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A MATHEMATICAL MODEL TO INVESTIGATE CONTACT DYNAMICS IN CONSTRAINED ROBOTS

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Abstract: *Contact dynamics problems arises in numerous applications in space and industry, as robotic welding, parts assembly, the operation of capturing a satellite in orbit or the general problem of a robotic manipulator grabbing and handling any object with its own dynamics. The theory developed here can also be applied to problems in which robots have to follow some prescribed patterns or trajectories when in contact with the environment (like in painting activities, for instance, or the ROKVISS experiment investigated at DLR). In this paper, the governing equations of motion of a system representative of a robotic manipulator in contact with a dynamic environment are derived. These equations are obtained through the Lagrangian formalism. The constraints are introduced into the governing equations for the case in which the bodies are in contact via the Lagrange multipliers. Some preliminary results (including profiles for the contact force) and recommendations are discussed.*

Keywords*: contact dynamics, robotic, space manipulator*

1. Introduction

There are several ways to deal with the problem of interaction between bodies. Impact dynamics and continuous contact between bodies can both be included in the mathematical model of the constrained problem, or just one of these effects can be considered. It depends, obviously, on the characteristics of the studied problem.

The investigations about the contact between bodies include (at least) two different kind of analysis (Pfeiffer and Glocker, 1996): one associated with the beginning of contact and one associated with its termination. In the first analysis, the distance between the bodies must be checked; in the second analysis, once the contact is established, the reaction (normal; compression) force between the bodies must be checked.

One of the hardest parts in the study of contact problems involves the different models that must be developed for contact and non-contact situations and the switching between these models when integrating the governing equations of motion (the different situations must be detected).

According to Figure 1, the system investigated here, the free end of the bar is allowed to move along the constraint represented by the mass named m_w. In some cases, m_w can be thought as a wall representing a constraint to the robotic system. All the movements occur in the horizontal plane in order to avoid additional effects induced by gravity. This consideration means no loss of generality. When contact occurs, impact and bouncing are also allowed to occur. The system is designed in such a way that the bar can turn 360^o but, in a part of its trajectory, contact with m_w is allowed to occur. The mass (m_s) in which the bar is pivoted is allowed to oscillate when excited by the movement of the bar (free or constrained). In the axis Z, passing through the connection between the bar and m_s (perpendicular to the paper sheet), there is a prescribed moment M_θ acting to turn the bar.

2. Geometric Model

The problem to be analyzed in this chapter is depicted in Figure 1. The dashed lines represent the position of the masses (m_s and m_w) in which the springs and dampers are free of forces, and the vertical arrows on the right side of these lines indicates the positive direction of the movement of these masses. The dotted line represents the position from which one starts to count θ. Point A represents a rotational joint, and an external moment, M_θ, acts on this point.

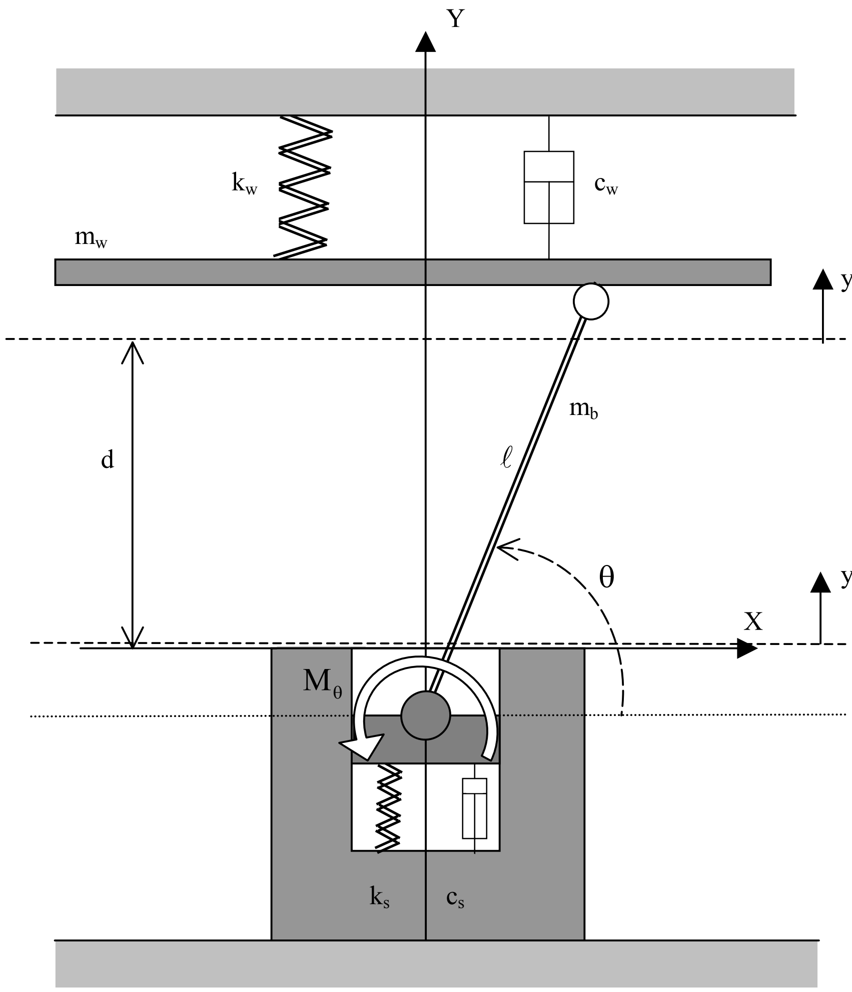

Figure 1 – Oscillating bar constrained by a body with properties like mass, stiffness and damping.

In physical terms, this system may represent a robot with a translational joint (mass m_s with its stiffness and damping) and a rotational joint (the oscillating bar as being one link of this robot); m_w then can be thought as an obstructing wall on the robot's trajectory or some object this robot must handle or interact with. In this same sense, M_θ can be thought as an external torque provided by a dc motor (rotational joint).

3. Derivation of the governing equations of motion

The kinetic energy of the system shown in Figure 1 is given by :

$$T = \frac{1}{2} I_{b,cm} \dot{\theta}^2 + \frac{1}{2} m_b |\dot{\mathbf{r}}_{cm}|^2 + \frac{1}{2} m_w |\dot{\mathbf{r}}_w|^2 + \frac{1}{2} m_s |\dot{\mathbf{r}}_s|^2 \tag{1}$$

where $I_{b,cm}$ represents the bar moment of inertia around its center of mass, θ represents the bar angular displacement, m_b represents the mass of the bar, $\mathbf{r}_{cm}$ represents the position vector that locates the bar center of mass, $\mathbf{r}_w$ represents the position vector that locates the center of mass of the wall and $\mathbf{r}_s$ represents the position vector that locates the support for the bar. All the vectors are referenced to the inertial reference frame, XY.

The vectors $\mathbf{r}_{cm}$, $\mathbf{r}_w$ and $\mathbf{r}_s$ are given by:

$$\mathbf{r}_{cm} = d_{Acmb} \cos\theta \, \mathbf{i} + \left(d_{Acmb} \sin\theta + y_s \right) \mathbf{j} \tag{2}$$

$$\mathbf{r}_w = (d + y_w)\,\mathbf{j} \tag{3}$$

$$\mathbf{r}_s = y_s\,\mathbf{j} \tag{4}$$

where **i** and **j** are unit vectors in the X and Y directions, respectively, and d_{Acmb} represents the distance from A to the center of mass of the bar.

Using (2) to (4), the velocities that appear in (1) are given by:

$$\dot{\mathbf{r}}_{cm} = -d_{Acmb}\dot{\theta}\sin\theta\,\mathbf{i} + \left(d_{Acmb}\dot{\theta}\cos\theta + \dot{y}_s\right)\mathbf{j} \tag{5}$$

$$\dot{\mathbf{r}}_w = \dot{y}_w\mathbf{j} \tag{6}$$

$$\dot{\mathbf{r}}_s = \dot{y}_s\mathbf{j} \tag{7}$$

Therefore, the kinetic energy given by (1) can be rewritten as:

$$T = \frac{1}{2}\left(I_{b,cm} + m_b d_{Acmb}^2\right)\dot{\theta}^2 + m_b d_{Acmb}\dot{\theta}\,\dot{y}_s\cos\theta + \frac{m_b + m_s}{2}\dot{y}_s^2 + \frac{m_w}{2}\dot{y}_w^2 \tag{8}$$

The Rayleigh function that accounts for dissipation of energy associated with the linear damping forces is given by

$$\begin{aligned} R &= \frac{1}{2}c_w\left|\dot{\mathbf{r}}_w\right|^2 + \frac{1}{2}c_s\left|\dot{\mathbf{r}}_s\right|^2 \\ &= \frac{1}{2}c_w\dot{y}_w^2 + \frac{1}{2}c_s\dot{y}_s^2 \end{aligned} \tag{9}$$

where c_w represents the damping coefficient associated with m_w, and c_s represents the damping coefficient associated with m_s.

The potential energy is given by

$$V = \frac{1}{2}k_w y_w^2 + \frac{1}{2}k_s y_s^2 \tag{10}$$

where k_w represents the stiffness coefficient associated with m_w, and k_s represents the stiffness coefficient associated with m_s.

The Lagrangian, L, is, therefore, given by :

$$\begin{aligned} L &= T - V \\ &= \frac{1}{2}\left(I_{b,cm} + m_b d_{Acmb}^2\right)\dot{\theta}^2 + m_b d_{Acmb}\dot{\theta}\,\dot{y}_s\cos\theta + \frac{m_b + m_s}{2}\dot{y}_s^2 + \frac{m_w}{2}\dot{y}_w^2 - \frac{k_w}{2}y_w^2 - \frac{k_s}{2}y_s^2 \end{aligned} \tag{11}$$

The condition for the beginning of contact is

$$d - y_s + y_w - \ell\sin\theta = 0 \tag{12}$$

The condition for the end of contact (in other words, for the transition between constrained and free movement) will be given later in this paper.

The Lagrange's equations, considering the constraints to the movement (Rosenberg,1977; Clough and Penzien,1975), are given by

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}_s}\right) - \frac{\partial L}{\partial y_s} + \frac{\partial R}{\partial \dot{y}_s} = \mathbf{F}_R\,\frac{\partial \mathbf{r}_{fe}}{\partial y_s} \tag{13}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}_w}\right)-\frac{\partial L}{\partial y_w}+\frac{\partial R}{\partial \dot{y}_w}=\mathbf{F}_R\frac{\partial \mathbf{r}_{fe}}{\partial y_w} \tag{14}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right)-\frac{\partial L}{\partial \theta}+\frac{\partial R}{\partial \dot{\theta}}=M_\theta+\mathbf{F}_R\frac{\partial \mathbf{r}_{fe}}{\partial \theta} \tag{15}$$

where $\mathbf{F}_R$ represents the vector of the reaction force at the constrained surface, considered here only through its normal component, $F_N \nabla\Phi$, with F_N representing the amplitude of the normal force. It is assumed that there are no friction forces involved. $\mathbf{r}_{fe}$ represents the vector that locates the free end of the bar. The quantity Φ represents the equation of the constrained surface given by

$$\Phi = d + y_w - Y = 0 \tag{16}$$

and $\nabla\Phi = \frac{\partial \Phi}{\partial X}\mathbf{i}+\frac{\partial \Phi}{\partial Y}\mathbf{j}$. The position of the free end of the bar is given by

$$\mathbf{r}_{fe} = \ell\cos\theta\mathbf{i}+\left(\ell\sin\theta + y_s\right)\mathbf{j} \tag{17}$$

where ℓ represents the total length of the bar. Using Equation (12), Φ and $\mathbf{r}_{fe}$ can also be written as

$$\Phi = Y - y_s - \ell\sin\theta = 0 \tag{18}$$

$$\mathbf{r}_{fe} = \ell\cos\theta\mathbf{i}+\left(d + y_w\right)\mathbf{j} \tag{19}$$

The term $\frac{\partial \mathbf{r}_{fe}}{\partial \alpha}$ (where $\alpha = y_s, y_w$ and θ) represents a vector that accounts for the variation of the free end position related to each one of the generalized coordinates considered. This variation is associated with the work developed by the constraint forces and this force (F_N), which appears in the right side of Equations (13) to (15), is (sometimes) also named as Lagrange multiplier.
Applying Lagrange's equations ((13) to (15)) and considering the expressions (18) and (19), the governing equations of motion are given by

$$\left(m_b + m_s\right)\ddot{y}_s + c_s\dot{y}_s + k_s y_s - m_b d_{Acmb}\dot{\theta}^2\sin\theta + m_b d_{Acmb}\ddot{\theta}\cos\theta + F_N = 0 \tag{20}$$

$$m_w\ddot{y}_w + c_w\dot{y}_w + k_w y_w - F_N = 0 \tag{21}$$

$$\left(I_{b,cm} + m_b d_{Acmb}^2\right)\ddot{\theta} + m_b d_{Acmb}\ddot{y}_s\cos\theta + F_N\ell\cos\theta = M_\theta \tag{22}$$

Equation (20) represents the governing equation of motion for y_s, Equation (21) represents the governing equation of motion for y_w, and Equation (22) represents the governing equation of motion for θ. Together with these equations, Equation (12) represents an additional relationship between the generalized coordinates y_s, θ and y_w when contact occurs. The set (12) and (20) to (22) provides four equations and four unknowns (y_s, θ, y_w and F_N) considering the constrained problem and three equations and three unknowns (y_s, θ and y_w) considering the unconstrained problem. In the unconstrained case, Equation (12) does not apply (one uses only equations (20) to (22)) and $F_N = 0$. The reaction force is not always present on the system.

4. The non-contact case

According to Figure 1, if the $d > 0$ contact is not allowed to occur and the dynamical system is governed by:

$$\left(m_b + m_s\right)\ddot{y}_s + c_s\dot{y}_s + k_s y_s - m_b d_{Acmb}\dot{\theta}^2\sin\theta + m_b d_{Acmb}\ddot{\theta}\cos\theta = 0 \tag{23}$$

$$\left(I_{b,cm} + m_b d_{Acmb}^2\right)\ddot{\theta} + m_b d_{Acmb}\ddot{y}_s\cos\theta = M_\theta \tag{24}$$

and the dynamics of mass m_w by

$$m_w\ddot{y}_w + c_w\dot{y}_w + k_w y_w = 0 \tag{25}$$

Equation (25) can be treated independently. The system of second order ordinary differential equations given by (23) and (24) is integrated using the fourth order Runge-Kutta algorithm. For this reason, it is convenient to rearrange Equations (23) and (24) in order to have only second order derivatives associated with one generalized coordinate in each one of them.

In matrix form, equations (23) and (24) are written as:

$$\begin{bmatrix} m_b + m_s & m_b d_{Acmb}\cos\theta \\ m_b d_{Acmb}\cos\theta & I_{b,cm} + m_b d_{Acmb}^2 \end{bmatrix}\begin{Bmatrix} \ddot{y}_s \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} c_s & 0 \\ 0 & 0 \end{bmatrix}\begin{Bmatrix} \dot{y}_s \\ \dot{\theta} \end{Bmatrix} + \begin{bmatrix} k_s & 0 \\ 0 & 0 \end{bmatrix}\begin{Bmatrix} y_s \\ \theta \end{Bmatrix} + \begin{Bmatrix} -m_b d_{Acmb}\dot{\theta}^2\sin\theta \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ M_\theta \end{Bmatrix} \tag{26}$$

The inverse of the nonlinear time varying inertia matrix is given by:

$$\begin{bmatrix} b_1 & b_2\cos\theta \\ b_2\cos\theta & b_3 \end{bmatrix}^{-1} = \frac{1}{b_1 b_3 - b_2^2\cos^2\theta}\begin{bmatrix} b_3 & -b_2\cos\theta \\ -b_2\cos\theta & b_1 \end{bmatrix} \tag{27}$$

where

$$b_1 = m_b + m_s \qquad b_2 = m_b d_{Acmb} \qquad b_3 = I_{b,cm} + m_b d_{Acmb}^2$$

Multiplying both sides of Equation (26) from the left by the inverse matrix (27) and writing the resulting equations back in scalar form results:

$$\ddot{y}_s + \left(\frac{b_3 c_s}{b_1 b_3 - b_2^2\cos^2\theta}\right)\dot{y}_s + \left(\frac{b_3 k_s}{b_1 b_3 - b_2^2\cos^2\theta}\right)y_s - \left(\frac{b_2 b_3\sin\theta}{b_1 b_3 - b_2^2\cos^2\theta}\right)\dot{\theta}^2 = -\left(\frac{b_2\cos\theta}{b_1 b_3 - b_2^2\cos^2\theta}\right)M_\theta \tag{28}$$

$$\ddot{\theta} - \left(\frac{b_2 c_s\cos\theta}{b_1 b_3 - b_2^2\cos^2\theta}\right)\dot{y}_s - \left(\frac{b_2 k_s\cos\theta}{b_1 b_3 - b_2^2\cos^2\theta}\right)y_s + \left(\frac{b_2^2\sin\theta\cos\theta}{b_1 b_3 - b_2^2\cos^2\theta}\right)\dot{\theta}^2 = \left(\frac{b_1}{b_1 b_3 - b_2^2\cos^2\theta}\right)M_\theta \tag{29}$$

If contact exists at least once and then the bodies separate, the behavior of m_w must also be monitored using Equation (25). The initial condition for y_w and $\dot{y}_w$, supposing m_w is initially at rest, are given by the beginning of contact (impact).

5. The contact case

In contact condition, for this problem, there is the loss of one degree of freedom. In other words, one of the variables is dependent of all the others. Considering the set of equations (20) to (22) and using (12) and its derivatives accordingly, the objective now is to eliminate one of the variables of this set and create a new set of equations with only three unknowns. The best choice is the elimination of the generalized coordinate y_w, which does not always belong to the system represented by the oscillating bar (Schäfer et al., 2004).

Solving equation (21) for F_N and substituting in (23) and (25) results in the two equations:

$$\left(m_b + m_s\right)\ddot{y}_s + c_s\dot{y}_s + k_s y_s - m_b d_{Acmb}\dot{\theta}^2\sin\theta + m_b d_{Acmb}\ddot{\theta}\cos\theta + m_w\ddot{y}_w + c_w\dot{y}_w + k_w y_w = 0 \tag{30}$$

$$\left(I_{b,cm} + m_b d_{Acmb}^2\right)\ddot{\theta} + m_b d_{Acmb}\ddot{y}_s\cos\theta + \left(m_w\ddot{y}_w + c_w\dot{y}_w + k_w y_w\right)\ell\cos\theta = M_\theta \tag{31}$$

In the following it is assumed that the free end of the bar and the wall are always in contact. Obviously, this is valid only for $e = 0$, i.e., for the fully plastic impact case. Separation will take place when the normal force is zero. For general values of the coefficient of restitution between 0 and 1, it is possible to have multiple impacts between the masses.

Therefore, for steady contact, the first and second time derivatives of equation (12) are given by:

$$-\dot{y}_s + \dot{y}_w - \ell\dot{\theta}\cos\theta = 0 \tag{32}$$

$$-\ddot{y}_s + \ddot{y}_w + \ell\dot{\theta}^2\sin\theta - \ell\ddot{\theta}\cos\theta = 0 \tag{33}$$

Solving for the time derivatives of y_w results in:

$$\dot{y}_w = \dot{y}_s + \ell\dot{\theta}\cos\theta \tag{34}$$

$$\ddot{y}_w = \ddot{y}_s - \ell\dot{\theta}^2\sin\theta + \ell\ddot{\theta}\cos\theta \tag{35}$$

Substituting (12), (34) and (35) in equations (30) and (31) results into a new set of governing equations of motion for the two coordinates y_s and θ given by:

$$\begin{aligned}&\left(m_b + m_s + m_w\right)\ddot{y}_s + \left(c_s + c_w\right)\dot{y}_s + \left(k_s + k_w\right)y_s - \left(m_b d_{Acmb} + m_w\ell\right)\dot{\theta}^2\sin\theta\\ &+\left(m_b d_{Acmb} + m_w\ell\right)\ddot{\theta}\cos\theta + c_w\ell\dot{\theta}\cos\theta + k_w\ell\sin\theta - k_w d = 0\end{aligned} \tag{36}$$

$$\begin{aligned}&\left(I_{b,cm} + m_b d_{Acmb}^2 + m_w\ell^2\cos^2\theta\right)\ddot{\theta} + \left(m_b d_{Acmb} + m_w\ell\right)\ddot{y}_s\cos\theta - m_w\ell^2\dot{\theta}^2\sin\theta\cos\theta\\ &+c_w\ell\dot{y}_s\cos\theta + c_w\ell^2\dot{\theta}\cos^2\theta + k_w\ell y_s\cos\theta + k_w\ell^2\sin\theta\cos\theta - k_w\ell d\cos\theta = M_\theta\end{aligned} \tag{37}$$

Of course, as soon as these two variables are known, the remaining variable, y_w, is also known through Equation (12). Equations (36) and (37) represent, respectively, the time behavior of the generalized coordinates y_s and θ during the contact condition. The same operations performed before for the unconstrained case will be performed in equations (36) and (37) in order to conveniently prepare them for numerical integration. After this operation, one has:

$$\begin{aligned}&\ddot{y}_s + \frac{1}{a_1 m_t + a_3\cos^2\theta}\Big(\; a_1\left(c_s + c_w\right)\dot{y}_s + a_1\left(k_s + k_w\right)y_s + a_1 c_w\ell\dot{\theta}\cos\theta + a_1 k_w\ell\sin\theta - a_1 a_2\dot{\theta}^2\sin\theta\\ &-m_b c_w\ell^2 d_{Acmb}\dot{\theta}\cos^3\theta\; - m_b k_w\ell^2 d_{Acmb}\sin\theta\cos^2\theta + m_b k_w d\ell d_{Acmb}\cos^2\theta + \ell\left(m_w\ell c_s - m_b d_{Acmb} c_w\right)\dot{y}_s\cos^2\theta\\ &+\ell\left(m_w\ell k_s - m_b d_{Acmb} k_w\right)y_s\cos^2\theta - a_1 k_w d\;\Big) \;=\; -\frac{a_2\cos\theta}{a_1 m_t + a_3\cos^2\theta}M_\theta\end{aligned} \tag{38}$$

$$\begin{aligned}&\ddot{\theta} + \frac{1}{a_1 m_t + a_3\cos^2\theta}\Big(\; c_w\ell\left(m_t\ell - a_2\right)\dot{\theta}\cos^2\theta + k_w\ell\left(m_t\ell - a_2\right)\sin\theta\cos\theta - k_w d\left(m_t\ell - a_2\right)\cos\theta\\ &\qquad+\left(a_2 m_b d_{Acmb} - m_w\ell\left(m_t\ell - a_2\right)\right)\dot{\theta}^2\sin\theta\cos\theta + \left(k_w\left(m_t\ell - a_2\right) - a_2 k_s\right)y_s\cos\theta\\ &\qquad+\left(c_w\left(m_t\ell - a_2\right) - a_2 c_s\right)\dot{y}_s\cos\theta\;\Big) \;=\; \frac{m_t}{a_1 m_t + a_3\cos^2\theta}M_\theta\end{aligned} \tag{39}$$

To follow the time behavior of the reaction force, F_N, Equation (21) can be rewritten using Equations (12), (34), (35), (38) and (39) to obtain:

$$
\begin{aligned}
F_N = \frac{1}{a_1 m_t + a_3\cos^2\theta} \Big(& c_w \ell\left[m_w \ell\left(m_b d_{Acmb} + a_2 - m_t \ell\right) + a_3\right]\dot{\theta}\cos^3\theta \\
& + k_w \ell\left[m_w \ell\left(m_b d_{Acmb} + a_2 - m_t \ell\right) + a_3\right]\sin\theta\cos^2\theta \\
& - m_w \ell\left[a_2 m_b d_{Acmb} - m_w \ell\left(m_t \ell - a_2\right) + a_3\right]\dot{\theta}^2\sin\theta\cos^2\theta \\
& - \left(m_w \ell\left[k_w\left(m_t \ell - a_2\right) - m_b d_{Acmb}\left(k_w + k_s\right)\right] - a_3 k_w\right)\cos^2\theta \\
& - a_1\left[m_w c_s - c_w\left(m_b + m_s\right)\right]\dot{y}_s - a_1\left[m_w k_s - k_w\left(m_b + m_s\right)\right]y_s \\
& + a_1 c_w \ell\left(m_b + m_s\right)\dot{\theta}\cos\theta + a_1 k_w \ell\left(m_b + m_s\right)\sin\theta \\
& + a_1 m_w\left(a_2 - m_t \ell\right)\dot{\theta}^2\sin\theta - k_w d\left[m_w \ell\left(m_b d_{Acmb} + a_2 - m_t \ell\right) + a_3\right]\cos^2\theta \\
& - \left(m_w \ell\left[c_w\left(m_t \ell - a_2\right) - m_b d_{Acmb}\left(c_w + c_s\right)\right] - a_3 c_w\right)\dot{y}_s\cos^2\theta \\
& + m_w\left(m_t \ell - a_2\right)M_\theta\cos\theta - a_1 k_w d\left(m_b + m_s\right) \Big)
\end{aligned}
\tag{40}
$$

It is evident that F_N depends on the impacting body velocities ($\dot{y}_s$ and $\dot{\theta}$) but also on the material properties of the body posed as constraint (m_w, k_w and c_w).

6. Numerical results: general case

The values of the parameters used in the numerical simulations that follows are presented in Table 1. The time step considered in the integration of the governing equations of motion is 0.001 s. The fourth order Runge-Kutta is the numerical integrator used. Three different cases are investigated which differ only in the value of the spring constant given to the wall. It is assumed that there is permanent compressive contact between the contacting bodies ($e = 0$), but no attractive one. Separation then takes place when the normal force goes to zero, i.e. when there will be a change from compressive to attractive force (taken zero here).

parameter	Case 1	Case 2	Case 3	unity
m_b	3.00	3.00	3.00	Kg
m_s	1.00	1.00	1.00	Kg
m_w	50.00	50.00	50.00	Kg
k_s	30.00	30.00	30.00	N/m
k_w	30.00	300.00	3000.00	N/m
c_s	0.10	0.10	0.10	Ns/m
c_w	20.00	20.00	20.00	Ns/m
ℓ	1.00	1.00	1.00	m
d	0.60	0.60	0.60	m
d_{Acmb}	0.50	0.50	0.50	m
M_θ	2.00	2.00	2.00	N m
$I_{b,cm}$	0.25	0.25	0.25	Kg m^2

Table 1 – Values used in the numerical simulations

The external torque, M_θ, is constant and equal to 2.00 Nm. This torque profile was chosen in order to make the system rotate always in the same direction and fulfill all of its possible 360^o turn. Any other kind of excitation (like a sinusoidal one with maximum amplitude of 180°, for instance) can be chosen without problem. In the simulation runs, the motion of the bar starts in $\theta = 0$.

The Figures 2 to 7 it is show the behaviour of the system for the Case 1 (……), Case 2 (- - - -) and Case 3 (——), where only the results associated with the first contact between the bodies are presented.

Figures 6 and 7 present the evolution of the relative distance between m_w and the tip of the bar, and the associated velocity. Figure 8 shows the time behavior of the contact force, F_N , between the tip of the bar and m_w for all the three cases. At the beginning of the first contact m_w is at rest. The amplitude of F_N jumps at the beginning of contact, from zero (no contact) to a value associated with the impact between the bodies and evolves with time according to the system states and properties. The value of F_N at impact does not necessarily represent the biggest value for the contact force during contact. A sudden change in velocity, when collision takes place, can be verified in Figure 7 for m_w.

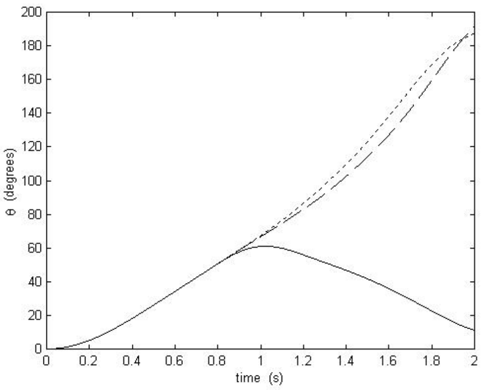

Figure 2 – Angular displacement of the bar

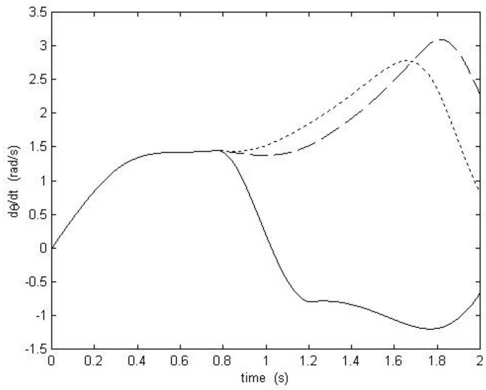

Figure 3 – Angular velocity of the bar

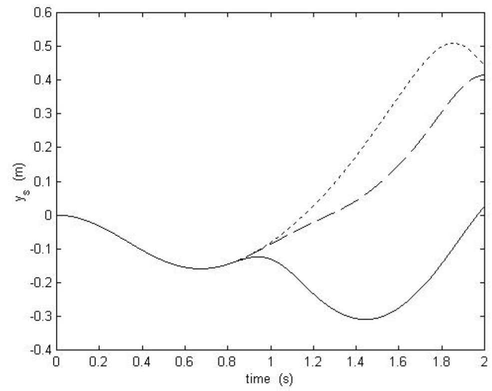

Figure 4 – Displacement of m_s

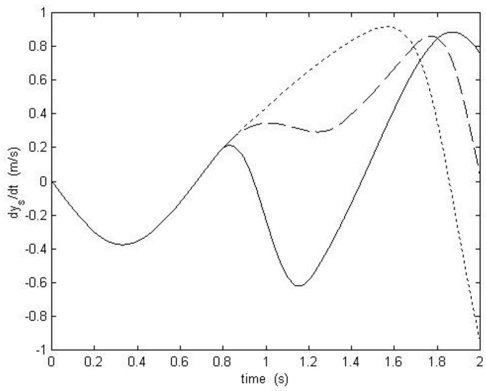

Figure 5 – Velocity of mass m_s

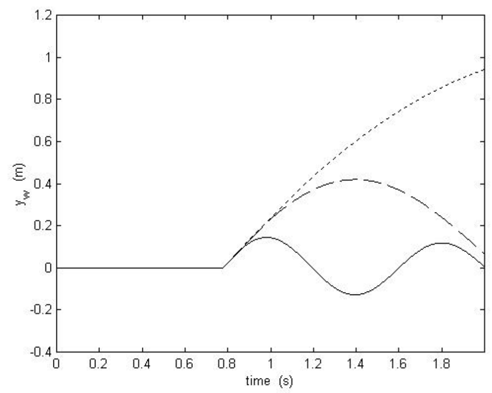

Figure 6 – Displacement of m_w

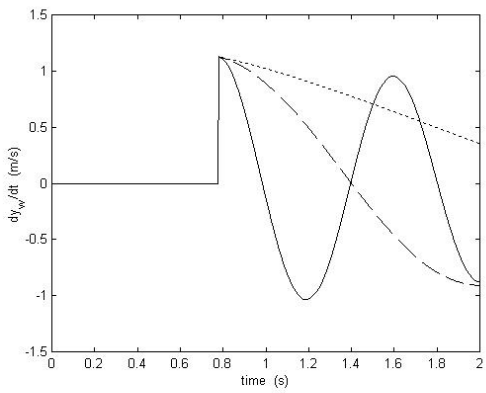

Figure 7 – Velocity of m_w

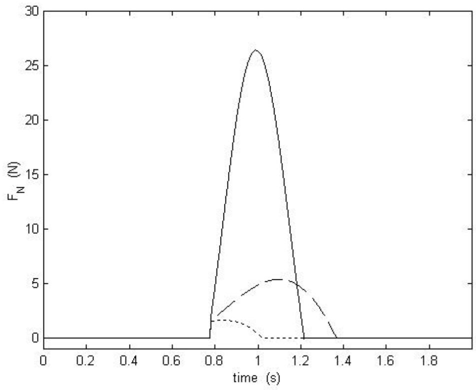

Figure 8 – The constraint (normal) force, F_N

In Cases 1 and 2 the bar overtakes the constraint and continue its rotating motion to the next quadrants. In Case 3, the bar does not overtake the first quadrant. In this case, the bar collides with the wall and goes backwards; the torque acting at point A pulls it back again and again against $m_{w.}$

7. Conclusions

The problem presented in this paper and the procedures developed for its analysis can be extended to many other situations. For instance, the case in which a robotic manipulator has to grab and handle an object with its own dynamics (as occurs in the capturing satellites scenario, for instance). The theory presented here can be applied to problems in which robots have to follow some prescribed patterns or trajectories when in contact with the environment (like in painting activities, for instance, or the ROKVISS experiment at DLR).

The necessity for changing from one set of governing equations to another represents a source of integration errors, since the integrators are faced with singularities (the system's states can change brusquely when impact occurs). In this work, the difficulty of solving a set of algebraic-differential equations (Equations (12), (20), (21) and (22)) is avoided by suitable differentiations and substitutions between the given equations. The set of equations that governs the system dynamics when the constraint condition is active is quite different from the one that governs the unconstrained movement of the system. One of these sets is always generating the initial states for the other. The number of degrees of freedom involved changes from one set of equations to the other.

8. Acknowledgements

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9. References

Clough, R. W., Penzien, J., 1975, "Dynamics of Structures", McGraw-Hill Kogakusha Ltd., Tokio, Japan.

Pfeiffer, F., Glocker, C.,1996, "Multibody dynamics with unilateral contacts", Wiley Series in Nonlinear Science, John Wiley and Sons, Inc.

Rosenberg, R. M., "Analytical Dynamics of Discrete Systems", Plenum Press, New York, 1977.

Schäfer, B., Rebele, B., Fenili, A., "Space Robotics Contact Dynamics Investigations and Numerical Simulations: ROKVISS", 15th CISM-IFToMM Symposium on Robot Design, Dynamics and Control, 2004.