A Mapping from Sequence-Pair to Rectangular Dissection *

H. Murata† K. Fujiyoshi† T. Watanabe† Y. Kajitani‡

†Japan Advanced Institute of Science and Technology, Ishikawa, Japan
‡Tokyo Institute of Technology, Tokyo, Japan

Abstract— A fundamental issue in floorplanning is in how to represent candidate solutions. Recently, a representation called *sequence-pair* is proposed [1]. Seq-pair is so general as to represent an area minimum placement, and also efficient because it does not represent any overlapping placement. However, seq-pair is not expressive enough since channels are not represented. This paper gives a mapping from seq-pair to rectangular dissection, which represents channels by line segments. Consequently, candidate arrangements of modules and channels are successfully represented with the generality and the efficiency inherited from the seq-pair.

I. Introduction

In the first stage of VLSI physical design, it is required to determine a rough arrangement of circuit components, such as modules and channels. A stochastic algorithm, such as simulated annealing or genetic algorithm, would be a good choice as an optimization algorithm since the problem is hard. To make a stochastic algorithm work effectively, a fundamental issue is in how to represent candidate arrangements, with enough generality and efficiency to cope with various design requirements.

Recently, Murata, Fujiyoshi, Nakatake, and Kajitani [1] proposed a representation called *sequence-pair*, which is a pair of module name sequences. For example, (abc, cab) is a seq-pair for module set $\{a, b, c\}$. For a seq-pair, they assigned a *HV-relation-set (HVRS)*, which is a set of horizontal (right of/left of) or vertical (above/below) relations for every module pair. For example, seq-pair (abc, cab) corresponds to HVRS $\{a$ is left of b, c is below a, c is below $b\}$. It is proved in the paper that a seq-pair always corresponds to a realizable HVRS, and there is a seq-pair whose HVRS can lead an area minimum placement. However, HVRS alone is not sufficient as a representation of candidate arrangements of components. Channel positions are also desired to be represented together.

A traditional method exists to represent channel positions together with module positions. It is the *rectangular-dissection*. (sometimes called *floorplan* in the literature [8].) However, known efficient representation techniques are limited for specific classes of rectangular-dissections, such as *slicing structure* [6].

To combine the merits of seq-pair and rectangular-dissection, it is desired to map a seq-pair to a rectangular-dissection. Observe from Fig. 2-(a) that a room with no module assignment, called the *empty room*, is necessary in the rectangular-dissection to keep the relational positions of modules. It worth to allow this empty room since it is essentially needed to represent an area optimal placement. However, introducing arbitrary many empty rooms results in arbitrary many line segments, which represent channels.

This paper gives a mapping from a seq-pair to a rectangular-dissection whose number of rooms is minimum among all the rectangular-dissections whose HVRSs are equivalent to the HVRS of the given seq-pair. Consequently, candidate arrangements of modules and channels are successfully represented with the generality and the efficiency inherited from the seq-pair.

The organization of this paper is as follows. Section II defines preliminary terms. Section III shows a necessary and sufficient condition that a seq-pair is mapped to a rectangular-dissection with no empty room. Section IV presents a procedure to output a rectangular-dissection with fewest empty rooms. Section V is for conclusion.

An early version of this paper is presented in [2].

II. Preliminary

A. HV-Relation-Set (HVRS)

A *HV-relation-set* for a set of modules is a set of horizontal (right of / left of) or vertical (above/below) relations for all module pairs. For example,

$$\{a \text{ is left of } b,\ c \text{ is below } a,\ c \text{ is below } b\}$$

is an HVRS for module set $\{a, b, c\}$. The cardinality of an HVRS is $\binom{n}{2}$, where n is the number of modules. The variety of HVRS is $4^{\binom{n}{2}}$.

A HVRS may or may not be realizable. The above example is realizable. A non-realizable example is : $\{a$ is left of b, b is left of c, c is left of $a\}$. A branch and bound approach [4] can be used to eliminate non-realizable HVRSs.

B. Sequence-Pair

A *seq-pair* is an ordered pair of Γ_+ and Γ_-, where each of Γ_+ and Γ_- is a sequence of names of given n modules. For example, $(\Gamma_+, \Gamma_-) = (abcd, bdac)$ is a seq-pair of module set $\{a, b, c, d\}$. If module x is the i'th module in Γ_+,

*This work was supported in part by Research Body CAD21.

ASP-DAC '97
0-89791-851-7$5.00

we denote $\Gamma_+(i) = x$, as well as $\Gamma_+^{-1}(x) = i$. Similar notation is used also for Γ_-. To help intuitive understanding, we use a notation such as

$$(\Gamma_+, \Gamma_-) = (\cdot\cdot\, a \cdot\cdot b \cdot\cdot, \cdot\cdot\, a \cdot\cdot b \cdot\cdot)$$

by which we mean

$$\Gamma_+^{-1}(a) < \Gamma_+^{-1}(b) \quad \text{and} \quad \Gamma_-^{-1}(a) < \Gamma_-^{-1}(b).$$

A seq-pair corresponds to an HVRS as follows [1]. For every module pair $\{a, b\}$, a is left of b (equivalently, b is right of a) if

$$(\Gamma_+, \Gamma_-) = (\cdot\cdot\, a \cdot\cdot b \cdot\cdot, \cdot\cdot\, a \cdot\cdot b \cdot\cdot).$$

Similarly, a is below b (equivalently, b is above a) if

$$(\Gamma_+, \Gamma_-) = (\cdot\cdot\, b \cdot\cdot a \cdot\cdot, \cdot\cdot\, a \cdot\cdot b \cdot\cdot).$$

For example, seq-pair $(abcd, bdac)$ implies HVRS: $\{b$ is below a, b is left of d, d is below c, a is left of c, d is below a, b is left of $c\}$.

The variety of HVRS represented by the seq-pair equals to the variety of the seq-pair, $(n!)^2$, thus drastically reduced from the original variety $4^{\binom{n}{2}}$, where n is the number of modules. Furthermore, seq-pair has the following property.

Property1 [1] The HVRS of every seq-pair is realizable. For any non-overlapping placement, there is a seq-pair whose HVRS is satisfied by the placement. $\square$

The HVRS of a seq-pair of n modules can be graphically understood by means of *oblique-grid*, defined as follows. Let $L_+(1), L_+(2), \cdots, L_+(n)$ be n parallel lines of slope $+1$ drawn on a plane, ordered from left. Let $L_-(1), L_-(2), \cdots, L_-(n)$ be n parallel lines of slope -1 drawn on the plane, also ordered from left. These $2n$ lines form a 45 degree oblique $n \times n$ grid, called the *oblique-gird*. The *oblique-grid-embedding* of a seq-pair (Γ_+, Γ_-) is the oblique-grid with each module name x written at the cross point of $L_+(\Gamma_+^{-1}(x))$ and $L_-(\Gamma_-^{-1}(x))$. Fig. 1-(a) shows the oblique-grid-embedding of seq-pair $(abcd, bdac)$. Using the oblique-grid-embedding, the HVRS of a seq-pair can be re-defined as: for each module x, the modules which are seen from x in the angle between -45 degree and 45 degree are right of x, the modules in the angle between 45 degree and 135 degree are above x, and so on.

The HVRS of a seq-pair is represented by a pair of directed acyclic graphs, called *horizontal-seq-pair-graph (H-SPG)* and *vertical-seq-pair-graph (V-SPG)*, defined as follows. For either graph, vertices uniquely correspond to modules and have the corresponding module names. The edge set of the H-SPG is constructed faithfully to the horizontal relations, from left to right, but eliminating the transitive edges. The edge set of the V-SPG is defined similarly from bottom to top. We sometime abbreviate the pair of H-SPG and V-SPG of a seq-pair to "SPGs".

Oblique-grid-embedding of a seq-pair with arrows additionally drawn corresponding to the edges of the SPGs is called the oblique-grid-embedding of the SPGs. Fig. 1-(b) shows an example, where the edges of the H-SPG are drawn using solid lines, and the edges of the V-SPG are drawn using dotted lines.

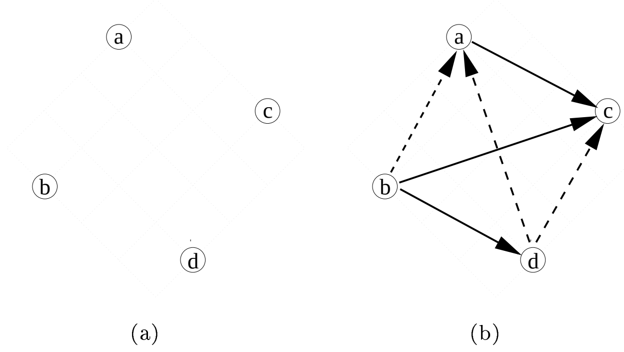


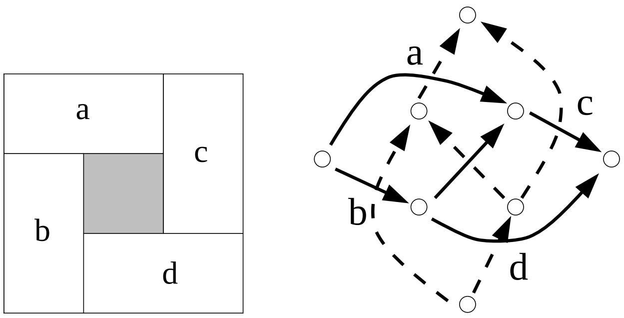
Fig. 1. (a) Seq-Pair $(abcd, bdac)$, (b) Horizontal-Seq-Pair-Graph (H-SPG) and Vertical-Seq-Pair-Graph (V-SPG). The edges of H-SPG are drawn in solid lines and the edges of V-SPG are drawn in dotted lines.

C. Rectangular-Dissection

A *rectangular-dissection* is a dissection of a rectangle into a set of rectangles, called *rooms*, with an injective assignment of modules to rooms (no two modules share a room.) An example is shown in Fig. 2-(a). Only T-intersections are used to form the dissection except for the four corners of the bounding rectangle. (Two T-intersections may form a cross shape as a degenerate case.) The bounding rectangle represents the chip, each room represents an area which is assignable to a module, and each line segment represents a channel. A room is said to be *occupied* if a module is assigned to the room, otherwise said to be *empty*. In Fig. 2-(a), the gray room at the center is empty and the other rooms are occupied. Empty rooms have been used to modify a rectangular-dissection incrementally[7].

A rectangular-dissection specifies relative positions of modules and channels as follows: If the right side of a room r_a and the left side of a room r_b are both on a same vertical line segment l_c, the module a assigned to the room r_a should be placed left of the channel c corresponding to the line segment l_c, and the module b assigned to the room r_b should be placed right of the channel c (horizontal relation). Notice that a horizontal relation between module pair a, b is transitively specified as: module a should be placed left of module b. Vertical relations are specified similarly using horizontal line segments.

The information of a rectangular-dissection is commonly represented by means of a pair of directed acyclic graphs [9, 5, 8], a *horizontal-rectangular-dissection-graph (H-RDG)* and a *vertical-rectangular-dissection-graph (V-RDG)*. Each vertical (horizontal) line segment corresponds to a vertex in the H-RDG (V-RDG) and each room corresponds to an edge (u, v) where u is the vertex corresponding to the left (bottom) side of the room and v is the vertex corresponding to the right (top) side of the room. We sometime abuse the word RDGs to denote the pair of H-RDG and V-RDG of a rectangular-dissection. Two rectangular-dissections are said to be equivalent if their



(a) (b)

Fig. 2. (a) Rectangular-Dissection, (b) Horizontal-Rectangular-Dissection-Graph (H-RDG) and Vertical-Rectangular-Dissection-Graph (V-RDG). H-RDG is drawn in solid lines and V-RDG is drawn in dotted lines.

RDGs (the two H-RDGs, as well as the two V-RDGs) are same. Fig. 2-(b) illustrates the RDGs of the rectangular-dissection shown in Fig. 2-(a). In the figure, the edges of H-RDG are drawn using solid lines, and the edges of V-RDG are drawn using dotted lines. An empty room corresponds to the anonymous edge in the figure.

H-RDG as well as V-RDG is a directed acyclic planar graph with possibly duplicated edges. Each RDG is polar, i.e. a directed acyclic graph with single source and single sink. Two polar graphs G_1 and G_2 are said to be in *polar-dual* relation if G_1 and G_2 become dual when an undirected edge from source to sink is added in each graph. From the configuration, the RDGs are in polar-dual relation. The reverse is also true since polar-dual graphs are known to be mapped to a rectangular-dissection [5].

Property2 Given two polar graphs G_1 and G_2, there exists a rectangular-dissection whose RDGs are G_1 and G_2, if and only if G_1 and G_2 are in polar-dual relation. $\Box$

When we construct a rectangular-dissection from H-RDG G_h and V-RDG G_v, we use the following procedure.

Procedure $\mathsf{ConstRD}(G_h, G_v)$
For a vertex $u \in V(G_h)$, $x(u)$ denotes the ordinal number of the vertex u in a topological order of the vertices in G_h. ($x(u)$ has a unique integer such that $x(u) < x(u')$ if there exists a path from u to u'). Similarly, $y(v)$ denotes the ordinal number of the vertex v in a topological order of the vertices in G_v. A pair of edges (e_h, e_v) is called a "cross" if $e_h(\in E(G_h))$ and $e_v(\in E(G_v))$ are in a dual relation. For each cross $((u_1, u_2), (v_1, v_2))$, draw a rectangle whose lower left corner is at $(x(u_1), y(v_1))$ and whose upper right corner is at $(x(u_2), y(v_2))$. (**Procedure ConstRD End**)

It is easily seen that ConstRD runs in $O(m)$ time, where $m = |E(G_h)| = |E(G_v)|$ which also equals to the number of rooms in the resultant rectangular-dissection.

D. Seq-Pair and Rectangular-Dissection

The major merit of the seq-pair and that of the rectangular-dissection is summarized as follows.

- The merit of the seq-pair is its efficiency in enumerating various HVRSs.
- The merit of the rectangular-dissection is its ability of representing the channels.

To have the two merits at the same time, the target of this paper is:

Target: To map a seq-pair to a rectangular-dissection.

The following three properties show a similarity of the seq-pair and the rectangular-dissection.

Property3 Given a seq-pair, for any two modules a and b, there is a path which connects a and b in H-SPG or in V-SPG, and not in both. $\Box$

Property4 In the H-RDG (V-RDG) of a rectangular-dissection, if there is a path from edge a to edge b, then the room a is left of (below) room b in the rectangular-dissection. $\Box$

Property5 Given a rectangular-dissection, for any two rooms a, b, there is a path which connects a and b in H-RDG or in V-RDG, and not in both. $\Box$

Property 3 and 4 are easily understood. See appendix for a proof of Property 5.

Property 4 and 5 imply that a rectangular-dissection, as well as a seq-pair, uniquely corresponds to an HVRS. Then, the correspondence between the seq-pair and the rectangular-dissection is in question. Next property can be easily derived from the result of [1].

Property6 For the HVRS T of any rectangular-dissection, there is unique seq-pair S whose HVRS is T. $\Box$

The reverse direction is essential to achieve our target. We have following observations.

Observation1 There is a seq-pair whose HVRS can only be represented by a rectangular-dissection with an empty room. $\Box$

$(abcd, bdac)$ is an example of such seq-pair whose HVRS can only be represented using an empty room. Fig. 1-(a) and Fig. 2-(a) illustrate the seq-pair and the corresponding rectangular-dissection.

Observation2 There is a set of modules whose area minimum placement can only be represented by a rectangular-dissection with an empty room. $\Box$

For instance, area minimum placement of four modules of sizes 3×2, 2×3, 3×3 and 2×4, can be represented essentially only by the rectangular-dissection shown in Fig. 2-(a). From Observation 1 and 2, it is our constraint that:

Constraint: The HVRS of a seq-pair should be preserved by the targeted mapping.

Observation3 For an HVRS, rectangular-dissection is not unique if arbitrary many empty rooms are allow to be introduced. $\Box$

Property7 For any rectangular-dissection, the number of line segments is equal to the number of rooms plus three. □

Property 7 can be proved by counting the number of room corners contributed by a line segment.

Recall that the line segments represent channels. Although the goodness about the number of channels might differ in several routing schemes, fewer number of channels is most likely preferred to avoid too many wire bends. Thus it is our criterion that:

Criterion: Minimize the number of rooms in the targeted mapping.

For an extreme counter situation, if n^2 rooms are acceptable for every seq-pair of n modules, it is known that one specific (fixed) rectangular-dissection, called *Bounded Sliceline Grid (BSG)*, is sufficient to represent the HVRS of an arbitrary seq-pair[3].

III. Rectangular-Dissection with No Empty Room

This section gives a procedure which maps a seq-pair to a rectangular-dissection without any empty room if the given seq-pair satisfies a condition. Then, the condition is revealed to be necessary and sufficient for eliminating the introduction of empty room. To describe the condition, we need to define two terms, *HV-cross* and *adjacent-cross*.

A. HV-cross and Adjacent-cross

Four modules a, b, c, d are said to form a *HV-cross* in a seq-pair $S = (\Gamma_+, \Gamma_-)$ if they satisfy the following three conditions in (Γ_+, Γ_-) or in (Γ_+, Γ'_-), where Γ'_- is the reverse of Γ_-.

- $(\cdot\cdot\, a \cdot\cdot\, b \cdot\cdot\, c \cdot\cdot\, d \cdot\cdot, \cdot\cdot\, c \cdot\cdot\, a \cdot\cdot\, d \cdot\cdot\, b \cdot\cdot)$
- There is no module x which satisfies

$$(\cdot\cdot\, a \cdot\cdot\, x \cdot\cdot\, d \cdot\cdot, \cdot\cdot\, a \cdot\cdot\, x \cdot\cdot\, d \cdot\cdot).$$

- There is no module x which satisfies

$$(\cdot\cdot\, b \cdot\cdot\, x \cdot\cdot\, c \cdot\cdot, \cdot\cdot\, c \cdot\cdot\, x \cdot\cdot\, b \cdot\cdot).$$

Fig. 3-(a) illustrates an HV-cross using oblique-grid. There is no module in the dark region because of the last two conditions in the definition. HV-cross is so called because it corresponds to a crossing between an edge in the H-SPG and an edge in the V-SPG in the oblique-grid-embedding of the SPGs.

If four modules a, b, c, d form an HV-cross and b and c are adjacent in Γ_+, and a and d are adjacent in $\Gamma_-(\Gamma'_-)$, the HV-cross is also called the *adjacent-cross*. The condition is illustrated in Fig. 3-(b).

Lemma1 If there is an HV-cross in a seq-pair S, then an adjacent-cross also exists in S.
(proof) The proof is by contradiction. Without loss of generality, let an HV-cross formed by four modules a, b, c

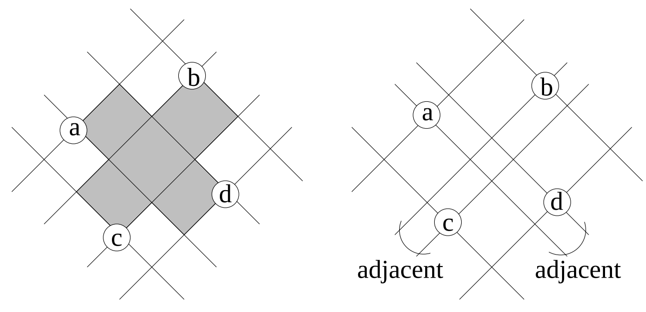


Fig. 3. (a) HV-cross (no module is in the dark region). (b) adjacent-cross (special case of HV-cross).

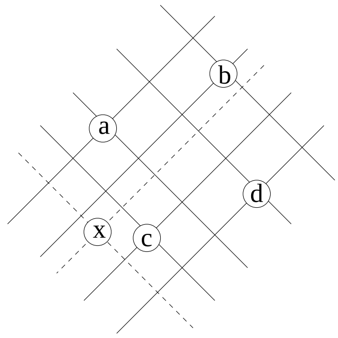


Fig. 4. Figure used in the proof of Lemma 1

and d be $S = (\Gamma_+, \Gamma_-) = (\cdot\cdot\, a \cdot\cdot\, b \cdot\cdot\, c \cdot\cdot\, d \cdot\cdot, \cdot\cdot\, c \cdot\cdot\, a \cdot\cdot\, d \cdot\cdot\, b \cdot\cdot)$. (See Fig. 4.) We can assume further that: (i) the distance between b and c in Γ_+ is minimum over all the HV-crosses in S; and (ii) among such HV-crosses, the distance between a and d in Γ_- is minimum.

If b and c are not adjacent in Γ_+, there is a module in between. Such modules are not between b and c in Γ_-, from the definition of HV-cross. In such modules, there is module x which satisfies one of the two cases:

- $S = (\cdot\cdot\, a \cdot\cdot\, b \cdot\cdot\, x \cdot\cdot\, c \cdot\cdot\, d \cdot\cdot, \cdot\cdot\, x \cdot\cdot\, c \cdot\cdot\, a \cdot\cdot\, d \cdot\cdot\, b \cdot\cdot)$ and a, b, x, d form an HV-cross, or
- $S = (\cdot\cdot\, a \cdot\cdot\, b \cdot\cdot\, x \cdot\cdot\, c \cdot\cdot\, d \cdot\cdot, \cdot\cdot\, c \cdot\cdot\, a \cdot\cdot\, d \cdot\cdot\, b \cdot\cdot\, x \cdot\cdot)$ and a, x, c, d form an HV-cross.

(Fig. 4 illustrates an example for the former case.) Either case contradicts to the assumption (i). Similarly, if a and d are not adjacent in Γ_-, a contradiction to the assumption (ii) is derived. Hence, a, b, c, d form an adjacent-cross. □

B. Procedure SeqPair–RDG

A procedure called SeqPair–RDG is presented to map a seq-pair to a pair of RDGs. From the resultant RDGs, a rectangular-dissection is obtained by the procedure ConstRD given in Section II. Fig. 5 illustrates the result of each step for input seq-pair $S = (abcde, becad)$. A hyper directed edge is denoted (V_i, V_o), where V_i is the input vertex set, and V_o is the output vertex set.

Procedure SeqPair–RDG

Input: Seq-pair $S = (\Gamma_+, \Gamma_-)$ which has no adjacent-cross.

Output: H-RDG G_{HP} and V-RDG G_{VP}.

(Step 1) Add four new modules s_h, t_h, s_v, t_v, called *phantom* modules, to the input seq-pair $S = (\Gamma_+, \Gamma_-)$ and obtain new seq-pair $S^\star = (t_v s_h \Gamma_+ t_h s_v, s_v s_h \Gamma_- t_h t_v)$. Construct H-SPG G_{HSP} and V-SPG G_{VSP} from $S^\star$.

(Step 2) Construct a horizontal hyper graph G_H and a vertical hyper graph G_V from G_{HSP} and G_{VSP} as follows. The vertex set of G_H and G_V are both equivalent to the vertex set of SPGs. A hyper edge (V_L, V_R) is in the edge set $E(G_H)$ if and only if the subgraph of G_{HSP} induced by $V_L \cup V_R$ is a maximal bipartite. The edge set $E(G_V)$ is similarly defined using G_{VSP}.

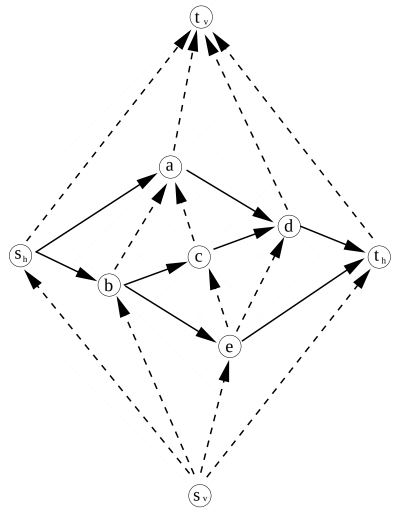
(Step 3) For G_H (also for G_V), construct a hyper graph G_{HP} (resp. G_{VP}) by converting all the hyper edges to the vertices and by converting all the vertices, except for the vertices corresponding to the phantom modules, to the edges. (**Procedure** SeqPair–RDG **End**)

C. Proof of SeqPair–RDG

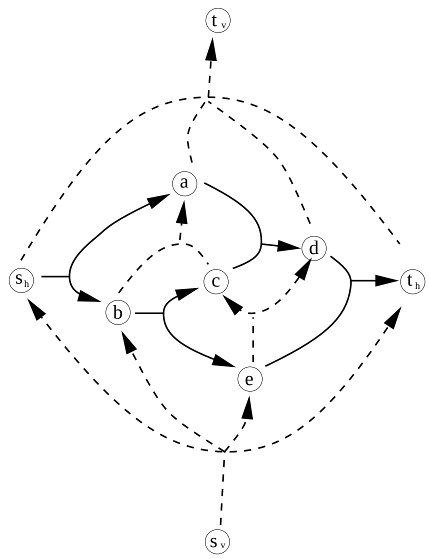
Theorem1 Let S be a seq-pair of n modules. If S does not include adjacent-cross, procedure SeqPair–RDG maps S to a pair of RDGs which correspond to a rectangular-dissection with no empty room such that the HVRS of the rectangular-dissection equals to the HVRS of S, in $O(n^2)$ time. □

From the resultant RDGs, a rectangular-dissection is obtained by ConstRD in $O(n)$ time. In the following, we prove this theorem.

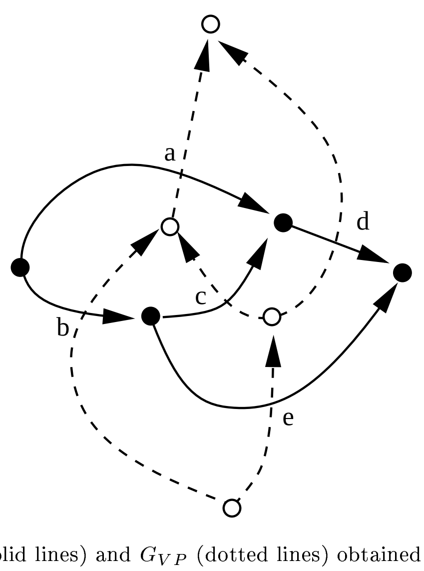
Lemma2 In **(Step 1)**, each edge of G_{HSP} (G_{VSP}) belongs to a unique maximal complete bipartite subgraph of G_{HSP} (G_{VSP}).
(proof) The proof is by contradiction. Assume an edge (a_1, b_1) belongs to two maximal complete bipartite subgraphs $G^1(V_i^1 \cup V_o^1, E^1)$ and $G^2(V_i^2 \cup V_o^2, E^2)$. Since G^1 and G^2 are both maximal complete bipartite graphs, there are two vertices $a_2 \in (V_i^1 \cup V_i^2)$ and $b_2 \in (V_o^1 \cup V_o^2)$ such that there is no edge (a_2, b_2) in $E(G_{HSP})$. The edges (a_1, b_1), (a_1, b_2) and (a_2, b_1) all exist in $E(G_{HSP})$. If a_1 and a_2 are in horizontal relation, then (a_1, b_1) or (a_2, b_1) becomes transitive. Hence a_1 and a_2 are in vertical relation. Without loss of generality, we assume a_1 is above a_2, i.e. $S = (\cdots a_1 \cdots a_2 \cdots, \cdots a_2 \cdots a_1 \cdots)$. Since there are edges (a_1, b_1) and (a_2, b_1), $S = (\cdots a_1 \cdots a_2 \cdots b_1 \cdots, \cdots a_2 \cdots a_1 \cdots b_1 \cdots)$.



(a) G_{HSP} (solid lines) and G_{VSP} (dotted lines) obtained in Step 1



(b) G_H (solid lines) and G_V (dotted lines) obtained in Step 2



(c) G_{HP} (solid lines) and G_{VP} (dotted lines) obtained in Step 3

Fig. 5. Snapshot of the procedure SeqPair–RDG

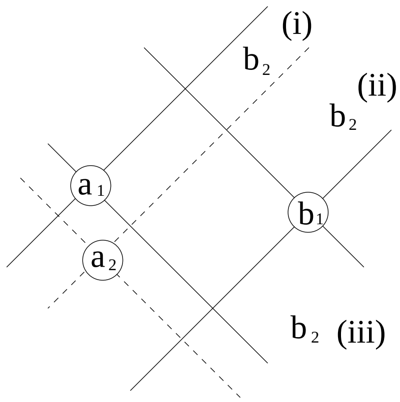


Fig. 6. Figure used in the proof of Lemma 2

Considering the fact that there is edge (a_1, b_2), the position of b_2 are exhaustively examined in the following. (See Fig. 6).

(i) $S = (\cdot\cdot\ a_1 \cdot\cdot b_2 \cdot\cdot a_2 \cdot\cdot b_1 \cdot\cdot, \cdot\cdot\ a_2 \cdot\cdot a_1 \cdot\cdot b_1 \cdot\cdot b_2 \cdot\cdot)$

In G_{VSP}, there is a path from a_2 to b_2. An edge in the path crosses to the edge $(a_1, b_1) \in E(G_{HSP})$, thus there is an HV-cross in S. This contradicts to the fact that S has no adjacent-cross (thus no HV-cross by Lemma 1).

(ii) $S = (\cdot\cdot\ a_1 \cdot\cdot a_2 \cdot\cdot b_2 \cdot\cdot b_1, \cdot\cdot\ a_2 \cdot\cdot a_1 \cdot\cdot b_1 \cdot\cdot b_2 \cdot\cdot)$

Since edge (a_2, b_2) does not exist in $E(G_{HSP})$, there is a vertex x which satisfies $S = (\cdot\cdot\ a_1 \cdot\cdot a_2 \cdot\cdot x \cdot\cdot b_2 \cdot\cdot b_1, \cdot\cdot\ a_2 \cdot\cdot x \cdot\cdot a_1 \cdot\cdot b_1 \cdot\cdot b_2)$ or $S = (\cdot\cdot\ a_1 \cdot\cdot a_2 \cdot\cdot x \cdot\cdot b_2 \cdot\cdot b_1, \cdot\cdot\ a_2 \cdot\cdot a_1 \cdot\cdot b_1 \cdot\cdot x \cdot\cdot b_2)$. The former case results in (a_2, b_1) being transitive, and the latter results in (a_1, b_2) being transitive, either contradicts to the definition of G_{HSP}.

(iii) $S = (\cdot\cdot\ a_1 \cdot\cdot a_2 \cdot\cdot b_1 \cdot\cdot b_2 \cdot\cdot, \cdot\cdot\ a_2 \cdot\cdot a_1 \cdot\cdot b_2 \cdot\cdot b_1 \cdot\cdot)$

Since there is no edge (a_2, b_2), there is module x which satisfies $S = (\cdot\cdot\ b_1 \cdot\cdot x \cdot\cdot b_2 \cdot\cdot, \cdot\cdot\ a_2 \cdot\cdot x \cdot\cdot a_1)$. Then, there is a path from x to a_1 in G_{VSP}. An edge in the path crosses to the edge $(a_2, b_1) \in E(G_{HSP})$, which is a contradiction.

(iv) The other cases are trivially impossible.

Hence, an edge in G_{HSP} belongs to a unique maximal bipartite subgraph of G_{HSP}. Similarly, the claim also holds for G_{VSP}. □

Lemma3 The pair of graphs G_{HP} and G_{VP} obtained by SeqPair–RDG are the RDGs.

(proof) In the following, we show the output is a pair of RDGs by converting the oblique-grid-embedding of $S^\star$. The modules in S are called *real* modules in contrast to the phantom modules.

In the oblique-grid embedding of $S^\star$ which is obtained in **(Step 1)**, any horizontal edge and vertical edge do not cross each other because $S^\star$ does not have HV-cross. (A cross between horizontal edges, or between vertical edges, is possible.)

In the HVRS of $S^\star$, phantom module s_h (t_h, s_v, t_v) is left of (right of, below, above, respectively) every real module. Hence all the vertices corresponding to real modules have at least one input edge and one output edge, both in G_{HSP} and in G_{VSP}.

In the hyper directed graphs G_H and G_V obtained in **(Step 2)**, the input degree and the output degree of each vertex are 0 or 1. From Lemma 2, the input degree and the output degree of the vertices which correspond to real modules are both 1, in either hyper graph. Further, any two edges do not cross each other if they are taken from distinct maximal complete bipartite subgraphs. Hence, G_H and G_V can be drawn without any crossing, as shown in Fig. 5-(b).

In **(Step 3)**, the conversion between hyper edges and vertices preserves the planarity, thus G_{HP} and G_{VP} are planar. The input degree and the output degree of G_{HP} and G_{VP} are 1. Hence, the two hyper graphs G_{HP} and G_{VP} are both ordinary graphs. Consequently, G_{HP} and G_{VP} are in polar-dual relation. From Property 2, they are RDGs. □

Lemma4 The HVRS of the RDGs obtained by SeqPair–RDG is equivalent to the HVRS of the input seq-pair.

(proof) In **(Step 1)**, a horizontal (vertical) relation is represented as a path between two vertices in G_{HSP} (G_{VSP}). For each path in G_{HSP} (G_{VSP}), the corresponding path exists in the hyper graph G_H (G_V) in **(Step 2)**, and also in the in G_{HP} (G_{VP}) in **(Step 3)**. No new relation is introduced in the resultant RDGs since the RDGs can not represent both horizontal and vertical relation for a module pair (Property 5). □

(**Proof of Theorem 1**)

Only the speed is proved in the following since other claims are already proved by Lemma 3 and 4.

In **(Step 1)**, SPGs are constructed faithfully to the HVRS, but eliminating the transitive edges, by its definition. For a module a, the set of all the modules $\{x_1, x_2, \ldots, x_m\}$ that are non-transitively right of module a can be computed in $O(n)$ time using the fact that they are in the form:

$$(\cdot\cdot\ x_m \cdot\cdot x_2 \cdot\cdot x_1 \cdot\cdot, \cdot\cdot\ a \cdot\cdot x_1 \cdot\cdot x_2 \cdot\cdot x_m).$$

Hence, SPGs can be constructed by $O(n^2)$ time.

(Step 2) can be done also in $O(n^2)$ time, proportional to the number of edges in SPGs, because each edge in SPGs belongs to a unique maximal bipartite in SPGs (Lemma 2).

It is obvious that the sum of the cardinality of the input vertex set and that of the output vertex set of all the hyper edges in G_H (G_V) is $O(n)$. Hence, **(Step 3)** can be done in $O(n)$ time.

Consequently, SeqPair–RDG can be done in $O(n^2)$ time. □

D. Necessary and Sufficient Condition

Theorem 1 shows that the absence of the adjacent-cross is sufficient for a seq-pair to be mapped to a rectangular-dissection without empty room. It is also necessary as follows.

Theorem2 A seq-pair can be mapped to a rectangular-dissection without introducing any empty room if and only if the seq-pair does not have an adjacent-cross.
(Proof) The condition is sufficient by Theorem 1. Let S be a seq-pair of n modules and S includes one or more adjacent-crosses. In the following, we show the HVRS of S is not equivalent to the HVRS of any rectangular-dissection with n rooms.

Let four modules a, b, c, d form an adjacent-cross in S. Without loss of generality, Let $S = (\cdots a \cdots bc \cdots d \cdots, \cdots b \cdots da \cdots c \cdots)$. The proof is by contradiction. Assume the relative module position of S is represented by a rectangular-dissection F without any empty room.

In the H-RDG of F, there are three paths;

(i) the path from the edge a to the edge c,

(ii) the path from the edge b to the edge c, and

(iii) the path from the edge b to the edge d.

For the path (ii), from the two facts "b and c are adjacent in Γ_+, and there is no anonymous edge in the H-RDG of F.", it is understood that edge b and edge c are directly connected by a vertex v. It implies that the vertex v is in the path (i) and also in the path (iii). Hence, there is a path from a to d (via v) in the H-RDG. This contradicts to the fact: a and d are in the vertical relation in the HVRS of S, thus not in the horizontal relation. □

IV. Rectangular-Dissection with Fewest Empty Rooms

In this section, we give a procedure which maps a seq-pair to a rectangular-dissection with fewest empty rooms. The maximum possible number of empty rooms is also presented.

A. Procedure RmAdjCross

Let $S = (\Gamma_+, \Gamma_-)$ be a seq-pair of n modules, which possibly includes adjacent-crosses. *Inserting* dummy module x into S is to add a new module x into Γ_+ and into Γ_-. "Adjacent-cross ab/cd" denotes an adjacent-cross such that a and b are adjacent in Γ_+ and c and d are adjacent in Γ_-. For example, $(\cdots d \cdots ab \cdots c \cdots, \cdots a \cdots cd \cdots b \cdots)$ and $(\cdots c \cdots ab \cdots d \cdots, \cdots b \cdots cd \cdots a \cdots)$ are such cases.

For a seq-pair which includes an adjacent-cross ab/cd, inserting a dummy module x at the *cross-point* of ab/cd indicates that inserting x between a and b in Γ_+ and between c and d in Γ_-. For example, when inserting dummy module x into $(\cdots d \cdots ab \cdots c \cdots, \cdots a \cdots cd \cdots b \cdots)$ at the cross-point of ab/cd, the resultant seq-pair will be $(\cdots d \cdots axb \cdots c \cdots, \cdots a \cdots cxd \cdots b \cdots)$.

Procedure RmAdjCross

Input: seq-pair $S = (\Gamma_+, \Gamma_-)$ which possibly has adjacent-crosses.

Output: seq-pair S which does not have adjacent-cross.

(Step 1) Find an adjacent-cross ab/cd in S. Insert dummy module x at the cross-point of ab/cd. Repeat the above process until no adjacent-cross exists.

(**Procedure** RmAdjCross **End**)

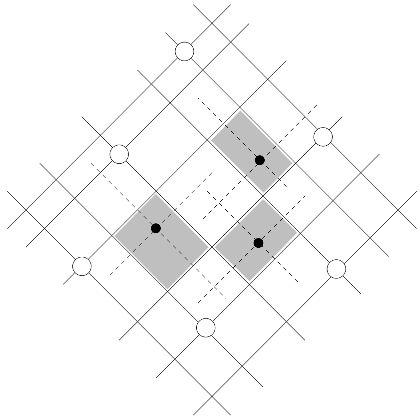


Fig. 7. Effect of the procedure RmAdjCross. Dummy modules (black dots) are inserted at the cross-points (dark region) of adjacent-crosses.

Fig. 7 illustrates the effect of the procedure. In the figure, white circles indicate the modules in the given seq-pair, which has three adjacent-crosses whose cross-points are remarked by dark color. The black dots indicates the dummy modules inserted by the procedure. It can be examined that the resultant seq-pair does not have any adjacent-cross.

B. Proof of RmAdjCross

Using the procedure RmAdjCross, the following theorem is proved in this section.

Theorem3 Let S be a seq-pair. Let F be a rectangular-dissection whose number of rooms is minimum over the rectangular-dissections whose HVRS is same to the HVRS of S. Such an F can be obtained by RmAdjCross followed by SeqPair–RDG and ConstRD, totally in $O(n^4)$ time. □

Lemma5 Let S be a seq-pair and k be the number of adjacent-crosses in S.

(1) RmAdjCross inserts k dummy modules and the number of adjacent-cross is made zero.

(2) The number of adjacent-cross can not be made zero by $k-1$ or less dummy modules.

(proof)

(1) Suppose a dummy module x is inserted at the cross-point of adjacent-cross ab/cd in S. Let the resultant seq-pair be S'. Then a, b, c, d do not form an adjacent-cross in S'. (The adjacent-cross is said to be removed.)

Assume a new adjacent-cross is created in S'. One of the four modules which form the new adjacent-cross is x. One of the other three modules is a, b, c or d. Without loss of generality, let a is the one. Let the other two be y and z. Neither of y nor z is a, b, c or d. The new adjacent-cross is then xa/yz. For the adjacent-cross xa/yz in S', there is an adjacent-cross ab/yz in S and it is removed in S'. If there are more new created adjacent-crosses (xa/yz), individual adjacent-crosses are removed (ab/yz). Therefore, the number of adjacent-crosses can be decreased by one by inserting a dummy module at the cross-point of an arbitrary adjacent-cross, which is exactly executed by RmAdjCross.

(2) Suppose there is a seq-pair $S^\dagger$ which does not include any adjacent-cross but includes only $k-1$ or less dummy modules. If we remove all the dummy modules from $S^\dagger$, the resultant seq-pair coincides with S. We remove the dummy modules one by one from $S^\dagger$, and stop when the number of adjacent-crosses is increased by two or more by removing the dummy module x. Then, if we insert x exactly at the position it has been existed, the number of adjacent-crosses should be decreased by two or more. We show this can not be happened, in the following.

Let a dummy module x be inserted to S and m adjacent-crosses be removed. When an adjacent-cross ab/cd is removed, (i) x is inserted between a and b in Γ_+, or (ii) x is inserted between c and d in Γ_-. Both of the conditions are true at most for one adjacent-cross. Thus at least $m-1$ adjacent-crosses satisfy either (i) or (ii). Let adjacent-cross ab/cd be one of those adjacent-crosses. Then, x and three modules from a, b, c, d form a new adjacent-cross in S' (such as xb/cd). This new created adjacent-cross (xb/cd) exists individually for all $m-1$ adjacent-crosses. Thus, the number of adjacent-crosses can be decreased at most by one by inserting one dummy module. □

Lemma6 Let S be a seq-pair. Let F be a rectangular-dissection whose number of rooms is minimum over the rectangular-dissections whose HVRS is same to the HVRS of S. Such an F can be obtained by RmAdjCross, followed by SeqPair–RDG and ConstRD

(proof) Since the seq-pair obtained by RmAdjCross does not include adjacent-cross, a rectangular-dissection F' is obtained by SeqPair–RDG and ConstRD. In the following, we show the number of rooms in F' equals to that of F. From Property 6, any rectangular-dissection with n modules, possibly has empty rooms, corresponds to unique seq-pair of n module names, preserving the HVRS. Then if we assign dummy modules to all the empty rooms in F, we have a unique seq-pair S' with modules corresponding to all the rooms including the empty rooms. From Theorem 2, S' does not have an adjacent-cross. The HVRSs of S and S' (with respect to the pre-existing modules) are same because they are same to the HVRS of F. Thus if we remove all the dummy modules from S', it coincides with S. Since the number of dummy modules inserted by RmAdjCross is minimum to remove all the dummy modules (Lemma 6), the number of rooms in F and that of F' is same. □

(**Proof of Theorem 3**)

Only the time complexity is proved in the following since the other claims are already proved by Lemma 6.

It is separately shown in the next section that the maximum number of adjacent-crosses is $O(n^2)$. For each adjacent-cross, RmAdjCross can identify the adjacent-cross in $O(n^2)$ time, and insert a dummy module in $O(n)$ time. Thus, RmAdjCross can be done in $O(n^4)$ time. Since the number of modules in the resultant seq-pair is $O(n^2)$, SeqPair–RDG runs in $O(n^4)$ time, and ConstRD runs in $O(n^2)$ time. □

The complexity of RmAdjCross can be improved to $O(n^2)$ if carefully implemented. However, the overall complexity is not improved because SeqPair–RDG dominates the total complexity.

C. Maximum Number of Empty Rooms

Theorem4 Let S be a seq-pair of n modules. Let F be a rectangular-dissection whose number of rooms is minimum over the rectangular-dissections whose HVRS is same to the HVRS of S. The maximum possible number of empty rooms in F is

$$\left\lceil \frac{n-2}{2} \right\rceil \left\lfloor \frac{n-2}{2} \right\rfloor .$$

(Proof) The proposition is true for $n \leq 3$. (No empty rooms are needed.) We assume $n \geq 4$ in the following.

Without loss of generality, we assume $\Gamma_+ = (1, 2, 3, \ldots, n)$ and $\Gamma_- = (a_1, a_2, a_3, \ldots, a_n)$. A necessary condition to form an adjacent-cross ab/cd is, a and b are adjacent in Γ_+, $c < \min(a, b)$, $d > \max(a, b)$, and $\Gamma_-^{-1}(c) < \Gamma_-^{-1}(d)$ if $b < a$, $\Gamma_-^{-1}(c) > \Gamma_-^{-1}(d)$ otherwise.

Thus, the number of empty rooms can not exceed

$$\sum_{i=2}^{n} \min(i-2, n-i) = \left\lceil \frac{n-2}{2} \right\rceil \left\lfloor \frac{n-2}{2} \right\rfloor .$$

Given n, the sequence-pair constructed as follows has exactly $\lceil (n-2)/2 \rceil \lfloor (n-2)/2 \rfloor$ adjacent-crosses. Γ_+ is constructed as $(1, 2, 3, \cdot, n)$. If n is even, then Γ_- is constructed as

$$\Gamma_-(i) = \begin{cases} \frac{n+1}{2} + (-1)^i (i - \frac{1}{2}) & \text{if } i \leq \frac{n}{2} \\ \frac{n+1}{2} + (-1)^i (n + \frac{1}{2} - i) & \text{otherwise} \end{cases} .$$

If $n = 4k + 1$ for some k, then

$$\Gamma_-(i) = \begin{cases} \frac{n}{2} + (-1)^i (i - \frac{1}{2}) & \text{if } i \leq \frac{n-1}{2} \\ \frac{n}{2} + 1 + (-1)^i (n + \frac{1}{2} - i) & \text{otherwise} \end{cases} .$$

If $n = 4k + 3$ for some k, then

$$\Gamma_-(i) = \begin{cases} \frac{n}{2} + (-1)^i (i - \frac{1}{2}) & \text{if } i \leq \frac{n+1}{2} \\ \frac{n}{2} + 1 + (-1)^i (n + \frac{1}{2} - i) & \text{otherwise} \end{cases} .$$

It is easily examined that the resultant seq-pair has $\lceil (n-2)/2 \rceil \lfloor (n-2)/2 \rfloor$ adjacent-crosses. □

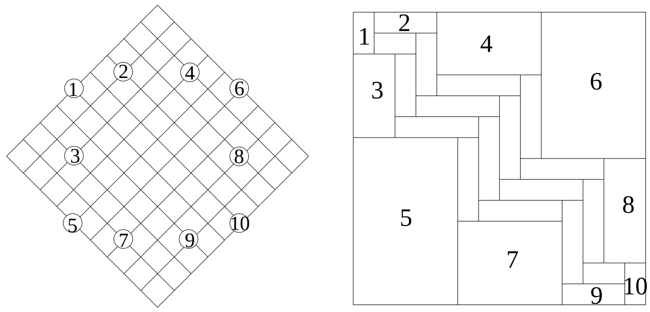


Fig. 8. Seq-pair with maximum adjacent-crosses (left) and its corresponding rectangular-dissection (right)

For example, for $n = 10$, the above construction results in:

$$(\Gamma_+, \Gamma_-) = (1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10,\ 5\ 7\ 3\ 9\ 1\ 10\ 2\ 8\ 4\ 6).$$

Fig. 8 shows the corresponding oblique-grid-embedding of the seq-pair and the corresponding rectangular-dissection with 16 empty rooms.

V. Conclusion

Recently, an elegant representation called sequence-pair [1] is proposed to represent candidate solutions for a placement problem. In spite of its efficiency in representing the modules positions, no information is provided for the channel positions. Such a channel information is added by this paper by giving a mapping from a seq-pair to a rectangular-dissection, which has been used to represent the channel positions together with the module positions.

The results of this paper is summarized as follows.

- Channels are additionally represented, without changing the information about module positions, thus the following two properties of the seq-pair are remain effective; an area minimum placement is represented, and no overlapping placement is represented.
- The number of channels are exactly minimized, which most likely minimizes the number of wire bends, later in the routing stage.
- The maximum possible number of empty rooms, which linearly corresponds to the maximum possible number of introduced channels, is presented.
- A necessary and sufficient condition of the seq-pair for not introducing any empty-room is presented.

Although the channels are represented, this paper does not give a technique to assign width to each channel. How to assign adequate widths to the channels remains hard, and would be solved heuristically.

Recent VLSI manufacturing technology increases the number of routing layers, consequently increases the importance of an "area router". Area routers typically do not require channels, however, they would still need some resources to control the wiring congestion. How to represent such a routing resources in the placement stage is another interesting problem.

References

[1] H. Murata, K. Fujiyoshi, S. Nakatake, and Y. Kajitani: "Rectangle-Packing-Based Module Placement," in IEEE International Conf. on Computer Aided Design, pp.472–479, 1995.

[2] T. Watanabe, H. Murata, K. Fujiyoshi, "An Optimal Floorplan with Module Relations Represented by a Sequence-Pair", 9th IEICE Circuit and Systems Karuizawa Workshop, pp.103-108, 1996.

[3] S. Nakatake, K. Fujiyoshi, H. Murata and Y. Kajitani, "Module Placement on BSG-Structure and IC Layout Applications", 1996 IEEE/ACM International Conference on Computer-Aided Design, Nov. 1996.

[4] H. Onodera and Y. Taniguchi and K. Tamaru, "Branch-and-Bound Placement for Building Block Layout", Proc. 28th ACM/IEEE Design Automation Conf., pp.433-439,1991.

[5] S. Wimer, I. Koren, I. Cederbaum, "Floorplans, Planar Graphs, and Layouts", IEEE Transaction on Circuits and Systems, Vol. 35, No. 3, pp.267–278, 1988.

[6] D.F.Wong and C.L.Liu: "A New Algorithm for Floorplan Designs," in Proc. 23rd ACM/IEEE Design Automation Conf., pp.101-107,1986.

[7] M.J.Ciesielski and E.Kinnen: "Digraph Relaxation for 2-Dimensional Placement of IC Blocks," in IEEE Trans. CAD, vol. CAD-6, No.1, pp.55-66, 1987.

[8] L.Stockmeyer: "Optimal Orientations of Cells in Slicing Floorplan Designs," in Info.and Control, vol.57, pp.91–101,1983.

[9] Sadiq M.Sait and Habib Youssef, "VLSI Physical Design Automation," IEEE Press,1995.

Appendix

Proof of Property 5

Let two graphs G and G' be polar-dual each other. Let source and sink of $G(G')$ be $s(s')$ and $t(t')$, respectively. A ***full-path*** of $G(G')$ is a path from $s(s')$ to $t(t')$ in $G(G')$.

For any two edges a and b, a full-path which includes both a and b exists either in G or G', and not exists in both G and G'.

(Proof)

Since G and G' are in polar-dual relation, the edge set of a full-path of $G(G')$ has one to one correspondence with a cut set of $G'(G)$.

If G has a full-path which includes both a and b, there is a cut set in G' which includes both a and b, hence G' does not have a full-path which includes both a and b.

In the following, we consider the case G does not have a full-path which includes both a and b. Let V_R be the subset of vertices in G consists of the vertices which is reachable from the outgoing vertex of a or the outgoing vertex of b. Let $V_{\overline{R}}$ be the rest. Since G is a directed acyclic graph, the incoming vertex of G and the incoming vertex of G' are both in $V_{\overline{R}}$. There is no edge from a vertex in V_R to a vertex in $V_{\overline{R}}$, hence the set of edges from a vertex in $V_{\overline{R}}$ to a vertex in V_R is a cut, and the cut includes both a and b. Therefore, G' has a full-path which includes both a and b. $\square$