A Linear Programming Method Based Optimal Power Flow Problem for Iraqi Extra High Voltage Grid (EHV)

Dr. Firas Mohammed Tuaimah
Assistant Professor
College of Engineering-University of Baghdad
Email: Dr.Firas@coeng.uobaghdad.edu.iq

Montather Fadhil Meteb
Department of Electrical Engineering
College of Engineering-University of Baghdad
Email: montather87@gmail.com

ABSTRACT

The objective of an Optimal Power Flow (OPF) algorithm is to find steady state operation point which minimizes generation cost, loss etc. while maintaining an acceptable system performance in terms of limits on generators real and reactive powers, line flow limits etc. The OPF solution includes an objective function. A common objective function concerns the active power generation cost. A Linear programming method is proposed to solve the OPF problem. The Linear Programming (LP) approach transforms the nonlinear optimization problem into an iterative algorithm that in each iteration solves a linear optimization problem resulting from linearization both the objective function and constrains. A computer program, written in MATLAB environment, is developed to represent the proposed method. The adopted program is applied for the first time on Iraqi 24 bus Extra High Voltage (EHV) network (400 kV). The required are data taken from the operation and control office, which belongs to the ministry of electricity.

Keywords: Optimal power flow, linear programming, active power dispatch.

السريان الأمثل للطاقة بأستخدام طريقة البرمجة الخطية لشبكة الضغط الفائق العراقية

د. فراس محمد طعيمة
أستاذ مساعد
كلية الهندسة/ جامعة بغداد

منتظر فاضل متعب
قسم هندسة الكهرباء
كلية الهندسة/ جامعة بغداد

الخلاصة

الهدف من الخوارزمية المقترحة لسريان القدرة الأمثل (OPF) هو العثور على نقطة عمل موحدة ومستقرة تؤدي إلى تقليل تكلفة التوليد، أو الخسائر مع الحفاظ على أداء مقبول لمجمل الشبكة من حيث الحدود المفروضة على القدرة الفعالة والغير فعالة المتولدة، وحدود سريان القدرة بخط النقل. الحل بأستخدام OPF يتضمن دالة هدف. ودالة الهدف الشائعة هي فيما يتعلق بكلفة توليد القدرة الفعالة، حيث تم اقتراح طريقة البرمجة الخطية (LP) لحلOPF. طريقة البرمجة الخطية (LP) تحول مسالة التحسين غير الخطية إلى خوارزمية تكرارية حيث في كل تكرار يحل مسالة التحسين الخطية الناتجة عن عملية التحويل إلى دالة خطية لكل من دالة الهدف والقيود. تمت كوين برنامج بأستخدام الحاسب الآلي، وكتب في بيئة MATLAB، لتمثيل الطريقة المقترحة.ويطبق البرنامج الذي اعتمد لأول مرة على شبكة الضغط الفائق العراقية (400 كيلوفولت) التي تتكون من 24 عقدة. حيث تم اخذ البيانات اللازمة من دائرة التشغيل والتحكم، التابعة الى وزارة الكهرباء العراقية.

الكلمات الرئيسية: السريان الأمثل للطاقة، البرمجة الخطية، نقل الطاقة الحقيقية

I. INTRODUCTION

Throughout the entire world, the electric power industry has undergone a considerable change in the past decade, and will continue to do so for the next several decades. In the past, the electric power industry has been either a government-controlled or a government-regulated Industry which existed as a monopoly in its service region. All people, businesses, and industries were required to purchase their power from the local monopolistic power company. This was not only a legal requirement, but a physical engineering requirement as well. It just did not appear feasible to duplicate the resources required to connect everyone to the power grid. Over the past decade, however, countries have begun to split up these monopolies in favor of the free market **Barkovich 1996, Morgan 1996**and **Rudnick 1996**.

Optimal Power Flow (OPF) solution methods have been developed over the years to meet this very practical requirement of power system operation **Acha 2000, El-Hawary 1986,Giacomoni 2010** and **Huneault 1991**.

The optimal power flow problem has been discussed since its introduction by Carpentier **Khaled 2008**. Because the OPF is a very large, non-linear mathematical programming problem, it has taken decades to develop efficient algorithms for its solution. Many different mathematical techniques have been employed for its solution. The majority of the techniques discussed in the literature use one of the following five methods **Alsac 1990, Dommel 1968, Sun 1984** and **Wood 1996**.

1. Lambda iteration method, also called the equal incremental cost criterion (EICC) method.
2. Gradient method.
3. Newton’s method.
4. Linear programming method.
5. Interior point method.

The Linear Programming (LP) approach transforms the nonlinear optimization problem into an iterative algorithm that in each iteration solves a linear optimization problem resulting from linearizing both the objective function and constrains **Alsac 1990, Chamorel 1983, Tareq 2008 and Ye Tao 2009**.

The large - scale application of LP - based methods has traditionally been limited to network constrained real and reactive dispatch calculations whose objectives are separable, comprising the sum of convex cost curves. The accuracy of calculation may be lost if the oversimplified approximation is adopted in LP - based OPF. The piecewise linear segmentation of the generator fuel cost curve should be good for avoiding this problem. The piecewise approach can fit an arbitrary curve convexly to any desired accuracy with a sufficient number of segments. Originally, a separable LP variable had to be used for each segment, with the resulting large problems with multi segments cost curve modeling were prohibitively time and storage consuming. The difficulty was alleviated considerably by a separable programming procedure that uses a single variable per cost curve, regardless of the number of the segments. However, the number of segments still affects the solution speed and precision, **Jizhong Zho 2009**.

This paper presents an LP-based OPF for generation cost minimization using as control variables the generator active power and generator voltage. It is intended to overcome the constraints of current LP-based OPF algorithms. The main problem of the current algorithms is the loss of accuracy of the linear approximation of the objective function when the changes of the control variables are not small enough. An attempt to address this issue consists of imposing limits to the deviation of control variables, **Alsac 1990** . Although this approach solves the problem, the convergence of the algorithm becomes very slow. The LP based OPF proposed in this paper improves the accuracy of the linear approximation of the objective function. The objective function is approximated by a piecewise linear function determined iteratively by segmented the objective function in each iteration.

2. PIECEWISE LINEAR APPROXIMATION OF OBJECTIVE FUNCTION

Assuming that the objective function is a quadratic characteristic, the objective function can be linearized by a piecewise linear approach. If the objective function is divided into *N* linear segments, the real power variable of each generator will also be divided into *N* variables. **Fig. 1** is an objective function with three linear segments. The corresponding slopes are b_1, b_2, and b_3, respectively **Jizhong Zho 2009**.

From **Fig. 1**, the generator power output variables for each segment can be presented as below:

$$P_{Gi\,min} \leq P_{Gi\,1} \leq P_{G1\,max} \tag{1}$$

$$P_{G1\,max} \leq P_{Gi\,2} \leq P_{G2\,max} \tag{2}$$

$$P_{G2\,max} \leq P_{Gi\,3} \leq P_{Gi\,max} \tag{3}$$

If $P_{Gi\,min}$ is selected as the initial generator output power, the incremental generator power outputs for each segment can be expressed as:

$$\Delta P_{Gi\,1} = P_{Gi\,1} - P_{Gi\,min} \tag{4}$$

$$\Delta P_{Gi\,2} = P_{Gi\,2} - P_{Gi\,1max} \tag{5}$$

$$\Delta P_{Gi\,3} = P_{Gi\,3} - P_{Gi\,2max} \tag{6}$$

Thus the constraint **Eq. (1)** to **Eq. (3)** become

$$0 \leq \Delta P_{Gi\,1} \leq P_{Gi\,1max} - P_{Gi\,min} \tag{7}$$

$$0 \leq \Delta P_{Gi\,2} \leq P_{Gi\,2max} - P_{Gi\,1max} \tag{8}$$

$$0 \leq \Delta P_{Gi\,3} \leq P_{Gi\,max} - P_{Gi\,2max} \tag{9}$$

The piecewise linear objective function becomes

$$\boldsymbol{F} = \sum_{i=1}^{NG} f_i(P_{Gi}) = \sum_{k=1}^{N} \sum_{i=1}^{NG} b_k\, \Delta P_{Gik} \tag{10}$$

Where;

N_G: The number of generators

$P_{Gi\,min}$: The minimal real power output at generator *i*

$P_{Gi\,max}$:The maximal real power output at generator *i*

3. MATHEMATICAL FORMULATION OF THE ALGORITHM

The objective function contains real power generation cost. Mathematically, it is formulated as follows:

$$\boldsymbol{F} = \sum_{i=1}^{NG} f_i(P_{Gi}) \tag{11}$$

Subject to

$$\sum_{i=1}^{NG} P_{Gi} = \sum_{k=1}^{ND} P_{Dk} + P_L \tag{12}$$

$$|P_{ij}| \leq P_{ij\,max} \quad ij \in N_T \tag{13}$$

$$P_{Gi\,min} \leq P_{Gi} \leq P_{Gi\,max} \quad i \in N_G \tag{14}$$

Where;
P_D : The real power load
P_{ij}: The power flow of transmission line *ij*
$P_{ij\,max}$: The power limits of transmission line *ij*
P_L: The network losses
f_i: The cost function of the generator *i*
N_T: The number of transmission lines
N_D: The number of loads
Since loads are constant for the given time, we can get the following expression through linearizing the real power balance equation:

$$\sum_{i=1}^{NG} \left(1 - \frac{\partial P_L}{\partial P_{Gi}}\right)\Big| P_{Gi}^0 \Delta P_{Gi} = 0 \tag{15}$$

The real power flow equation of a branch can be written as follows:

$$P_{ij} = V_i^2 G_{ij} - V_i V_j (\, G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij} \,) \tag{16}$$

Where;
P_{ij}: The sending end real power on transmission branch *ij*
V_i: The node voltage magnitude of node *i*
δ_{ij}: The difference of node voltage angles between the sending end and receiving end of the line *ij*
B_{ij}: The susceptance of transmission branch *ij*
G_{ij}: The conductance of transmission branch *ij*

Through linearizing Eq. **(16),** we get the incremental branch power expression as below:

$$\Delta P_{ij} = -\, V_i^0 V_j^0 \, (-G_{ij} \sin \delta_{ij}^0 \Delta\delta_{ij} + B_{ij} \cos \delta_{ij}^0 \Delta\delta_{ij}) \tag{17}$$

In a high - voltage power network, the value of δ_{ij} is very small, and the following approximate equations are easily obtained:

$$\sin\delta_{ij} \cong 0 \tag{18}$$

$$\cos\delta_{ij} \cong 1 \tag{19}$$

In addition, assume that the magnitudes of all bus voltages are the same and equal to 1.0 p.u. Furthermore, suppose the reactance of the branch is much bigger than the resistance of the branch, so that we can neglect the resistance of the branch. Thus,

$$G_{ij} = \frac{R_{ij}}{R_{ij}^2 + X_{ij}^2} \approx 0 \tag{20}$$

$$B_{ij} = -\frac{X_{ij}}{R_{ij}^2 + X_{ij}^2} \approx -\frac{X_{ij}}{X_{ij}^2} \approx -\frac{1}{X_{ij}} \tag{21}$$

Substituting **Eq. (18)** to **Eq. (21)** in to **Eq. (17)** , we get

$$\Delta P_{ij} = -B_{ij}\Delta\delta_{ij} = -B_{ij}(\Delta\delta_i - \Delta\delta_j) = \frac{(\Delta\delta_i - \Delta\delta_j)}{X_{ij}} \tag{22}$$

The above equation can also be written in matrix form, i.e.,

$$\Delta Pb = B'\Delta\delta \tag{23}$$

Where the elements of the susceptance matrix B′ are

$$B'_{ij} = B_{ij} = -\frac{1}{X_{ij}} \tag{24}$$

$$B'_{ii} = -\sum_{\substack{j=1 \\ j\neq i}}^{n} B_{ij} \tag{25}$$

The bus power injection equation can be written as

$$P_{Gi} - P_{Di} = V_i \sum_{j=1}^{n} V_j \left(G_{ij}\cos\delta_{ij} + B_{ij}\sin\delta_{ij} \right) \tag{26}$$

Since the load demand is constant, the linearization expression of **Eq.(26)** can be written as below:

$$\Delta P_{Gi} = V_i^0 \sum_{j=1}^{n} V_j^0 \left(-G_{ij}\sin\delta_{ij}^0\Delta\delta_{ij} + B_{ij}\cos\delta_{ij}^0\Delta\delta_{ij}\right) \tag{27}$$

$$= V_i^0 \sum_{j=1}^{n} V_j^0 \left(-G_{ij}\sin\delta_{ij}^0 + B_{ij}\cos\delta_{ij}^0\right)\Delta\delta_{ij} \tag{28}$$

The above equation can also be written in the following matrix form

$$\Delta P_G = H\Delta\delta \tag{29}$$

Eq. (29) stands for the relationship between the incremental generator output power (except for the generator that is taken as slack unit) and the incremental bus voltage angle. Matrix *H* can also be simplified by using **Eq. (18)** and **Eq. (21).**

According to **Eq. (23)** and **Eq. (29),** we can get the direct linear relationship between the incremental branch power flow and incremental generator output power, i.e.,

$$\Delta P_b = B'\Delta\delta = B'H^{-1}\Delta P_G = D\,\Delta P_G \tag{30}$$

Where

$$D=B'H^{-1} \tag{31}$$

It is also called as the linear sensitivity of the branch power flow with respect to the generator power output.

Thus the linear expression of the branch power flow constraints can be written as:

$$|D\ \Delta P_G| \le \Delta P_{b\ max} \tag{32}$$

The element of the matrix $\Delta P_{b\ max}$ is the incremental power flow limit $\Delta P_{ij\ max}$ of the branch *ij*, i.e.,

$$\Delta P_{ij\ max}=P_{ij\ max}-P_{ij}^{0} \tag{33}$$

The incremental form of the generator output power constraint is , **Jizhong Zho 2009**

$$P_{Gi\ min}-P_{Gi}^{0}\le \Delta P_{Gi}\le P_{Gi\ max}-P_{Gi}^{0} \qquad i \in N_G \tag{34}$$

4. THE PROPOSED METHOD IMPLEMENTATION

The above mentioned method for solving optimal power by LP uses an iterative technique to obtain the optimal solution, so it is also called a successive linear programming (SLP) method. The solution procedures of SLP for optimal power flow are summarized below:

Step1. Select the set of initial control variables

Step2. Solve the power flow problem to obtain a feasible solution that satisfies the power balance equality constraint.

Step3. Linearize the objective function and inequality constraints around the power flow solution and formulate the LP problem. Then solve the LP problem and obtain the optimal incremental control variables ΔP_{Gi}.

Step4. Update the control variables :

$$P_{Gi}^{k+1}=P_{Gi}^{k}+\Delta P_{Gi}$$

Step5. Obtain the power flow solution with updated control variables.

Step6. Check the convergence. If ΔP_{Gi} in Step 4.Are bellow the user-defined tolerance, the solution converges. Otherwise, go to Step 3.

Fig. 2 shows the flowchart of linear programming method Based optimal power flow problem.

5. CASE STUDY

The Iraqi 400kV (EHV) network shown in **Fig. 3** was chosen to implement the proposed LP algorithm for OPF.

The Iraqi EHV network consists of 24 bus bars, 38 transmission lines and 11 generating stations. Two operational case studies for the Iraqi network were chosen to be studied by this paper for optimal power flow solution. These two case studies are with cheap and expensive international fuel price conditions.

All the data for this work was taken from the Iraqi operation and control office that belongs to the ministry of electricity.

Table 1 indicates transmission system parameters in p.u. / km (at a base of 100 MVA) for the three types of the transmission lines used in the Iraqi network.

6. RESULTS

The algorithm described in this paper has been coded in MATLAB (R2008a) language. The performance of the algorithm is illustrated considering for a state of load of the operation of the Iraqi power system. The results obtained from using Linear Programming (LP) method are compared with the results obtained from power flow solution using Newton-Raphson method. It is worth mentioning that the distribution of loads on the power plants identified for Newton-Raphson so that we get less losses. There are four power plants run on two types of fuel to generate electric power, so we compared the results when they operate on the cheap fuel type and expensive fuel type. **Fig.4** shows the voltage magnitude in per unit for each bus when cheap fuel price is used to generate power in power plants that operate on two types of fuel and for the two algorithms (Newton-Raphson and Linear Programming).

Fig. 5 shows the generation of each plant for Newton-Raphson power flow solution compared with Linear Programming method when cheap fuel price is used, **Fig. 6** shows the production cost when cheap fuel price is used

Fig.7 shows the voltage magnitude for each bus when expensive fuel price is used to generate power in power plants that operate on two types of fuel and for the two algorithms (Newton-Raphson and Linear Programming).

Fig. 8 shows the generation of each plant for Newton-Raphson power flow solution compared with Linear Programming method when expensive fuel price is used.
Fig. 9 shows the variation of production cost through optimization using Linear Programming method (LP).

7. CONCLUSION

The Linear Programming (LP) algorithm is used for the first time on the Iraqi Extra High Voltage (EHV 400kV) Grid for optimal power flow to minimize the active power generation cost. paper has presented a LP based.

The problem constraints are the coupled linearized power flow equations and the system variable limits. A piecewise linear approximation of the objective function is built by adding iteratively a tangent cut in each iteration.
It can be also note that the results of the production cost are significantly decreased when using Linear Programming with the results derived in the case of Newton–Raphson. From **Table 2** there is about 30.16% decrease in the production costs when using cheap fuel type, whereas there is about 28.2% decrease in production costs when using expensive fuel type as given in **Table 3**.

REFERENCES

Acha, E., Ambriz-Pe´rez, H., Fuerte-Esquivel, C.R , 2000, *Advanced Trans-former Control Modeling in An Optimal Power Flow Using Newton's Method, IEEE Trans. Power Systems* 15(1), pp. 290–298.

Alsac O., J. Bright, M. Prais and B. Stott, , 1990, *Further Developments in LP-Based Optimal Power Flow*, IEEE Transactions on Power Systems, Vol. 5(3), pp. 697-711.

Barkovich B.R. and D. Hawk, 1996, *Charting a New Course in California, IEEE Spectrum*, Vol. 33, No. 7, July, pp. 26-31.

KhaledZehar& Samir Sayah, 2008, *Optimal Power Flow with Environmental Constraint using a Fast Successive Linear Programming Algorithm*, Application to the Algerian power system, Elsevier, Volume 49, Issue 11, November pp. 3362–3366.

Chamorel P-A., A. J. Germond, 1983, *Optimal Voltage and Reactive Power Control in an Interconnected Power System with Linear Programming*, CIGRE Study Committee 38, Montreaux.

Dommel H. W. and W. F. Tinney, 1968, *Optimal Power Flow Solutions, IEEE Transactions on Power Apparatus and Systems*, Vol. PAS-87, October, pp. 1866-1876.

El-Hawary, M.E., Tsang, D.H., 1986, *The Hydro-Thermal Optimal Power Flow, A Practical Formulation and Solution Technique Using Newton's Approach,* IEEE Trans. Power Systems PWRS-1(3), pp. 157–167,.

Huneault, M., Galiana, F.D., A, , 1991, *Survey of the Optimal Power Flow Literature, IEEE Trans. Power Systems* volume:6, Issue:2, pp. 762–770.

JizhongZho, 2009, *Optimization Of Power System Operation, Institute Of Electrical and Electronics Engineers John Wiley & Sons, Inc., Hoboken, New Jersey*.

Tareq A. Al-Muhawesh&Isa S. Qamber, 2008,*The Established Mega Watt Linear Programming-Based Optimal Power Flow Model Applied to The Real Power 56-Bus System In Eastern Province of Saudi Arabia, Elsevie*r, Volume 33, Issue 1, January, pp. 12–21.

Giacomoni A.M.&Wollenberg B.F.,2010, *Linear Programming Optimal Power Flow Utilizing A Trust Region Method, IEEE, North American Power Symposium (NAPS)* .

Morgan M.G. and S. Talukdar, 1996 ,*Nurturing R&D, IEEE Spectrum*, Vol. 33, No.7, July pp. 32-33.

Ye Tao,A , 2009, *Sequential Linear Programming Algorithm for Security-Constrained Optimal Power Flow,* IEEE, North American Power Symposium (NAPS) Conference**.**

Rudnick H., 1996, *Pioneering Electricity Reform in South America, IEEE Spectrum,* Vol. 33, No. 8, August pp. 38-44.

Sun D.I., B. Ashley, B. Brewer, A. Hughes and W. F. Tinney, 1984, *Optimal Power Flow by Newton Approach, IEEE Transactions on Power Apparatus and Systems*, Vol. PAS-103, October pp. 2864-2880.

Wood J. and B. F. Wollenberg, 1996, *Power Generation Operation and Control, New York, NY*: John Wiley & Sons, Inc., pp. 39-517,.

9. ABBREVIATIONS AND SYMBOLS

EHV	Extra high voltage
LP	Linear programming
OPF	Optimal power flow
B_{ii}	Self susceptance
B_{ij}	Mutual susceptance

F_i	The generator fuel cost function
G_{ii}	Self conductance
G_{ij}	Mutual conductance
P_{Di}	Active load power at bus i
P_{Gi}	Active power generated by unit i
$P_{Gi\,min}$	Minimum generator active power limit
$P_{Gi\,max}$	Maximum generator active power limit
V_i	Bus voltage at bus i
ΔP	Change in active power
$\Delta\delta$	Change in voltage angle
ΔV	Change in voltage magnitude
δ_i	Bus voltage angle at bus i

Table1. Iraqi transmission line system parameters.

Conductor Type	R p.u. / km	X p.u. / km	B p.u. / km
AAAC	0.00002167	0.000197	0.005837
ACSS	0.0000228	0.0001868	0.005784
ACSD	0.0000228	0.0001897	0.005962

Table 2. Results when cheap fuel price is used.

	NR	LP
Total Active Gen.	4188.97 [MW]	4192.63 [MW]
Total Reactive Gen.	11.74 [Mvar]	63.99 [Mvar]
Total Active Load	4177 [MW]	4177 [MW]
Total Reactive Load	1998 [Mvar]	1998 [Mvar]
Total Active Loss	11.967 [MW]	15.633 [MW]
Total Reactive Loss	104.97 [Mvar]	136.20 [Mvar]
ProductionCost ($/h)	688960	481158.11

Table 3. Results when expensive fuel price is used.

	NR	LP
Total Active Gen.	4188.97 [MW]	4195.76 [MW]
Total Reactive Gen.	11.74 [Mvar]	91.23 [Mvar]
Total Active Load	4177 [MW]	4177 [MW]
Total Reactive Load	1998 [Mvar]	1998 [Mvar]
Total Active Loss	11.967 [MW]	18.756 [MW]
Total Reactive Loss	104.97 [Mvar]	164.64 [Mvar]
Production Cost ($/h)	974630	699679.68

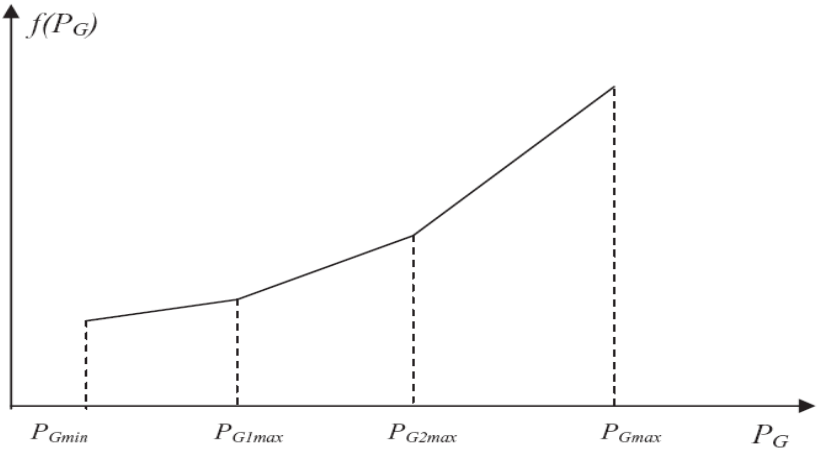

Figure 1. Piecewise linear objective function.

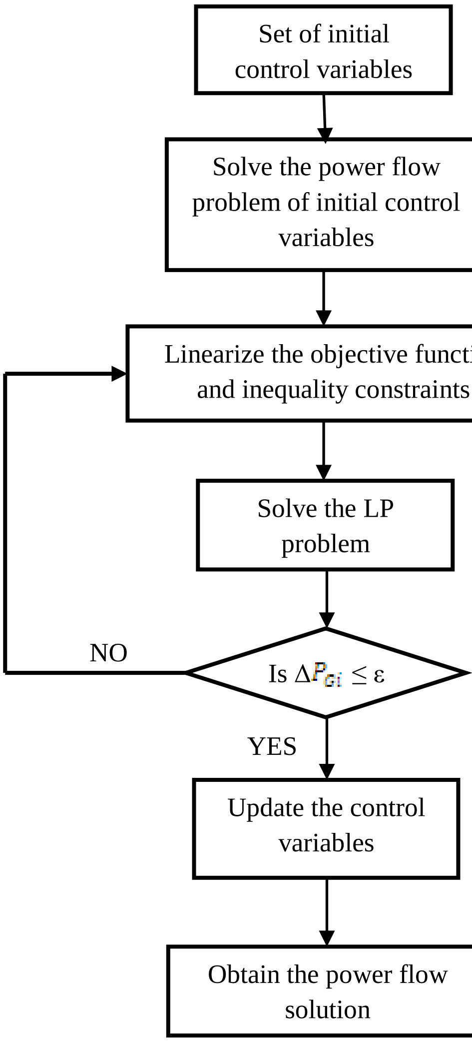

Figure 2. The flowchart of linear programming method based-optimal power flow problem**.**

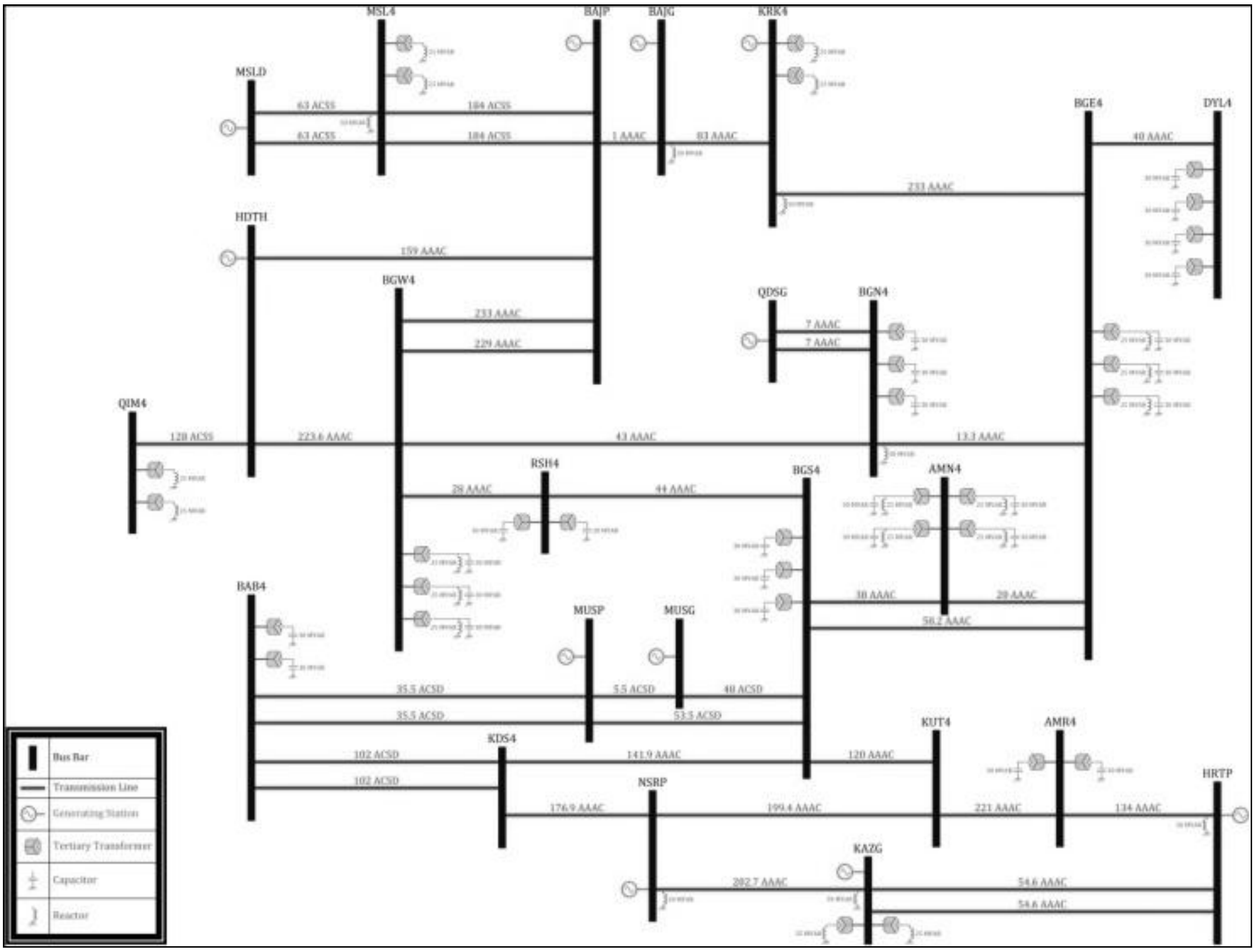

Figure 3. Single line diagram of the Iraqi network 400kV.

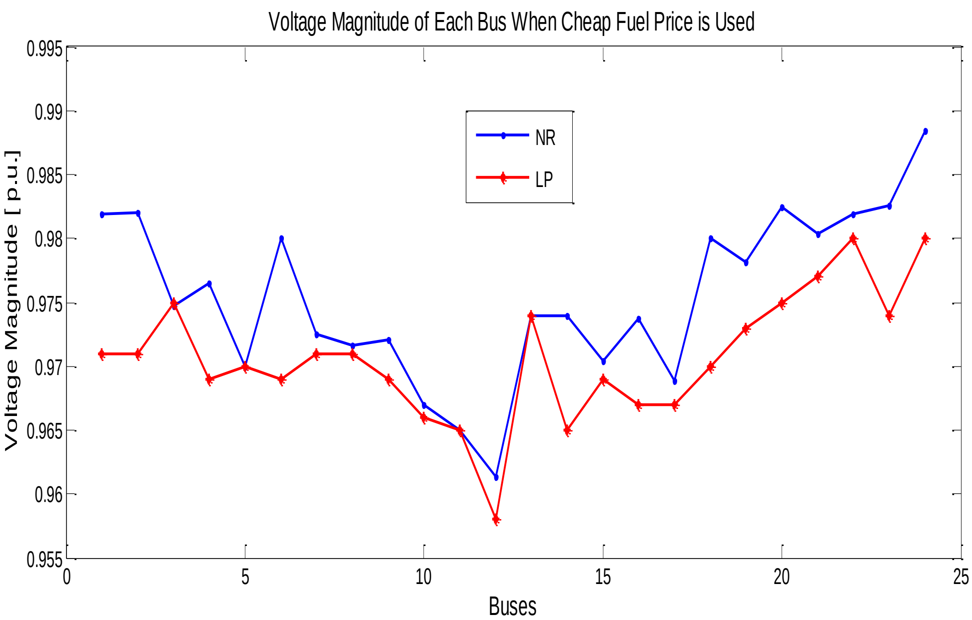

Figure 4. Voltage magnitude at each bus with cheap fuel price.

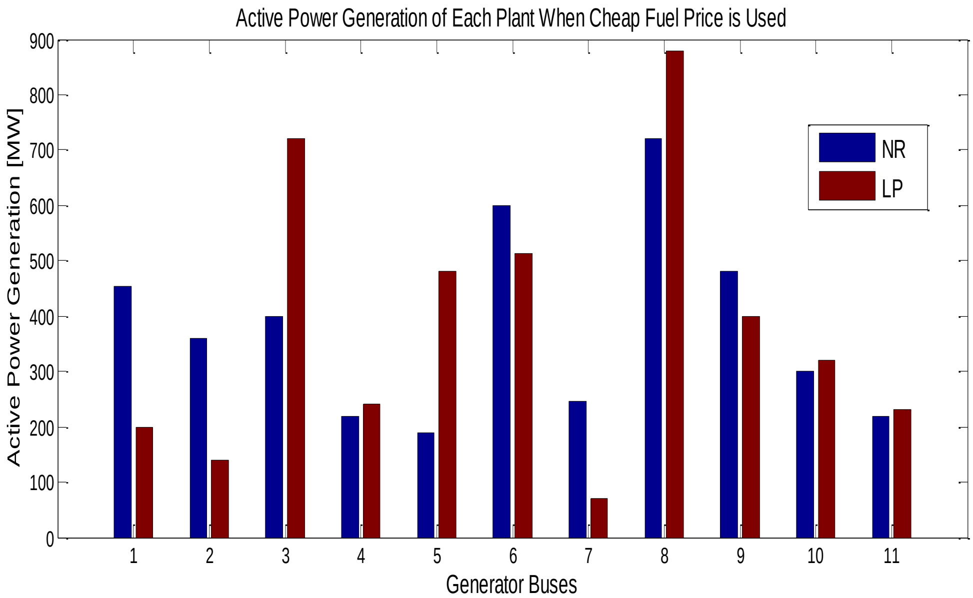

Figure 5. Active power generation for each plant with cheap fuel price.

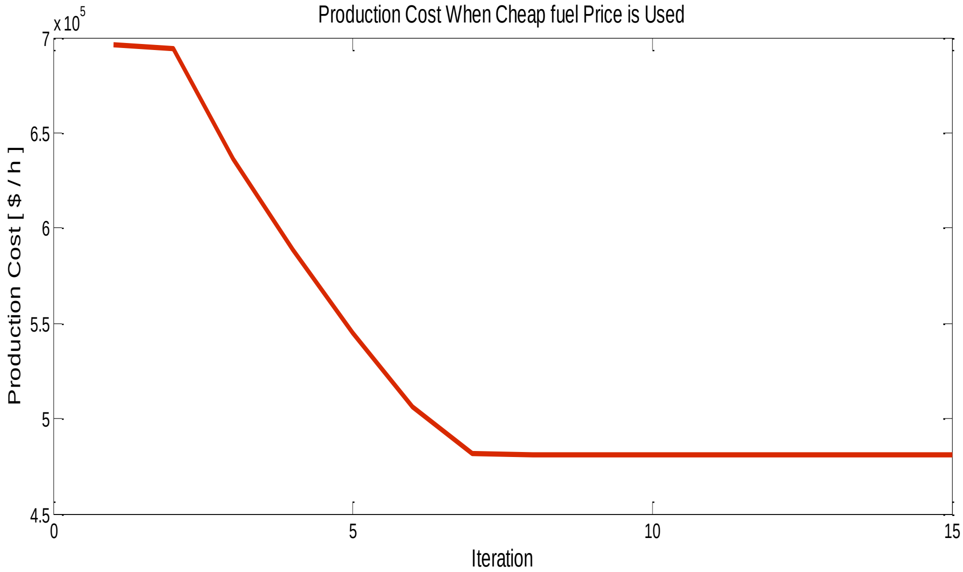

Figure 6. Production cost when cheap fuel price is used.

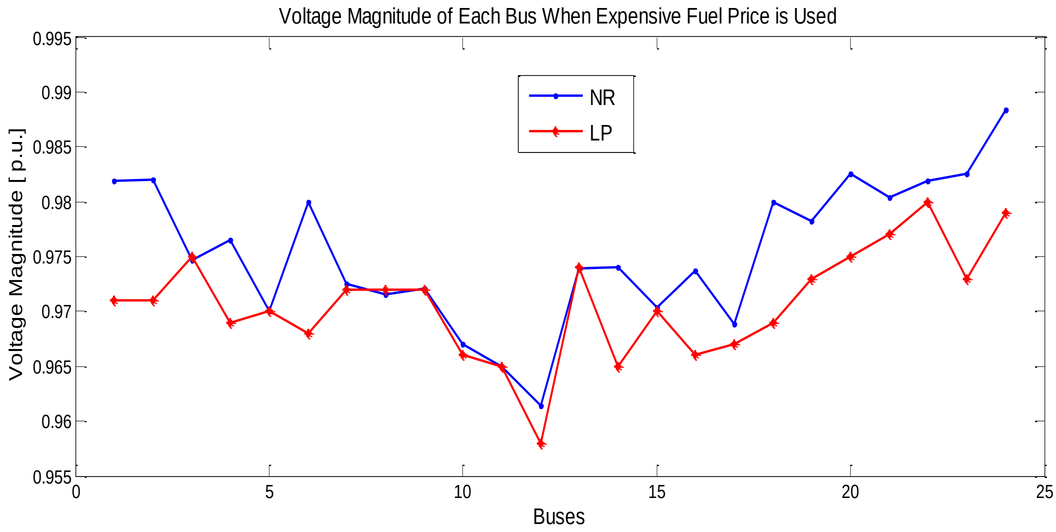

Figure 7. Voltage magnitude at each Bus with expensive fuel price.

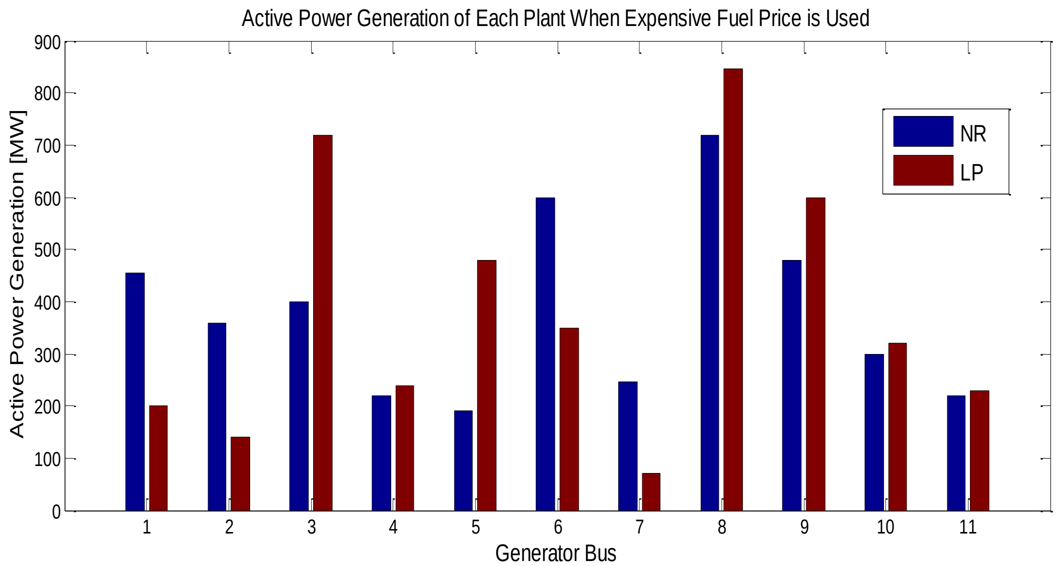

Figure 8. Active power generation for each plant with expensive fuel price.

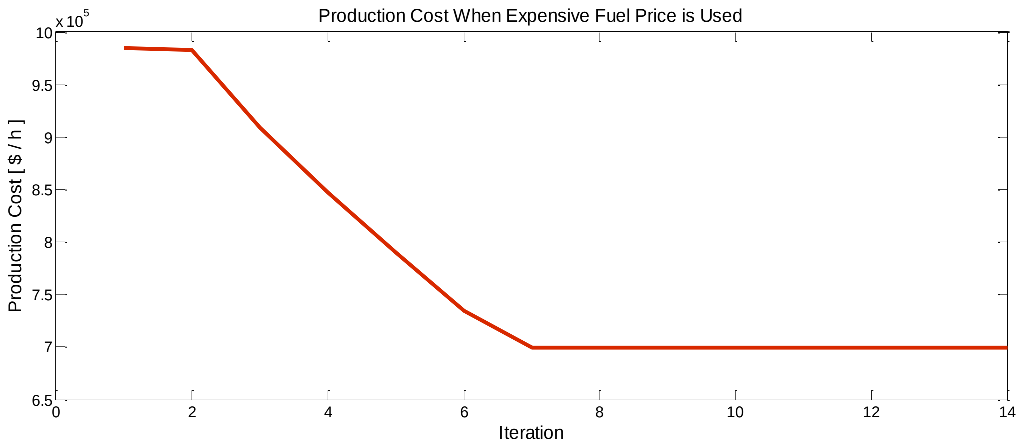

Figure 9. Production cost when expensive fuel price is used.