Mon. Not. R. astr. Soc. (1980) 191, 855–861

A lower limit to the magnetic field in Cassiopeia-A

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Received 1979 November 8; in original form 1979 March 8

Summary. The magnetic field strength in the radio-emitting shell of the supernova remnant Cassiopeia-A should be greater than 8×10^{-5} G if the bremsstrahlung by the electrons responsible for the non-thermal radio emission is not to exceed the upper limits to the gamma-ray emissivity set by the recent observations. This field strength is shown to be too large to be generated by a mere compression of the interstellar field by the supernova shock but must arise due to magnetohydrodynamic instabilities in the expanding shell. Gamma-ray generation through inverse-Compton scattering of photons in the surrounding H II region is also briefly discussed.

1 Introduction

Soon after it was recognized that the radio emission of supernova remnants (SNR) is mainly due to synchrotron radiation, Shklovskii (1960) suggested a possible equipartition between the energy densities of relativistic particles and the magnetic field, and predicted the rate of decrease of the radio luminosity of young SNR during their expansion. This prediction was verified qualitatively for the bright remnant Cassiopeia-A. On the other hand, van der Laan (1962) considered the radio emission as arising from the compression of the interstellar field and cosmic-ray electrons in the hydrodynamic shock between the expanding envelope and the interstellar medium. This theory explained the observations of old remnants like the Cygnus loop, but did not attempt to account for the strong emission from young SNR. More recently, Gull (1973a, b) has discussed the evolution of the structure and radio emission of SNRs in terms of a more elaborate fluid dynamical model and he concludes that the expanding shell is subject to various magnetohydrodynamic instabilities which cause it to become turbulent. The resulting shearing motions tangle up and amplify the ambient magnetic fields to equipartition values. Following the suggestion of Scott & Chevalier (1975) that, in such regions, particles could be accelerated to high energies by the Fermi-mechanism, an analytic solution has been obtained for the evolution of the synchrotron luminosity that satisfactorily explains all the radio data on Cassiopeia-A, as well as the general features of galactic SNR (Cowsik 1979). The strength of the magnetic field is an important parameter in all these considerations and a direct estimate of its magnitude would be of significance in clarifying the relevant issues.

Such a direct estimate of the magnetic field in SNR is made possible by the following considerations. Current ideas regarding stellar nucleosynthesis and the evolution of supernova progenitors (Arnett 1975) lead us to expect that the ejected material in SNR should consist of several solar masses of matter that is considerably enriched in heavy elements and, further, that H II regions would be associated with the remnants. Indeed, recent optical and X-ray observations of Cassiopeia-A provide definite support to these expectations (van den Bergh & Dodd 1970; Gorenstein, Harnden & Tucker 1974). A diagnostic tool is provided by the gamma rays generated through bremsstrahlung in the ambient matter and through inverse-Compton scattering of the photons associated with the remnant, by the relativistic electrons responsible for the synchrotron emission. The observed gamma-ray flux would define an upper limit to the relativistic electron density which would then demand a minimal magnetic field to generate the observed radio intensities. In the following sections of this paper we apply this general concept to the remnant Cassiopeia-A.

2 Summary of the observations

The SNR Cassiopeia-A is believed to have originated in a Type II supernova event (Minkowski 1968) about 300 yr ago, at a distance of 2.8 kpc (van den Bergh 1971). It is the brightest radio object in the sky and the high-resolution map of Rosenberg (1970) at 5 GHz indicates that almost all the radio emission arises from a shell of outer radius 130 arcsec and thickness 30 arcsec. The absolute radio flux is rather well measured over about three decades in frequency, with rms errors less than 5 per cent at any point. The power spectrum that describes the radio flux density between 38 MHz and 16 GHz is given by

$$F_\nu = (3181 \pm 26) \times 10^{-26} \nu^{-(0.792 \pm 0.006)} \ \mathrm{W\,m^{-2}\,Hz^{-1}} \tag{1}$$

at the epoch 1965.0 (Dent, Aller & Olsen 1973).

The radio-emitting shell is roughly coincident with an optical nebulosity consisting of small condensations of two distinct types characterized by either slow or rapid motions. The slowly moving condensations have electron densities of $\sim 7 \times 10^3\,\mathrm{cm^{-3}}$ and the relative abundance of nitrogen in these knots is enhanced by a factor of ~ 10 with respect to the so-called universal abundances. The rapidly moving condensations, on the other hand, have somewhat higher densities of $\sim 3 \times 10^4\,\mathrm{cm^{-3}}$, high temperature, high state of ionization and an over-abundance of oxygen and heavier elements. The total amount of matter in these optical knots is estimated to be $\sim 6 M_\odot$ by Gorenstein *et al.* (1974) on the basis of the difference between the ion and electron temperatures.

The study of X-ray emission from Cassiopeia-A is also very useful in fixing the mass of the material ejected during the supernova outburst. The X-ray emission originates from a shell which overlaps the radio-emitting region (Fabian, Zarnecki & Culhane 1973) and the spectrum of X-rays can be fitted by a two-component thermal bremsstrahlung process (Davison, Culhane & Mitchell 1976). From the measured luminosity of the high-temperature component alone, Pravdo *et al.* (1976) deduce the density of matter in the shell to be $\sim 4.5\,\mathrm{H\,atom\,cm^{-3}}$ corresponding to an ejected mass $\gtrsim 4.6 M_\odot$. Consistent with this is the interpretation of the X-ray data in terms of a reverse shock model by McKee (1974), who obtains an ejected mass of $\sim 6 M_\odot$.

Further support for the large amount of ejected matter comes from stellar-evolution calculations. The abundances in the fast-moving knots inferred by Peimbert (1971) were shown by Arnett (1975) to indicate a value of $22 M_\odot$ for the mass of the supernova progenitor. More recently the improved abundance determination of Chevalier & Kirshner

(1978) has been shown by Lamb (1978) to constrain the mass of the progenitor to $>9\,M_\odot$. The lack of any observable stellar remnant indicates that most of the mass was ejected as debris in the supernova event (Kamper & van den Bergh 1976).

Attempts to observe Cassiopeia-A in high-energy gamma-rays have yielded only upper limits on its emission so far. Using the telescope aboard *SAS-2*, Fichtel *et al.* (1975) place a 3σ upper limit of 1.1×10^{-6} photon cm^{-2} s^{-1} on the integral flux of gamma-rays above 100 MeV. In addition, Cassiopeia-A was not detected as a discrete source of gamma-rays by the *COS-B* telescope. On the basis of the sensitivity for detection of sources in the same galactic longitude region (Hermsen *et al.* 1977), we may infer a slightly better limit,

$$J_\gamma(>100\,\mathrm{MeV}) < 10^{-6}\ \mathrm{photon\ cm^{-2}\,s^{-1}}. \qquad (2)$$

3 Derivation of a lower limit to the magnetic field strength

First, we assume that the radio emission is synchrotron radiation generated by an isotropic distribution of relativistic electrons spiralling in a random magnetic field of strength H (gauss). Following Ginzburg & Syrovatskii (1964), the emissivity described in equation (1) leads to a spectral density of electrons

$$\begin{aligned} N_\mathrm{e}(E) &= 7.44\times10^{4}(R^2/V)\,H^{-1.792}E^{-2.584}\ \mathrm{cm^{-3}\,MeV^{-1}}, \quad \text{for} \quad E \geqslant 0.85H^{-1/2} \qquad (3)\\ &= 0, \quad \text{for} \quad E < 0.85H^{-1/2} \end{aligned}$$

Here R (cm) is the distance to the remnant, V(cm^3) the volume of the radio-emitting region and E (MeV) is the electron energy. These electrons would generate gamma-rays through bremsstrahlung in the ambient matter. Using the cross-sections in the weak-screening limit given by Hayakawa (1969), the integral flux of gamma-rays is given by

$$\begin{aligned} J_{\gamma,\mathrm{brems}}(>E_\gamma) &= \frac{M}{V}\left\langle\frac{Z(Z+1)}{A}\right\rangle 1.27\times10^{11}H^{-1}\ln(3.32H^{-1/2})\\ &\quad\times[1+1.584\{\ln(0.85H^{-1/2})-\ln E_\gamma\}], \quad \text{for} \quad E_\gamma \leqslant 0.85H^{-1/2};\\ &= \frac{M}{V}\left\langle\frac{Z(Z+1)}{A}\right\rangle 9.85\times10^{10}\ln\left(\frac{E_\gamma}{0.256}\right)E_\gamma^{-1.584}H^{-1.792} \quad \text{for} \quad E_\gamma \geqslant 0.85H^{-1/2}. \qquad (4) \end{aligned}$$

Further, if the H II region associated with the remnant generates a background of photons (mostly Lyman-α with an energy of 10.2 eV per photon) in the radio-emitting region, then inverse-Compton scattering by the relativistic electrons will yield an integral flux of high-energy gamma-rays

$$J_{\gamma,\mathrm{IC}}(>E_\gamma) = 2.85\times10^{-14}n_\mathrm{ph}H^{-1.792}E_\gamma^{-0.792} \quad \text{for} \quad E_\gamma \geqslant 1.1\times10^{-4}H^{-1}, \qquad (5)$$

where n_ph(cm^{-3}) is the density of the Ly-α photons (Ginzburg & Syrovatskii 1964).

Now, to set a strict lower limit on the strength of the magnetic field, we will initially consider only the gamma-rays generated by bremsstrahlung in the ejecta, and neglect contributions of bremsstrahlung in the swept-up interstellar matter and inverse-Compton scattering of ambient photons. From the observations presented earlier, the volume V of the region-emitting radio waves is 3.8×10^{56} cm^3, and a reasonable lower limit to the mass M of the ejecta in the volume is $5\,M_\odot$ ($\approx 10^{34}$ g). From the estimates of elemental abundances (Chevalier & Kirshner 1978), we obtain the value of $\langle Z(Z+1)/A\rangle$ to be ~4.3. The integral gamma-ray spectrum, calculated using equation (4) for several values of the ambient magnetic field, is shown in Fig. 1. It is immediately noticed that, unless the nebular field is

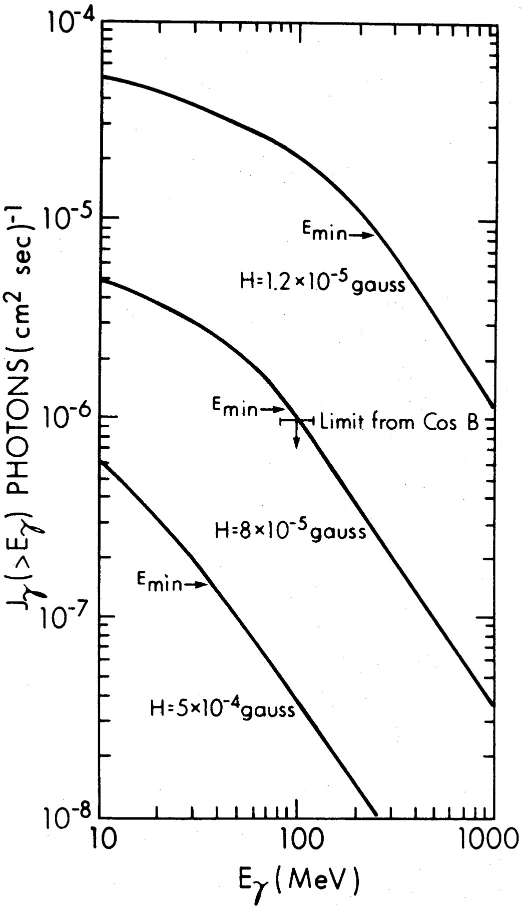

Figure 1. Integral flux of gamma-rays from Cassiopeia-A due to bremsstrahlung in the ejected matter, for various values of the magnetic field. The electron spectrum must extend down to E_{min} or to still smaller energies, so that the synchrotron spectrum may not show too much curvature.

larger than 8×10^{-5} G, the expected bremsstrahlung gamma-ray flux will exceed the observational upper limit of $< 10^{-6}\,\text{cm}^{-2}\,\text{s}^{-1}$ at 100 MeV. This leads us to conclude that the magnetic field in Cassiopeia-A is indeed stronger than 8×10^{-5} G.

4 Discussion

4.1 EVIDENCE FOR FIELD-AMPLIFICATION

The lower limit to the field of $\sim 8 \times 10^{-5}$ G derived in Section 2 clearly exceeds the value of 1.2×10^{-5} G expected as a result of compression of typical interstellar fields of $\sim 3 \times 10^{-6}$ G, by the supernova shock (van der Laan 1962). However, to demonstrate conclusively that an amplification of the interstellar field has occurred over and beyond a simple adiabatic compression, we must discuss the consequences of an initially strong ambient interstellar field whose compression in the shock might have given a field strength consistent with our lower limit.

We may proceed by making the reasonable assumption that the strength of the ambient interstellar field, H_{amb}, is correlated with the density of the ambient interstellar matter according to the frozen-in condition $H_{\text{amb}} = H_0 n_{\text{H}}^{2/3}$, where $H_0 \approx 3 \times 10^{-6}$ G for a typical interstellar density $\bar{n}_{\text{H}} \approx 1$ H atom cm^{-3}. Let the actual field in the SNR, H, be a factor η larger than H_{amb}, i.e. $H = \eta H_0 n_{\text{H}}^{2/3}$. If we wish to choose a high value of H_{amb}, then we should also include in the estimates of the γ-ray spectra the bremsstrahlung in the correspondingly large amount of interstellar matter swept up with the field. The mass of the swept-up matter in the shell of Cassiopeia-A is about $0.56 n_{\mathrm{H}} M_\odot$ which, when included with $5 M_\odot$ of the ejecta, leads to the constraining equation

$$J_\gamma(>100\,\mathrm{MeV}) = 3.56\times10^{-4}\eta^{-1.792} n_{\mathrm{H}}^{-1.195} + 1.86\times10^{-5}\eta^{-1.792} n_{\mathrm{H}}^{-0.195} \leqslant 10^{-6}\,\text{gamma-ray cm}^{-2}\,\mathrm{s}^{-1}. \tag{6}$$

The values of the amplification factor η consistent with the above equation are listed in Table 1 for various assumed values of H_{amb}. Since compression of magnetic fields in strong shocks can increase their strength at most by a factor of about 4, the high values of η shown in Table 1 are evidence for amplification of the field by secondary processes. If, in fact, we choose n_{H} to be between 1 and 2 in accord with the X-ray data, we get $\eta \approx 20$, about five times larger than the value for adiabatic compression.

The high value of the magnetic field derived earlier cannot be attributed to any pulsar, since there is no evidence of pulsation at either radio (Reifenstein, Brundage & Staelin 1969), optical (Horowitz, Papaliolios & Carleton 1971) or X-ray (Holt *et al.* 1973) frequencies. Further, both radio (Rosenberg 1970) and X-ray maps (Charles, Culhane & Fabian 1977) fail to show any evidence of enhanced emission from the central regions of the nebula.

A satisfactory explanation for the large field in the remnant can be sought on the basis of several theoretical ideas reviewed by Caswell & Lerche (1979). Notice that the measurements show the radio shell of Cassiopeia-A to be moving considerably slower than the optical knots, indicating that the faster-moving ejected matter is catching up with the radio shell, generating turbulence and thereby contributing to the maintenance of a strong magnetic field (Bell 1977). Alternatively, the increasing mass in the radio shell could be prolonging the Rayleigh–Taylor instability which drives the turbulence and consequently amplifies the field, as in the model by Gull (1973a). Even though the limit on the magnetic field derived in this paper is lower than the 'equipartition field' of 4×10^{-4} G, it still clearly indicates that these processes of magnetohydrodynamic field amplification are at work in the remnant.

4.2 INVERSE-COMPTON GAMMA-RAYS

A more definitive determination of the magnetic field than that presented here would be possible by improved measurements of the actual spectrum of high-energy gamma-rays from

Table 1. The amplification factor, η, by which the ambient magnetic field, H_{amb}, is boosted, for various values of the interstellar density, n_{H}, around Cassiopeia-A. H is the value of the amplified field.

n_{H} (cm^{-3})	H_{amb} (10^{-5} G)	η	H (10^{-5} G)
1	0.3	27.3	8.2
2	0.48	17.7	8.4
3	0.62	13.8	8.6
5	0.88	10.3	9.1
7	1.1	8.6	9.5
10	1.4	7.2	10
20	2.2	5.4	12
30	2.9	4.6	13
50	4.1	4.0	16
70	5.1	3.7	19
100	6.5	3.4	22
200	10	3.0	31

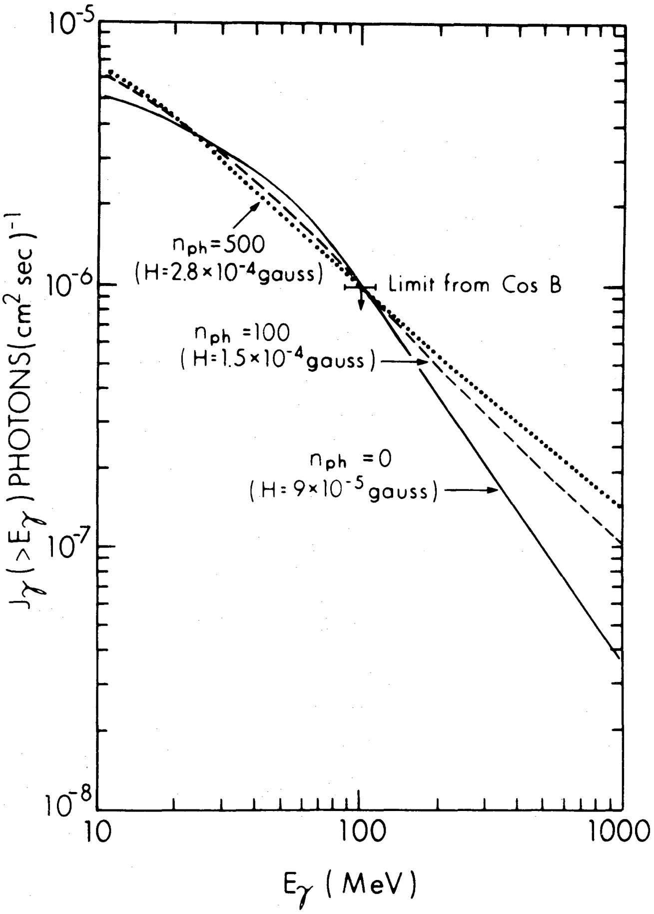

Figure 2. The expected spectral shape of gamma rays for various values of the Ly α photon density $n_{\mathrm{ph}}(\mathrm{cm}^{-3})$. Besides the inverse-Compton effect, bremsstrahlung in $5M_\odot$ of ejected matter and in $3M_\odot$ of swept-up interstellar matter is included.

the remnant. In interpreting any such spectrum it is to be recognized that inverse-Compton scattering of the Lyman α photons from the H II region associated with the remnant could contribute significantly to the gamma-ray flux. To estimate this contribution we assume diffusive propagation of the Ly α photons and derive a photon density $n_{\mathrm{ph}} \sim 250\,\mathrm{cm}^{-3}$ for typical values of the opacity (Kaplan & Pikelner 1970) and the estimated value of $8\,\mathrm{cm}^{-3}$ for the electron density (Peimbert 1971). The actual density of the photons could, of course, be much less, due to capture by partially ionized heavy elements in the supernova shell. Because of this uncertainty, we treat n_{ph} as a free parameter and show the total gamma-ray spectrum due to inverse-Compton and the bremsstrahlung processes in Fig. 2. With an actual determination of both the absolute intensities and the spectral shape of the gamma-rays, a clear separation of the relative contribution of the two processes can be achieved.

Acknowledgments

One of us (RC) thanks Professor R. M. Walker and the McDonnell Center for the Space Sciences, Washington University, St Louis, Missouri, for hospitality during the preparation of this manuscript.

References

Arnett, W. T., 1975. *Astrophys. J.*, **195**, 727.

Bell, A. R., 1977. *Mon. Not. R. astr. Soc.*, **179**, 573.

Caswell, J. L. & Lerche, I., 1979. Preprint.

Charles, P. A., Culhane, J. L. & Fabian, A. C., 1977. *Mon. Not. R. astr. Soc.*, **178**, 307.

Chevalier, R. A. & Kishner, R. P., 1978. *Astrophys. J.*, **219**, 931.

Cowsik, R., 1979. *Astrophys. J.*, **227**, 856.

Davison, P. J. N., Culhane, J. L. & Mitchell, R. J., 1976. *Astrophys. J.*, **206**, L37.

Dent, W. H., Aller, H. D. & Olsen, E. T., 1974. *Astrophys. J.*, **188**, L11.

Fabian, A. C., Zarnecki, J. C. & Culhane, J. L., 1973. *Nature Phys. Sci.*, **242**, 18.

Fichtel, C. E., Hartman, R. C., Kniffen, D. A., Thompson, D. J., Bignami, G. F., Ogelman, H. B., Ozel, M. E. & Tiimer, T., 1975. *Astrophys. J.*, **189**, 163.

Ginzburg, V. L. & Syrovatskii, S. I., 1964. *The Origin of Cosmic Rays*, Pergamon Press, Oxford.

Gorenstein, P., Harnden, F. R. & Tucker, W. H., 1974. *Astrophys. J.*, **192**, 661.

Gull, S. F., 1973a. *Mon. Not. R. astr. Soc.*, **161**, 47.

Gull, S. F., 1973b. *Mon. Not. R. astr. Soc.*, **162**, 135.

Hayakawa, S., 1969. *Cosmic Ray Physics*, Wiley Interscience, London.

Hermsen, W. *et al.* (Caravanne Collaboration), 1977. *Nature*, **269**, 494.

Holt, S. S., Boldt, E. A., Serlemitsos, P. J. & Brisken, A. F., 1973. *Astrophys. J.*, **180**, 169.

Horowitz, P., Papaliolios, C. & Carleton, N. P., 1971. *Astrophys. J.*, **163**, L5.

Kamper, K. & van den Bergh, S., 1976. *Astrophys. J. Suppl.*, **32**, 351.

Kaplan, S. A. & Pikelner, S. B., 1970. *The Interstellar Medium*, Harvard University Press.

Lamb, S. A., 1978. *Astrophys. J.*, **220**, 186.

McKee, C. F., 1974. *Astrophys. J.*, **188**, 335.

Minkowski, R., 1968. *Nebulae and Interstellar Matter*, p. 623, eds Middlehurst, B. M. & Aller, L. H., University of Chicago Press.

Peimbert, M., 1971. *Astrophys. J.*, **170**, 261.

Pravdo, S. H., Becker, R. H., Boldt, E. A., Holt, S. S., Rothschild, R. E., Serlemitsos, P. J. & Swank, J. H., 1976. *Astrophys. J.*, **206**, L41.

Reifenstein, E. C., Brundage, W. D. & Staelin, D. H., 1969. *Astrophys. J.*, **156**, L125.

Rosenberg, I., 1970. *Mon. Not. R. astr. Soc.*, **151**, 109.

Scott, J. S. & Chevalier, R. A., 1975. *Astrophys. J.*, **197**, L5.

Shklovskii, I. S., 1960. *Soviet Astr. A.J.*, **4**, 243.

van den Bergh, S., 1971. *Astrophys. J.*, **165**, 457.

van den Bergh, S. & Dodd, W. W., 1970. *Astrophys. J.*, **162**, 485.

van der Laan, H., 1962. *Mon. Not. R. astr. Soc.*, **124**, 125.