A Linear Contribution Factor Model of Distribution Reliability Indices and Its Applications in Monte Carlo Simulation and Sensitivity Analysis

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***Abstract*—This research work presents a linear contribution factor model (LCFM) between a component's contribution to reliability indices and the failure rate of the component in distribution systems. This linear model can be applied to risk analysis (based on Monte Carlo simulation) and sensitivity analysis. Traditional approaches for both analyses require many repetitions of reliability index assessment (RIA). If this linear model is applied to either risk analysis or sensitivity analysis, only one RIA is needed and the computation is tremendously simplified.**

***Index Terms*—Distribution systems, Monte Carlo simulation, reliability index, risk analysis, sensitivity analysis.**

I. Reliability Index Assessment

Reliability indices are measures of distribution system reliability. They are broadly adopted by many regulatory boards and utilities. These indices include SAIFI, SAIDI, $\mathrm{MAIFI_E}$, MAIFI, CAIDI, and others. This research work uses SAIFI, SAIDI, and $\mathrm{MAIFI_E}$ for illustrative purpose. Generally, the same approach can be applied to the other indices.

Reliability index assessment (RIA) is time-consuming. Fig. 1 shows the total calculation time of reliability indices SAIFI, SAIDI, and $\mathrm{MAIFI_E}$, with respect to system sizes. The quadratic-like curve in the figure not only shows RIA is time-consuming, but also implies that the calculation time may increase considerably fast if larger systems are applied in the future. Next, the reason of this quadratic-like relation is briefly discussed.

RIA takes n steps for a system consisting of n components [1], [2]. Within each step, a searching scheme is employed to identify all affected upstream and downstream components due to the failure of an individual component. For all affected components, impacts are from protection system response, upstream restoration, downstream restoration, etc. [2]. The number of affected components within each step may be roughly related to n. In other words, there may be kn components affected, where k is a factor between 0 and 1. For example, in a system with 100 components, to evaluate the impact of the failure of one component, ten components are affected. For a similar, but larger system with 500 components, 50 components may be affected since the "tree" may be deeper. That is, each feeder or lateral may have more components. In both cases, k equals $0.1 (= 10/100 = 50/500)$. Since each step in RIA takes kn substeps, a full RIA should take kn^2 steps to complete.

Some high-level analyses such as risk analysis (based on Monte Carlo simulation) [2], [3] and sensitivity analysis [2], [4] usually are based on RIA. Traditional approaches may require many iterations of RIA, and therefore, are very time-consuming. This research work presents a linear contribution factor model (LCFM) that can be applied to simplify Monte Carlo simulation and sensitivity analysis. With the LCFM, both analyses can be tremendously expedited, since the simplified approaches require only one full RIA and all the remaining work can be done in just a one-step summation.

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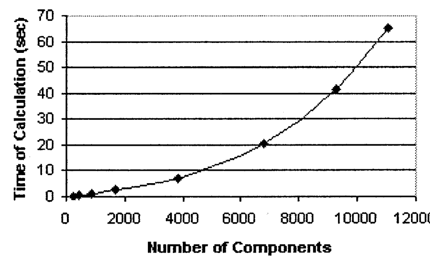

Fig. 1. Running time of reliability index assessment.

II. Linear Contribution Factor Model of Reliability Indices

SAIFI, SAIDI, or $\mathrm{MAIFI_E}$ can be written as the sum of contributions from each individual component to SAIFI, SAIDI, or $\mathrm{MAIFI_E}$, respectively. This is given as follows:

$$SAIFI = \Sigma(SAIFI_i^C) \tag{1}$$

$$SAIDI = \Sigma(SAIDI_i^C) \tag{2}$$

$$MAIFI_{\mathrm{E}} = \Sigma(MAIFI_{\mathrm{E}\,i}^C) \tag{3}$$

where

- $SAIFI_i^C$ contribution to SAIFI from component i;
- $SAIDI_i^C$ contribution to SAIDI from component i;
- $MAIFI_{\mathrm{E}\,i}^C$ contribution to $MAIFI_{\mathrm{E}}$ from component i.

Next, $SAIFI_i^C$, $SAIDI_i^C$, and $MAIFI_{\mathrm{E}\,i}^C$ can be written as

$$SAIFI_i^C = \lambda_i \cdot \frac{S_i}{n} \tag{4}$$

$$SAIDI_i^C = \lambda_i \cdot \frac{D_i}{n} = \lambda_i \cdot \frac{\left(\sum_{j=1}^{S_i} d_{ij}\right)}{n} \tag{5}$$

$$MAIFI_{\mathrm{E}\,i}^C = \lambda_i \cdot \frac{T_i}{n} \tag{6}$$

where

- λ_i failure rate of component i;
- S_i number of customers experiencing sustained interruption due to a failure of component i;
- D_i sum of customer interruption durations due to a failure of component i;

i	S_i/n
1	0.0063
2	0.0108
3	0.0042
…	…
n	0.0051

(A)

i \ j	1	2	3	…	n
1	NA	0.0142	0.0082	…	0.0095
2	0.0142	NA	0.0150	…	0.0159
3	0.0082	0.0150	NA	…	0.0051
…	…	…	…	NA	…
n	0.0095	0.0159	0.0051	…	NA

(B)

Fig. 2. Vector and matrix for $\mathrm{N}-1$ and $\mathrm{N}-2$ contingencies.

d_{ij} interruption duration for customer j due to a failure of component i, $j=1,2,\ldots,S_i$;

T_i number of customers experiencing temporary interruption (event) due to a failure of component i;

n total number of customers.

This work has the following assumptions

1) mean time to repair (MTTR) and mean time to switch (MTTS) of all components remain unchanged;
2) the system configuration is not changed.

First, consider $\mathrm{N}-1$ contingencies (i.e., only one component fails at a time). With the above assumptions, S_i, D_i, and T_i for each component are constant. S_i, D_i, and T_i can be obtained with a similar computational approach discussed in the previous work [2] to calculate SAIFI, SAIDI, and $\mathrm{MAIFI_E}$. Therefore, for component i, $SAIFI_i^C$, $SAIDI_i^C$, and $MAIFI_{\mathrm{E}}^C{}_i$ are linearly related to the failure rate of this component. The contribution factors are (S_i/n), (D_i/n), and (T_i/n), respectively. A vector can be created to represent the contribution factor for each index. Fig. 2(A) shows the vector of (S_i/n). Each entry in the vector corresponds to the (S_i/n) of a component.

Next, consider $\mathrm{N}-2$ contingencies. With the assumption previously mentioned, a similar linear relation can be obtained as well. For simultaneous failures of component i and j, $SAIFI_{ij}^C$ is linearly related to the product of λ_i and λ_j. In this case, a matrix can be created to hold the contribution of SAIFI from component i and j, as shown in Fig. 2(B). This matrix is symmetric without diagonal entries. As such, only the upper or lower triangle part needs to be stored to save space. Higher order contingencies can be implemented with multidimensional arrays, though they are difficult to visualize. Also, similar vectors and matrices can be obtained for SAIDI and $\mathrm{MAIFI_E}$.

This linear contribution factor model (LCFM) can be applied to simplify risk analysis (based on Monte Carlo simulation) and sensitivity analysis, since both respect the two assumptions previously mentioned. In the following sections, the simplified approaches for Monte Carlo simulation and sensitivity analysis are presented with the consideration of $\mathrm{N}-1$ contingencies. Also, only SAIFI is considered for simplicity. Similar approaches can be easily applied to $\mathrm{N}-2$ contingencies and other reliability indices like SAIDI and $\mathrm{MAIFI_E}$.

III. Application in Monte Carlo Simulation

Risk analysis [2], [3] is a commonly used high-level, reliability-based analysis, which identifies the risk of poor reliability. Since analytical methods such as functional convolution usually are not feasible, nonsequential Monte Carlo simulation is commonly employed. The simulation starts with the identification of the constant failure rate λ of each component. Since a component with a constant failure rate follows a Poisson process [2], [3], the probability of its failing i times in a specific year $p(i)$ and the associated density function $P(i)$ are given by

$$p(i) = \lambda^i \cdot \frac{e^{-\lambda}}{i!} \tag{7}$$

$$P(i) = \sum_{k=0}^{i} p(k). \tag{8}$$

Next, a random number r is generated in the range of [0,1] to determine the actual number of failures (ANF) during a specific year. For example, given $\lambda = 0.3$, the ANF is 0 if r is in [0, $0.7408 = \mathrm{P}(0)$], 1 if r is in (0.7408, $0.9631 = \mathrm{P}(1)$], and so on. Once the ANF for each component is obtained for a sample year, a RIA is carried out to get SAIFI.

To obtain a precise result for risk analysis, a large number of sample years are simulated. Each simulated sample year takes the above approach. Finally, the results are fitted into a probability distribution curve and statistical measures are treated in a rigorous manner [2], [3]. Since each sample year requires a RIA, 500 iterations of RIA are executed if there are 500 sample years. It should be noted that the RIA for a sample year could be completed about 5–10 times faster than a regular RIA, since many components may have 0 ANF and can be skipped. However, the accrued running time could still be hours and potentially grow very fast for larger systems.

A simplified approach for Monte Carlo simulation based on LCFM is described as follows.

1. Run a full RIA to obtain SAIFI contribution vector of (S_i/n).
2. For the j^{th} sample year ($j = 1 \sim \mathrm{Number\ of\ sample\ years}$).
 2.1. Use the random number approach to generate the ANF_{ij} for component i.
 2.2. Use **(4)** and then **(1)** to compute SAIFI for this year.

After the initial RIA, the simplified approach only needs one-step execution of (4) and (1) for each sample year, as opposed to execution of a RIA in the traditional approach. This tremendously reduces the running time. For a test system with 9200 components, this reduces the running time for 500 sample years from 40 min to about 50 s.

It should be noted that the simplified approach produces the same results (no error) as the traditional approach, if both approaches use the same randomly generated ANF_{ij} for component i in sample year j. The reason is that the simplified approach does not change the mathematical kernel of Monte Carlo simulation.

IV. Application in Sensitivity Analysis

Sensitivity analysis [2], [4] is another high-level, reliability-based analysis, which identifies the change of reliability indices if the failure rates of components in set $\{\mathrm{Sc}\}$ are changed. In other words, it measures the sensitivity of reliability indices to the failure rates of a set of components. The sensitivity of SAIFI is given by

$$s = \frac{100 * \left[\frac{(SAIFI_1 - SAIFI_0)}{SAIFI_0}\right]}{\Delta\lambda} \tag{9}$$

where

$SAIFI_0$ system SAIFI of the base case;
$SAIFI_1$ new system SAIFI with the updated failure rates;
$\Delta\lambda$ change of failure rates in % for all components in $\{Sc\}$.

The traditional approach evaluates the new SAIFI by completing a RIA based on the new failure rates of the components in $\{\mathrm{Sc}\}$. If there are x different scenarios of $\{\mathrm{Sc}\}$ to be analyzed, the time-consuming RIA will be run for x times. Based on LCFM, however, this approach can be speeded up by the following steps.

1. Run a full RIA to obtain SAIFI contribution vector of (S_i/n).
2. Compute original SAIFI with **(4)** and then **(1)**.
3. For each scenario of sensitivity analysis
 3.1. Compute new SAIFI with new component failure rates using **(4)** and then **(1)**.
 3.2. Compute the sensitivity with **(9)**.

After the initial RIA, each new scenario can be performed instantly, as opposed to a RIA for each scenario in the traditional approach. For the previous test system with 9200 components, the simplified approach takes less than a few milliseconds for every new scenario, while the traditional approach takes about 43 s. If there are many scenarios to run, the running time can be reduced tremendously.

Similar to the LCFM application in Monte Carlo simulation, the simplified and traditional approaches for sensitivity analysis produce the same results, because both approaches have the same mathematical kernel.

V. Conclusions

The research work presents a linear model between a component's contribution to reliability indices and the failure rate of the component. When it is applied to risk analysis (based on Monte Carlo simulation) and sensitivity analysis, only one full RIA is required. The remaining work can be completed in just a one-step summation of reliability contributions from all components. This tremendously reduces the running time by 98% for risk analysis, and from 43 s to a few milliseconds for sensitivity analysis.

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