A long-lived horseshoe companion to the Earth

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ABSTRACT

We present a dynamical investigation of a newly found asteroid, 2010 SO16, and the discovery that it is a horseshoe companion of the Earth. The object's absolute magnitude ($H = 20.7$) makes this the largest object of its type known to-date. By carrying out numerical integrations of dynamical clones, we find that (a) its status as a horseshoe is secure given the current accuracy of its ephemeris, and (b) the time spent in horseshoe libration with the Earth is several times 10^5 yr, two orders of magnitude longer than determined for other horseshoe asteroids of the Earth. Further, using a model based on Hill's approximation to the three-body problem, we show that, apart from the low eccentricity which prevents close encounters with other planets or the Earth itself, its stability can be attributed to the value of its Jacobi constant far from the regime that allows transitions into other co-orbital modes or escape from the resonance altogether. We provide evidence that the eventual escape of the asteroid from horseshoe libration is caused by the action of planetary secular perturbations and the stochastic evolution of the eccentricity. The questions of its origin and the existence of as-yet-undiscovered co-orbital companions of the Earth are discussed.

Key words: methods: analytical – methods: numerical – celestial mechanics – Earth – minor planets, asteroids: general – minor planets, asteroids: individual: 2010 SO16.

1 INTRODUCTION

Horseshoes and tadpoles comprise the two classical solutions to the special case of co-orbital motion in the context of the restricted three-body problem. Both types of motion have been studied extensively, both analytically and numerically, partly due to their wide applicability to quasi-stable configurations in the real Solar system. Horseshoe-type orbits appear to be less prevalent than tadpoles, owing perhaps to their different stability properties. An often-cited example of objects horseshoeing with each other are the Saturnian satellites Janus and Epimetheus. In addition, numerical integrations of the motion of near-Earth asteroids (NEAs) 54509 YORP and 2002 AA29 show that they are currently horseshoeing with the Earth (Connors et al. 2002; Brasser et al. 2004). Confirmation of the status of NEA 2001 GO2 as the third Earth horseshoe will have to await further refinement of its orbit (Brasser et al. 2004). The libration periods and lifetimes of these Earth horseshoes are typically a few hundred and several thousand years, respectively.

Here we demonstrate the horseshoe dynamics of a newly discovered NEA, 2010 SO16. We find that, in contrast to the previous cases, the lifetime of horseshoe libration between 2010 SO16 and the Earth is substantially higher, of the order of 10^5 yr. We attribute this to two factors: its low eccentricity, moderate inclination orbit and its energy state deep into the horseshoe regime. We discuss the issue of its origin and briefly comment on the possible existence of additional objects of this type.

2 THE ASTEROID

2010 SO16, hereafter referred to as 'SO16', was discovered on 2010 September 17 by the *WISE* Earth-orbiting observatory (Obs. Code C51) and subsequently followed up by ground-based telescopes. As of 2010 December 3, its orbit has been determined by 40 observations spanning a data arc of 75 d (MPEC 2010-X02). The orbital elements and their 1σ statistical uncertainties as given by the NEODYS orbital information service[1] at that date (http://newton.dm.unipi.it/neodys/) are shown in Table 1. Its minimum orbit intersection distance (MOID) with respect to the Earth is 0.027 au and its absolute magnitude H is 20.7 implying a diametre of 200–400 m depending on the object's unknown albedo.

3 VERIFYING THE OBJECT'S DYNAMICAL STATE

Following astrometry of SO16 by the Spacewatch II telescope (Obs. Code 291) on November 18, and while the obs. arc length was 62 d, the nominal orbit solution as given by the NEODYS ephemeris information system had a semimajor axis (a) uncertainty of $\sigma_a = 1.6 \times 10^{-5}$ au while the difference between the value of this orbital element and the corresponding value for the Earth was $\Delta a \sim 4 \times 10^{-4}$ au. Both are small compared to the range of possible values

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[1] The data base was compiled by A. Milani, M. E. Sansaturio, G.-F. Gronchi, O. Arratia, S. Chesley, N. Ronci and the OrbFit Consortium.

Table 1. Orbital elements of 2010 SO16 at JD 245 5400.5.

Element	Value	1σ uncert.
a (au)	1.000 39	9.961e-6
e	0.075 188	3.284e-6
i (°)	14.536	0.000 8793
ω (°)	108.283	0.003 205
Ω (°)	40.523	0.001 306
M (°)	137.831	0.004 636

for Δa that allow co-orbital motion ($\Delta a_{\max} \sim 10^{-2}$ au) suggesting that SO16 is a 1:1 resonant librator.

We numerically integrated this nominal orbit, as well as the one subsequently published following additional observations obtained on December 1 (Table 1), for $\pm 10^5$ yr using the RADAU algorithm implemented within the MERCURY integration package (Chambers 1999). The model of the Solar system used in the integrations included all eight major planets (initial state vectors for which were taken from the HORIZONS ephemeris service; Giorgini et al. 1996) with the gravity of the Moon treated by adding its mass to that of the Earth. In both integrations, the object persisted as a horseshoe librator of the Earth. Fig. 1(a) shows the cartesian motion of the asteroid for a period of 2×10^5 yr, averaged over one orbital period, on the ecliptic plane. The opening angle of the horseshoe is $\sim 25°$ and the half-amplitude of variation in a is $\sim 4 \times 10^{-3}$ au. The period of the horseshoe libration is ~ 350 yr. SO16 is currently at
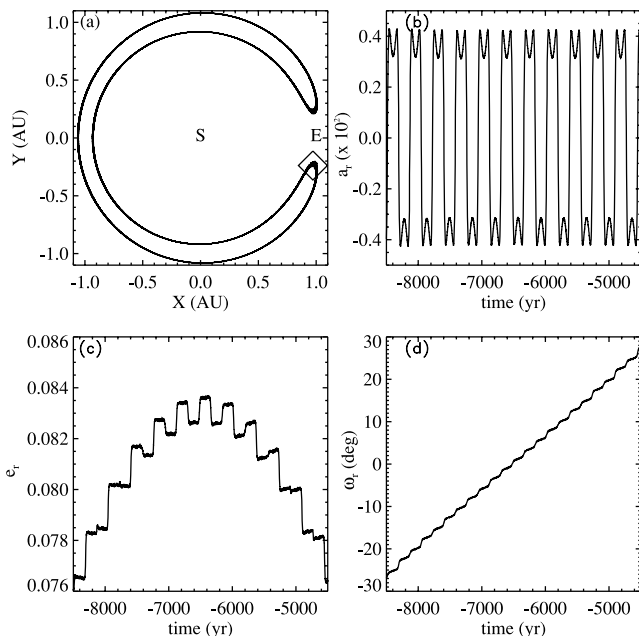

Figure 1. (a) The guiding centre trajectory of NEO 2010 SO16 in a cartesian ecliptic heliocentric frame corotating with the Earth from an integration of the asteroid's nominal orbit solution (Table 1) and for an interval of 2×10^5 yr centred on 2000 January 1.5 UT. The radial extent of the horseshoe has been exaggerated by a factor of 20 for clarity. The positions of the Sun and the Earth are denoted by the letters 'S' and 'E', respectively. The diamond indicates the position of the asteroid at 2010 January 1.5 UT. (b) Relative semimajor axis showing the alternation between two values, one interior and the other exterior to the Earth's, typical of horseshoe libration. (c) and (d) Relative eccentricity and argument of pericentre, respectively. Note the step-wise behaviour owing to the object's encounter with the Earth every ~ 175 yr.

the turning point of the horseshoe that is lagging behind the Earth in its orbital motion, approaching it at 0.13 au in mid-May every year and at <0.2 au until 2016. It will remain as an evening object in the sky for several decades to come.

To test this result further, and explore the likely lifetime of the object in the horseshoe state, we integrated dynamical clones of SO16. Starting conditions for these were generated by varying the asteroid's semimajor axis a, mean anomaly M, eccentricity e and argument of pericentre ω by increments equal to their 1σ (a) and 1.5σ (M, e, ω) ephemeris uncertainties, respectively. Nine different values were used for the semimajor axis and three for the other elements (inclusive of the nominal values), resulting in 243 clones which were integrated with MERCURY for $\pm 10^5$ yr with respect to the present. All clones remained in horseshoe libration with respect to the Earth.

4 ANALYSIS OF THE DYNAMICS

4.1 Theoretical framework

The existence of horseshoe trajectories within the circular restricted three body problem was first demonstrated by Brown (1911). The realization that two recently discovered satellites of Saturn, Janus and Epimetheus, were engaged in mutual horseshoe libration (Dermott & Murray 1981b), provided new impetus for theoretical modelling of the dynamics.

Dermott & Murray (1981a) investigated quasi-circular symmetric horseshoe orbits. In the symmetric regime, the initial separation in semimajor axis between the orbits (the impact parameter) and the eccentricity are adiabatic invariants of the motion, recovering their initial values following two consecutive encounters. In that work, it was also demonstrated numerically that the assumption of symmetry is valid up to $|a_0| \simeq 0.74\,\epsilon$. Here, ϵ is the mass parameter, a fundamental scaling constant in Hill's approximation – where the indirect perturbation is ignored and only the interaction potential is considered during the encounter – equal to $(\mu/3)^{1/3}$ where μ is the mass ratio between the secondary (or secondaries) and the primary. The quantity a_0 is the relative semimajor axis far from encounter, equivalent to the Jacobi constant for planar, circular orbits. As the symmetry is not exactly preserved, the horseshoe configuration has a finite lifetime of $T/\mu^{5/3}$ where T is the orbital period of the secondary (e.g. for the Earth, this expression evaluates to $\sim 1.6 \times 10^9$ yr). Hénon & Petit (1986) formally demonstrated that the eccentricity receives an impulse of order $\exp(-8\pi\mu/3a_0^3)$ during an encounter and hence is an adiabatic invariant of the motion (see also Namouni 1999). For small to moderate eccentricities ($\lesssim \epsilon$) the assumption of symmetry is valid if $\phi_{\min} \gg a_0$ where $\phi_{\min}$, related to the impact parameter by

$$\phi_{\min} = 8\mu/3a_0^2, \tag{1}$$

is the angular separation between the two bodies at encounter.

In the context of Hill's three-body problem (Hill 1878), the evolution of the asteroid's orbit can be studied by averaging over the fast epicyclic motion (Dermott & Murray 1981a; Hénon & Petit 1986; Namouni 1999). Namouni (1999) derived an expression that describes the slow variation of the guiding centre:

$$a_r^2 = a_0^2 - \frac{4\mu}{3\pi} \int_{-\pi}^{\pi} \left[(e_r \cos u + a_r)^2 + 4(l_r - e_r \sin u)^2 \right.$$
$$\left. + I_r^2 \sin(u + \omega_r)^2 \right]^{-1/2} du, \tag{2}$$

where the subscript 'r' denotes the relative elements as defined in that work and $l_r = \phi/2$. The constant a_0 is related to the Jacobi

constant H_r and the conserved Poincaré action $K_r = 1 - \sqrt{1 - e_r^2} \cos I_r$:

$$a_0^2 = \frac{8}{3}(K_r - H_r) \qquad (3)$$

assuming a planetary mean motion of unity.

In our case, $a_r \ll e_r$ and we can write

$$a_r^2 = a_0^2 - \frac{4\mu}{3\pi e_r} \int_{-\pi}^{\pi} \left[\cos^2 u + 4(\bar{l} - \sin u)^2 \right.$$
$$\left. + (\bar{K} - 1)\sin(u + \omega_r)^2 \right]^{-1/2} \mathrm{d}u, \qquad (4)$$

where $\bar{K} = 1 + I_r^2/e_r^2$ and $\bar{l} = l_r/e_r$. The second term on the right-hand side of this equation is the ponderomotive potential S scaled by $8\mu/3e_r$ (equation 29 of Namouni 1999). It is π-periodic in ω_r and symmetric with respect to $\bar{l} = 0$. It was also shown in that paper that averaging the co-orbital potential over the phase $\bar{l}$ (or l_r) allows one to obtain analytically the secular evolution of e_r, I_r, ω_r and Ω_r if the orbits are non-collisional. In fact, the regularity of the secular dynamics persists at large eccentricity and inclination if Hill's equations are replaced with those of the circular restricted three body problem.

4.2 Short-term evolution

In the case of SO16, it is expected that the orbital elements will evolve in the short term due to the co-orbital resonance and in the long term due to secular perturbations. To understand the dynamical evolution of SO16 we must first test the assumptions under which the results of the previous section are valid. The condition that $\phi_{min} \gg a_0$ is satisfied as $|a_0| \leq \epsilon$; however, e_r and I_r are large relative to the mass parameter.

In Fig. 1 we show the variation of a_r, e_r and ω_r for several co-orbital cycles. We observe impulsive step-wise changes in e_r at the same time that a_r changes sign upon encounter with the Earth. These step-wise changes cancel each other out over a full libration cycle (two consecutive encounters) so that the original eccentricity is recovered. This behaviour of impulsive changes, also seen in the plot for ω_r as well as the evolution of I_r and Ω_r (not shown here), is similar to that observed in the integrations of Dermott & Murray (1981a) for symmetric orbits. It is superposed on a slower, gradual evolution of these orbital elements. These observations indicate that SO16 currently occupies the symmetric regime of co-orbital motion as discussed in Section 4.1 and that the secular motion is regular at these time-scales.

Therefore, the formulation presented in Section 4.1 may be used as a starting point in our effort to explain the pertinent characteristics of 2010 SO16's long-term orbital evolution. Note that, as the Earth's orbital eccentricity becomes significant within the time-scales considered in our integrations (Quinn, Tremaine & Duncan 1991), its contribution to e_r cannot be neglected.

By evaluating equation (4) at JD 245 5197.5 (2010 January 1.0 UT) when $l_r = -13°8/2$, $e_r = 0.0645$, $I_r = 14°5$, $\omega_r = 118°8$ and setting $a_r = 0$, we find $a_0 = 0.544\epsilon$. The corresponding angle from equation (1) is $15°5$, in good agreement given that the true orbit has a non-negligible eccentricity and inclination.

In panel (a) of Fig. 2 we observe an oscillation of ϕ_{min} with a period of $\sim 10^4$ yr and between $14°3$ (at $t = -2 \times 10^3$ yr) and $12°5$ (at $t = +3 \times 10^3$ yr). This is due to the contribution of e_r in equation (4) which oscillates with the same period. This element, together with ω_r, form a conjugate pair of variables which, as our integrations show, oscillate between two states; one where $e_r = $
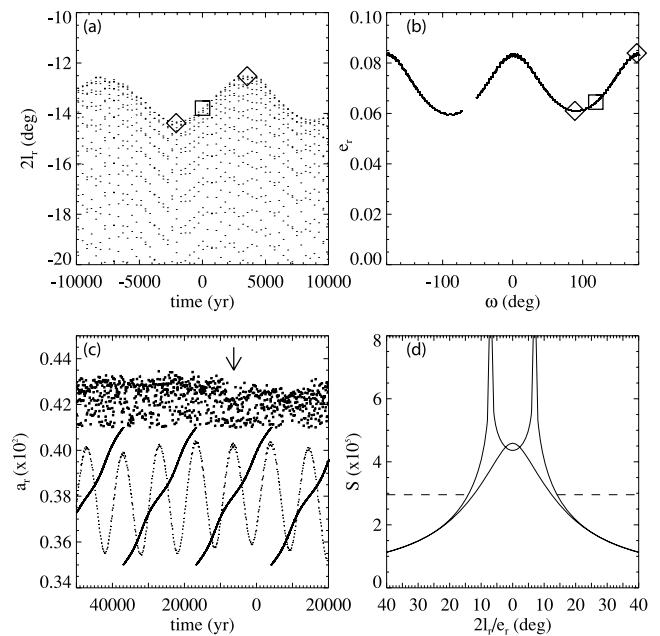

Figure 2. Evolution of the nominal orbit of 2010 SO16 (Table 1) for time-scales of the order of 10^4 yr showing key features of the dynamics. (a) The Earth-trailing branch of the relative mean longitude $2l_r$ between SO16 and the planet. Note the $\sim 10^4$ yr periodicity. The rectangle indicates the value of $2l_r$ at JD 245 5197.5. (b) Evolution of the relative eccentricity (e_r) as a function of the relative argument of pericentre (ω_r). The diamonds in this and the previous panel indicate the extreme states referred to in Section 4.2. (c) The evolution of the libration width of the relative semimajor axis a_r (for clarity, only values $\geq +0.41\epsilon$ are shown) through a jump, indicated by the arrow, at $t = -6.3 \times 10^3$ yr. The time evolution of e_r and ω_r is superimposed on the plot as the oscillating and slanted curves, respectively. (d) The form of the scaled ponderomotive potential $(8\mu/3e_r)S$ corresponding to the two snapshots in $e_r - \omega_r$ evolution highlighted in (b) and Section 4.2. The horizontal dashed line indicates the current state of SO16's horseshoe trajectory as quantified by the constant a_0^2. This is to be compared with the two values of the extremum at $\bar{l} = 0$ referred to in the text.

$e_{r,max} = 0.083$ and $\omega_r = k\pi$ and the other where $e_r = e_{r,min} = 0.061$ and $\omega_r = k\pi + \pi/2$ (panel b of Fig. 2). This cycle, hereafter referred to as the eccentricity cycle, by itself defines the object's closest possible distance to the Earth as $\simeq 0.12$ au. Two such cycles are completed for every full circulation of ω_r. This behaviour is characteristic of the horseshoe state (cf. fig. 12 of Namouni 1999) and demonstrates the regularity of the secular evolution of this asteroid's orbit over at least 2×10^4 yr or $\simeq 10^2$ horseshoe encounters with the Earth. Over the same period, the inclination I_r and action K_r remain essentially constant and equal to $\sim 14°$ and ~ 0.0340. If we substitute these values in equation (4) we find $\phi_{min} = 14°1$ and $12°4$ for the low-e and high-e states, respectively, in excellent agreement with the simulations.

The horseshoe trajectory of the guiding centre appears to widen at $t \simeq -6.3 \times 10^3$ yr (panel c of Fig. 2). This coincides with e_r reaching its maximum. However, similar behaviour is not observed at the next e_r maximum ($t = +3 \times 10^3$ yr) even though the same value is attained. Inspection of our simulations shows that this increase takes place for 10–15 per cent of our clones, indicating that it is not deterministic. Its size is $\Delta a_r \simeq 0.01\epsilon$. Nothing similar is observed in the time history of the other elements. Its impulsive nature indicates that this asteroid's orbit evolves towards the boundary of the symmetric horseshoe regime over time.

The ultimate fate of the horseshoe depends on the relative evolution of a_0 and the ponderomotive potential S. Since $e_r < I_r$ the extremum of the scaled ponderomotive potential at $\bar{l} = 0$ changes from a maximum to a minimum when $\omega_r > \omega_0(\bar{K})$ (Namouni 1999, see panel d of Fig. 2). This threshold evaluates to $30°$ for the average e_r and I_r of this asteroid's orbit. In any case, the horseshoe will transition to, or form a compound orbit with, a quasi-satellite orbit or a passing orbit when $a_0^2 \simeq (8\mu/3e_r)S$. For example, and for the e_r–ω_r behaviour observed in the integrations, the value of the extremum varies from $(0.661\epsilon)^2$ to $(0.678\epsilon)^2$ (see panel d of Fig. 2). Hence, an increase in a_0 of $\gtrsim 0.12\epsilon$ is required in order for the horseshoe to be disrupted.

4.3 Long-term evolution

To explore further the question of the horseshoe's likely survival lifetime and the mechanisms which might limit it, we generated additional clones by sampling the semimajor axis with a step of 0.1σ out to $\pm 1.4\sigma$ and then at ± 2, ± 3 and $\pm 4\sigma$. This new set of 35 clones was integrated for $\pm 2 \times 10^6$ yr. We regard the integrations in the past and in the future as separate trials. Out of this set of 70 trials, 50 persisted as horseshoes for 2–5×10^5 yr, four for $< 2 \times 10^5$ yr and the remaining 16 for $> 5 \times 10^5$. Eight clones persisted for $> 10^6$ yr while two remained in horseshoe libration until the end of the integration. The shortest lifetime observed was 1.2×10^5 yr. In Fig. 3 we show two typical cases of a clone that is stable and one that escapes within 10^6 yr. In panel (a), the width of the libration in a_r remains essentially constant over this period. At the same time, the behaviour of the eccentricity remains regular,
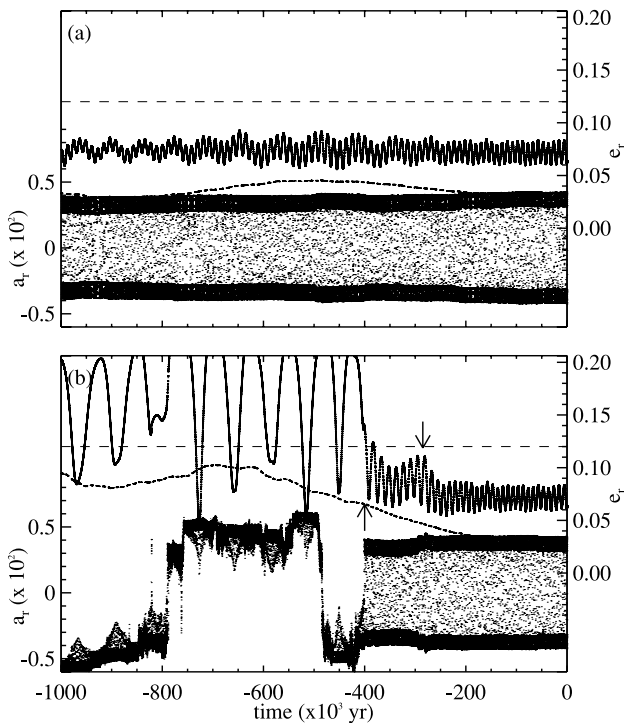
staying well below the collision threshold (dashed horizontal line; see below for explanation) despite subtle changes (e.g. the $\sim 5 \times 10^4$ yr modulation that sets in towards the middle of the plot). Interestingly, K_r also evolves in a regular fashion over this period, reaching a maximum value of ~ 0.05 near the middle of the time interval. The time-scale of variation implies that this evolution is due to secular forcing by the planets, acting to change the asteroid's (and the Earth's) orbital elements over $> 5 \times 10^4$ yr time-scales.

On the other hand, the behaviour of a_r and e_r of the clone shown in panel (b) changes markedly at $t = -2.8 \times 10^5$ and -4.0×10^5 yr. In the first instance, we observe an increase of both the average eccentricity and the amplitude of the secular cycle while the width of the horseshoe decreases slightly by $0.04\,\epsilon$. In the second instance, a further increase of the amplitude of the eccentricity cycle leads to a transition into a passing orbit. In both of these instances, the change occurs over 1–2 eccentricity cycles so no clear correlation with particular phases of the cycle – of the sort evident in Fig. 2(c) – can be made. Establishing such a correlation would require a thorough statistical analysis that is outside the scope of the paper. However, K_r evolves smoothly through the change (i.e. no jumps are observed), suggesting that they are dynamical features of the restricted Sun–Earth–asteroid problem. On the other hand, the time-scale over which these events occur, similar to that governing the evolution of K_r, implies that planetary secular perturbations also play a role. The transition itself occurs because the maximum of the scaled potential S is reduced to the point of reaching parity with a_0^2. In physical terms, the eccentricity becomes large enough (~ 0.12; indicated by the dashed horizontal line) to allow close encounters (< 0.01 au) with Earth while the inclination acts to stabilize the transition. Afterwards, the eccentricity becomes high enough to allow close encounters with Venus and the evolution is similar to that of other high-e, high-I co-orbitals of the Earth (Christou 2000).

5 COMPARISON WITH OTHER EARTH HORSESHOE ASTEROIDS

There are currently three NEAs known to follow horseshoe trajectories with respect to the Earth (Table 2; also Brasser et al. 2004). Of those, 2002 AA29 is the most similar to 2010 SO16, mainly due to its low eccentricity and moderate inclination. Its value of H implies a likely size of a few tens of metres. Hence, it is an order of magnitude smaller than SO16.

The orbital eccentricities of 54509 YORP and 2001 GO2 allow close encounters with the Earth. These do not necessarily eject the asteroids from the co-orbital resonance but instead they effect their transition into another mode of libration or circulation. This state is shared by other objects in so-called compound and/or transition orbits such as 3753 Cruithne, the first of its class to be recognized as such (Wiegert, Innanen & Mikkola 1997, 1998; Namouni 1999; Namouni, Christou & Murray 1999; Christou 2000). The opening angles $2\phi_{min}$ of GO2 and 54509 are wider than those of SO16 implying an energy state deeper into the horseshoe domain. However, the contribution of their higher eccentricities in equation (4) places them closer to the transition regime (see panel d of Fig. 2) than SO16 and accounts, at least in part, for their shorter lifetimes – a few times 10^3 yr – as horseshoe companions of the Earth.

6 DISCUSSION

The existence of this long-lived horseshoe raises the twin questions of its origin and whether objects in similar orbits are yet

Figure 3. Evolution of two dynamical clones of 2010 SO16 for 10^6 yr in the past. Panel (a) shows a clone that persists as a horseshoe of the Earth for this period. Panel (b) shows a horseshoe that is disrupted at $t = -4 \times 10^5$ yr following two impulsive changes in e_r (indicated by the arrows). The dashed horizontal line indicates the critical value of the eccentricity (~ 0.12) as discussed in the text. The evolution of the Poincaré action K_r, in the same scale as e_r, is denoted by the dashed curve.

Table 2. Known Earth horseshoe asteroids. Data for 2002 AA29 and 2001 GO2 were taken from Connors et al. (2002) and Brasser et al. (2004). Information for 3753 Cruithne (Wiegert et al. 1998) is also listed as this object is the archetype of the class of transient co-orbitals of the Earth. Parameters of 54509's horseshoe libration were determined by numerical integration of nine dynamical clones of that asteroid. Additional information was taken from Margot & Nicholson (2003).

Designation	$a_{\rm r,max}$ (ϵ)	e	I (°)	Obs. data arc[a] (d)	Lifetime (yr)	$2\phi_{\min}$ (°)	Libration period (yr)	H
2010 SO16	0.42	0.07	14.5	75	>120 000	25	350	20.7
2002 AA29	0.70	0.01	10.7	735	2000	∼8	190	24.1
2001 GO2 [b]	0.68	0.17	4.6	5	≤2500	36	200	23.5
54509 YORP	0.59	0.23	1.6	1100	≤2200	54	200	22.6
3753 Cruithne[c]	0.23	0.51	19.8	4200	5500	N/A	770	15.5

[a] At time of demonstration of horseshoe state.
[b] Obs. arc length 5 d.
[c] Compound orbit.

to be found. The object's Earth-like orbit makes a direct or indirect origin in the main belt an unlikely, although not impossible, proposition (Bottke et al. 2000; Brasser & Wiegert 2008). Another plausible source is the Earth–Moon system. Margot & Nicholson (2003) have suggested that 54509 may have originated within the Earth–Moon system. SO16's current orbit does not provide such a direct dynamical pathway to the Earth–Moon system as in that case, although the situation is likely to change within a time-scale of several times 10^5 yr. A third possibility is that the object originated near the asteroid–Earth–Sun L4 or L5 equilibrium point as a tadpole librator. Tabachnik & Evans (2000) showed that Earth tadpoles can persist for 5×10^7 yr in both high ($24° < I < 34°$) and low ($I < 16°$) inclination orbits and at typical eccentricities of ∼0.06. Extrapolating their results to 5×10^9 yr time-scales, they found it conceivable that a small fraction of a postulated initial population of Earth tadpoles can survive to the present. In order for this scenario to apply to SO16, it must have been a tadpole until very recently. However, the work of Tabachnik & Evans does not take into account the action of the Yarkovsky effect, which mobilizes the semimajor axes of NEAs at typical rates of 10^{-9} au yr^{-1} (Chesley et al. 2008). As the radial half-width of Earth's tadpole region is $\simeq \sqrt{8\mu/3} \sim 2.8 \times 10^{-3}$ au (Murray & Dermott 1999), it may be difficult for SO16 to survive as a tadpole companion of the Earth for more than a few times 10^6 yr. On the other hand, a study of the stability of Trojans of Mars has shown that the Yarkovsky acceleration does not necessarily destabilize large Trojan companions of the planets (Scholl, Marzari & Tricarico 2005) and this result may also apply here. In any case, observational determination of the asteroid's basic properties – size, spectral class and spin state – will be extremely useful in clarifying the situation. Subject to the successful determination of said parameters, SO16 may be a suitable test target for the direct detection of the Yarkovsky acceleration as an object a few hundred metres across that makes frequent close encounters with the Earth during the next decade (Vokrouhlický et al. 2005).

Finally, regarding the possibility of existence of other bodies in similar orbits, the case for Earth-based reconnaissance of the Trojan regions of the Earth has already been made (Evans & Tabachnik 2000; Wiegert, Innanen & Mikkola 2000). For relatively bright targets such as SO16, the main limiting factor is likely to be the minimum solar elongation that can be reached by an observational survey. As typical horseshoe libration periods are of the order of hundreds of years, completing the census of large ($H < 21$) terrestrial horseshoe or tadpole companions would likely require a space-based platform in a heliocentric orbit interior to the Earth's.

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