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A Local Fractional Elzaki Transform Decomposition Method for the Nonlinear System of Local Fractional Partial Differential Equations

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Abstract: In this paper, the nonlinear system of local fractional partial differential equations is solved via local fractional Elzaki transform decomposition method. The local fractional Elzaki decomposition transform method combines a local fractional Elzaki transform and the Adomian decomposition method. Applications related to the nonlinear system of local fractional partial differential equations are presented.

Keywords: local fractional derivative; Elzaki transform; Adomian decomposition method

1. Introduction

With the rapid advancement of human knowledge, there is an urgent need to improve the basic definitions in the various scientific fields. One of these new developments in differential equations is the use of new concepts with local and non-local fractional differential calculus. These concepts have been rapidly applied in various branches of mathematics, physics and engineering [1–13].

Recently, local fractional calculus has been a considerably important topic in both mathematics and engineering [14–23]. Researchers have been used the Adomian decomposition method (ADM) [24] to solve local fractional differential equations, local fractional partial differential equations and local fractional integral equations [25–28]. Local fractional derivative operators with some known transforms, such as the Young–Laplace transform and the Sumudu transform, are combined with a decomposition method. A few of these methods are the local fractional Laplace decomposition method [29], the local fractional Sumudu decomposition method [30,31] and the Yang–Laplace decomposition method [32]. Thus, local fractional differential equations or local fractional partial differential equations are solved by these methods.

This paper analyzes the non-differentiable solutions to the fractional form of the coupled nonlinear system of partial differential equations, including the local fractional derivative (LFD), with the aid of a novel, efficient method. There have been few studies about the solutions of local fractional partial differential equations in the literature [25–32]. We carried out such study, in order to find an effective method among the existing methods in the literature for the existing solutions of such equations, and to gain a new perspective for future studies. The novelty of this paper is to define a local fractional Elzaki transform and give its properties. Additionally, we suggest a local fractional Elzaki transform decomposition method (LFETDM), which is constructed from the local fractional Elzaki transform and the Adomian decomposition method. We note that the local fractional Elzaki transform decomposition method is the same as the Yang–Laplace decomposition method. The nonlinear system of local fractional partial differential equations is solved by LFETDM. The originality of this study is to propose a new hybrid method for obtaining the analytical or numerical solutions of a nonlinear system of local fractional partial differential equations.

2. Preliminaries

In this section, we give the concepts of local fractional derivatives and integrals and polynomial functions on Cantor sets.

Definition 1. *Let the function $f(x) \in C_\alpha(a, b)$ [33,34]. If there are*

$$|f(x) - f(x_0)| < \varepsilon^\alpha, \ 0 < \alpha \leq 1, \tag{1}$$

where there is $|x - x_0| < \delta$, for $\varepsilon > 0$ and $\varepsilon \in \mathbb{R}$, then $f(x)$ is local, fractional and continuous.

Definition 2. *Let the function $f(x) \in C_\alpha(a, b)$. The local fractional integral of $f(x)$ of order α in the interval $[a, b]$ is defined by [35,36]*

$$_aI_b^{(\alpha)} f(x) = \frac{1}{\Gamma(1 + \alpha)} \int_a^b f(t)(dt)^\alpha = \frac{1}{\Gamma(1 + \alpha)} \lim_{\Delta t \to 0} \sum_{j=0}^{N-1} f(t_j)(\Delta t_j)^\alpha, \tag{2}$$

where the partitions of the interval $[a, b]$ are denoted as $(t_j, t_{j+1}), j = 0, \ldots, N - 1, t_0 = a$ and $t_N = b$ with $\Delta t_j = t_{j+1} - t_j$ and $\Delta t = max\{\Delta t_0, \Delta t_1, \Delta t_j, \ldots\}$.

Definition 3. *Let the function $f(x) \in C_\alpha(a, b)$ [36,37]. The local fractional derivative of $f(x)$ of order α in the interval $[a, b]$ is defined by*

$$\frac{d^\alpha f(x_0)}{dx^\alpha} = \frac{\Delta^\alpha(f(x) - f(x_0))}{(x - x_0)^\alpha}, \tag{3}$$

where

$$\Delta^\alpha(f(x) - f(x_0)) \cong \Gamma(1 + \alpha)[f(x) - f(x_0)]. \tag{4}$$

The local fractional partial differential operator of order α ($0 < \alpha \leq 1$) is given by [36,37]

$$\frac{\partial^\alpha u(x_0, t)}{\partial t^\alpha} = \frac{\Delta^\alpha(u(x_0, t) - u(x_0, t_0))}{(t - t_0)^\alpha}, \tag{5}$$

where

$$\Delta^\alpha(u(x_0, t) - u(x_0, t_0)) \cong \Gamma(1 + \alpha)[u(x_0, t) - u(x_0, t_0)]. \tag{6}$$

Theorem 1. *(Local fractional Laplace transform of local fractional derivative) [38,39].*
Let $L_\alpha\{f(x)\} = f_s^{L,\alpha}(s)$. Then, we have

$$L_\alpha\{f^\alpha(x)\} = s^\alpha L_\alpha\{f(x)\} - f(0). \tag{7}$$

3. Local Fractional Elzaki Transform

The local fractional Elzaki transform (LFET) is proposed, and some properties of this transform are analyzed. We use the concepts of polynomial functions on Cantor sets.

If there is a new transform operator $^{LF}E_\alpha : f(x) \to F(u)$, so that Equation (8) is valid.

$$^{LF}E_\alpha\{f(x)\} = {}^{LF}E_\alpha\left[\sum_{k=0}^\infty a_k x^{\alpha k}\right] = \sum_{k=0}^\infty \Gamma(1 + \alpha k) a_k v^{\alpha(k+2)}. \tag{8}$$

As a classical example, we can use

$$^{LF}E_\alpha\left[\frac{x^\alpha}{\Gamma(1 + \alpha)}\right] = v^{3\alpha}.$$

Definition 4. *The local fractional Elzaki transform of $f(x)$ of order α is defined by*

$$^{LF}E_\alpha\{f(x)\} = F_\alpha(v) =: \frac{v^\alpha}{\Gamma(1+\alpha)} \int_0^\infty E_\alpha\left(-v^{-\alpha}x^\alpha\right)f(x)(dx)^\alpha,\ 0 < \alpha \le 1. \tag{9}$$

where the integral converges and $v^\alpha \in \mathbb{R}^\alpha$.
The inverse local fractional Elzaki transform of $f(x)$ of order α is defined by

$$^{LF}E_\alpha{}^{-1}\{F_\alpha(v)\} = f(x),\ 0 < \alpha \le 1. \tag{10}$$

Theorem 2. *(linearity)*
Let $^{LF}E_\alpha\{f(x)\} = F_\alpha(v)$ and $^{LF}E_\alpha\{g(x)\} = G_\alpha(v)$. Then, we have

$$^{LF}E_\alpha\{f(x) + g(x)\} = F_\alpha(v) + G_\alpha(v). \tag{11}$$

Proof. By using Equation (9), we have

$$^{LF}E_\alpha\{f(x) + g(x)\} = \frac{v^\alpha}{\Gamma(1+\alpha)} \int_0^\infty E_\alpha\left(-v^{-\alpha}x^\alpha\right)(f+g)(x)(dx)^\alpha \tag{12}$$

$$= \frac{v^\alpha}{\Gamma(1+\alpha)} \int_0^\infty E_\alpha\left(-v^{-\alpha}x^\alpha\right)f(x)(dx)^\alpha + \frac{v^\alpha}{\Gamma(1+\alpha)} \int_0^\infty E_\alpha\left(-v^{-\alpha}x^\alpha\right)g(x)(dx)^\alpha \tag{13}$$

$$= F_\alpha(v) + G_\alpha(v). \tag{14}$$

The proof is completed. $\square$

Theorem 3. *(Local fractional Laplace–Elzaki duality).*
Let $L_\alpha\{f(x)\} = f_s^{L,\alpha}(s)$ and $^{LF}E_\alpha\{f(x)\} = F_\alpha(v)$. Then, we get

$$^{LF}E_\alpha\{f(x)\} = v^\alpha L_\alpha\left\{f\left(\frac{1}{x}\right)\right\}, \tag{15}$$

$$L_\alpha\{f(x)\} = s^\alpha\, {}^{LF}E_\alpha\left\{f\left(\frac{1}{s}\right)\right\}. \tag{16}$$

Proof. By using the definitions of the local fractional Laplace and Elzaki transforms, then we directly obtain these results. $\square$

Theorem 4. *(Local fractional Elzaki transform of the local fractional derivative).*
Let $^{LF}E_\alpha\{f(x)\} = F_\alpha(v)$. Then, we get

$$^{LF}E_\alpha\left\{\frac{d^\alpha f(x)}{dx^\alpha}\right\} = \frac{F_\alpha(v)}{v^\alpha} - v^\alpha f(0). \tag{17}$$

Proof. By using Equations (7)–(15), the local fractional Elzaki transform of the local fractional derivative of $f(x)$ is obtained as

$$^{LF}E_\alpha\{A(x)\} = v^\alpha L_\alpha\left\{A\left(\frac{1}{x}\right)\right\} = v^\alpha\left[\frac{1}{v^\alpha}L_\alpha\left\{f\left(\frac{1}{x}\right)\right\} - f(0)\right] = L_\alpha\left\{f\left(\frac{1}{x}\right)\right\} - v^\alpha f(0)$$
$$= \frac{F_\alpha(v)}{v^\alpha} - v^\alpha f(0), \tag{18}$$

where

$$A(x) = \frac{d^\alpha f(x)}{dx^\alpha}.$$

The proof is completed. □

By the direct result of Equation (17), the generalization of Equation (17) is obtained. Let $^{LF}E_\alpha\{f(x)\} = F_\alpha(v)$. Then, we get

$$^{LF}E_\alpha\left\{\frac{d^{n\alpha}f(x)}{dx^{n\alpha}}\right\} = \frac{F_\alpha(v)}{v^{n\alpha}} - \left[\sum_{k=0}^{n-1} v^{(2-n+k)\alpha}f^{(k\alpha)}(0)\right]. \tag{19}$$

After substituting $n = 2$ into Equation (19), we have

$$^{LF}E_\alpha\left\{\frac{d^{2\alpha}f(x)}{dx^{2\alpha}}\right\} = \frac{F_\alpha(v)}{v^{2\alpha}} - f(0) - v^\alpha f^{(\alpha)}(0). \tag{20}$$

4. Local Fractional Elzaki Transform Decomposition Method

Consider the general nonlinear system as a local fractional derivative [33]:

$$\begin{cases} \frac{\partial^a X}{\partial \tau^a} + \frac{\partial^a X}{\partial x^a} + N_{\alpha,1}(X,T) + R_{\alpha,1}(X,T) = \varphi(x,\tau), \\ \frac{\partial^a T}{\partial \tau^a} + \frac{\partial^a T}{\partial x^a} + N_{\alpha,2}(X,T) + R_{\alpha,2}(X,T) = \psi(x,\tau), \end{cases} \tag{21}$$

where $\frac{\partial^\alpha}{\partial(.)^\alpha}$ represents the linear local fractional derivative operator of order α, $R_{\alpha,1}$ and $R_{\alpha,2}$ are the linear local fractional operators, $N_{\alpha,1}$ and $N_{\alpha,2}$ are the nonlinear local fractional operators and $\varphi(x,\tau), \psi(x,\tau)$ are two unknown functions.

An analytical solution of this system is obtained by the following steps.

Step 1 If the local fractional Elzaki transform (LFET) is applied to both sides of each equation in system (21), then it is obtained as

$$\begin{cases} ^{LF}E_\alpha\left[\frac{\partial^a X}{\partial \tau^a}\right] + ^{LF}E_\alpha\left[\frac{\partial^a X}{\partial x^a}\right] + ^{LF}E_\alpha[N_{\alpha,1}(X,T)] + ^{LF}E_\alpha[R_{\alpha,1}(X,T)] = ^{LF}E_\alpha[\varphi(x,\tau)], \\ ^{LF}E_\alpha\left[\frac{\partial^a T}{\partial \tau^a}\right] + ^{LF}E_\alpha\left[\frac{\partial^a T}{\partial x^a}\right] + ^{LF}E_\alpha[N_{\alpha,2}(X,T)] + ^{LF}E_\alpha[R_{\alpha,2}(X,T)] = ^{LF}E_\alpha[\psi(x,\tau)]. \end{cases} \tag{22}$$

If the differential property of Elzaki transform is applied, then we have

$$\begin{cases} ^{LF}E_\alpha[X] = v^{2\alpha}X(x,0) + v^\alpha\left[^{LF}E_\alpha[\varphi(x,\tau)]\right] - ^{LF}E_\alpha\left[\frac{\partial^a X}{\partial x^a} + N_{\alpha,1}(X,T) + R_{\alpha,1}(X,T)\right], \\ ^{LF}E_\alpha[T] = v^{2\alpha}T(x,0) + v^\alpha\left[^{LF}E_\alpha[\psi(x,\tau)]\right] - ^{LF}E_\alpha\left[\frac{\partial^a T}{\partial x^a} + N_{\alpha,2}(X,T) + R_{\alpha,2}(X,T)\right]. \end{cases} \tag{23}$$

If the inverse LFET is applied to both sides of each system Equation (23), then it is obtained as

$$\begin{cases} X = ^{LF}E_\alpha^{-1}\left(v^{2\alpha}X(x,0)\right) + ^{LF}E_\alpha^{-1}\left(v^\alpha\left[\left(^{LF}E_\alpha[\varphi(x,\tau)]\right)\right]\right) - ^{LF}E_\alpha^{-1}\left(v^\alpha\left[\left(^{LF}E_\alpha\left[\frac{\partial^a X}{\partial x^a} + N_{\alpha,1}(X,T) + R_{\alpha,1}(X,T)\right]\right)\right]\right), \\ T = ^{LF}E_\alpha^{-1}\left(v^{2\alpha}T(x,0)\right) + ^{LF}E_\alpha^{-1}\left(v^\alpha\left[\left(^{LF}E_\alpha[\psi(x,\tau)]\right)\right]\right) - ^{LF}E_\alpha^{-1}\left(v^\alpha\left[\left(^{LF}E_\alpha\left[\frac{\partial^a T}{\partial x^a} + N_{\alpha,2}(X,T) + R_{\alpha,2}(X,T)\right]\right)\right]\right). \end{cases} \tag{24}$$

Step 2 Applying the Adomian decomposition method [40], we can show the two unknown functions X and T as infinite series.

$$X(u,t) = \sum_{n=0}^\infty X_n(u,t), \tag{25}$$

$$T(u,t) = \sum_{n=0}^\infty T_n(u,t), \tag{26}$$

and the nonlinear terms are decomposed as

$$\begin{cases} N_{\alpha,1}(X,T) = \sum\limits_{n=0}^{\infty} A_n, \\ N_{\alpha,2}(X,T) = \sum\limits_{n=0}^{\infty} B_n, \end{cases} \tag{27}$$

where A_n and B_n are Adomian polynomials [20].

If Equations (25)–(27) are substituted into Equation (24), then we obtain

$$\begin{cases} \sum\limits_{n=0}^{\infty} X_n(x,\tau) = {}^{LF}E_\alpha^{-1}\left(v^{2\alpha}X(x,0)\right) + {}^{LF}E_\alpha^{-1}\left(v^\alpha\left[\left({}^{LF}E_\alpha[\varphi(x,\tau)]\right)\right]\right) - {}^{LF}E_\alpha^{-1}\left(v^\alpha\left[\left({}^{LF}E_\alpha\left[\frac{\partial^a}{\partial x^a}\left(\sum\limits_{n=0}^{\infty}X_n(x,\tau)\right) + \sum\limits_{n=0}^{\infty}A_n + R_{\alpha,1}\left(\sum\limits_{n=0}^{\infty}X_n, \sum\limits_{n=0}^{\infty}T_n\right)\right]\right)\right]\right), \\ \sum\limits_{n=0}^{\infty} T_n(x,\tau) = {}^{LF}E_\alpha^{-1}\left(v^{2\alpha}T(x,0)\right) + {}^{LF}E_\alpha^{-1}\left(v^\alpha\left[\left({}^{LF}E_\alpha[\psi(x,\tau)]\right)\right]\right) - {}^{LF}E_\alpha^{-1}\left(v^\alpha\left[\left({}^{LF}E_\alpha\left[-\frac{\partial^{2a}}{\partial x^{2a}}\left(\sum\limits_{n=0}^{\infty}T_n(x,\tau)\right) + \sum\limits_{n=0}^{\infty}B_n + R_{\alpha,2}\left(\sum\limits_{n=0}^{\infty}X_n, \sum\limits_{n=0}^{\infty}T_n\right)\right]\right)\right]\right). \end{cases} \tag{28}$$

If both sides of Equation (28) are compared, then we obtain

$$X_0(x,\tau) = {}^{LF}E_\alpha^{-1}\left(v^{2\alpha}X(x,0)\right) + {}^{LF}E_\alpha^{-1}\left(v^\alpha\left[\left({}^{LF}E_\alpha[\varphi(x,\tau)]\right)\right]\right), \tag{29}$$

$$X_1(x,\tau) = -{}^{LF}E_\alpha^{-1}\left(v^\alpha\left[\left({}^{LF}E_\alpha\left[\frac{\partial^a X_0}{\partial x^a} + A_0 + R_{\alpha,1}(X_0,T_0)\right]\right)\right]\right), \tag{30}$$

$$X_2(x,\tau) = -{}^{LF}E_\alpha^{-1}\left(v^\alpha\left[\left({}^{LF}E_\alpha\left[\frac{\partial^a X_1}{\partial x^a} + A_1 + R_{\alpha,1}(X_1,T_1)\right]\right)\right]\right), \tag{31}$$

$$X_3(x,\tau) = -{}^{LF}E_\alpha^{-1}\left(v^\alpha\left[\left({}^{LF}E_\alpha\left[\frac{\partial^a X_2}{\partial x^a} + A_2 + R_{\alpha,1}(X_2,T_2)\right]\right)\right]\right), \tag{32}$$

$$T_0(x,\tau) = {}^{LF}E_\alpha^{-1}\left(v^{2\alpha}T(x,0)\right) + {}^{LF}E_\alpha^{-1}\left(v^\alpha\left[\left({}^{LF}E_\alpha[\psi(x,\tau)]\right)\right]\right), \tag{33}$$

$$T_1(x,\tau) = -{}^{LF}E_\alpha^{-1}\left(v^\alpha\left[\left({}^{LF}E_\alpha\left[\frac{\partial^a T_0}{\partial x^a} + B_0 + R_{\alpha,2}(X_0,T_0)\right]\right)\right]\right), \tag{34}$$

$$T_2(x,\tau) = -{}^{LF}E_\alpha^{-1}\left(v^\alpha\left[\left({}^{LF}E_\alpha\left[\frac{\partial^a T_1}{\partial x^a} + B_1 + R_{\alpha,2}(X_1,T_1)\right]\right)\right]\right), \tag{35}$$

$$T_3(x,\tau) = -{}^{LF}E_\alpha^{-1}\left(v^\alpha\left[\left({}^{LF}E_\alpha\left[\frac{\partial^a T_2}{\partial x^a} + B_2 + R_{\alpha,2}(X_2,T_2)\right]\right)\right]\right), \tag{36}$$

and so on.

Step 3 The analytical solution of (X,T) of the system (21) is obtained as

$$\begin{cases} X(x,\tau) = \lim\limits_{N\to\infty} \sum\limits_{n=0}^{N} X_n(x,\tau), \\ T(x,\tau) = \lim\limits_{N\to\infty} \sum\limits_{n=0}^{N} T_n(x,\tau). \end{cases} \tag{37}$$

5. Applications

In this section, the proposed method for solving a nonlinear system of LFPDEs (LFETDM) is applied.

Example 1. *Consider the following coupled nonlinear system of local fractional Burger equations [33]:*

$$\begin{cases} \frac{\partial^a X}{\partial \tau^a} - \frac{\partial^{2a} X}{\partial x^{2a}} - 2XX_x^\alpha + (XT)_x^\alpha = 0, \\ \frac{\partial^a T}{\partial \tau^a} - \frac{\partial^{2a} T}{\partial x^{2a}} - 2TT_x^\alpha + (XT)_x^\alpha = 0, \end{cases} \quad 0 < \alpha \le 1, \tag{38}$$

with the initial conditions

$$X(x,0) = sin_\alpha(x^\alpha), T(x,0) = sin_\alpha(x^\alpha). \tag{39}$$

If we apply the LFET to both sides of each equation of the system Equation (38),

$$\begin{cases} {}^{LF}E_\alpha[X(x,\tau)] = \left[v^{2\alpha}sin_\alpha(x^\alpha)\right] - v^\alpha\left[\left({}^{LF}E_\alpha\left[-\frac{\partial^{2a}X}{\partial x^{2a}} - 2XX_x^\alpha + (XT)_x^\alpha\right]\right)\right], \\ {}^{LF}E_\alpha[T(x,\tau)] = \left[v^{2\alpha}sin_\alpha(x^\alpha)\right] - v^\alpha\left[\left({}^{LF}E_\alpha\left[-\frac{\partial^{2a}T}{\partial x^{2a}} - 2TT_x^\alpha + (XT)_x^\alpha\right]\right)\right]. \end{cases} \tag{40}$$

By applying the inverse LFET to both sides of each equation in Equation (40), we have

$$\begin{cases} X(x,\tau) = sin_\alpha(x^\alpha) - {}^{LF}E_\alpha{}^{-1}\left(v^\alpha\left[\left({}^{LF}E_\alpha\left[-\frac{\partial^{2a}X}{\partial x^{2a}} - 2XX_x^\alpha + (XT)_x^\alpha\right]\right)\right]\right), \\ T(x,\tau) = sin_\alpha(x^\alpha) - {}^{LF}E_\alpha{}^{-1}\left(v^\alpha\left[\left({}^{LF}E_\alpha\left[-\frac{\partial^{2a}T}{\partial x^{2a}} - 2TT_x^\alpha + (XT)_x^\alpha\right]\right)\right]\right). \end{cases} \tag{41}$$

If the Adomian decomposition method is applied, then each function of the solution (X, T) is decomposed as an infinite series

$$\begin{cases} X(x,\tau) = \sum\limits_{n=0}^{\infty} X_n(x,\tau), \\ T(x,\tau) = \sum\limits_{n=0}^{\infty} T_n(x,\tau), \end{cases} \tag{42}$$

and the nonlinear terns can be decomposed as

$$XX_x^{(\alpha)} = \sum_{n=0}^{\infty} A_n(X), \tag{43}$$

$$(XT)_x^{(\alpha)} = \sum_{n=0}^{\infty} B_n(X,T), \tag{44}$$

and

$$TT_x^{(\alpha)} = \sum_{n=0}^{\infty} C_n(T). \tag{45}$$

If Equations (42)–(45) are substituted into Equation (41), then we have

$$\begin{cases} \sum\limits_{n=0}^{\infty} X_n(x,\tau) = sin_\alpha(x^\alpha) - {}^{LF}E_\alpha{}^{-1}\left(v^\alpha\left[\left({}^{LF}E_\alpha\left[-\frac{\partial^{2a}}{\partial x^{2a}}\left(\sum\limits_{n=0}^{\infty} X_n(x,\tau)\right) - 2\sum\limits_{n=0}^{\infty} A_n(X) + \sum\limits_{n=0}^{\infty} B_n(X,T)\right]\right)\right]\right), \\ \sum\limits_{n=0}^{\infty} T_n(x,\tau) = sin_\alpha(x^\alpha) - {}^{LF}E_\alpha{}^{-1}\left(v^\alpha\left[\left({}^{LF}E_\alpha\left[-\frac{\partial^{2a}}{\partial x^{2a}}\left(\sum\limits_{n=0}^{\infty} T_n(x,\tau)\right) - 2\sum\limits_{n=0}^{\infty} C_n(T) + \sum\limits_{n=0}^{\infty} B_n(X,T)\right]\right)\right]\right). \end{cases} \tag{46}$$

By comparing both sides of Equation (46), we obtain

$$X_0(x,\tau) = sin_\alpha(x^\alpha), \tag{47}$$

$$X_1(x,\tau) = -{}^{LF}E_\alpha{}^{-1}\left(v^\alpha\left[\left({}^{LF}E_\alpha\left[-\frac{\partial^{2a}X_0}{\partial x^{2a}} - 2A_0(X) + B_0(X,T)\right]\right)\right]\right), \tag{48}$$

$$X_2(x,\tau) = -{}^{LF}E_\alpha{}^{-1}\left(v^\alpha\left[\left({}^{LF}E_\alpha\left[-\frac{\partial^{2a}X_1}{\partial x^{2a}} - 2A_1(X) + B_1(X,T)\right]\right)\right]\right), \tag{49}$$

$$X_3(x,\tau) = -{}^{LF}E_\alpha{}^{-1}\left(v^\alpha\left[\left({}^{LF}E_\alpha\left[-\frac{\partial^{2a}X_2}{\partial x^{2a}} - 2A_2(X) + B_2(X,T)\right]\right)\right]\right), \tag{50}$$

$$T_0(x,\tau) = sin_\alpha(x^\alpha), \tag{51}$$

$$T_1(x,\tau) = -{}^{LF}E_\alpha{}^{-1}\left(v^\alpha\left[\left({}^{LF}E_\alpha\left[-\frac{\partial^{2a}T_0}{\partial x^{2a}} - 2C_0(T) + B_0(X,T)\right]\right)\right]\right), \tag{52}$$

$$T_2(x,\tau) = -{}^{LF}E_\alpha{}^{-1}\left(v^\alpha\left[\left({}^{LF}E_\alpha\left[-\frac{\partial^{2a}T_1}{\partial x^{2a}} - 2C_1(T) + B_1(X,T)\right]\right)\right]\right), \tag{53}$$

$$T_3(x,\tau) = -{}^{LF}E_\alpha{}^{-1}\left(v^\alpha\left[\left({}^{LF}E_\alpha\left[-\frac{\partial^{2a}T_2}{\partial x^{2a}} - 2C_2(T) + B_2(X,T)\right]\right)\right]\right), \tag{54}$$

and so on.

The first few components of $A_n(X)$, $B_n(X,T)$ and $C_n(T)$ polynomials [41] are obtained as

$$A_0(X) = X_0 X_{0,x}{}^{(\alpha)}, \tag{55}$$

$$A_1(X) = X_0 X_{1,x}{}^{(\alpha)} + X_1 X_{0,x}{}^{(\alpha)}, \tag{56}$$

$$A_2(X) = X_0 X_{2,x}{}^{(\alpha)} + X_2 X_{0,x}{}^{(\alpha)} + X_1 X_{1,x}{}^{(\alpha)}, \tag{57}$$

$$B_0(X,T) = (X_0 T_0)_x{}^{(\alpha)}, \tag{58}$$

$$B_1(X,T) = (X_0 T_1 + X_1 T_0)_x{}^{(\alpha)}, \tag{59}$$

$$B_2(X,T) = (X_1 T_1 + X_0 T_2 + X_2 T_0)_x{}^{(\alpha)}, \tag{60}$$

$$C_0(X) = T_0 T_{0,x}{}^{(\alpha)}, \tag{61}$$

$$C_1(T) = T_0 T_{1,x}{}^{(\alpha)} + T_1 T_{0,x}{}^{(\alpha)}, \tag{62}$$

$$C_2(T) = T_0 T_{2,x}{}^{(\alpha)} + T_2 T_{0,x}{}^{(\alpha)} + T_1 T_{1,x}{}^{(\alpha)}, \tag{63}$$

and so on.

According to Equations (47)–(54) and Equations (55)–(63), the first terms of LFETDM of the system Equation (38) are obtained as

$$X_0(x,\tau) = sin_\alpha(x^\alpha), \tag{64}$$

$$X_1(x,\tau) = -sin_\alpha(x^\alpha)\frac{\tau^\alpha}{\Gamma(1+\alpha)}, \tag{65}$$

$$X_2(x,\tau) = sin_\alpha(x^\alpha)\frac{\tau^{2\alpha}}{\Gamma(1+2\alpha)}, \tag{66}$$

$$X_3(x,\tau) = -sin_\alpha(x^\alpha)\frac{\tau^{3\alpha}}{\Gamma(1+3\alpha)}, \tag{67}$$

and

$$T_0(x,\tau) = sin_\alpha(x^\alpha), \tag{68}$$

$$T_1(x,\tau) = -sin_\alpha(x^\alpha)\frac{\tau^\alpha}{\Gamma(1+\alpha)}, \tag{69}$$

$$T_2(x,\tau) = sin_\alpha(x^\alpha)\frac{\tau^{2\alpha}}{\Gamma(1+2\alpha)}, \tag{70}$$

$$T_3(x,\tau) = -sin_\alpha(x^\alpha)\frac{\tau^{3\alpha}}{\Gamma(1+3\alpha)}, \tag{71}$$

and so on.

Thus, the local fractional series solutions of $X(x,\tau)$ and $T(x,\tau)$ are, respectively, obtained as

$$\begin{cases} X(x,\tau) = \sum_{n=0}^{\infty} X_n(x,\tau), \\ T(x,\tau) = \sum_{n=0}^{\infty} T_n(x,\tau), \end{cases} \tag{72}$$

Therefore, the local fractional series solutions of the system Equation (38) can be written in the form:

$$\begin{cases} X(x,\tau) = sin_\alpha(x^\alpha) \lim_{N\to\infty} \sum_{n=0}^{N} \frac{(-1)^n (\tau)^{n\alpha}}{\Gamma(1+n\alpha)}, \\ T(x,\tau) = sin_\alpha(x^\alpha) \lim_{N\to\infty} \sum_{n=0}^{N} \frac{(-1)^n (\tau)^{n\alpha}}{\Gamma(1+n\alpha)}. \end{cases} \tag{73}$$

If Equation (73) is rewritten in a closed form, then the non-differentiable solutions of $X(x,\tau)$ and $T(x,\tau)$ are obtained as

$$\begin{cases} X(x,\tau) = sin_\alpha(x^\alpha) E_\alpha(-\tau^\alpha), \\ T(x,\tau) = sin_\alpha(x^\alpha) E_\alpha(-\tau^\alpha). \end{cases} \tag{74}$$

After substituting $\alpha = 1$ into Equation (74), we obtain

$$\begin{cases} X(x,\tau) = \sin(x) e^{-\tau}, \\ T(x,\tau) = \sin(x) e^{-\tau}. \end{cases} \tag{75}$$

Equation (66) can supply the exact solutions of Equation (38). Additionally, the LFETDM solutions of Equation (38) are same as both solutions obtained in [33] by the Yang–Laplace decomposition method and in [42] by the natural decomposition method.

The graph of the LFETDM solution for $alpha = 1$ and the exact solution of the system (38) is shown in Figure 1.
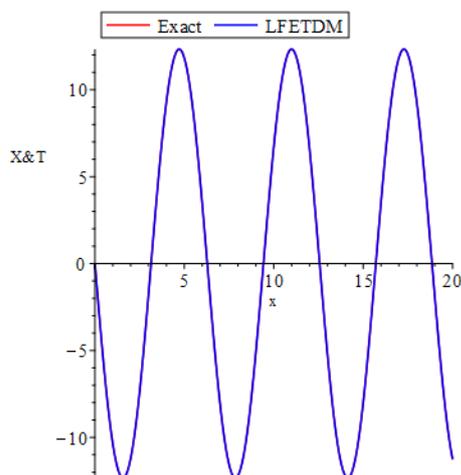
Figure 1. A comparison of the LFETDM solution and the exact solution of the system (38).

The graph of LFETDM solutions for $alpha = 0.9$, $alpha = 0.8$, $alpha = 0.7$, $alpha = 0.6$ and the exact solution of the system (38) is shown in Figure 2.
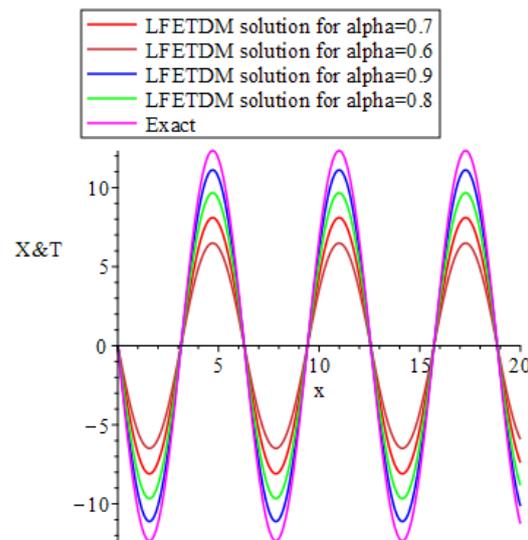
Figure 2. A comparison of the LFETDM solutions for *alpha* = 0.9, *alpha* = 0.8, *alpha = 0.7, alpha = 0.6* and the exact solution of the system (38).

Example 2. *Consider the following coupled nonlinear system of local fractional KdV equations* [39]:

$$\begin{cases} \frac{\partial^a X}{\partial \tau^a} + \frac{\partial^{3a} X}{\partial x^{3a}} + 2XX_x^\alpha + 2TX_x^\alpha = 0, \\ \frac{\partial^a T}{\partial \tau^a} + \frac{\partial^{3a} T}{\partial x^{3a}} + 2TT_x^\alpha + 2XT_x^\alpha = 0, \end{cases} \quad 0 < \alpha \le 1, \tag{76}$$

with the initial conditions

$$X(x,0) = E_\alpha(-x^\alpha), T(x,0) = -E_\alpha(-x^\alpha), \tag{77}$$

If we apply the LFET to both sides of each equation of the system in Equation (76),

$$\begin{cases} {}^{LF}E_\alpha[X(x,\tau)] = \left[v^{2\alpha}E_\alpha(-x^\alpha)\right] - {}^{LF}E_\alpha{}^{-1}\left(v^\alpha\left[\left({}^{LF}E_\alpha\left[\frac{\partial^{3a} X}{\partial x^{3a}} + 2XX_x^\alpha + 2TX_x^\alpha\right]\right)\right]\right), \\ {}^{LF}E_\alpha[T(x,\tau)] = \left[-v^{2\alpha}E_\alpha(-x^\alpha)\right] - {}^{LF}E_\alpha{}^{-1}\left(v^\alpha\left[\left({}^{LF}E_\alpha\left[\frac{\partial^{3a} T}{\partial x^{3a}} + 2TT_x^\alpha + 2XT_x^\alpha\right]\right)\right]\right). \end{cases} \tag{78}$$

By applying the inverse LFET to both sides of each equation in Equation (78), then we have

$$\begin{cases} X(x,\tau) = E_\alpha(-x^\alpha) - {}^{LF}E_\alpha{}^{-1}\left(v^\alpha\left[\left({}^{LF}E_\alpha\left[\frac{\partial^{3a} X}{\partial x^{3a}} + 2XX_x^\alpha + 2TX_x^\alpha\right]\right)\right]\right), \\ T(x,\tau) = -E_\alpha(-x^\alpha) - {}^{LF}E_\alpha{}^{-1}\left(v^\alpha\left[\left({}^{LF}E_\alpha\left[\frac{\partial^{3a} T}{\partial x^{3a}} + 2TT_x^\alpha + 2XT_x^\alpha\right]\right)\right]\right). \end{cases} \tag{79}$$

If the Adomian decomposition method is applied, then each function of the solution (X, T) is decomposed as an infinite series:

$$\begin{cases} X(x,\tau) = \sum_{n=0}^{\infty} X_n(x,\tau), \\ T(x,\tau) = \sum_{n=0}^{\infty} T_n(x,\tau), \end{cases} \tag{80}$$

and the nonlinear terms can be decomposed as

$$XX_x^\alpha = \sum_{n=0}^{\infty} A_n(X), \tag{81}$$

$$TX_x^\alpha = \sum_{n=0}^{\infty} B_n(X,T), \tag{82}$$

$$TT_x^\alpha = \sum_{n=0}^{\infty} C_n(T), \tag{83}$$

and

$$XT_x^\alpha = \sum_{n=0}^{\infty} D_n(X,T). \tag{84}$$

If Equations (80)–(84) are substituted in Equation (79), then we have

$$\begin{cases} \sum\limits_{n=0}^{\infty} X_n(x,\tau) = E_\alpha(-x^\alpha) - {}^{LF}E_\alpha^{-1}\left(v^\alpha\left[\left({}^{LF}E_\alpha\left[\frac{\partial^{3a}}{\partial x^{3a}}\left(\sum\limits_{n=0}^{\infty} X_n(x,\tau)\right) + 2\sum\limits_{n=0}^{\infty} A_n(X) + 2\sum\limits_{n=0}^{\infty} B_n(X,T)\right]\right)\right]\right), \\[3mm] \sum\limits_{n=0}^{\infty} T_n(x,\tau) = -E_\alpha(-x^\alpha) - {}^{LF}E_\alpha^{-1}\left(v^\alpha\left[\left({}^{LF}E_\alpha\left[\frac{\partial^{3a}}{\partial x^{3a}}\left(\sum\limits_{n=0}^{\infty} T_n(x,\tau)\right) + 2\sum\limits_{n=0}^{\infty} C_n(T) + 2\sum\limits_{n=0}^{\infty} D_n(X,T)\right]\right)\right]\right). \end{cases} \tag{85}$$

By comparing both sides of Equation (85), we obtain

$$X_0(x,\tau) = E_\alpha(-x^\alpha), \tag{86}$$

$$X_1(x,\tau) = -{}^{LF}E_\alpha^{-1}\left(v^\alpha\left[\left({}^{LF}E_\alpha\left[\frac{\partial^{3a}X_0}{\partial x^{3a}} + 2A_0(X) + 2B_0(X,T)\right]\right)\right]\right), \tag{87}$$

$$X_2(x,\tau) = -{}^{LF}E_\alpha^{-1}\left(v^\alpha\left[\left({}^{LF}E_\alpha\left[\frac{\partial^{3a}X_1}{\partial x^{3a}} + 2A_1(X) + 2B_1(X,T)\right]\right)\right]\right), \tag{88}$$

$$X_3(x,\tau) = -{}^{LF}E_\alpha^{-1}\left(v^\alpha\left[\left({}^{LF}E_\alpha\left[\frac{\partial^{3a}X_2}{\partial x^{3a}} + 2A_2(X) + 2B_2(X,T)\right]\right)\right]\right), \tag{89}$$

$$T_0(x,\tau) = -E_\alpha(-x^\alpha), \tag{90}$$

$$T_1(x,\tau) = -{}^{LF}E_\alpha^{-1}\left(v^\alpha\left[\left({}^{LF}E_\alpha\left[\frac{\partial^{3a}T_0}{\partial x^{3a}} + 2C_0(T) + 2D_0(X,T)\right]\right)\right]\right), \tag{91}$$

$$T_2(x,\tau) = -{}^{LF}E_\alpha^{-1}\left(v^\alpha\left[\left({}^{LF}E_\alpha\left[\frac{\partial^{3a}T_1}{\partial x^{3a}} + 2C_1(T) + 2D_1(X,T)\right]\right)\right]\right), \tag{92}$$

$$T_3(x,\tau) = -{}^{LF}E_\alpha^{-1}\left(v^\alpha\left[\left({}^{LF}E_\alpha\left[\frac{\partial^{3a}T_2}{\partial x^{3a}} + 2C_2(T) + 2D_2(X,T)\right]\right)\right]\right), \tag{93}$$

and so on.

The first few components of $A_n(X)$, $B_n(X,T)$, $C_n(T)$ and $D_n(X,T)$ polynomials [39] are obtained as

$$A_0(X) = X_0 X_{0,x}{}^{(\alpha)}, \tag{94}$$

$$A_1(X) = X_0 X_{1,x}{}^{(\alpha)} + X_1 X_{0,x}{}^{(\alpha)}, \tag{95}$$

$$A_2(X) = X_0 X_{2,x}{}^{(\alpha)} + X_2 X_{0,x}{}^{(\alpha)} + X_1 X_{1,x}{}^{(\alpha)}, \tag{96}$$

$$B_0(X,T) = T_0 X_{0,x}{}^{(\alpha)}, \tag{97}$$

$$B_1(X,T) = T_0 X_{1,x}{}^{(\alpha)} + T_1 X_{0,x}{}^{(\alpha)}, \tag{98}$$

$$B_2(X,T) = T_0 X_{2,x}{}^{(\alpha)} + T_1 X_{1,x}{}^{(\alpha)} + T_2 X_{0,x}{}^{(\alpha)}, \tag{99}$$

$$C_0(X) = T_0 T_{0,x}{}^{(\alpha)}, \tag{100}$$

$$C_1(T) = T_0 T_{1,x}{}^{(\alpha)} + T_1 T_{0,x}{}^{(\alpha)}, \tag{101}$$

$$C_2(T) = T_0 T_{2,x}{}^{(\alpha)} + T_2 T_{0,x}{}^{(\alpha)} + T_1 T_{1,x}{}^{(\alpha)}, \tag{102}$$

$$D_0(X, T) = X_0 T_{0,x}{}^{(\alpha)}, \tag{103}$$

$$D_1(X, T) = X_0 T_{1,x}{}^{(\alpha)} + X_1 T_{0,x}{}^{(\alpha)}, \tag{104}$$

$$D_2(X, T) = X_0 T_{2,x}{}^{(\alpha)} + X_1 T_{1,x}{}^{(\alpha)} + X_2 T_{0,x}{}^{(\alpha)}, \tag{105}$$

and so on.

According to Equations (86)–(93) and Equations (94)–(105), the first terms of LFETDM of the system in Equation (76) are obtained as

$$X_0(x, \tau) = E_\alpha(-x^\alpha), \tag{106}$$

$$X_1(x, \tau) = E_\alpha(-x^\alpha) \frac{\tau^\alpha}{\Gamma(1+\alpha)}, \tag{107}$$

$$X_2(x, \tau) = E_\alpha(-x^\alpha) \frac{\tau^{2\alpha}}{\Gamma(1+2\alpha)}, \tag{108}$$

$$X_3(x, \tau) = E_\alpha(-x^\alpha) \frac{\tau^{3\alpha}}{\Gamma(1+3\alpha)}, \tag{109}$$

and

$$T_0(x, \tau) = -E_\alpha(-x^\alpha), \tag{110}$$

$$T_1(x, \tau) = -E_\alpha(-x^\alpha) \frac{\tau^\alpha}{\Gamma(1+\alpha)}, \tag{111}$$

$$T_2(x, \tau) = -E_\alpha(-x^\alpha) \frac{\tau^{2\alpha}}{\Gamma(1+2\alpha)}, \tag{112}$$

$$T_3(x, \tau) = -E_\alpha(-x^\alpha) \frac{\tau^{3\alpha}}{\Gamma(1+3\alpha)}, \tag{113}$$

and so on.

Thus, the local fractional series solutions of $X(x, \tau)$ and $T(x, \tau)$ are, respectively, obtained as

$$\begin{cases} X(x, \tau) = \sum\limits_{n=0}^{\infty} X_n(x, \tau), \\ T(x, \tau) = \sum\limits_{n=0}^{\infty} T_n(x, \tau), \end{cases} \tag{114}$$

Therefore, the local fractional series solutions of the system in Equation (76) can be written in the form

$$\begin{cases} X(x, \tau) = E_\alpha(-x^\alpha) \lim\limits_{N \to \infty} \sum\limits_{n=0}^{N} \frac{\tau^{n\alpha}}{\Gamma(1+n\alpha)}, \\ T(x, \tau) = -E_\alpha(-x^\alpha) \lim\limits_{N \to \infty} \sum\limits_{n=0}^{N} \frac{\tau^{n\alpha}}{\Gamma(1+n\alpha)}. \end{cases} \tag{115}$$

If Equation (115) is rewritten in a closed form, then the non-differentiable solutions of $X(x, \tau)$ and $T(x, \tau)$ are obtained as

$$\begin{cases} X(x, \tau) = E_\alpha(-x^\alpha) E_\alpha(\tau^\alpha), \\ T(x, \tau) = -E_\alpha(-x^\alpha) E_\alpha(\tau^\alpha). \end{cases} \tag{116}$$

By substituting $\alpha = 1$ into Equation (116), we obtain

$$\begin{cases} X(x, \tau) = e^{-x+\tau}, \\ T(x, \tau) = -e^{-x+\tau}. \end{cases} \tag{117}$$

Equation (117) provides the exact solutions of Equation (76). Additionally, the LFETDM solutions of Equation (76) are the same as the solutions obtained in [42] by the LFRDTM.

Example 3. *Consider the following coupled nonlinear system of local fractional partial differential equations [33]. For $0 < \alpha \leq 1$,*

$$
\begin{cases}
X_\tau{}^{(\alpha)} + T_x{}^{(\alpha)} Z_\nu{}^{(\alpha)} - T_\nu{}^{(\alpha)} Z_x{}^{(\alpha)} = -X, \\
T_\tau{}^{(\alpha)} + X_x{}^{(\alpha)} Z_\nu{}^{(\alpha)} + X_\nu{}^{(\alpha)} Z_x{}^{(\alpha)} = T, \\
Z_\tau{}^{(\alpha)} + X_x{}^{(\alpha)} T_\nu{}^{(\alpha)} + X_\nu{}^{(\alpha)} T_x{}^{(\alpha)} = Z,
\end{cases}
\tag{118}
$$

with the initial conditions

$$
X(x, \nu, 0) = E_\alpha(x^\alpha + \nu^\alpha), \ T(x, \nu, 0) = E_\alpha(x^\alpha - \nu^\alpha), Z(x, \nu, 0) = E_\alpha(-x^\alpha + \nu^\alpha).
\tag{119}
$$

If we apply the LFET to both sides of each equation of the system in Equation (118),

$$
\begin{cases}
{}^{LF}E_\alpha[X(x, \nu, \tau)] = [\nu^{2\alpha} E_\alpha(x^\alpha + \nu^\alpha)] - \nu^\alpha \left[\left({}^{LF}E_\alpha \left[T_x{}^{(\alpha)} Z_\nu{}^{(\alpha)} - T_\nu{}^{(\alpha)} Z_x{}^{(\alpha)} + X \right] \right) \right], \\
{}^{LF}E_\alpha[T(x, \nu, \tau)] = [\nu^{2\alpha} E_\alpha(x^\alpha - \nu^\alpha)] - \nu^\alpha \left[\left({}^{LF}E_\alpha \left[X_x{}^{(\alpha)} Z_\nu{}^{(\alpha)} + X_\nu{}^{(\alpha)} Z_x{}^{(\alpha)} - T \right] \right) \right], \\
{}^{LF}E_\alpha[Z(x, \nu, \tau)] = [\nu^{2\alpha} E_\alpha(-x^\alpha + \nu^\alpha)] - \nu^\alpha \left[\left({}^{LF}E_\alpha \left[X_x{}^{(\alpha)} T_\nu{}^{(\alpha)} + X_\nu{}^{(\alpha)} T_x{}^{(\alpha)} - Z \right] \right) \right].
\end{cases}
\tag{120}
$$

By applying the inverse LFET on both sides of each equation of Equation (120), we have

$$
\begin{cases}
X(x, \nu, \tau) = E_\alpha(x^\alpha + \nu^\alpha) - {}^{LF}E_\alpha{}^{-1} \left(\nu^\alpha \left[\left({}^{LF}E_\alpha \left[T_x{}^{(\alpha)} Z_\nu{}^{(\alpha)} - T_\nu{}^{(\alpha)} Z_x{}^{(\alpha)} + X \right] \right) \right] \right), \\
T(x, \nu, \tau) = E_\alpha(x^\alpha - \nu^\alpha) - {}^{LF}E_\alpha{}^{-1} \left(\nu^\alpha \left[\left({}^{LF}E_\alpha \left[X_x{}^{(\alpha)} Z_\nu{}^{(\alpha)} + X_\nu{}^{(\alpha)} Z_x{}^{(\alpha)} - T \right] \right) \right] \right), \\
Z(x, \nu, \tau) = E_\alpha(-x^\alpha + \nu^\alpha) - {}^{LF}E_\alpha{}^{-1} \left(\nu^\alpha \left[\left({}^{LF}E_\alpha \left[X_x{}^{(\alpha)} T_\nu{}^{(\alpha)} + X_\nu{}^{(\alpha)} T_x{}^{(\alpha)} - Z \right] \right) \right] \right).
\end{cases}
\tag{121}
$$

By applying the Adomian decomposition method (ADM), each function of the solution (X, T, Z) is decomposed as an infinite series

$$
\begin{cases}
X(x, \nu, \tau) = \sum_{n=0}^{\infty} X_n(x, \nu, \tau), \\
T(x, \nu, \tau) = \sum_{n=0}^{\infty} T_n(x, \nu, \tau), \\
Z(x, \nu, \tau) = \sum_{n=0}^{\infty} Z_n(x, \nu, \tau),
\end{cases}
\tag{122}
$$

and the nonlinear terns can be decomposed as

$$
T_x{}^{(\alpha)} Z_\nu{}^{(\alpha)} = \sum_{n=0}^{\infty} A_n(T, Z), T_\nu{}^{(\alpha)} Z_x{}^{(\alpha)} = \sum_{n=0}^{\infty} A_n{}'(T, Z),
\tag{123}
$$

$$
X_x{}^{(\alpha)} Z_\nu{}^{(\alpha)} = \sum_{n=0}^{\infty} B_n(X, Z), X_\nu{}^{(\alpha)} Z_x{}^{(\alpha)} = \sum_{n=0}^{\infty} B_n{}'(X, Z),
\tag{124}
$$

$$
X_x{}^{(\alpha)} T_\nu{}^{(\alpha)} = \sum_{n=0}^{\infty} C_n(X, T), X_\nu{}^{(\alpha)} T_x{}^{(\alpha)} = \sum_{n=0}^{\infty} C_n{}'(X, T).
\tag{125}
$$

If Equations (122)–(125) are substituted into Equation (121), then we have

$$
\begin{cases}
\sum\limits_{n=0}^{\infty} X_n(x,\nu,\tau) = E_\alpha(x^\alpha + \nu^\alpha) - {}^{LF}E_\alpha^{-1}\left(\nu^\alpha\left[\left({}^{LF}E_\alpha\left[\sum\limits_{n=0}^{\infty} A_n(T,Z) - \sum\limits_{n=0}^{\infty} A_n{}'(T,Z) + \sum\limits_{n=0}^{\infty} X_n(x,\nu,\tau)\right]\right)\right]\right), \\[4pt]
\sum\limits_{n=0}^{\infty} T_n(x,\nu,\tau) = E_\alpha(x^\alpha - \nu^\alpha) - {}^{LF}E_\alpha^{-1}\left(\nu^\alpha\left[\left({}^{LF}E_\alpha\left[\sum\limits_{n=0}^{\infty} B_n(X,Z) + \sum\limits_{n=0}^{\infty} B_n{}'(X,Z) - \sum\limits_{n=0}^{\infty} T_n(x,\nu,\tau)\right]\right)\right]\right), \\[4pt]
\sum\limits_{n=0}^{\infty} Z_n(x,\nu,\tau) = E_\alpha(-x^\alpha + \nu^\alpha) - {}^{LF}E_\alpha^{-1}\left(\nu^\alpha\left[\left({}^{LF}E_\alpha\left[\sum\limits_{n=0}^{\infty} C_n(X,T) + \sum\limits_{n=0}^{\infty} C_n{}'(X,T) - \sum\limits_{n=0}^{\infty} Z_n(x,\nu,\tau)\right]\right)\right]\right).
\end{cases}
\tag{126}
$$

By comparing both sides of Equation (126), we obtain

$$
X_0(x,\nu,\tau) = E_\alpha(x^\alpha + \nu^\alpha),
\tag{127}
$$

$$
X_1(x,\nu,\tau) = -{}^{LF}E_\alpha^{-1}\left(\nu^\alpha\left[\left({}^{LF}E_\alpha\left[A_0(T,Z) - A_0{}'(T,Z) + X_0(x,\nu,\tau)\right]\right)\right]\right),
\tag{128}
$$

$$
X_2(x,\nu,\tau) = -{}^{LF}E_\alpha^{-1}\left(\nu^\alpha\left[\left({}^{LF}E_\alpha\left[A_1(T,Z) - A_1{}'(T,Z) + X_1(x,\nu,\tau)\right]\right)\right]\right),
\tag{129}
$$

$$
X_3(x,\nu,\tau) = -{}^{LF}E_\alpha^{-1}\left(\nu^\alpha\left[\left({}^{LF}E_\alpha\left[A_2(T,Z) - A_2{}'(T,Z) + X_2(x,\nu,\tau)\right]\right)\right]\right),
\tag{130}
$$

$$
T_0(x,\nu,\tau) = E_\alpha(x^\alpha - \nu^\alpha),
\tag{131}
$$

$$
T_1(x,\nu,\tau) = -{}^{LF}E_\alpha^{-1}\left(\nu^\alpha\left[\left({}^{LF}E_\alpha\left[B_0(X,Z) - B_0{}'(X,Z) + T_0(x,\nu,\tau)\right]\right)\right]\right),
\tag{132}
$$

$$
T_2(x,\nu,\tau) = -{}^{LF}E_\alpha^{-1}\left(\nu^\alpha\left[\left({}^{LF}E_\alpha\left[B_1(X,Z) - B_1{}'(X,Z) + T_1(x,\nu,\tau)\right]\right)\right]\right),
\tag{133}
$$

$$
T_3(x,\nu,\tau) = -{}^{LF}E_\alpha^{-1}\left(\nu^\alpha\left[\left({}^{LF}E_\alpha\left[B_2(X,Z) - B_2{}'(X,Z) + T_2(x,\nu,\tau)\right]\right)\right]\right),
\tag{134}
$$

$$
Z_0(x,\nu,\tau) = E_\alpha(-x^\alpha + \nu^\alpha),
\tag{135}
$$

$$
Z_1(x,\nu,\tau) = -{}^{LF}E_\alpha^{-1}\left(\nu^\alpha\left[\left({}^{LF}E_\alpha\left[C_0(X,T) - C_0{}'(X,T) + Z_0(x,\nu,\tau)\right]\right)\right]\right),
\tag{136}
$$

$$
Z_2(x,\nu,\tau) = -{}^{LF}E_\alpha^{-1}\left(\nu^\alpha\left[\left({}^{LF}E_\alpha\left[C_1(X,T) - C_1{}'(X,T) + Z_1(x,\nu,\tau)\right]\right)\right]\right),
\tag{137}
$$

$$
Z_3(x,\nu,\tau) = -{}^{LF}E_\alpha^{-1}\left(\nu^\alpha\left[\left({}^{LF}E_\alpha\left[C_2(X,T) - C_2{}'(X,T) + Z_2(x,\nu,\tau)\right]\right)\right]\right),
\tag{138}
$$

and so on.

The first few components of $A_n(T,Z)$, $B_n(X,Z)$ and $C_n(X,T)$ polynomials [41] are obtained as

$$
A_0(T,Z) = T_{0x}{}^{(\alpha)} Z_{0\nu}{}^{(\alpha)},
\tag{139}
$$

$$
A_1(T,Z) = T_{1x}{}^{(\alpha)} Z_{0\nu}{}^{(\alpha)} + T_{0x}{}^{(\alpha)} Z_{1\nu}{}^{(\alpha)},
\tag{140}
$$

$$
A_2(T,Z) = T_{0x}{}^{(\alpha)} Z_{2\nu}{}^{(\alpha)} + T_{2x}{}^{(\alpha)} Z_{0\nu}{}^{(\alpha)} + T_{1x}{}^{(\alpha)} Z_{1\nu}{}^{(\alpha)},
\tag{141}
$$

$$
B_0(X,Z) = X_{0x}{}^{(\alpha)} Z_{0\nu}{}^{(\alpha)},
\tag{142}
$$

$$
B_1(X,Z) = X_{1x}{}^{(\alpha)} Z_{0\nu}{}^{(\alpha)} + X_{0x}{}^{(\alpha)} Z_{1\nu}{}^{(\alpha)},
\tag{143}
$$

$$
B_2(X,Z) = X_{0x}{}^{(\alpha)} Z_{2\nu}{}^{(\alpha)} + X_{2x}{}^{(\alpha)} Z_{0\nu}{}^{(\alpha)} + X_{1x}{}^{(\alpha)} Z_{1\nu}{}^{(\alpha)},
\tag{144}
$$

$$
C_0(X,T) = X_{0x}{}^{(\alpha)} T_{0\nu}{}^{(\alpha)},
\tag{145}
$$

$$
C_1(X,T) = X_{1x}{}^{(\alpha)} T_{0\nu}{}^{(\alpha)} + X_{0x}{}^{(\alpha)} T_{1\nu}{}^{(\alpha)},
\tag{146}
$$

$$
C_2(X,T) = X_{0x}{}^{(\alpha)} T_{2\nu}{}^{(\alpha)} + X_{2x}{}^{(\alpha)} T_{0\nu}{}^{(\alpha)} + X_{1x}{}^{(\alpha)} T_{1\nu}{}^{(\alpha)},
\tag{147}
$$

and so on.

Additionally, the polynomials $A_n{}'$, $B_n{}'$ and $C_n{}'$ are similarly computed.

According to Equations (127)–(138) and Equations (139)–(147), the first terms of the local fractional Elzaki transform decomposition method of the system Equation (118) are obtained as

$$X_0(x, v, \tau) = E_\alpha(x^\alpha + v^\alpha), \tag{148}$$

$$X_1(x, v, \tau) = -E_\alpha(x^\alpha + v^\alpha)\frac{\tau^\alpha}{\Gamma(1+\alpha)}, \tag{149}$$

$$X_2(x, v, \tau) = E_\alpha(x^\alpha + v^\alpha)\frac{\tau^{2\alpha}}{\Gamma(1+2\alpha)}, \tag{150}$$

$$X_3(x, v, \tau) = -E_\alpha(x^\alpha + v^\alpha)\frac{\tau^{3\alpha}}{\Gamma(1+3\alpha)}, \tag{151}$$

$$T_0(x, v, \tau) = E_\alpha(x^\alpha - v^\alpha), \tag{152}$$

$$T_1(x, v, \tau) = E_\alpha(x^\alpha - v^\alpha)\frac{\tau^\alpha}{\Gamma(1+\alpha)}, \tag{153}$$

$$T_2(x, v, \tau) = E_\alpha(x^\alpha - v^\alpha)\frac{\tau^{2\alpha}}{\Gamma(1+2\alpha)}, \tag{154}$$

$$T_3(x, v, \tau) = E_\alpha(x^\alpha - v^\alpha)\frac{\tau^{3\alpha}}{\Gamma(1+3\alpha)}, \tag{155}$$

and

$$Z_0(x, v, \tau) = E_\alpha(-x^\alpha + v^\alpha), \tag{156}$$

$$Z_1(x, v, \tau) = E_\alpha(-x^\alpha + v^\alpha)\frac{\tau^\alpha}{\Gamma(1+\alpha)}, \tag{157}$$

$$Z_2(x, v, \tau) = E_\alpha(-x^\alpha + v^\alpha)\frac{\tau^{2\alpha}}{\Gamma(1+2\alpha)}, \tag{158}$$

$$Z_3(x, v, \tau) = E_\alpha(-x^\alpha + v^\alpha)\frac{\tau^{3\alpha}}{\Gamma(1+3\alpha)}, \tag{159}$$

and so on.

Thus, the local fractional series solutions of $X(x, v, \tau)$, $T(x, v, \tau)$ and $Z(x, v, \tau)$ are, respectively, obtained as

$$\begin{cases} X(x, v, \tau) = \sum\limits_{n=0}^{\infty} X_n(x, v, \tau), \\ T(x, v, \tau) = \sum\limits_{n=0}^{\infty} T_n(x, v, \tau), \\ Z(x, v, \tau) = \sum\limits_{n=0}^{\infty} Z_n(x, v, \tau), \end{cases} \tag{160}$$

Therefore, the local fractional series solutions of the system Equation (118) can be written in the form

$$\begin{cases} X(x, v, \tau) = E_\alpha(x^\alpha + v^\alpha)\lim\limits_{N\to\infty}\sum\limits_{n=0}^{N}\frac{(-1)^n\tau^{n\alpha}}{\Gamma(1+n\alpha)}, \\ T(x, v, \tau) = E_\alpha(x^\alpha - v^\alpha)\lim\limits_{N\to\infty}\sum\limits_{n=0}^{N}\frac{\tau^{n\alpha}}{\Gamma(1+n\alpha)}, \\ Z(x, v, \tau) = E_\alpha(-x^\alpha + v^\alpha)\lim\limits_{N\to\infty}\sum\limits_{n=0}^{N}\frac{\tau^{n\alpha}}{\Gamma(1+n\alpha)}. \end{cases} \tag{161}$$

If Equation (161) is rewritten in a closed form, then the non-differentiable solutions of $X(x, v, \tau)$, $T(x, v, \tau)$ and $Z(x, v, \tau)$ are obtained as

$$\begin{cases} X(x, v, \tau) = E_\alpha(x^\alpha + v^\alpha)E_\alpha(-\tau^\alpha), \\ T(x, v, \tau) = E_\alpha(x^\alpha - v^\alpha)E_\alpha(\tau^\alpha), \\ Z(x, v, \tau) = E_\alpha(-x^\alpha + v^\alpha)E_\alpha(\tau^\alpha). \end{cases} \tag{162}$$

Additionally, Equation (163) is obtained from Equation (162) as [32]

$$\begin{cases} X(x, v, \tau) = E_\alpha(x^\alpha + v^\alpha - \tau^\alpha), \\ T(x, v, \tau) = E_\alpha(x^\alpha - v^\alpha + \tau^\alpha), \\ Z(x, v, \tau) = E_\alpha(-x^\alpha + v^\alpha + \tau^\alpha). \end{cases} \tag{163}$$

By substituting $\alpha = 1$ into Equation (163), it is obtained as

$$\begin{cases} X(x, v, \tau) = e^{x+v-\tau}, \\ T(x, v, \tau) = e^{x-v+\tau}, \\ Z(x, v, \tau) = e^{-x+v+\tau}. \end{cases} \tag{164}$$

The solutions in Equation (164) are the exact solutions of Equation (118). Additionally, the LFETDM solutions in Equation (163) are the same as the solution obtained in [32] by the Yang–Laplace decomposition method and in [41] by the natural decomposition method.

6. Discussion

It has observed from the solutions of a coupled nonlinear system of local fractional Burger equations that LFETDM, LFRDTM, the Yang–Laplace decomposition method and the natural decomposition method give the same results. Additionally, Figures 1 and 2 show the results obtained by using LFETDM for Example 1. Besides, it has been observed that the solutions decrease as the alpha values move away from 1. Additionally, we saw from the solutions of a coupled nonlinear system of local fractional partial differential equations that LFETDM, the Yang–Laplace decomposition method, LFRDTM and the natural decomposition method give the same results. It is seen that the overall structure of the curve obtained from Maple software differs for different values of fractional order αlpha for Example 1.

7. Conclusions

The nonlinear systems of local fractional partial differential equations have been solved by the local fractional Elzaki transform decomposition method (LFETDM). Additionally, Figures 1 and 2 show the results obtained by using LFETDM for Example 1. It has been seen that the solutions decrease as the alpha values move away from 1. Additionally, it has observed from the non-differentiable solutions of a coupled nonlinear system of local fractional partial differential equations that LFETDM, the Yang–Laplace decomposition method, LFRDTM and the natural decomposition method give the same results. In addition, it was seen that the overall structure of the graph obtained from Maple software differed for different values of fractional order αlpha. It has been shown that LFETDM is an effective algorithm. Additionally, it was shown that this algorithm provides the solution in a series form which converges quickly and successfully to the exact solution. Thus, it is concluded that LFETDM is reliable, effective and powerful in obtaining the analytical solutions for different classes of linear and nonlinear local fractionals of ordinary and partial differential equations.

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