕ_r' , it is assumed that $\phi_r' = \phi_p' = 43^\circ$. By interpolating between $c/\gamma H$ values, the variation of F_t with r_u is shown in Fig. 13, which reveals a fairly linear dependence. With this plot, the limiting equilibrium condition is found to take place at $r_u = 0.54$. This r_u value is not substantially different from that at the condition of full saturation with groundwater flow parallel to the slope, a state not difficult to attain during a wet season.

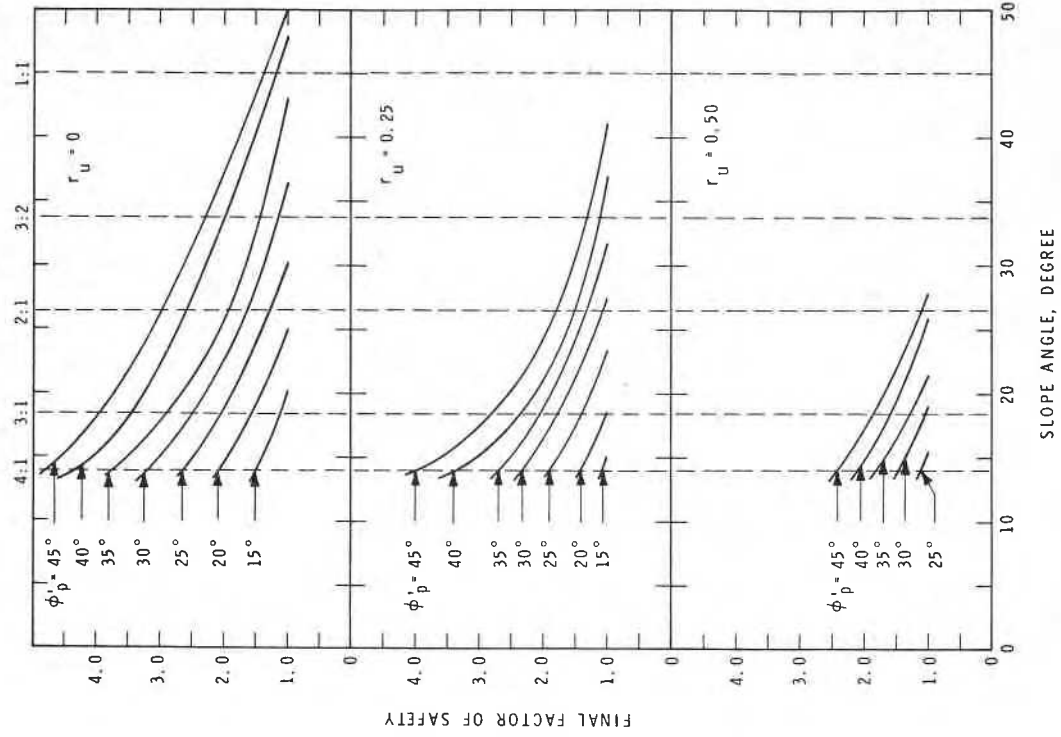
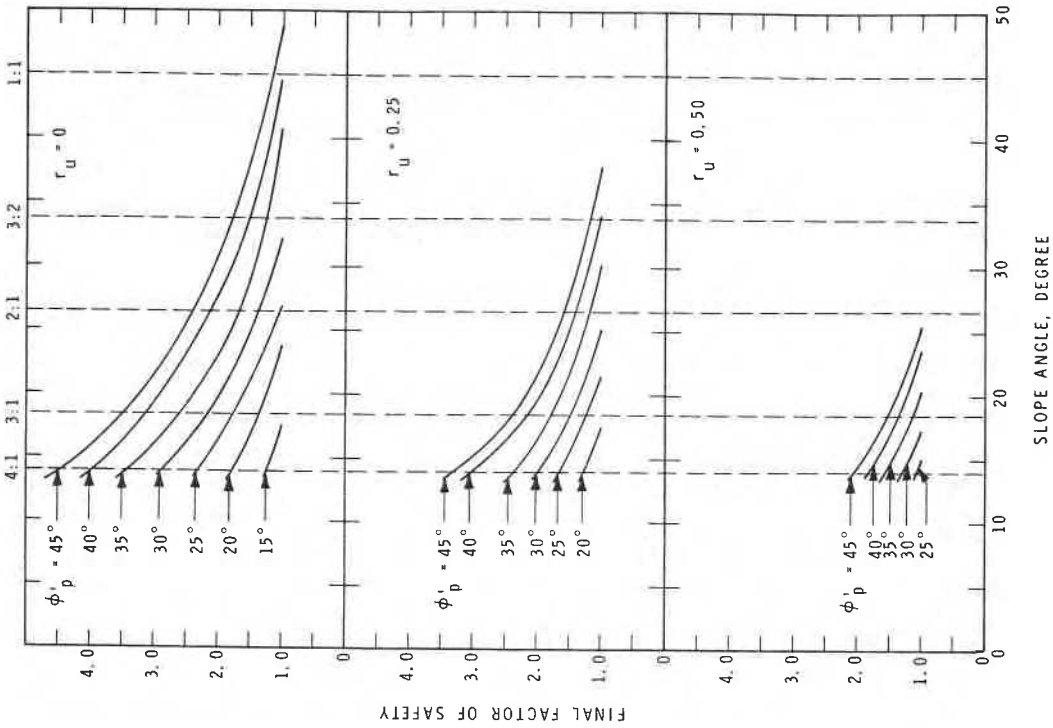
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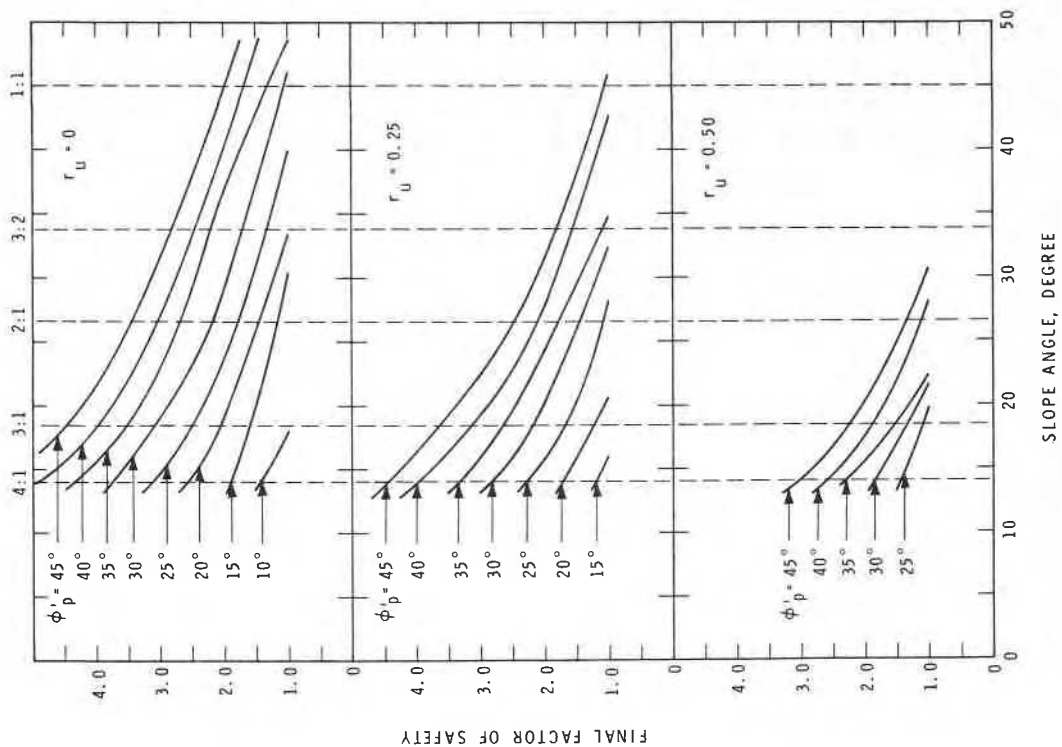
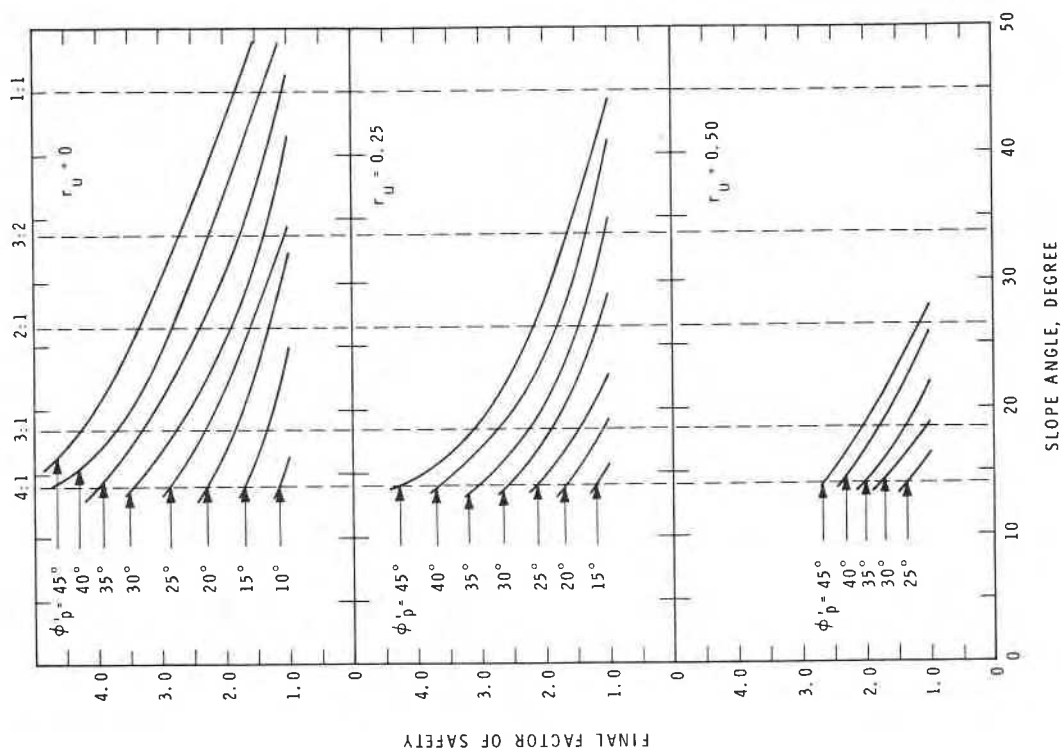
It is recognized that there are soils whose post-peak strengths are not characterised by $c_r' = 0$ and $\phi_r' = \phi_p'$ and therefore the stability charts presented should not be applied to them. Instead, the procedure outlined in the sections on method of analysis through final safety factor should be employed. It is beyond the scope of this paper to furnish stability charts for all cases.

One of the advantages of the proposed method is that a solution can be obtained even when the initial stress state is unknown. The initial stresses existing in a natural slope depend on how and when it was formed. They are also dependent on the various processes operative throughout the geological history of the slope. The present state of the art does not permit a precise evaluation of all these factors. Estimation of the initial stresses is, therefore, a rather dubious exercise; however, an accurate estimation is essential to an adequate assessment of slope stability using the more sophisticated finite element method. The alternative method proposed in this paper bypasses this difficulty and yet produces reasonable results as supported by the analysis of well documented case records.

Summary and Conclusion

A limit equilibrium method has been proposed for analyzing the stability of slopes susceptible to the process of progressive failure. The method is based on the consideration of force and moment equilibrium from which local failure can be detected. Once local failure takes place at a certain point, the post-peak shear resistance is called into play and from there on, that point remains in a

Fig. 8. Stability charts, $c'/\gamma H = 0.05$.Fig. 7. Stability charts, $c'/\gamma H = 0.025$.

Fig. 10. Stability charts, $c'/\gamma H = 0.10$.Fig. 9. Stability charts, $c'/\gamma H = 0.075$.

FINAL FACTOR OF SAFETY

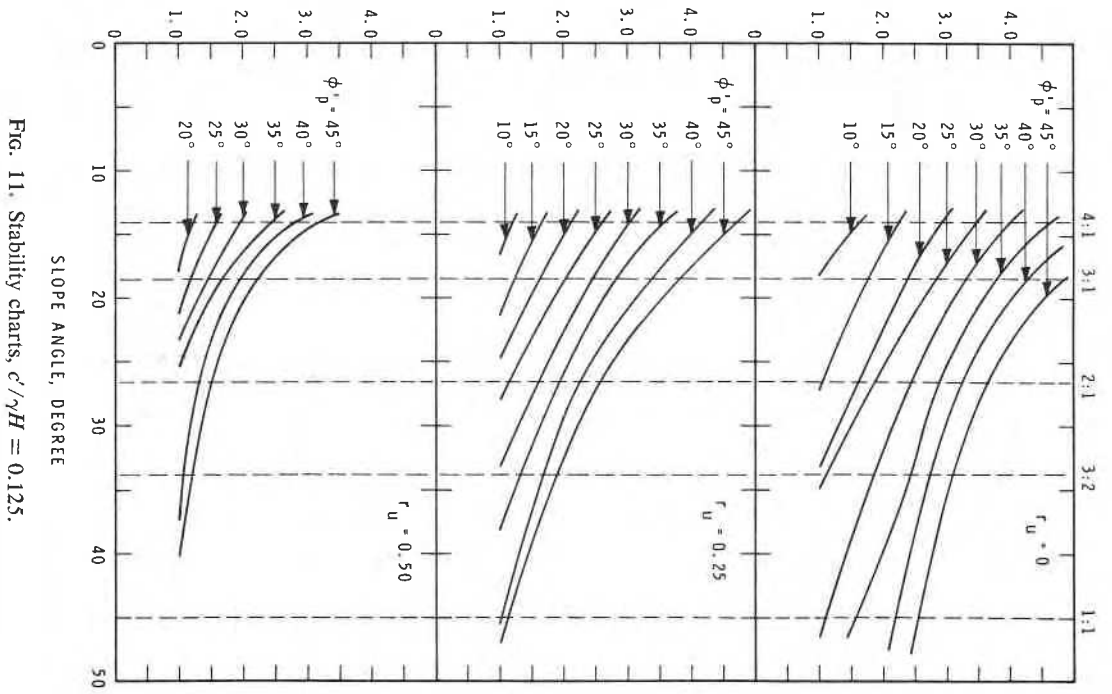


Fig. 11. Stability charts, $c'/\gamma H = 0.125$.

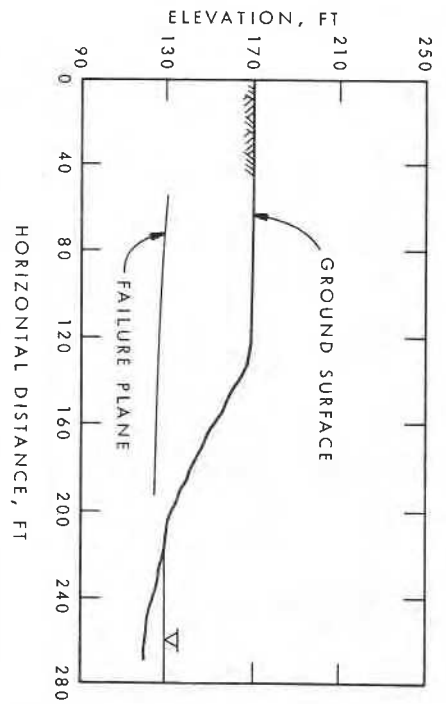


Fig. 12. Slope profile at Rockcliffe landslide (after Eden and Mitchell 1970)

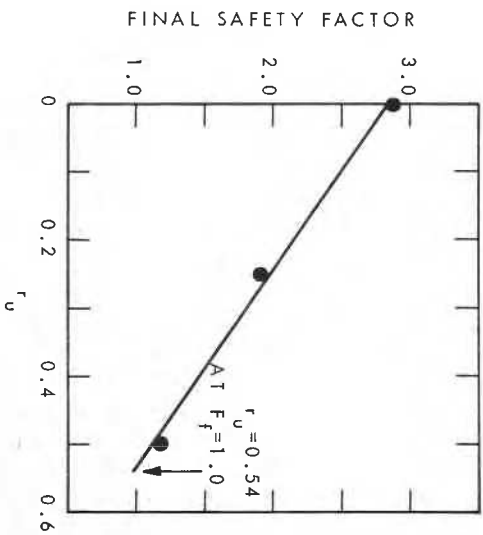


Fig. 13. Variation of final safety factor with pore-water pressure parameter (Rockcliffe landslide).

limiting equilibrium condition. Complete failure of the whole slope, however, will not occur until the limiting condition is reached throughout the entire slip surface. For a stable slope, an overall final factor of safety is determined in terms of the actual available reserve of strength of the soil. This reserve of strength is available only from the portion of soil that has not suffered local failure.

With the proposed method, three well documented case records have been re-examined. Although these records cannot be satisfactorily explained by the conventional limit equilibrium method using either peak or post-peak strength, they can be successfully analyzed by the proposed method.

Employing a post-peak strength characterized by an unchanged angle of internal friction and zero cohesion, a series of stability charts have been established. These charts permit a rapid evaluation of the safety of existing slopes and, at the same time, provide direct information for designing a safe slope for given soil properties and slope height.

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Notation

- b = width of a slice
- c' = effective cohesion
- d = vertical distance between the line of application of an interslice force and the base of the slice
- E, X = horizontal and vertical components of an interslice force
- F_f = final factor of safety
- h = height of a slice
- P', u = effective normal force and pore pressure acting at the base of a slice
- S = mobilized shear stress at the base of a slice
- S_p, S_r = peak and post-peak shear strengths at the base of a slice
- W = weight of a slice
- α = inclination of the base of the slice
- β = slope angle
- γ = bulk density of soil
- γ_w = density of water
- γ_w = density of water
- ϕ' = effective angle of internal friction
- θ = inclination of the resultant of an interslice force
- i = subscript denoting the i th slice or sidewall
- p = subscript denoting peak value
- r = subscript denoting post-peak value