Geophys. J. R. astr. Soc. (1970) **20**, 149–157.

A Matrix Method for Interpreting Oceanic Magnetic Anomalies

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(Received 1969 December 15)

Summary

This paper presents a matrix method for deducing the distribution of magnetization within a specified layer which causes a given magnetic anomaly. It extends the earlier method given by Bott by allowing irregular upper and lower surfaces for the magnetic layer. The method is particularly applicable to interpretation of oceanic magnetic anomalies in terms of a magnetization distribution within Layer 2 of the oceanic crust. It provides a method for applying the Vine–Matthews hypothesis to regions of irregular topography. The method is applied to a profile across the Sheba Ridge, Gulf of Aden.

1. Introduction

Strip-like magnetic anomalies of several hundred gamma amplitude were first discovered off the California coast (Raff & Mason 1961; Mason & Raff 1961) and are typical of much of the oceanic regions. The strip-like anomalies tend to be parallel to the crest of mid-ocean ridges. The normal interpretation of the anomalies is in terms of the hypothesis of Vine & Matthews (1963) which is itself dependent on the idea of ocean-floor spreading. Following the Vine–Matthews hypothesis, the study of oceanic magnetic anomalies has made outstandingly important contributions to our knowledge of the history of formation of ocean basins in relation to continental drift, and to determining the time-scale of reversals of the Earth's magnetic field over the last 70 My and more.

Vine (1966) and Heirtzler *et al.* (1968) have used the indirect method of magnetic interpretation to show that typical magnetic profiles of several hundreds of kilometres length can be simulated by a series of two-dimensional rectangular blocks of alternating magnetic polarity which are symmetrical about the ocean ridge crests. These blocks nominally represent Layer 2 of the oceanic crust, which is 2 km thick on average: indeed the main source of the anomalies must lie within Layer 2 because Layer 1 consists of effectively non-magnetic sediments and Layer 3 is too deep to explain the anomalies without unacceptably high and irregular variations in magnetization (Bott 1967). A particular sequence of blocks representing the past time-scale of reversals of the geomagnetic field over the last 70 My has been shown by Heirtzler *et al.* (1968) to give good agreement with observations across the ocean ridge system at widely separated places, provided different relative spreading rates are assumed.

The indirect simulation techniques of these authors have provided an adequate first step in the study of oceanic magnetic anomalies. However, as more seismic evidence relating to the upper and lower boundaries of Layer 2 becomes available, the interpretation of oceanic magnetic anomalies can be treated by the direct method of determining the distribution of magnetization given the shape of the source body and the direction of magnetization. This problem gives rise to a linear integral equation (Bott 1967) which can be solved to quite acceptable accuracy if it is replaced by a set of linear equations which are solved by matrix or other methods. This direct approach has the following advantages over the indirect method: (1) it is entirely objective, (2) variation in the shape of the magnetic layer can readily be incorporated, and (3) it is unaffected by the dip of the magnetic field (eyeball methods are particularly difficult where the field dips at about 45°).

This matrix method was first presented by Bott (1967) where it was used to interpret a profile across the Juan de Fuca ridge assuming the anomaly was caused by magnetization within a horizontal Layer 2. More recently, Emilia & Bodvarsson (1969) have presented a version of the method which approximates Layer 2 by a series of rectangular blocks of variable depth range. In this paper we present another modification of the method which is now capable of incorporating irregular variation of the upper and lower surfaces of the magnetic layer.

2. Direct magnetic interpretation using matrix methods

The matrix method applied to two-dimensional magnetic interpretation is as follows. We assume the geometry of the magnetic body from seismic or other evidence. The direction of magnetization is generally assumed to be parallel to the average geocentric dipole field. We now wish to evaluate the distribution of intensity of magnetization within a specified layer from the observed magnetic profile. To do this, the layer is split up into a series of two-dimensional blocks each possessing uniform magnetization (Fig. 1). The observed anomaly at each point on the surface can be expressed as the sum of the contributions from the individual blocks:

$$A_i = \sum_{j=1}^{m} K_{ij} J_j \tag{1}$$

where A_i ($i = 1$ to n) is the observed anomaly at point i, J_j is the intensity of magnetization of block j ($j = 1$ to m), and K_{ij} is the magnetic anomaly at point i caused by block j for an intensity of magnetization of unity. If $m = n$, the above set of equations can be solved directly for J_j provided K is not singular. If $m < n$, then the overdetermined set of equations can be solved by least-squares or by some other method for minimizing a function of the residuals.

Bott (1967) applied the method in two ways. The *first approach* was the straightforward method of solution for a two-dimensional magnetic profile in a single matrix operation, by direct solution or using least-squares. Two-dimensional rectangular blocks of constant size and depth range were used. More recently, Emilia & Bodvarsson (1969) have adapted the method to incorporate rectangular blocks of varying depth range. The *second approach* is applicable to long profiles to be

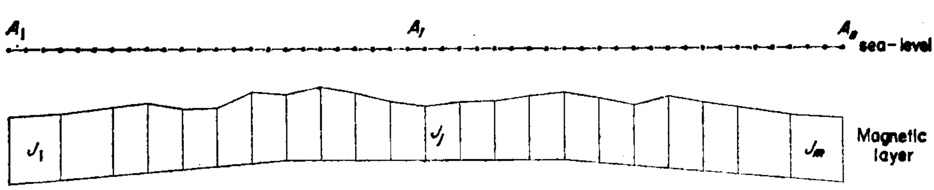


FIG. 1. Model used for the interpretation of oceanic magnetic anomalies.

interpreted in terms of identical rectangular blocks forming a horizontal layer. The method uses a single matrix inversion applied to only a limited length of the profile to produce a series of weighting factors which, when convolved with the observed anomalies, would yield the magnetization of the block at the centre of the part of the profile involved. By successive applications of the convolution, the distribution of magnetization could be determined for all the blocks except those near the ends of the profile. This method was applied with success to the Juan de Fuca ridge, where the assumption of a horizontal Layer 2 is acceptable.

3. The direct method using trapezoidal blocks

The availability of increasingly powerful computing facilities has encouraged us to develop further the straightforward method of Bott (1967) and Emilia & Bodvarsson (1969), by which all the data for a given magnetic profile is inverted to give the underlying magnetization distribution in a single operation. This method is very flexible as it allows block shapes other than simple rectangles of constant depth range to be used. This allows irregular topography at the top of Layer 2 and varying depth to the base to be incorporated. Such an approach is desirable if magnetic profiles over oceanic ridges are to be studied with maximum advantage.

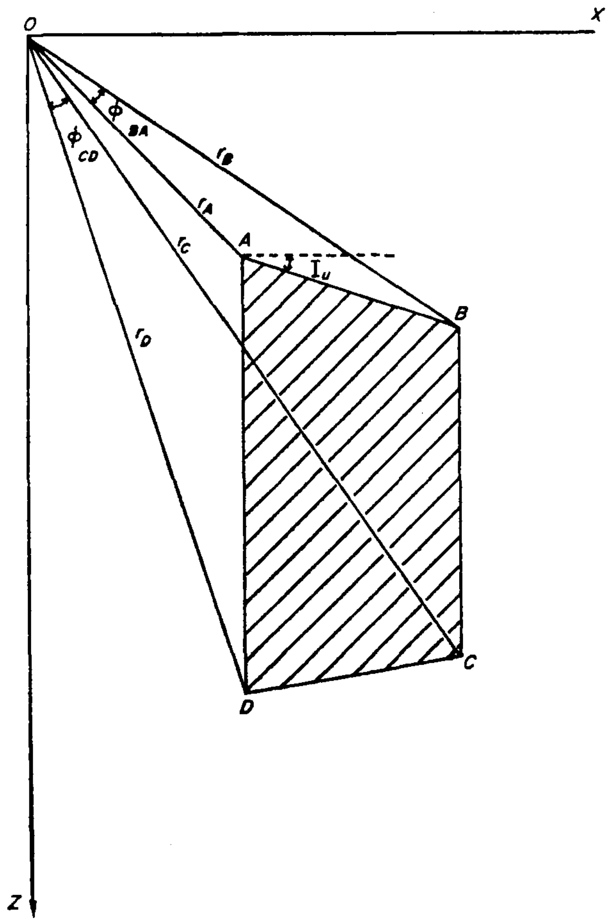


Fig. 2. Geometry of the basic two-dimensional model unit (ABCD) in relation to field point O.

The basic model unit adopted as being most suitable is the two-dimensional trapezoidal block with vertical sides (Fig. 2). Let us suppose the trapezoidal block in Fig. 2 is the jth block and that the origin O is the position at which the ith observed anomaly value is measured. The x-axis points in the direction of the magnetometer profile, perpendicular to the strike, and the z-axis points vertically downwards. Let us assume that the block posessses unit magnetization with dip of I_m and azimuth of α_m measured from the positive x-axis (in either sense). Let the measured magnetic anomaly component at O have a dip of I_e and an azimuth α_e. Then $K_{ij} J_j$ is the magnetic anomaly at O caused by the block. Let the dip of the upper surface of the block be I_u and that of the lower surface be I_l, both being measured downwards from the positive x-axis. It can be shown by elementary methods that

$$K_{ij} = 2F \cos I_u \{\sin (I_u + \beta) \ln(r_B/r_A) - \phi_{BA} \cos (I_u + \beta)\}$$
$$-2F \cos I_l \{\sin (I_l + \beta) \ln(r_C/r_D) - \phi_{CD} \cos (I_l + \beta)\} \quad (2)$$

where

$$\beta = \arctan (\tan I_m/\cos \alpha_m) + \arctan (\tan I_e/\cos \alpha_e),$$
$$F = (\sin^2 I_m + \cos^2 I_m \cos^2 \alpha_m)^{\frac{1}{2}} (\sin^2 I_e + \cos^2 I_e \cos^2 \alpha_e)^{\frac{1}{2}}$$

and $r_A, r_B, r_C, r_D, \phi_{BA}$ and ϕ_{CD} are defined in Fig. 2.

The first computational stage is to evaluate the terms K_{ij} for all i and j. This results in a matrix K which is square if $m = n$ but rectangular otherwise ($m < n$). J_j can then be determined by solving (1). The formal solution is $J = K^{-1} A$ for $m = n$ and $J = (K^T K)^{-1} K^T A$ for $m < n$, where K^T is the transpose of K (Bott 1967). In the present work, the solution of the equations (1) is accomplished using the IBM scientific subroutines SIMQ or LLSQ. The results have been found to be consistently satisfactory. The square matrix $(K^T K)$ is singular if the blocks together form a complete horizontal layer extending to infinity, and the method breaks down unless the magnetization of one of the blocks is specified. In no other cases has the matrix to be inverted been found to be singular.

Two precautions need to be taken. Firstly, the width of the blocks should not be less than about 0·6 times the depth to their upper surfaces; otherwise random errors in the observations of the order of ± 2 gamma produce unacceptably large fluctuations in the resulting values of intensity of magnetization. Narrower blocks could, however, be used if the observations are smoothed to filter out all wavelengths less than about 0·8 times the depth of sea. If undoctored interpretations involving narrower blocks are desired, either (1) the accuracy of observations after diurnal correction has to be substantially improved, or (2) observations can be made nearer the seabed (Luyendyk, Mudie & Harrison 1968). This limitation on the width of blocks is not imposed by this particular method but by the inherent properties of fields derived from a potential which satisfies Laplace's equation.

Secondly, a linear regional gradient should be subtracted from the observations. It may also be desirable on long profiles to filter out the long wavelength Fourier components, because quite small observational errors of long wavelength (caused by the diurnal variation or otherwise) produce large disturbances of the same wavelength in the interpreted pattern of magnetization.

We are in the process of extending the above described method in two ways. (1) Instead of using vertical boundaries between the blocks, we are attempting interpretations using blocks with boundaries which slope outwards or inwards in relation to the crest of the ocean ridge. (2) We are incorporating the effect of three-dimensional topography into the method, thereby providing a more realistic approach to certain ocean ridges.

4. Application of the technique

As an example, the above method has been applied to a total-field magnetometer profile across the Sheba Ridge in the Gulf of Aden observed by Matthews, Williams & Laughton (1967). The profile extends between 13° 49·2′ *N*, 55° 45·0′ E and 15° 54·6′ N, 56° 28·7′ E. It is perpendicular to the local median valley of the ridge and to the trend of the magnetic anomalies, justifying two-dimensional interpretation. The linear regional gradient along the profile has been computed by least squares, and has been removed prior to interpretation. Because of the general lack of sediment and the rugged relief of the seabed, it has been assumed that the bathymetry along the profile represents the upper surface of the magnetic Layer 2 rocks. The rough bathymetry suggests that allowance for it should considerably improve the reliability of the resulting interpretation.

The form of the lower surface of Layer 2 is not known. Two possible configurations of it have been assumed as follows: (1) its base has been assumed to be horizontal at 5 km depth (Fig. 3(a)); and (2) it has been assumed to slope away from the ridge crest so that Layer 2 retains an approximately uniform thickness (Fig. 3(b)). The computer method described above was then used to estimate directly the distribution of magnetization within Layer 2 required to cause the observed anomaly profile. The observed profile has been sampled at 1·15 km interval yielding a total of 213 field-point values along the profile. Layer 2 has been subdivided into 106 two-dimensional trapezoidal blocks of 2·3 km width. As there are more field-points than blocks, the least-squares version of the method has been used.

The results of the computations (Fig. 3) show that both models give accurate simulation of the observed profile. Residual anomaly values do not exceed 62γ and are in general appreciably less than this (R.M.S. value is $\pm 12\gamma$). The largest residuals occur where the upper surface of Layer 2 is shallowest near the crest of the ridge. These residuals could be reduced by using narrower blocks. However, much narrower blocks could not be used with justification over the deeper portions of the profile unless one was sure that individual magnetic observations had been corrected accurately for daily variation. Otherwise, the instability inherent in any form of downward continuation would cause unacceptably large fluctuations in the interpreted magnetization values of the adjacent blocks.

Both models shown in Fig. 3 equally well account for the observed magnetic anomalies with acceptable accuracy. Both models show groups of blocks alternating between more positive and more negative magnetization values. The boundaries between adjacent groups of blocks are marked by abrupt changes in magnetization, reaching 0·01 e.m.u. cm^{-3} near the ridge crest. Both models also show a central zone of positive magnetization beneath the axial rift zone of the ridge, on either side of which there is some degree of symmetry in the pattern, although this is far from perfect.

The main difference between the two models shown in Fig. 3 is that the intensities of magnetization beneath the central part of the profile are higher in the model which has the thinner Layer 2 beneath the crest (as expected). The magnetic anomalies do not otherwise distinguish between the two hypotheses. However, the sloping base to the layer is to be preferred on other grounds. It is in better accord with the ocean floor-spreading hypothesis and with seismic refraction observations in various parts of the oceans. Our interpretation suggests that the strongest contrasts in magnetization occur beneath the axial part of the ridge, unlike the situation for the Juan de Fuca Ridge (Bott 1967) or for the Eltanin traverse across the East Pacific Rise (Emilia & Bodvarsson 1969).

Both computed models in Fig. 3 show a conspicuous long wavelength component of magnetization. This causes the magnetization values beneath the flanks of the ridge to be depressed so that they are predominantly negative. This may possibly be a real effect but it would be more convincing if the values were predominantly positive because of induced magnetization. A more probable explanation is that lack of correction for the diurnal variation has introduced a spurious long-wavelength component in the observed anomaly profile. This would be erroneously interpreted in terms of an underlying magnetization distribution of the same wavelength. The amplitude would be proportional to that of the anomaly component and also to the wavelength. Thus the influence on the interpreted magnetization values becomes

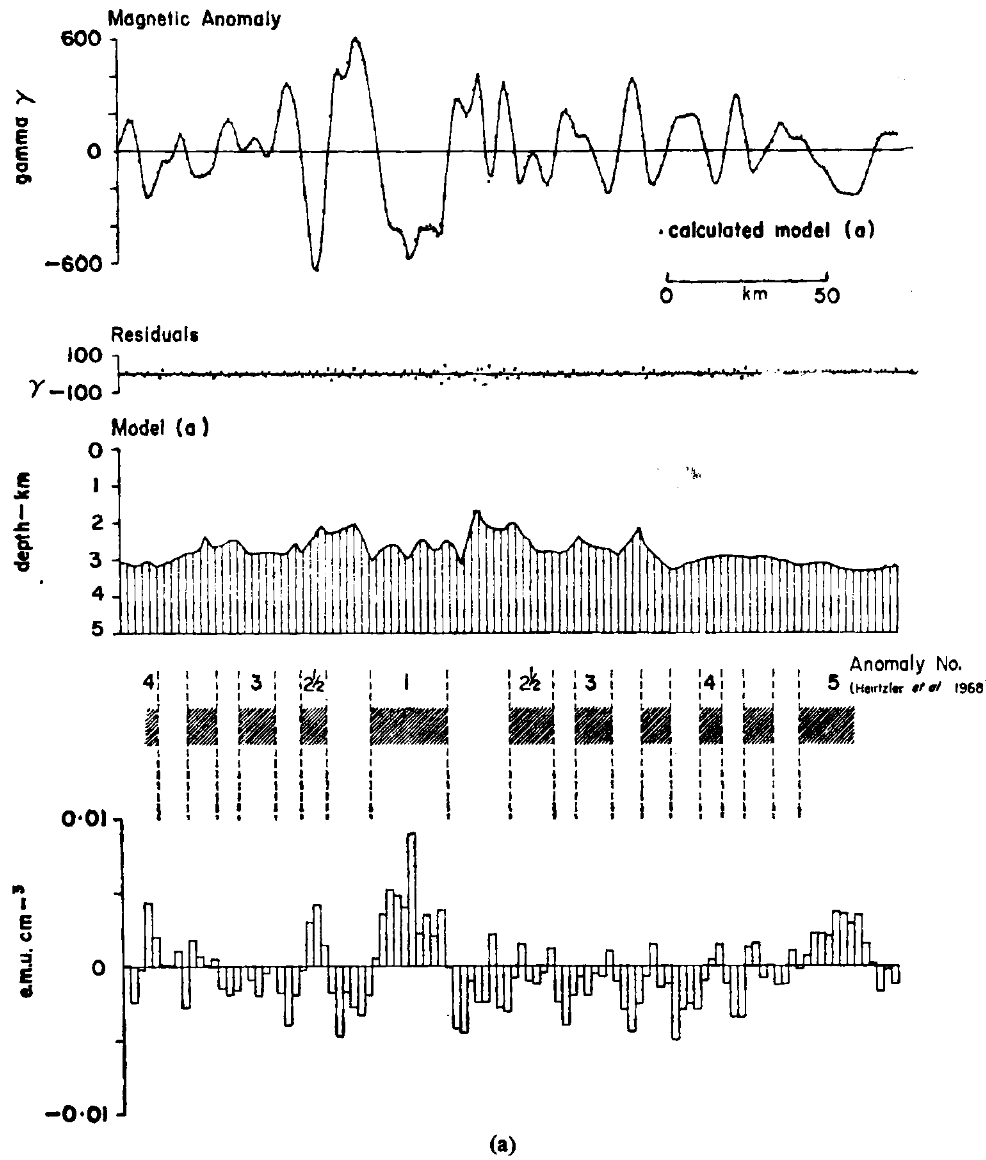
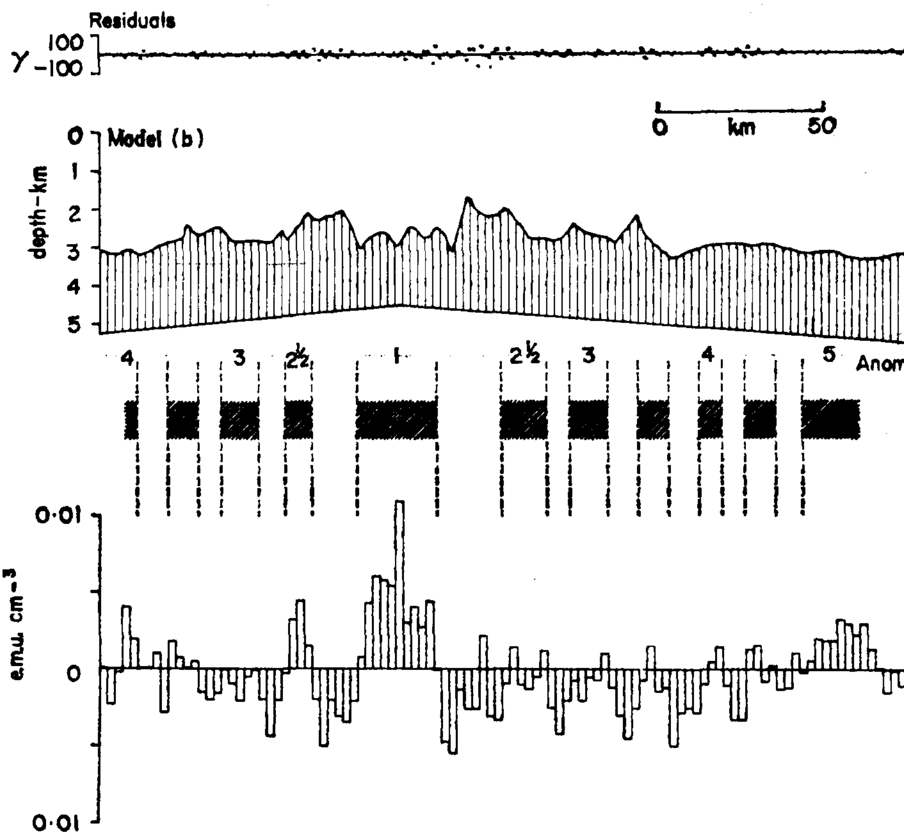


FIG. 3. (a) The interpretation of magnetic profile IJ (Matthews, Williams & Laughton 1967) across the Sheba ridge in terms of a distribution of magnetization confined to Layer 2 of the oceanic crust, assuming the base of Layer 2 to be horizontal. (b) Interpretation of the profile of Fig. 3(a) assuming a sloping interface for the base of Layer 2.



(b)

progressively more acute for longer wavelengths. This difficulty can be overcome by determining the long wavelength components by Fourier analysis of the profile and by removing these before interpretation is carried out. This has been done, and the result for the model with a horizontal base is shown in Fig. 4. The most obvious difference between Figs 3(a) and 4 is that removal of the long wavelengths causes the alternating groups of magnetization to be more clearly differentiated on the basis of algebraic sign.

The interpretation shown in Fig. 4 is in agreement with the Vine–Matthews hypothesis. The positive and negative groups of magnetization in Figs 3 and 4 have been provisionally correlated with the numbering sequence of reversals of the geomagnetic field established by Heirtzler *et al.* (1968). The correlation suggests a spreading rate of about 1 cm y^{-1} for both limbs, a value which is consistent with other work in the area (Laughton, Jones & Whitmarsh 1970). The pattern is not quite as regular as theory would predict, but errors in interpretation could be caused by deviation of the bathymetry from a true two-dimensional structure and by vertical variations in magnetization or other irregularities associated with the rough ridge bathymetry.

The above interpretation demonstrates the importance of obtaining as accurate magnetometer observations at sea as possible, if these are to be used for quantitative interpretation. In particular, both short-period and long-period time variations of the geomagnetic field ought to be removed as accurately as possible. The resulting short wavelength inaccuracies in measurement limit the narrowness of the blocks which can be used in interpretation without introduction of unacceptably high

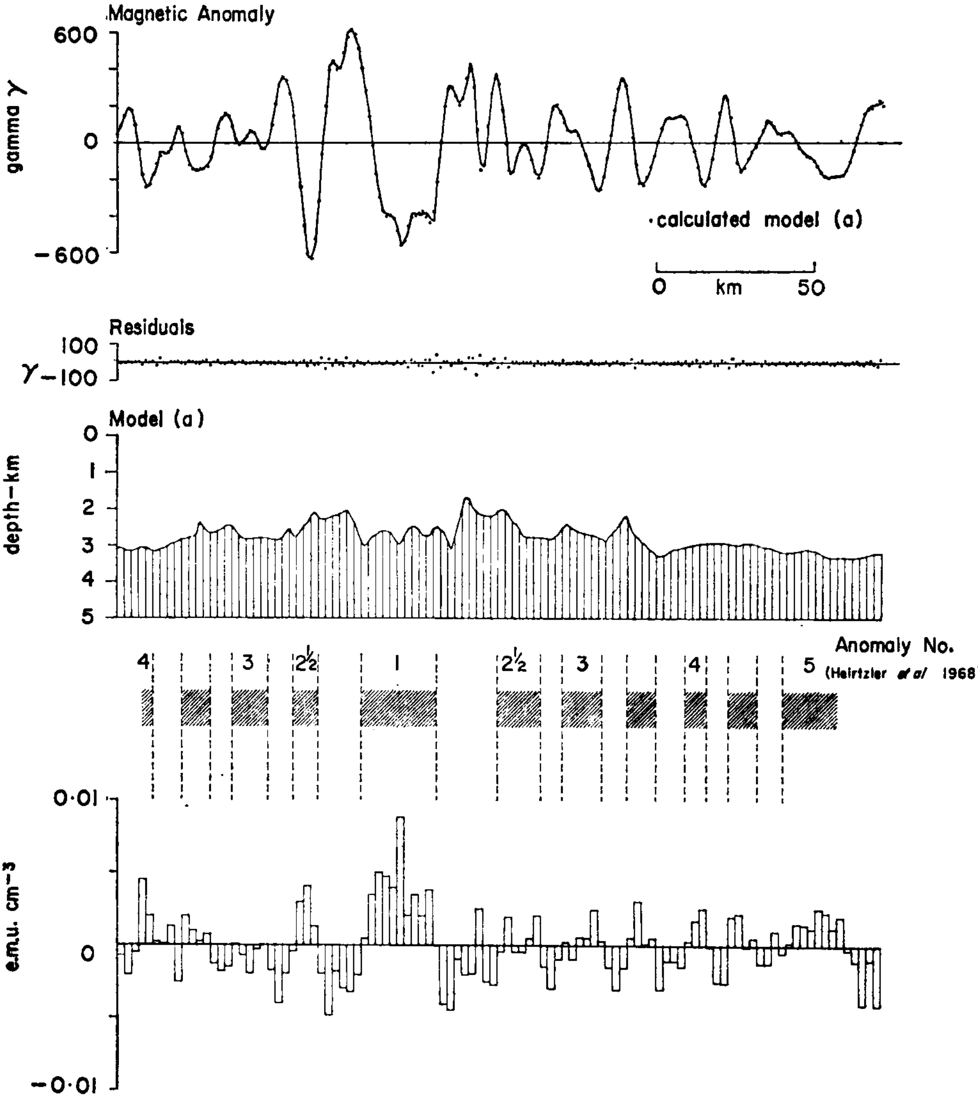


FIG. 4. Interpretation of ' filtered ' magnetic profile across the Sheba ridgein terms of a distribution of magnetization confined to Layer 2. The base of Layer 2 is assumed to be horizontal at 5 km depth.

fluctuations in magnetization; long wavelength inaccuracies introduce spurious long wavelength magnetization distributions which may mask real effects of interest.

A listing of the program, written in PL1 language for an IBM 360/67 computer, can be obtained by writing to the authors.

Acknowledgments

We are grateful to Dr M. T. Jones of the National Institute of Oceanography for permission to use magnetic observations obtained by *R.R.S. Discovery* in 1967. Computations have been done on the Northumbrian Universities Multiple Access Computer (IBM 360/67).

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