MONASH

UNIVERSITY

AUSTRALIA

A LOCALLY MOST MEAN POWERFUL BASED SCORE TEST FOR ARCH AND GARCH REGRESSION DISTURBANCES

John H.H. Lee and Maxwell L. King

Working Paper No. 9/91

October 1991

DEPARTMENT OF ECONOMETRICS

ISSN 1032-3813

ISBN 0 86746 942 0

A LOCALLY MOST MEAN POWERFUL BASED SCORE TEST FOR ARCH AND GARCH REGRESSION DISTURBANCES

John H.H. Lee and Maxwell L. King

Working Paper No. 9/91

October 1991

DEPARTMENT OF ECONOMETRICS, FACULTY OF ECONOMICS COMMERCE & MANAGEMENT

MONASH UNIVERSITY, CLAYTON, VICTORIA 3168, AUSTRALIA.

A LOCALLY MOST MEAN POWERFUL BASED SCORE TEST FOR ARCH AND GARCH REGRESSION DISTURBANCES

by

John H. H. Lee and Maxwell L. King
Department of Econometrics,
Monash University
Clayton, Victoria 3168,
Australia.

ABSTRACT

This paper considers the twin problems of testing for ARCH and GARCH disturbances in the linear regression model. A feature of these testing problems, ignored by the standard Lagrange multiplier test, is that they are one-sided in nature. A test which exploits this one-sided aspect is constructed based on the sum of the scores. Its small-sample size and power properties under both normal and leptokurtic disturbances are investigated via a Monte Carlo experiment. The results indicate that the new test typically has superior power to two versions of the Lagrange multiplier test and possibly also more accurate asymptotic critical values.

KEY WORDS: Autoregressive conditional heteroscedasticity; Generalized autoregressive conditional heteroscedasticity; Lagrange multiplier test; Leptokurtic regression disturbances; Monte Carlo experiment; Power.

1. INTRODUCTION

There has been considerable interest in conditionally heteroscedastic disturbance processes since Engle (1982) introduced the autoregressive conditional heteroscedasticity (ARCH) disturbance model. The ARCH model and its various derivatives, especially the generalized ARCH (GARCH) model introduced by Bollerslev (1986), have been particularly popular and useful in modelling the disturbance behaviour of regression models of monetary and financial variables. These models provide an attractive alternative to the difficult process of modelling time-varying disturbance variances using exogenous variables. They also recognize that disturbance variances can evolve over time based on past information. This gives rise to conditional heteroscedasticity as opposed to unconditional heteroscedasticity. Extensive surveys of this literature are given by Engle and Bollerslev (1986) and Bollerslev, Chou, Jayaraman and Kroner (1990).

To date there has been comparatively little emphasis in this literature on testing for the presence of ARCH and GARCH disturbances. Engle (1982) recommended the use of the Lagrange multiplier (LM) test for ARCH disturbances. Bollerslev (1986) observed that a difficulty with constructing the LM test for GARCH disturbances is that the block of the information matrix, whose inverse is required, is singular. Lee (1990) shows how this difficulty can be by-passed and finds that the LM tests for GARCH and ARCH disturbances are identical. Although, as we shall see, both testing problems are one-sided in nature, the LM test fails to exploit this and therefore may lack power. One way to make use of the one-sided nature of the problem would be to derive the Kuhn-Tucker test which is a one-sided version of the LM test introduced by Gourieroux, Holly and Monfort (1982). Unfortunately, the asymptotic distribution of the Kuhn-Tucker test under the null hypothesis is a probability mixture of chi-squared distributions and the degenerate distribution at zero which makes it a very unattractive test to apply.

Recently, SenGupta and Vermeire (1986) introduced the class of locally most mean powerful (LMMP) unbiased tests for multiparameter testing problems. These tests maximize the mean slope of the power hypersurface in the neighbourhood of the null hypothesis. King and Wu (1990) derived LMMP tests for one-sided multiparameter testing problems. The test statistic is based on the sum of scores and, as King and Wu point out, suggests an alternative form of the LM or score test for one-sided testing problems.

The aim of this paper is to derive an LMMP-based score (LBS) test for the presence of ARCH and GARCH disturbances in the linear regression model. We also investigate the small-sample properties of the new test and compare them with those of two versions of the LM test by conducting a Monte Carlo experiment. Both normal and leptokurtic pseudo-random errors are used in the experiment, the results of which suggest that typically the LBS test has better power than either version of the LM test.

The paper is organized as follows. The regression model with ARCH or GARCH disturbances and the LM and LBS tests are introduced in the next section. The experimental design of the Monte Carlo study and its results are discussed in Section 3. The more popular version of the LM test and the LBS test are found to provide conflicting inferences when applied to a model for weekly silver prices in Section 4. Some concluding remarks are made in the final section.

2. THEORY

The regression model with ARCH disturbances for the dependent variable y_t can be written as

$$y_t = x_t'b + \varepsilon_t , \qquad t = 1,\dots,n , \tag{1}$$

where

$$\varepsilon_t|\psi_{t-1} \sim IN(0,\sigma_t^2) , \tag{2}$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \dots + \alpha_q \varepsilon_{t-q}^2 , \tag{3}$$

in which ψ_t is the information set available at time t, x_t is a k×1 vector of observations on lagged endogenous and exogenous variables included in ψ_{t-1}, and b and α are unknown parameter vectors. The conditional variance of the disturbance term, ε_t, is σ_t^2 which is a function of past squared disturbance terms up to a lag of q. To ensure that the conditional variance is strictly positive for all realizations of ε_t, (3) requires that the parameter space be restricted to $\alpha_0 > 0$ and $\alpha_i \geq 0$ for $i = 1,\dots,q$. A further requirement for finite unconditional variance is that

$$\sum_{i=1}^{q} \alpha_i < 1 .$$

The GARCH regression model is given by (1) and (2), with the conditional variance equation (3) generalized to

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 . \tag{4}$$

To ensure that σ_t^2 is strictly positive for all realizations of ε_t, requires $\alpha_0 > 0$, $\alpha_i \geq 0$ and $\beta_j \geq 0$ for $i = 1,\dots,q$, $j = 1,\dots,p$. For finite unconditional variance we require

$$\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j < 1.$$

Our interest is in the twin problems of testing for the presence of ARCH or GARCH disturbances in the linear regression model. In testing for an ARCH effect, (3), the problem can be parameterized as testing

$$H_{01} : \alpha_1 = \alpha_2 = \ldots = \alpha_q = 0$$

against the alternative

$$H_{a1} : \alpha_i \geq 0, \quad i = 1,\ldots,q \text{ , with at least one strict inequality,}$$

in the context of (1), (2) and (3). In the case of testing for a GARCH effect, (4), the problem is one of testing

$$H_{02} : \alpha_1 = \alpha_2 = \ldots = \alpha_q = \beta_1 = \ldots = \beta_p = 0$$

against the alternative

$H_{a2} : \alpha_i \geq 0,\ \beta_j \geq 0,\quad i = 1,\ldots,q\ ,\ j = 1,\ldots,p,$ with at least one strict inequality,

in the context of (1), (2) and (4).

If e_t, $t = 1,\ldots,n$, represent the ordinary least squares (OLS) residuals from (1) and $\hat{\sigma}^2$ is the maximum likelihood estimator of $\sigma^2 = \alpha_0$, the disturbance variance under H_{01} (or equivalently H_{02}), then the LM test statistic against ARCH disturbances has the form

$$LM_{ARCH} = f^{0\prime} W(W'W)^{-1}W'f^0/2 \qquad (5)$$

where

$$W' = \left[w_{q+1} \vdots \ldots\ldots \vdots w_n \right] ,$$

$$w_t = \left(1, e_{t-1}^2, \ldots\ldots, e_{t-q}^2 \right) ,$$

$$f^{0\prime} = \left(e_{q+1}^2 \Big/ \hat{\sigma}^2 - 1, \ldots\ldots, e_n^2 \Big/ \hat{\sigma}^2 - 1 \right) .$$

This statistic is of the same form as Breusch and Pagan's (1979) LM test for heteroscedasticity in the disturbances of (1). Lee (1990) has demonstrated that it is also the LM test of H_{02} against H_{a2}; i.e., it can be viewed as the LM test against both ARCH and GARCH disturbances. Under normality, which is assumed here, it can be shown that plim $f^{0\prime}f^0/(n-q) = 2$. Thus an asymptotically equivalent statistic is

$$(n-q)f^{0\prime}W(W'W)^{-1}W'f^0 \Big/ f^{0\prime}f^0 = (n-q)R^2 \qquad (6)$$

where R^2 is the squared multiple correlation between f^0 and W which is the R^2 from the regression of e_t^2 on an intercept and q consecutive lagged values of e_t^2. Both test statistics have an asymptotic chi-squared distribution with q degrees of freedom under H_0.

The LM test is applied as an asymptotic test. Its small-sample properties against ARCH disturbances have been investigated by Engle, Hendry and Trumble (1985), Luukkonen, Saikkonen and Terasvirta (1988), Bollerslev and Wooldridge (1988), Diebold and Pauly (1989) and Gregory (1989). The typical finding of these studies is that the actual size of the LM test is generally less than its nominal size. This is consistent with results reported by Breusch and Pagan (1979) and Godfrey (1978) for the LM test for disturbance heteroscedasticity which is a function of exogenous variables. In other words, the nominal size of the test tends to overestimate the true probability of a Type I error in finite samples. The small-sample power of the test is not unreasonable, but as already noted, the LM test fails to take into account the one-sided nature of the testing problem. An LM-type test which uses this information may result in a significant improvement in small-sample power. When $q = 1$ in the ARCH model, Engle, Hendry and Trumble (1985) have noted that such a test can be based on the square-root of (5) or (6) with an appropriate choice of sign. Based on Lee (1990), this test is also a one-sided LM test for GARCH(1,1) disturbances. We shall now consider a LMMP-based generalization for $q \geq 2$, namely the LBS test.

Suppose we wish to test $\bar{H}_0 : \theta = 0$ based on x, which is an $n \times 1$ random vector with probability density function $f(x|\theta)$, where θ is a $p \times 1$ vector of unknown parameters. When $p = 1$, it is well-known (see for example Ferguson (1967) or Cox and Hinkley (1974)) that the locally best test of $\bar{H}_0$ against $\bar{H}_a^+ : \theta > 0$ has critical regions of the form

$$\left.\frac{\partial \ell n f(x|\theta)}{\partial \theta}\right|_{\theta=0} > c_1 \tag{7}$$

where c_1 is a suitably chosen constant. Observe that the LHS of (7) is the score evaluated at $\bar{H}_0$. This result gives a power justification to the LM or score test (see for example, Cox and Hinkley (1974) and King and Hillier (1985)). There have been a number of attempts to generalize it to $p \geq 2$. Recently, SenGupta and Vermeire (1986) introduced the class of LMMP tests which maximize the mean curvature of the power function in the neighbourhood of $\bar{H}_0$. King and Wu (1990) demonstrated that the LMMP test of $\bar{H}_0$ against

$$\bar{H}_a^+ : \theta_1 \geq 0, \ldots, \theta_p \geq 0, \qquad \theta \neq 0,$$

has the form

$$\sum_{i=1}^{p} \left.\frac{\partial \ell n f(x|\theta)}{\partial \theta_i}\right|_{\theta=0} > c_2$$

where c_2 is an appropriate constant. This test is based on the sum of scores evaluated at $\bar{H}_0$. Its similarity to the LM test suggests we can use the familiar asymptotic theory of the LM test to derive asymptotic critical values.

Most testing problems in econometrics involve nuisance parameters and the problems under study in this paper are no exception. Suppose θ is partitioned as $\theta = (\theta_1', \theta_2')'$ where θ_1 is $p_1 \times 1$ and θ_2 is $p_2 \times 1$ such that $p_1 + p_2 = p$. Consider testing $\bar{H}_0 : \theta_1 = 0$ against $\bar{H}_a^+ : \theta_{1i} \geq 0$, $i = 1, \ldots, p_1$, $\theta_1 \neq 0$ when θ_2 is an unknown vector of nuisance parameters. If the value of θ_2 was known, then a LMMP test of $\bar{H}_0$ is given by

$$s = \sum_{i=1}^{p_1} \left. \frac{\partial \ell n f(x|\theta)}{\partial \theta_{1i}} \right|_{\theta=(0',\theta_2')'} > c_3 .$$

King and Wu observed that the LM approach suggests that the unknown θ_2 should be replaced by its maximum likelihood estimate under H_0, denoted by $\hat{\theta}_2$. Let $\mathcal{J}^{11}$ denote the upper $p_1 \times p_1$ block of the inverse of the information matrix and let $\hat{s}$ and $\hat{\mathcal{J}}^{11}$ denote s and $\mathcal{J}^{11}$, respectively, evaluated at $\hat{\theta} = (0', \hat{\theta}_2')'$. King and Wu observed that a one-sided score test could be based on rejecting $\bar{H}_0$ against $\bar{H}_a^+$ for large values of

$$\hat{s} \Big/ \left(\ell' \left(\hat{\mathcal{J}}^{11} \right)^{-1} \ell \right)^{1/2} \tag{8}$$

assuming an asymptotic N(0,1) distribution under H_0 where ℓ is the $p_1 \times 1$ vector of ones. In contrast to the Kuhn-Tucker test, this one-sided test, which we call the LBS test, can be applied with ease because of its N(0,1) asymptotic distribution.

The form of (8) for testing an ARCH(q) process, namely testing H_{01} against H_{a1}, in the context of (1), (2) and (3), can readily be derived. The log likelihood function for this model is

$$L = \text{constant} - \frac{1}{2} \sum_{t=q+1}^{n} \log \sigma_t^2 - \frac{1}{2} \sum_{t=q+1}^{n} \varepsilon_t^2 \Big/ \sigma_t^2 \tag{9}$$

where

$$\varepsilon_t = y_t - x_t'b ,$$

$$\sigma_t^2 = (1, z_t')\alpha ,$$

and

$$z_t' = \left(\varepsilon_{t-1}^2, \varepsilon_{t-2}^2, \ldots, \varepsilon_{t-q}^2\right) , \quad t = q+1, \ldots, n .$$

The score associated with α_i, for $i = 1, \ldots, q$, is

$$\frac{\partial L}{\partial \alpha_i} = \frac{1}{2} \sum_{t=q+1}^{n} \left[\varepsilon_t^2/\sigma_t^2 - 1 \right] \varepsilon_{t-i}^2 \Big/ \sigma_t^2 . \tag{10}$$

The information matrix is block diagonal. We only need be concerned with that block associated with α which we denote by $\mathcal{I}_{\alpha\alpha'}$. Let $\theta = (b', \alpha')'$. If $\hat{\theta}$ denotes the maximum likelihood estimator of θ under H_0 then

$$\hat{\theta} = \left(\hat{b}', \hat{\sigma}^2, 0'\right)'$$

where $\hat{b}$ is the OLS estimator of b in (1), $\hat{\sigma}^2$ is the usual maximum likelihood estimator of σ^2 assuming independent $N(0, \sigma^2)$ disturbances and 0 is the $q \times 1$ vector of zeros.

One can show that under the restricted estimates, which in this case are the OLS estimates,

$$\hat{\mathcal{I}}_{\alpha\alpha'} = \frac{1}{2\hat{\sigma}^4} \begin{bmatrix} (n-q) & \Sigma_t \hat{z}_t' \\ \Sigma_t \hat{z}_t & \Sigma_t \hat{z}_t \hat{z}_t' \end{bmatrix}$$

in which $\hat{z}_t' = \left(e_{t-1}^2, e_{t-2}^2, \ldots, e_{t-q}^2\right)$. The block inverse associated with $\alpha_1, \alpha_2, \ldots, \alpha_q$, the parameters under test, is

$$\mathcal{J}^{11}_{\alpha\alpha'} = \left\{\frac{1}{2\hat{\sigma}^4}\left[\Sigma_t\hat{z}_t\hat{z}_t' - \Sigma_t\hat{z}_t\Sigma_t\hat{z}_t' \Big/ (n-q)\right]\right\}^{-1} .$$

Thus King and Wu's LBS test is based on rejecting H_{01} for large values of

$$s_{ARCH} = \frac{\Sigma_t\left(e_t^2 \Big/ \hat{\sigma}^2 - 1\right)\sum_{i=1}^{q} e_{t-i}^2}{\left\{2\ell'\left[\Sigma_t\hat{z}_t\hat{z}_t' - \Sigma_t\hat{z}_t\Sigma_t\hat{z}_t' \Big/ (n-q)\right]\ell\right\}^{1/2}} \qquad (11)$$

where ℓ is the q×1 vector of ones.

We now turn our attention to testing against a GARCH(p,q) process, namely testing H_{02} against H_{a2}, in the context of (1), (2) and (4). The log likelihood function for this model is similar to (9), but the conditional variance is now given by (4) which can be written as

$$\sigma_t^2 = (1, z_t', h_t')v$$

where z_t' is defined as above,

$$h_t' = \left(\sigma_{t-1}^2, \; \ldots, \; \sigma_{t-p}^2\right), \qquad t = r+1, \; \ldots., \; n ,$$

$v' = (\alpha', \beta')$ and

$r = \max(p,q)$.

The score associated with α_i is equivalent to (10) and the score associated with β_j, for $j = 1,\ldots,p$, is

$$\frac{\partial L}{\partial \beta_j} = \frac{1}{2}\sum_{t=r+1}^{n}\left[\varepsilon_t^2 / \sigma_t^2 - 1\right]\sigma_{t-j}^2 / \sigma_t^2. \qquad (12)$$

Here $\theta = (b', v')'$, and

$$\hat{\theta} = \left(\hat{b}', \hat{\sigma}^2, 0'\right)'$$

where $\hat{b}'$ and $\hat{\sigma}^2$ are defined as above and 0 is the (p+q) x 1 vector of zeros.

When (12) is evaluated at $\theta = \hat{\theta}$ under H_{02}, its value is zero. Therefore $\hat{s}$, the sum of the scores evaluated at $\theta = \hat{\theta}$, is precisely that for the LBS test against an ARCH(q) process. The final step is to derive the asymptotic variance of $\hat{s}$ under H_{02}. Because H_{01} and H_{02} are equivalent, it follows that the required asymptotic variance is that of $\hat{s}$ for the test against an ARCH(q) process. This implies that the LBS test against a GARCH(p,q) process is equivalent to the LBS test against an ARCH(q) process, i.e. based on (11). This result is similar to that for the LM test as shown by Lee (1990). It is not surprising since both the LM and LBS tests are based on locally optimal testing principles.

Finally, it is straightforward to show that the LBS test against GARCH(1,1) disturbances is identical to the one-sided LM test for ARCH(1) disturbances based on the square root of (5) with an appropriate choice of sign.

3. MONTE CARLO EXPERIMENTS

3.1 Experimental Design

A Monte Carlo study was conducted to investigate and compare the small-sample size and power properties of the LBS test and both versions of the LM test in the context of (1), (2) and (3), as well as in the context of (1), (2) and (4). We shall use LM1 and LM2 to denote the LM test based on (5) and (6), respectively. The first part of the study involved the use of the Monte Carlo method to estimate appropriate five per cent critical values for all three tests. This allows the estimated powers of the three tests to be compared at approximately the same significance level. The second part involved a comparison of estimated sizes and powers based on asymptotic critical values.

The first four design matrices used in the comparisons involved

$$x_t = (1, m_t) ,$$

where m_t is generated as

$$m_t = \delta m_{t-1} + \nu_t, \qquad \nu_t \sim IN(0,4), \qquad t = 1,\ldots,n ,$$

with δ taking the values 0, 0.8, 1.0 and 1.02 for design matrices X1, X2, X3 and X4, respectively. In each case, m_t is generated artificially and then held fixed from iteration to iteration. These regressor choices are influenced by Engle, Hendry and Trumble's (1985) Monte Carlo study. The four δ values cover four types of economic data, namely white noise, autoregressive, random walk and explosive processes. The remaining three design matrices used were:

X5: (k = 2). A constant and the monthly value weighted market index for the Sydney Stock Market computed by the Centre for Research in Finance at the Australian Graduate School of Management and commencing 1978(1).

X6: (k = 2). A constant and the quarterly Australian CPI commencing 1959(1).

X7: (k = 3). X6 augmented by adding the CPI lagged one quarter.

The disturbance term, ε_t, in the linear regression model, (1), whose conditional variance is time-varying according to either an ARCH process or a GARCH process can be written as

$$\varepsilon_t = \eta_t \sigma_t \qquad (13)$$

where η_t is i.i.d. with $E(\eta_t) = 0$ and $Var(\eta_t) = 1$. We used (13) to generate the disturbances for the Monte Carlo study.

As both tests are based on OLS residuals from (1), they are invariant to the value of b. They are also invariant to α_0. Without loss of generality, α_0 and b_i, $i = 1,\ldots,k$, were set equal to one. For testing white noise disturbances against ARCH disturbances, the sizes and powers of both tests were estimated using the Monte Carlo method with y_t generated by (1), (2) and (3) and for the following combinations of parameters:

$$q = 2,$$
$$\alpha_1 = 0,\ 0.2,\ 0.4,\ 0.6,$$
$$\alpha_2 = 0,\ 0.2,\ 0.4,$$
$$n = 20,\ 50,\ 100, \quad \text{for X1 - X5}$$

and

$$n = 20,\ 50,\ 80, \quad \text{for X6 - X7.}$$

The ARCH(2) disturbances were generated using (13) by

$$\varepsilon_t = \eta_t\left[\, 1 + \alpha_1\varepsilon_{t-1}^2 + \alpha_2\varepsilon_{t-2}^2 \,\right]^{1/2},$$

where $\eta_t \sim N(0,1)$. The α_1 and α_2 values were chosen so as not to violate the boundary condition that $\alpha_1 + \alpha_2 < 1$ except for the combination $\alpha_1 = 0.6$ and $\alpha_2 = 0.4$ which falls on this boundary.

For testing white noise disturbances against GARCH disturbances, the y_t are generated by (1), (2), and (4) and for the following combinations of parameters:

$$p = 1, \quad q = 1,$$
$$\alpha_1 = 0,\ 0.05,\ 0.1,\ 0.4,$$
$$\beta_1 = 0,\ 0.3,\ 0.6,\ 0.9,$$

and n is the same as when testing against ARCH disturbances. The GARCH(1,1) disturbances can be generated using (13) by

$$\varepsilon_t = \eta_t \left[1 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \right]^{1/2} ,$$

where $\eta_t \sim N(0,1)$. These particular α_1 and β_1 values were chosen because in most empirical applications of this model the estimates of α_1 are commonly small whereas the estimates of β_1 are typically larger. However, some combinations of these chosen α_1 and β_1 values violate the boundary condition of $\alpha_1 + \beta_1 < 1$. The results of these combinations are not presented except when they fall on the boundary.

One thousand replications were used throughout. Where required, pseudo-random N(0,1) variates were generated using the uniform random number generator intrinsic RAN on a VAX11-780 computer and then transforming to N(0,1) using Ripley's (1987) polar transformation. All tests were conducted at the nominal significance level of 0.05. There is evidence in the literature that suggests that in some applications, $\varepsilon_t | \psi_{t-1}$ may follow distributions with fatter tails than the normal distribution (see, for example, Baillie and Bollerslev (1989) and Baillie and DeGennaro (1990)). We therefore investigated the robustness of the tests to non-normality by repeating the whole experiment with pseudo-random values of ε_t generated from a symmetric distribution with a kurtosis of six. In other words, leptokurtic disturbances. These disturbances were generated using Ramberg and Schmeiser's (1972,1974) algorithm, namely

$$r(p) = \lambda_1 + \left\{ p^{\lambda_3} - (1-p)^{\lambda_4} \right\} \Big/ \lambda_2, \qquad 0 \le p \le 1,$$

where r(p) is the generated pseudo-random variate, p is a uniform pseudo-random variate, λ_1 is a location parameter, λ_2 is a scale parameter and λ_3 and λ_4 are shape parameters. The tables of λ values provided by Ramberg, Tadikamalla, Dudewicz and Mykytka (1979) allow one to use appropriate λ values that give pseudo-random variates with the required first four moments.

3.2 The Results

Table 1 reports the estimated sizes of both LM tests and the LBS test when asymptotic critical values at the nominal level of five per cent are used. The reported sizes are applicable when testing against ARCH(2), GARCH(1,2) or GARCH(2,2) disturbances.

We see that all estimated sizes are less than the nominal size when the disturbances are normal. These results for the LM test are consistent with results reported by Engle, Hendry and Trumble (1985) and Bollerslev and Wooldridge (1988). As might be expected, the estimated sizes of the LBS test are similar to those for the LM tests. However, it does appear that when $n = 100$, the LBS test has sizes closer to the nominal size than do the LM tests.

For leptokurtic disturbances, the actual sizes of all three tests are smaller than the nominal size when $n = 20$. In contrast, for $n = 80$ and $n = 100$, the LM1 test has sizes significantly above the nominal level while those of the LM2 test are below 0.05. The estimated sizes for the LBS test are above 0.05, but 95 per cent confidence intervals include the nominal value. It would appear that sizes of the LM2 and LBS tests are somewhat robust to leptokurtic errors.

We shall now discuss the estimated powers of the three tests based on empirically derived critical values against ARCH(2) disturbances. Results for the X1 and X5 design matrices are presented in Tables 2 and 3, respectively. Those for the remaining design matrices show reasonably similar patterns and so are not given. They are available from the authors.

The powers of all three tests increase as α_1 and α_2 move away from H_{01} and also as n increases, *ceteris paribus*. The LM1 test based on (5) is almost always more powerful than the LM2 test which uses the more approximate $(n-q)R^2$ formula. Exceptions are rare for normal disturbances and occur on three occasions when n = 50 for $\alpha_1 = 0.2$ and α_2 = 0.0, 0.2. As might be expected, differences appear to decline as n increases. There are more exceptions for leptokurtic errors. These occur only when n = 100 and typically when $\alpha_1 = 0.2$.

The most powerful test is almost always the LBS test. The only exceptions occur when n = 80 or 100 and $\alpha_2 = 0.0$ for normal errors and X1, X2 and X3 with n = 20 for $\alpha_1 = 0.2$, $\alpha_2 = 0.0$ and leptokurtic errors. For the great majority of these exceptions, the LBS test is slightly less powerful than the LM1 test but more powerful than the LM2 test. The power advantage of the LBS test is typically greatest away from the $\alpha_2 = 0.0$ boundary to the H_{a1} parameter space. The results clearly demonstrate that a distinct improvement in power results from replacing the popular LM2 test with the LBS test.

A comparison of the powers for normal disturbances with those for leptokurtic disturbances reveals that all tests typically have lower powers under leptokurtic disturbances. This finding is consistent with the results of Bollerslev and Wooldridge (1988). It is noticeable that for larger samples, the typical decline in power going from normal to leptokurtic disturbances is much greater for the LM tests than the LBS test. This suggests that the LBS test is more robust to departures from normality in large samples.

With respect to the estimated powers of the three tests based on asymptotic critical values against ARCH(2) disturbances, the results for X5 are presented in Table 4. Those for the remaining design matrices show reasonably similar patterns and are available from the authors.

In most cases, the LBS test is more powerful than the LM tests, especially away from H_{01}, away from the boundary $\alpha_2 = 0.0$ and when n is large. This is true for both normal and leptokurtic disturbances. On the other hand, when n is small, the powers of the LM tests dominate those of the LBS test along $\alpha_2 = 0.0$ and close to H_{01}. Also, the power of the LM1 test, in the case of large n and leptokurtic disturbances, appears to be competitive relative to the LBS test. This is easily explained by the higher than nominal sizes of the LM1 test. It serves as a reminder that care is needed in interpreting these results.

We now turn to the results for testing against GARCH(1,1) disturbances. Table 5 reports the estimated sizes of the three tests when asymptotic critical values at the five per cent level are used. The reported sizes are applicable when testing against GARCH(1,1) or ARCH(1) disturbances.

Again we find that all estimated sizes are less than the nominal size when the disturbances are normal. In this case, the estimated sizes of the LBS test are almost always closer to 0.05 than those of the LM tests. This is also true for leptokurtic errors when n = 20 and n = 50. For n = 80 and n = 100, the change from normal to leptokurtic errors appears to have no effect on the estimated sizes of the LM2 tests while increasing those of the LM1 and LBS tests.

We shall now discuss the estimated powers of the three tests against GARCH(1,1) disturbances based on empirically derived critical values. Results for X4 and X6 are presented in Tables 6 and 7, respectively. Those for the remaining design matrices are available from the authors.

The powers of all three tests almost always increase as α_1 increases and as n increases, *ceteris paribus*. But the powers of these tests are hardly affected by increasing β_1. In fact, there typically is a drop in power as β_1 increases to 0.9, *ceteris paribus*. Otherwise, for fixed n, X and α_1, powers seem reasonably constant as β_1 changes although for n = 100 it does seem that powers increase with β_1. These results are true regardless of whether the disturbances are normal or leptokurtic. They suggest that the power of each of the tests comes from detecting a non-zero value of α_1. This is not surprising given that each of the tests can be derived as a test for ARCH(1) disturbances.

The LM1 test is typically more powerful than the LM2 test. Exceptions are less frequent as α_2 increases, as n increases and when normal errors are replaced by leptokurtic errors. The most powerful test is almost always the LBS test, particularly for larger samples and also for leptokurtic errors. A large number of exceptions occur when n = 20 and for normal distributions. In these circumstances, they only occur when α_1 = 0.05 or 0.10 and typically when β_1 = 0.9 although there is a greater frequency of exceptions for the X4 and X6 design matrices. Overall, the results show that using the LBS test almost always results in an improvement in power, particularly over the LM2 test, when $n \geq 50$.

Finally, we briefly consider the estimated powers against GARCH(1,1) disturbances based on asymptotic critical values at the five per cent level. Table 8 reports these results for X3. It is noticeable that the LBS test, with a few minor exceptions when n = 20, is always more powerful than the LM tests. This is partly because it has higher size than the LM tests and partly because of its typically better power. Another feature is that the estimated powers of the LM tests and frequently those of the LBS test are always below the nominal size when α_1 = 0.05, 0.1 and n = 20.

4. AN ILLUSTRATIVE EXAMPLE

This section reports the application of the LBS test to an empirical example discussed by Watson and Engle (1985). They focussed on the problem of testing for a varying regression coefficient in a model for weekly gold and silver prices. The underlying model is

$$R_t = b_1 + b_2 r_t + \varepsilon_t \tag{14}$$

where R_t is the one period holding yield on the metal and r_t is the risk-free rate of return assumed known by the agents at the beginning of the period. Using 208 weekly observations on gold and silver prices over the period 1975-1979 and the return on 90-day U.S. Treasury bills with one week remaining until maturity as r_t, their test rejected constancy of b_2 for gold prices but not for silver prices. An identical result was obtained by King (1987) using an alternative test.

We focus on Watson and Engle's estimated model for silver prices which is

$$R_t = \underset{(51.5)}{0.6} + \underset{(9.3)}{0.75} r_t + e_t .$$

The standard errors, given in parentheses, indicate a poor fit which may be due to conditional heteroscedasticity of some form. When the LM test for ARCH(2) (or equivalently GARCH(1,2) or GARCH(2,2)) disturbances is applied using the popular form (6); i.e., 206 times R^2 from the regression of e_t^2 on an intercept, e_{t-1}^2 and e_{t-2}^2; one gets a test statistic value of 3.721. When compared against the ninety-five percentile of the $\chi^2(2)$ distribution, namely 5.991, this test suggests one should not reject the null hypothesis of well-behaved disturbances.

On the other hand, when the LBS test statistic against ARCH(2) (or equivalently GARCH(1,2) or GARCH(2,2)) disturbances is calculated using (11) with q = 2, we get a value of 2.628. When compared with the N(0,1) distribution, this suggests clear rejection of the null hypothesis.

The conflict in the results of the two tests might be explained by the superior small-sample power of the LBS test. It would seem wise to ignore the outcome of the LM test and assume conditional heteroscedasticity of some form in the disturbances of (14).

5. CONCLUDING REMARKS

This paper considered the twin problems of testing for ARCH and GARCH disturbances in the linear regression model. A feature of these testing problems is that they are one-sided in nature. This aspect is ignored by the standard LM test for ARCH disturbances proposed by Engle (1982). We took up the suggestion of King and Wu (1990) and constructed tests based on the sum of scores. In the absence of nuisance parameters, such tests are LMMP.

The small-sample properties of the new tests were compared with those of two standard versions of the LM test in a Monte Carlo experiment. The results suggest that typically the new test has better power than either version of LM test while its asymptotic critical values appear to be at least as accurate. All three tests seem to be reasonably robust to leptokurtic disturbances.

When testing against GARCH disturbances, all three tests seem insensitive to the magnitude of the β parameters. A topic for future research is to construct and investigate the properties of tests that are sensitive to different β values. Such tests might be based on a mixture of scores and higher-order derivatives of the log-likelihood function.

ACKNOWLEDGMENTS

This paper has benefited from constructive comments from Kim Sawyer and participants of a seminar at Victoria University, Wellington, a Monash Financial Econometrics Symposium and at the 1991 Australasian Meetings of the Econometric Society, University of New South Wales, Sydney.

REFERENCES

Baillie, R. T., and Bollerslev, T. (1989), " The Message in Daily Exchange Rates : A Conditional-Variance Tale," *Journal of Business and Economic Statistics*, 7, 297 - 305.

Baillie, R. T., and DeGennaro, P. P. (1990), " The Impact of Delivery Terms on Stock Return Volatility," *Journal of Financial Services Research*, 3, 55 - 76.

Bollerslev, T. (1986), "Generalized Autoregressive Conditional Heteroskedasticity," *Journal of Econometrics*, 31, 307-327.

Bollerslev, T., Chou, R. Y., Jayaraman, N., and Kroner, K. F. (1990), "ARCH Modelling in Finance : A Selective Review of the Theory and Empirical Evidence, with Suggestions for Future Research," unpublished manuscript, Department of Economics, Northwestern University.

Bollerslev, T., and Wooldridge, J. M. (1988), "Quasi-Maximum Likelihood Estimation of Dynamic Models with Time-Varying Covariances," Working Paper no. 505, Department of Economics, MIT.

Breusch, T. S., and Pagan, A. R. (1979), "A Simple Test for Heteroscedasticity and Random Coefficient Variation," *Econometrica*, 47, 1287 - 1294.

Cox, D. R., and Hinkley, D. V. (1974), *THEORETICAL STATISTICS*, London, Chapman and Hall.

Diebold, F. X., and Pauly, P. (1989), "Small Sample Properties of Asymptotically Equivalent Tests for Autoregressive Conditional Heteroskedasticity," *Statistische Hefte*, 30, 105 - 131.

Engle, R. F. (1982), "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of U. K. Inflation," *Econometrica*, 50, 987 - 1008.

Engle, R. F., and Bollerslev, T. (1986), "Modelling the Persistence of Conditional Variances," *Econometric Reviews*, 5, 1 - 50.

Engle, R. R., Hendry, D. F., and Trumble, D. (1985), "Small Sample Properties of ARCH Estimators and Tests," *Canadian Journal of Economics*, 18, 66 - 93.

Ferguson, T. S. (1967), *MATHEMATICAL STATISTICS : A DECISION THEORETIC APPROACH*, New York, Academic Press.

Godfrey, L. G. (1978), "Testing for Multiplicative Heteroskedasticity," *Journal of Econometrics*, 8, 227 - 236.

Gourieroux, C., Holly, A., and Monfort, A. (1982), "Likelihood Ratio Test, Wald Test and Kuhn-Tucker Test in Linear Models with Inequality Constraints on Regression Parameters," *Econometrica*, 50, 63 - 80.

Gregory, A. W. (1989), "A Non-Parametric Test for Autoregressive Conditional Heteroskedasticity : A Markov-Chain Approach," *Journal of Business and Economic Statistics*, 7, 107 - 115.

King, M.L. (1987), "An Alternative Test for Regression Coefficient Stability," *Review of Economics and Statistics*, 69, 379-381.

King, M. L., and Hillier, G. H. (1985), "Locally Best Invariant Tests of the Error Covariance Matrix of the Linear Regression Model," *Journal of the Royal Statistical Society B*, 47, 98 - 102.

King, M. L., and Wu, P. X. (1990), "Locally Optimal One-Sided Tests for Multiparameter Hypotheses," , presented in the 6th World Congress of the Econometric Society, Barcelona, Working Paper 2/90, Department of Econometrics, Monash University.

Lee, J. H. H. (1990), "A Lagrange Multiplier Test for GARCH Models," unpublished manuscript, Department of Econometrics, Monash University.

Luukkonen, R., Saikkonen, P. and Terasvirta, T. (1988), "Testing Linearity in Univariate Time Series Models," *Scandanavian Journal of Statistics*, 15, 161 - 175.

Ramberg, J. S., and Schmeiser, B. W. (1972), "An Approximate Method for Generating Symmetric Random Variables," *Communications of the Association for Computing Machinery*, 15, 987 - 990.

Ramberg, J. S., and Schmeiser, B. W. (1974), "An Approximate Method for Generating Asymmetric Random Variables," *Communications of the Association for Computing Machinery*, 17, 78 - 87.

Ramberg, J. S., Tadikamalla, P. R., Dudewicz, E. J., and Mykytka, E. F. (1979), "A Probability Distribution and Its Uses in Fitting Data," *Technometrics*, 21, 201 - 215.

Ripley, B. D. (1987), *STOCHASTIC SIMULATION*, New York, J. Wiley.

SenGupta, A., and Vermeire, L. (1986), "Locally Optimal Tests for Multiparameter Hypotheses," *Journal of the American Statistical Association*, 81, 819 - 825.

Watson, M.W., and Engle, R.F. (1985), "Testing for Regression Coefficient Stability with a Stationary AR(1) Alternative," *Review of Economics and Statistics*, 67, 341-346.

TABLE 1

Estimated sizes of the LM and LBS tests against ARCH(2) (or equivalently GARCH(1,2) or GARCH(2,2)) disturbances based on asymptotic critical values at the 5% nominal level.

NORMAL ERRORS

X-matrix	LM1	LM2	LBS	LM1	LM2	LBS	LM1	LM2	LBS
	n = 20			n = 50			n = 100		
X1	0.011	0.028	0.016	0.020	0.022	0.021	0.030	0.025	0.029
X2	0.014	0.024	0.012	0.020	0.015	0.020	0.028	0.026	0.033
X3	0.016	0.029	0.012	0.018	0.019	0.019	0.024	0.025	0.028
X4	0.012	0.022	0.008	0.022	0.019	0.019	0.024	0.028	0.031
X5	0.012	0.021	0.010	0.021	0.021	0.016	0.024	0.022	0.032
	n = 20			n = 50			n = 80		
X6	0.010	0.023	0.010	0.020	0.020	0.019	0.036	0.035	0.036
X7	0.014	0.023	0.009	0.016	0.019	0.014	0.033	0.037	0.034

LEPTOKURTIC ERRORS

X-matrix	LM1	LM2	LBS	LM1	LM2	LBS	LM1	LM2	LBS
	n = 20			n = 50			n = 100		
X1	0.019	0.021	0.007	0.063	0.034	0.038	0.088	0.038	0.061
X2	0.025	0.024	0.013	0.056	0.031	0.040	0.080	0.035	0.052
X3	0.028	0.021	0.009	0.057	0.036	0.045	0.087	0.039	0.057
X4	0.021	0.020	0.014	0.060	0.035	0.044	0.091	0.035	0.059
X5	0.014	0.019	0.006	0.066	0.037	0.043	0.081	0.039	0.063
	n = 20			n = 50			n = 80		
X6	0.023	0.020	0.013	0.059	0.037	0.046	0.084	0.033	0.047
X7	0.018	0.020	0.011	0.057	0.037	0.046	0.085	0.033	0.050

TABLE 2

Estimated powers for X1 of the LM and LBS tests against ARCH(2) disturbances using empirically derived critical values at the 5% level.

α_1	α_2	n	NORMAL ERRORS			LEPTOKURTIC ERRORS		
			LM1	LM2	LBS	LM1	LM2	LBS
0.2	0.0	20	0.085	0.073	0.094	0.092	0.087	0.091
		50	0.171	0.183	0.208	0.147	0.120	0.178
		100	0.357	0.324	0.333	0.243	0.249	0.298
0.2	0.2	20	0.101	0.092	0.160	0.123	0.098	0.144
		50	0.293	0.284	0.409	0.260	0.211	0.344
		100	0.543	0.518	0.661	0.426	0.410	0.570
0.2	0.4	20	0.147	0.118	0.227	0.144	0.119	0.198
		50	0.457	0.439	0.595	0.362	0.292	0.460
		100	0.787	0.737	0.849	0.598	0.601	0.734
0.4	0.0	20	0.131	0.104	0.165	0.136	0.108	0.152
		50	0.369	0.358	0.415	0.278	0.218	0.327
		100	0.690	0.634	0.644	0.493	0.466	0.549
0.4	0.2	20	0.156	0.120	0.237	0.152	0.125	0.198
		50	0.473	0.440	0.607	0.373	0.310	0.469
		100	0.805	0.765	0.869	0.627	0.608	0.736
0.4	0.4	20	0.194	0.146	0.289	0.172	0.146	0.246
		50	0.607	0.547	0.737	0.461	0.374	0.587
		100	0.894	0.872	0.940	0.734	0.715	0.865
0.6	0.0	20	0.171	0.135	0.234	0.165	0.147	0.194
		50	0.555	0.496	0.576	0.398	0.322	0.442
		100	0.861	0.801	0.842	0.659	0.615	0.698
0.6	0.2	20	0.211	0.149	0.307	0.182	0.156	0.245
		50	0.626	0.555	0.733	0.482	0.401	0.576
		100	0.919	0.881	0.936	0.755	0.727	0.856
0.6	0.4	20	0.246	0.175	0.357	0.210	0.174	0.282
		50	0.720	0.632	0.822	0.566	0.468	0.678
		100	0.958	0.923	0.978	0.837	0.796	0.922

TABLE 3

Estimated powers for X5 of the LM and LBS tests against ARCH(2) disturbances using empirically derived critical values at the 5% level.

α_1	α_2	n	NORMAL ERRORS			LEPTOKURTIC ERRORS		
			LM1	LM2	LBS	LM1	LM2	LBS
0.2	0.0	20	0.092	0.080	0.093	0.085	0.066	0.102
		50	0.175	0.170	0.206	0.148	0.130	0.172
		100	0.362	0.333	0.344	0.247	0.262	0.284
0.2	0.2	20	0.102	0.096	0.159	0.106	0.078	0.146
		50	0.287	0.265	0.409	0.248	0.213	0.331
		100	0.561	0.517	0.648	0.419	0.433	0.558
0.2	0.4	20	0.138	0.120	0.222	0.131	0.097	0.212
		50	0.457	0.418	0.589	0.357	0.304	0.470
		100	0.779	0.735	0.852	0.610	0.613	0.732
0.4	0.0	20	0.141	0.122	0.161	0.128	0.093	0.155
		50	0.378	0.353	0.405	0.267	0.225	0.322
		100	0.686	0.638	0.639	0.490	0.482	0.539
0.4	0.2	20	0.157	0.128	0.234	0.141	0.100	0.214
		50	0.478	0.436	0.603	0.358	0.301	0.475
		100	0.808	0.763	0.865	0.618	0.621	0.735
0.4	0.4	20	0.184	0.161	0.296	0.170	0.117	0.273
		50	0.606	0.550	0.744	0.456	0.387	0.590
		100	0.904	0.858	0.941	0.747	0.726	0.858
0.6	0.0	20	0.189	0.158	0.247	0.160	0.125	0.196
		50	0.546	0.493	0.579	0.388	0.329	0.440
		100	0.865	0.804	0.842	0.645	0.630	0.697
0.6	0.2	20	0.207	0.163	0.320	0.167	0.123	0.273
		50	0.621	0.556	0.732	0.469	0.396	0.579
		100	0.920	0.873	0.938	0.755	0.739	0.840
0.6	0.4	20	0.233	0.182	0.368	0.200	0.149	0.313
		50	0.715	0.640	0.811	0.559	0.463	0.683
		100	0.961	0.925	0.979	0.832	0.804	0.912

TABLE 4

Estimated powers for X5 of the LM and LBS tests against ARCH(2) disturbances based on aymptotic critical values at the 5% level.

α_1	α_2	n	NORMAL ERRORS			LEPTOKURTIC ERRORS		
			LM1	LM2	LBS	LM1	LM2	LBS
0.2	0.0	20	0.030	0.044	0.020	0.033	0.022	0.021
		50	0.112	0.116	0.116	0.178	0.113	0.153
		100	0.286	0.272	0.291	0.349	0.229	0.307
0.2	0.2	20	0.039	0.053	0.043	0.049	0.024	0.040
		50	0.219	0.207	0.284	0.287	0.186	0.308
		100	0.476	0.467	0.592	0.527	0.378	0.591
0.2	0.4	20	0.062	0.077	0.078	0.066	0.040	0.070
		50	0.365	0.342	0.442	0.398	0.278	0.441
		100	0.722	0.693	0.808	0.712	0.561	0.757
0.4	0.0	20	0.060	0.076	0.042	0.071	0.036	0.037
		50	0.292	0.279	0.285	0.312	0.206	0.294
		100	0.622	0.589	0.597	0.608	0.433	0.565
0.4	0.2	20	0.063	0.083	0.081	0.072	0.042	0.066
		50	0.394	0.352	0.467	0.401	0.281	0.449
		100	0.746	0.724	0.831	0.711	0.577	0.754
0.4	0.4	20	0.095	0.101	0.113	0.087	0.056	0.094
		50	0.523	0.467	0.623	0.509	0.359	0.557
		100	0.871	0.830	0.930	0.819	0.677	0.868
0.6	0.0	20	0.095	0.091	0.084	0.097	0.051	0.070
		50	0.476	0.411	0.471	0.428	0.294	0.412
		100	0.823	0.770	0.808	0.745	0.589	0.713
0.6	0.2	20	0.106	0.101	0.124	0.096	0.064	0.104
		50	0.556	0.476	0.613	0.524	0.359	0.545
		100	0.901	0.851	0.926	0.831	0.697	0.855
0.6	0.4	20	0.128	0.122	0.147	0.111	0.075	0.124
		50	0.646	0.561	0.723	0.611	0.430	0.649
		100	0.940	0.900	0.972	0.891	0.771	0.927

TABLE 5

Estimated sizes of the LM and LBS tests against ARCH(1) or GARCH(1,1) disturbances based on asymptotic critical values at the 5% nominal level.

NORMAL ERRORS

X-matrix	LM1	LM2	LBS	LM1	LM2	LBS	LM1	LM2	LBS
	n = 20			n = 50			n = 100		
X1	0.011	0.023	0.026	0.029	0.037	0.044	0.027	0.029	0.034
X2	0.013	0.019	0.026	0.026	0.034	0.041	0.030	0.032	0.033
X3	0.013	0.022	0.025	0.025	0.034	0.041	0.028	0.028	0.034
X4	0.010	0.018	0.025	0.026	0.034	0.044	0.025	0.026	0.030
X5	0.011	0.023	0.021	0.021	0.031	0.045	0.028	0.028	0.031
	n = 20			n = 50			n = 80		
X6	0.011	0.021	0.025	0.026	0.032	0.045	0.029	0.032	0.045
X7	0.010	0.021	0.028	0.026	0.035	0.041	0.028	0.033	0.043

LEPTOKURTIC ERRORS

X-matrix	LM1	LM2	LBS	LM1	LM2	LBS	LM1	LM2	LBS
	n = 20			n = 50			n = 100		
X1	0.009	0.011	0.023	0.040	0.029	0.043	0.053	0.029	0.067
X2	0.017	0.021	0.032	0.043	0.028	0.054	0.059	0.032	0.073
X3	0.024	0.023	0.034	0.042	0.026	0.048	0.050	0.023	0.069
X4	0.014	0.017	0.023	0.043	0.026	0.047	0.051	0.028	0.072
X5	0.018	0.018	0.027	0.041	0.026	0.050	0.054	0.029	0.064
	n = 20			n = 50			n = 80		
X6	0.015	0.017	0.031	0.042	0.025	0.048	0.059	0.032	0.065
X7	0.023	0.022	0.043	0.044	0.026	0.053	0.057	0.034	0.063

TABLE 6

Estimated powers for X4 of the LM and LBS tests against GARCH(1,1) disturbances using empirically derived critical values at the 5% level.

α_1	β_1	n	NORMAL ERRORS			LEPTOKURTIC ERRORS		
			LM1	LM2	LBS	LM1	LM2	LBS
0.05	0.0	20	0.067	0.055	0.068	0.071	0.061	0.075
		50	0.084	0.085	0.102	0.085	0.069	0.092
		100	0.105	0.086	0.138	0.123	0.104	0.129
0.05	0.3	20	0.068	0.059	0.065	0.072	0.062	0.077
		50	0.087	0.091	0.103	0.084	0.073	0.094
		100	0.108	0.090	0.143	0.124	0.106	0.130
0.05	0.6	20	0.068	0.060	0.063	0.069	0.064	0.077
		50	0.088	0.091	0.102	0.086	0.076	0.097
		100	0.110	0.094	0.144	0.127	0.108	0.132
0.05	0.9	20	0.056	0.055	0.054	0.058	0.060	0.065
		50	0.075	0.071	0.088	0.081	0.075	0.087
		100	0.106	0.097	0.142	0.119	0.103	0.125
0.1	0.0	20	0.082	0.068	0.078	0.085	0.072	0.097
		50	0.124	0.118	0.141	0.123	0.109	0.145
		100	0.208	0.172	0.259	0.199	0.179	0.209
0.1	0.3	20	0.082	0.066	0.080	0.091	0.073	0.097
		50	0.124	0.118	0.140	0.128	0.110	0.148
		100	0.218	0.185	0.263	0.208	0.183	0.214
0.1	0.6	20	0.073	0.068	0.079	0.093	0.081	0.095
		50	0.128	0.119	0.140	0.130	0.110	0.149
		100	0.215	0.191	0.261	0.202	0.180	0.214
0.1	0.9	20	0.056	0.063	0.055	0.066	0.070	0.066
		50	0.097	0.097	0.106	0.115	0.106	0.127
		100	0.243	0.207	0.278	0.195	0.176	0.203
0.4	0.0	20	0.175	0.158	0.193	0.184	0.158	0.209
		50	0.437	0.387	0.462	0.371	0.331	0.421
		100	0.745	0.701	0.790	0.631	0.580	0.646
0.4	0.3	20	0.167	0.147	0.182	0.177	0.158	0.195
		50	0.442	0.413	0.477	0.384	0.342	0.445
		100	0.759	0.714	0.803	0.656	0.598	0.673
0.4	0.6	20	0.159	0.143	0.176	0.160	0.140	0.170
		50	0.446	0.381	0.473	0.395	0.346	0.448
		100	0.800	0.722	0.841	0.669	0.607	0.686

TABLE 7

Estimated powers for X6 of the LM and LBS tests against GARCH(1,1) disturbances using empirically derived critical values at the 5% level.

α_1	β_1	n	NORMAL ERRORS			LEPTOKURTIC ERRORS		
			LM1	LM2	LBS	LM1	LM2	LBS
0.05	0.0	20	0.065	0.054	0.060	0.066	0.063	0.068
		50	0.086	0.087	0.099	0.083	0.078	0.090
		80	0.089	0.080	0.102	0.099	0.084	0.111
0.05	0.3	20	0.064	0.056	0.062	0.067	0.064	0.068
		50	0.088	0.088	0.102	0.083	0.085	0.093
		80	0.092	0.083	0.105	0.104	0.085	0.112
0.05	0.6	20	0.065	0.057	0.061	0.065	0.066	0.067
		50	0.087	0.089	0.098	0.085	0.080	0.097
		80	0.095	0.090	0.105	0.105	0.088	0.112
0.05	0.9	20	0.054	0.051	0.050	0.057	0.060	0.061
		50	0.079	0.071	0.083	0.083	0.083	0.088
		80	0.091	0.090	0.095	0.093	0.090	0.102
0.1	0.0	20	0.075	0.064	0.083	0.081	0.072	0.086
		50	0.125	0.114	0.141	0.123	0.114	0.144
		80	0.154	0.146	0.186	0.160	0.136	0.177
0.1	0.3	20	0.071	0.066	0.081	0.078	0.074	0.088
		50	0.125	0.116	0.136	0.127	0.117	0.147
		80	0.162	0.153	0.186	0.162	0.143	0.181
0.1	0.6	20	0.073	0.067	0.073	0.077	0.078	0.085
		50	0.127	0.115	0.139	0.133	0.117	0.151
		80	0.164	0.156	0.197	0.155	0.141	0.173
0.1	0.9	20	0.050	0.062	0.051	0.062	0.061	0.063
		50	0.097	0.096	0.108	0.116	0.114	0.126
		80	0.150	0.144	0.185	0.146	0.127	0.166
0.4	0.0	20	0.181	0.156	0.201	0.182	0.154	0.205
		50	0.439	0.390	0.468	0.375	0.343	0.422
		80	0.623	0.590	0.659	0.490	0.434	0.514
0.4	0.3	20	0.172	0.157	0.191	0.176	0.153	0.201
		50	0.446	0.415	0.486	0.384	0.358	0.445
		80	0.638	0.595	0.686	0.497	0.447	0.525
0.4	0.6	20	0.170	0.154	0.185	0.149	0.141	0.174
		50	0.450	0.383	0.473	0.391	0.359	0.452
		80	0.683	0.607	0.710	0.550	0.461	0.575

TABLE 8

Estimated powers for X3 of the LM and LBS tests against GARCH(1,1) disturbances based on asymptotic critical values at the 5% nominal value.

α_1	β_1	n	NORMAL ERRORS			LEPTOKURTIC ERRORS		
			LM1	LM2	LBS	LM1	LM2	LBS
0.05	0.0	20	0.018	0.024	0.034	0.034	0.028	0.047
		50	0.056	0.063	0.089	0.070	0.040	0.091
		100	0.062	0.062	0.104	0.125	0.072	0.160
0.05	0.3	20	0.017	0.026	0.034	0.032	0.028	0.047
		50	0.059	0.071	0.091	0.070	0.039	0.092
		100	0.063	0.065	0.106	0.127	0.073	0.162
0.05	0.6	20	0.015	0.025	0.032	0.032	0.030	0.047
		50	0.062	0.072	0.091	0.068	0.040	0.089
		100	0.064	0.070	0.111	0.130	0.073	0.157
0.05	0.9	20	0.013	0.023	0.022	0.023	0.028	0.036
		50	0.054	0.060	0.077	0.067	0.040	0.083
		100	0.070	0.068	0.090	0.117	0.066	0.154
0.1	0.0	20	0.024	0.034	0.049	0.042	0.032	0.054
		50	0.093	0.100	0.128	0.110	0.068	0.137
		100	0.152	0.145	0.213	0.199	0.127	0.251
0.1	0.3	20	0.024	0.035	0.048	0.042	0.033	0.055
		50	0.093	0.100	0.131	0.109	0.072	0.141
		100	0.159	0.148	0.215	0.202	0.132	0.256
0.1	0.6	20	0.023	0.033	0.043	0.040	0.035	0.055
		50	0.093	0.100	0.132	0.107	0.072	0.142
		100	0.161	0.155	0.224	0.205	0.130	0.254
0.1	0.9	20	0.013	0.025	0.027	0.022	0.030	0.047
		50	0.067	0.080	0.099	0.101	0.071	0.124
		100	0.185	0.174	0.237	0.196	0.128	0.245
0.4	0.0	20	0.097	0.094	0.142	0.101	0.074	0.151
		50	0.371	0.364	0.448	0.340	0.260	0.409
		100	0.674	0.658	0.739	0.633	0.498	0.683
0.4	0.3	20	0.091	0.095	0.131	0.112	0.082	0.149
		50	0.383	0.373	0.454	0.350	0.254	0.428
		100	0.699	0.675	0.762	0.662	0.505	0.711
0.4	0.6	20	0.072	0.095	0.108	0.093	0.074	0.128
		50	0.383	0.333	0.458	0.364	0.280	0.429
		100	0.746	0.670	0.805	0.671	0.525	0.722