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A MATRIX REPRESENTATION FOR ASSOCIATIVE ALGEBRAS. II

BY

JACQUES LEWIN[1]

ABSTRACT. The results of part I of this paper are applied to show that if F is a free algebra over the field K and W is a subset of F which is algebraically independent modulo the commutator ideal $[F, F]$, then W again generates a free algebra. On the way a similar theorem is proved for algebras that are free in the variety of K-algebras whose commutator ideal is nilpotent of class n.

It is also shown that if L is a Lie algebra with universal enveloping algebra F, and U, V are ideals of L, then $FUF \cdot FVF \cap L = [U \cap V, U \cap V]$. This is used to extend the representation theorem of part I to free Lie algebras.

I. Introduction. In this paper we give some further applications of the matrix representation theorem proved in part I [7]. We first prove a general result communicated to us by George Bergman: If R is a ring and I an ideal of R which is free as a right R-module, and $x \in R$, then x is a zero divisor in R/I if and only if it is a zero divisor in R/I^n. Next, let $[F, F]$ be the commutator ideal of the free algebra F. The algebra $R = F/[F, F]^n$ is a free algebra in the variety (in the sense of universal algebra) of algebras whose commutator ideal is nilpotent of class at most n. We show that a subset Z of R freely generates a (relatively) free subalgebra if, and only if, Z is algebraically independent modulo $[R, R]$. Since $\bigcap_{i=1}^{\infty} [F, F]^i = 0$, this allows us to show that a subset of F which is algebraically independent modulo $[F, F]$ generates a free subalgebra of F. In another direction, if U, V are ideals of a Lie algebra L, and U_F, V_F are the corresponding ideals in the universal associative enveloping algebra F, we show that $U_F V_F \cap L = [U \cap V, U \cap V]$. If L is a free Lie algebra, the matrix representation [4] for F with kernel $U_F V_F$ then gives a representation for L with kernel $[U \cap V, U \cap V]$.

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II. Regular elements. The results of this section are due to G. Bergman.

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Lemma 1. *Let R be a ring, $f: M \to N$ a homomorphism of right R-modules, $f': M' \to N'$ its restriction to an appropriate pair of submodules, and J a left ideal of R. Suppose that the induced maps $M/M' \to N/N'$ and $M/MJ \to N/NJ$ are injective, and that N and J are flat as (right, resp. left) R-modules. Then the induced map $M/M'J \to N/N'J$ is also injective*

Proof. Injectivity of $M/M' \to N/N'$ and flatness of J give the injectivity of the top arrow in the commutative diagram:

$$\begin{array}{ccc} (M/M')\otimes J & \longrightarrow & (N/N')\otimes J \\ \downarrow & & \downarrow \\ MJ/M'J & \longrightarrow & NJ/N'J. \end{array}$$

Here to construct the left-hand descending arrow, we tensor $M' \to M \to M/M' \to 0$ with J and conclude that $(M/M')\otimes J \cong (M \otimes J)/\mathrm{Im}(M' \otimes J)$, then use the natural map $M \otimes J \to MJ$ to induce a map $M \otimes J/\mathrm{Im}(M' \otimes J) \otimes MJ/M'J$. The right-hand map is similarly obtained, but because N is flat, the map $N \otimes J \to NJ$ is an isomorphism, hence so is $N \otimes J/\mathrm{Im}(N' \otimes J) \to NJ/N'J$, and we see that the right-hand descending map is an isomorphism. As the top arrow is 1-1, so is the diagonal composite map, hence so is the left-hand descending map. As the latter is onto, it too is an isomorphism; hence the bottom map is injective.

Combining this with the injectivity of $M/MJ \to N/NJ$, we get the desired injectivity of $M/M'J \to N/N'J$. □

Lemma 2. *Let R be a ring (with 1), M a generator in the category of left R-modules, and J a two-sided ideal of R. If $x \in R$ acts as a left-zero-divisor on R/J, then it acts as a zero-divisor on M/MJ.*

Proof. Since M is a generator, some direct sum $N = M \oplus \cdots \oplus M$ has a direct summand isomorphic to R. Hence N/JN has a summand isomorphic to R/J. Hence x acts as a zero divisor on $N/JN = M/JM \oplus \cdots \oplus M/JM$. Hence x acts as a zero divisor on M/JM. □

Theorem 1. *Let R be a ring, x an element of R, and I a two-sided ideal of R. Then:*

(i) *If I is flat as a left R-module, and x is not a left zero-divisor on R/I, then x is not a left zero-divisor on R/I^n for any n.*

(ii) *If I is a generator of the category of left R-modules, and x is a left zero-divisor on R/I, then x is a left zero-divisor on R/I^n for all n.*

(iii) *If R is a free associative algebra over a field (and I arbitrary), then for all $n > 0$, x is a zero-divisor in R/I if and only if it is a zero-divisor in R/I^n.*

Proof. (i) and (ii) are proved by induction on n. For (i), apply Lemma 1, with $M = N = R$, f = left multiplication by x, $M' = N' = I^{n-1}$, and $J = I$. For (ii), apply Lemma 2 with $M = I$, $J = I^{n-1}$. We get (iii) from (i) and (ii) and the symmetric statements on the right, and the result that in a free associative algebra R, any right or left ideal I is free as a (right or left) R-module (Cohn [4]). □

III. Free subalgebras. Recall from [7] that if $F = K\langle X\rangle$ is a free K-algebra, $r \to r'$, $r \to r''$ K-homomorphisms of F into K-algebras R', R'' with kernels U, V respectively and $T = T(F; R', R'')$ a free (R', R'')-bimodule on generators $\delta(x)$ $(x \in X)$, then the map $x \to \begin{pmatrix} x'' & 0 \\ \delta(x) & x' \end{pmatrix}$ extends to a homomorphism $\mu: F \to \begin{pmatrix} R'' & 0 \\ T & R' \end{pmatrix}$ with kernel UV. The induced map $\delta: F \to T$ is a derivation. If we consider T as a right $R'^{\mathrm{opp}} \otimes R''$ bimodule, then we define, for each x, partial derivatives $\partial/\partial x: F \to R'^{\mathrm{opp}} \otimes R''$ by the formula $\delta(f) = \Sigma_{x \in X}\, \delta(x)(\partial f/\partial x)$. It is easily verified that if $m \in F$ is a monomial $m = x_1 x_2 \cdots x_n$ $(x_i \in X)$, then

$$\frac{\partial m}{\partial x} = \sum_{x_i = x} (x_1' \cdots x_{i-1})' \otimes (x_{i+1} \cdots x_n)''. \tag{1}$$

Let now $A = R' = R'' = F/[F, F]$, with $[F, F]$ the commutator ideal of F. Since $[F, F]^2$ is the kernel of μ, μ induces an injection $\bar{\mu}: \bar{F} = F/[F, F]^2 \to \begin{pmatrix} A & 0 \\ T & A \end{pmatrix}$ and $\partial/\partial x$ induces a partial derivative $\bar{\partial}/\bar{\partial} x: \bar{F} \to A \otimes A$. Each $\bar{\partial}/\bar{\partial} x$, when restricted to $[\bar{F}, \bar{F}]$ is an A-bimodule homomorphism. An element $w \in [\bar{F}, \bar{F}]$ is zero if and only if for all x, $\bar{\partial} w/\bar{\partial} x = 0$. Let ν be the natural homomorphism $\nu: A \otimes_K A \to A \otimes_A A = A$. For $m_1, m_2 \in \bar{F}$, then

$$\frac{\bar{\partial}}{\bar{\partial} x}(m_1 m_2 - m_2 m_1) = \frac{\bar{\partial} m_1}{\bar{\partial} x} m_2' - m_2' \frac{\bar{\partial} m_1}{\bar{\partial} x} + \left(m_1' \frac{\bar{\partial} m_2}{\bar{\partial} x} - \frac{\bar{\partial} m_2}{\bar{\partial} x} m_1' \right).$$

Since ν acts by removing tensor signs, the composed map $(\bar{\partial}/\bar{\partial} x)\nu$ is zero at $m_1 m_2 - m_2 m_1$. Since $(\bar{\partial}/\bar{\partial} x)\nu$ is an A-bimodule homomorphism on $[\bar{F}, \bar{F}]$ it follows that $(\bar{\partial}/\bar{\partial} x)\nu$ is zero on $[\bar{F}, \bar{F}]$, and thus can be considered as a derivation $d/dx': A = \bar{F}/[\bar{F}, \bar{F}] \to A$. It is immediate from (1) that if we write x' for the coset $x + [F, F]$, then, d/dx' is indeed the usual partial derivative with respect to x' in the polynomial ring $A = K[X']$.

Lemma 3. *Let $A = R' = R_1 = F/[F, F]$ and $R'' = R_d = F/[F, F]^d$. Let $w_1, \cdots, w_n$ be elements of $F = K\langle x_1, \cdots, x_n\rangle = K\langle X\rangle$ such that $w_1', \cdots, w_n'$ are algebraically independent in $R_1 = K[X']$. Then the Jacobian matrix $\Gamma_d = (\partial w_i/\partial x_j)$ is regular over $R_1 \otimes R_d$.*

Proof. We first treat the case $d = 1$.

Let $\nu_*: (A \otimes_K A)_* \to A_*$ be the map induced by ν on $n \times n$ matrices. Then, by the foregoing discussion, $\Gamma_1 \nu_* = (dw_i'/dx_j')$, the Jacobian matrix of the elements w_i' of A. The independence of $w_i', \cdots, w_n'$ insures that $\Gamma_1 \nu_*$ is regular in A_n. Thus $\Gamma_1 \nu_*$ has a nonzero determinant. Since ν is a ring homomorphism,

$(\det \Gamma_1)\nu = \det(\Gamma_1 \nu_*)$. So Γ_1 also has a nonzero determinant, and, hence, is regular.

Let now π^d be the natural map $\pi^d\colon R_1 \otimes_K F \to R_1 \otimes_K R_d$, and let π^d_* be the map induced on $n \times n$ matrices. Then the kernel of π^d is $R_1 \otimes [F, F]^d$, so that Ker π^d_* is $(R_1 \otimes [F, F]^d)_* = ((R_1 \otimes_K [F, F])_*)^d$. Now, since F is a free algebra, $[F, F]$ is free as a left module and it follows that $R_1 \otimes [F, F]$ is free as a left $R_1 \otimes F$-module (with basis $1 \otimes (F$-basis for $[F, F]))$. Thus, when we go to $n \times n$ matrices, $(R_1 \otimes [F, F])_*$ is free as a left $(R_1 \otimes F)_*$ module. Now if $\partial_0/\partial_0 x$ are the partial derivatives with values in $R_1^{\mathrm{opp}} \otimes F = R_1 \otimes F$, then $\Gamma_d = (\partial_0 w_i/\partial_0 x)\pi^d_*$, $\Gamma_1 = (\partial_0 w_i/\partial_0 x)\pi^1_*$. We may now apply Theorem 1 with $R = (R_1 \otimes F)_*$ and $I = (R_1 \otimes [F, F])_*$ to deduce from the first part of the proof that since Γ_1 is left regular, so is Γ_d. The same argument works just as well qua right modules, so Γ_d is also right regular. □

Let now $\mathfrak{V}_d$ be the variety of all K-algebras whose commutator ideal is nilpotent of class d. (This is the class of all algebras which satisfy the polynomial identity $\prod_{i=1}^d [x_i, y_i]z_i = 0$.) Then $R_d = F/[F, F]^d$ is a free algebra in this variety, and we say that a subset W of R_d freely generates R_d if every relation of W is a polynomial identity of R_d. We may ask which subalgebras of R_d are again $\mathfrak{V}_d$-free.

Theorem 2. *Let $R_d = F/[F, F]^d$. A subset of R_d $\mathfrak{V}_d$-freely generates a $\mathfrak{V}_d$-free subalgebra if and only if it is algebraically independent modulo $[R_d, R_d]$.*

Proof. We first consider the case where F is finitely generated, $F = K\langle x_1, \cdots, x_n\rangle$ and W consists of precisely n elements, $w_1 + [F, F]^d, \cdots, w_n + [F, F]^d$, which are algebraically independent modulo $[R_d, R_d]$. From the discussion at the beginning of this section, R_d is embedded in the algebra

$$\begin{pmatrix} R_{d-1} & 0 \\ T & R_1 \end{pmatrix},$$

T a free (R_1, R_{d-1})-bimodule with basis $\delta(x_1), \cdots, \delta(x_n)$, via the map

$$x_i + [F, F]^d \to \begin{pmatrix} x_i + [F, F]^{d-1} & 0 \\ \delta(x_i) & x_i + [F, F] \end{pmatrix}.$$

If $w_i + [F, F]^d$ corresponds to

$$\begin{pmatrix} w_i + [F, F]^{d-1} & 0 \\ t_i & w_i + [F, F] \end{pmatrix}$$

then, considering T as a right $R_1 \otimes R_{d-1}$ module, $(t_1, \cdots, t_n) = (\delta(x_1), \cdots, \delta(x_n))\Gamma_{d-1}$. Since, by Lemma 3, Γ_{d-1} is regular, $\{t_1, \cdots, t_n\}$ is again a basis for a free $R_1 \otimes R_{d-1}$ module. Now, $w_1 + [F, F], \cdots, w_n + [F, F]$ are algebraically independent, so the map $x_i + [F, F] \to w_i + [F, F]$ extends to an injection $R_1 \to R_1$ and we may assume by induction that $x_i + [F, F]^{d-1} \to w_i + [F, F]^{d-1}$ also extends to an injection $R_{d-1} \to R_{d-1}$. The freedom of the t_i's now insures that the map $x_i + [F, F]^d \to w_i + [F, F]^d$ (which extends to an endomorphism of R_d by the freedom of R_d) extends to an injection.

Now for the general case. It is clear that for a subset Z of R_d to freely generate a $\mathfrak{C}_d$-free subalgebra it is sufficient that every finite subset of Z freely generates a free subalgebra. Consider $\{z_1, \cdots, z_n\}$. Then, for some integer s, $\{z_1, \cdots, z_n\}$ is in the subalgebra S of R_d generated by $x_1 + [F, F]^d, \cdots, x_s + [F, F]^d$. Since $z_1 + [R_d, R_d], \cdots, z_n + [R_d, R_d]$ are algebraically independent, we can find $z'_{n+1}, \cdots, z'_s$ in S such that $\{z_1, \cdots, z_n, z'_{n+1}, \cdots, z'_s\}$ is still algebraically independent modulo $[S, S]$. From the special case above we deduce that $\{z_1, \cdots, z_n, z'_{n+1}, \cdots, z'_s\}$, and hence $\{z_1, \cdots, z_n\}$, freely generates a free subalgebra.

For the converse, suppose $z_1, \cdots, z_n \in R_d$ are not algebraically independent modulo $[R_d, R_d]$. Then there is a nonzero element $p(x_1, \cdots, x_n) \notin [F, F]$ such that $p(z_1, \cdots, z_n) \in [R_d, R_d]$. Thus $p(z_1, \cdots, z_n)^d = 0$. If $z_1, \cdots, z_n$ freely generate a $\mathfrak{C}_d$-free subalgebra, then $p(x_1, \cdots, x_n)^d$ is an identity of R, a contradiction since all the identities of R_d are in the commutator ideal of F. □

Note the close resemblance between Theorem 2 and the following result of G. Baumslag [2, Theorem 2]:

Let G be a group which is free in the variety $\mathfrak{S}_d$ of groups that are solvable of derived length at most d. A subset of G $\mathfrak{S}_d$-freely generates an $\mathfrak{S}_d$-free group if and only if it is linearly independent module $[G, G]$, the commutator subgroup of G.

In one direction, Theorem 2 can be extended to (absolutely) free algebras.

Theorem 3. *Let W be a subset of the free algebra $F = K\langle X\rangle$ which is algebraically independent modulo $[F, F]$. Then W freely generates a free subalgebra of F.*

Proof. By extending W if necessary, we may assume that W is indexed by the elements of X. Consider the endomorphism α of F defined by $x \to w_x$, and suppose $0 \neq u \in \operatorname{Ker} \alpha$. Choose d such that $u \notin [F, F]^d$. (This can be done since $\bigcap_{k=1}^{\infty} [F, F]^k = 0$.) Then α induces a map α_d: $F/[F, F]^d \to F/[F, F]^d$ which, by the previous theorem, is injective. However, $(u + [F, F]^d)\alpha_d = u\alpha + [F, F]^d = 0$, so that $u \in [F, F]^d$, a contradiction. Thus $\operatorname{Ker} \alpha = 0$ and we are done. □

IV. Lie algebras. In this section, we prove a representation theorem for Lie algebras.

Throughout what follows, L will be a Lie algebra over a field K, and A its universal enveloping algebra. We denote Lie multiplication by $[\ ,\ ]$, so that it agrees with the commutator operation in A. We must begin with some results on the relation between L and A.

If B is a K-basis for L, and we totally order B, then an *ascending monomial* will mean an expression $x_1x_2 \cdots x_n$ (representing an element of A), with $n \geq 0$, $x_i \in B$, and $x_1 \leq x_2 \leq \cdots \leq x_n$. By the Poincaré-Birkhoff-Witt theorem, the ascending monomials form a K-basis for A. There is an explicit reduction procedure for expressing any monomial in B as a K-linear combination of ascending monomials (cf. Jacobson [5, Chapter V], also Bergman [3, §4]). If $y = y_1y_2 \cdots y_m$ is a monomial and $y_i > y_{i+1}$ for some i, we write:

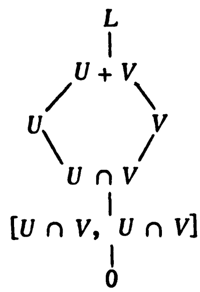
$$
\begin{aligned}
y &= y_1 \cdots y_iy_{i+1} \cdots y_m \\
&= (y_1 \cdots y_{i+1}y_i \cdots y_m) + (y_1 \cdots [y_i, y_{i+1}] \cdots y_m),
\end{aligned}
\tag{2}
$$

and then express $[y_i, y_{i+1}] \in L$ as a K-linear combination of elements of B. Thus, y is a linear combination of monomials of smaller length, $m - 1$, and a monomial which is closer to being ascending. So this process, applied to the terms of any expression, will eventually terminate in a linear combination of ascending monomials, the "canonical form" for the element in question.

Now suppose U is a Lie ideal in L. Let us first choose a K-basis $B(U)$ for U, then extend this to a basis B of L. Suppose a monomial y in B has at least one factor y_j belonging to $B(U)$. Then we claim that a reduction (2) takes y to a linear combination of monomials which again have a factor in $B(U)$. Indeed, if $j \neq i$, $i + 1$ in (2), this is trivial, while if $j = i$ or $i + 1$, then $[y_i, y_{i+1}]$ again lies in U because U is an ideal, so $[y_i, y_{i+1}]$ is a linear combination of elements of $B(U)$; the assertion is then clear.

Let $U_A \subseteq A$ denote AUA, the ideal of A generated by U. The elements of U_A are by definition those that can be written as linear combinations of (arbitrary) monomials having a factor in $B(U)$. It follows from the above observations that the canonical form for an element of U_A will still be as a linear combination of such monomials. In particular, since L is the subspace of A spanned by monomials of length 1, we have $U_A \cap L = U$. This can also be seen directly from the Poincaré-Birkhoff-Witt theorem applied to L/U, which has universal enveloping algebra A/U_A. But the analogous result we shall now obtain for *two* ideals is not so trivial.

Let U and V be two Lie ideals of L. Let us construct the basis B for L as follows. First choose a basis $B([U \cap V, U \cap V])$ for the ideal $[U \cap V, U \cap V] \subseteq L$; extend this to a basis $B(U \cap V)$ of the ideal $U \cap V$; extend $B(U \cap V)$ on the one hand to a basis $B(U)$ of U, and on the other hand to a basis $B(V)$ of V, and note that $B(U) \cup B(V)$ will form a basis of the ideal $U + V$ (cf. diagram).



We extend $B(U) \cup B(V)$ to a basis $B = B(L)$ of L.

We choose any total ordering of B subject to the condition that all elements of $B(U)$ not lying in $B(U \cap V)$ should be < all elements of $B(U \cap V)$, which should in turn be < all other elements of $B(V)$.

Now put $U_A = AUA$ and $V_A = AVA$. We wish to study the ideal $U_A V_A = AUAVA$. For this purpose, let S denote the set of all monomials on B which either (i) involve an element of $B(U)$ to the left of (i.e., preceding, possibly with some other terms in between) an element of $B(V)$, or (ii) involve an element of $B([U \cap V, U \cap V])$. Clearly every member of S, and hence every linear combination of members of S represents an element of $U_A V_A$, and conversely, $U_A V_A$ is spanned by the elements represented by monomials in S.

We now claim that any reduction of the form (2) will take a monomial $y \in S$ to a linear combination of monomials in S. It is clear from earlier observations that if y satisfies condition (ii), so will all the terms in the expression to which it reduces. Suppose, on the other hand, that y has a term $y_j \in B(U)$ to the left of a term $y_k \in B(V)$ (i.e., $j < k$). If neither j nor k belongs to $\{i, i+1\}$ (in the notation of (2)), the result is trivial, and if only one of them does, it is straightforward, like our earlier result. So suppose, finally, that $j = i$, $k = i + 1$. In order for the reduction (2) to be applicable, the element $y_j \in B(U)$ must be greater, under our ordering, than $y_k \in B(V)$. By the conditions on our ordering, this can only happen if y_j and y_k both lie in $B(U \cap V)$. In this case, when we apply (2) the term of length m resulting contains the sequence $y_k y_j$, and hence still has a member of $B(U)$ (actually a member of $B(U \cap V)$) to the left of a member of $B(V)$ (ditto), while $[y_j, y_k] \in [U \cap V, U \cap V]$, so the monomials of length $m-1$ all involve terms in $B([U \cap V, U \cap V])$.

It follows that $U_A V_A$ consists of those elements whose canonical representation involves only monomials from S. Intersecting with L, spanned by the monomials of length 1, we get:

Theorem 4. *Let U, V be ideals of the Lie algebra L. Let A be the universal* *associative enveloping algebra of* L *and let* $U_A = AUA$, $V_A = AVA$ *be the ideals generated by* U *and* V *respectively in* A. *Then* $U_A V_A \cap L = [U \cap V, U \cap V]$. □

Note that the improper ideal L of L induces the augmentation ideal $J = L_A \subseteq A$.

Corollary. *If* U *is an ideal of* L, *and* J *the augmentation ideal of* A, *then,* $[U, U] = U_A J \cap L = U_A^2 \cap L = JU_A \cap L$. □

If L is the free Lie algebra on the set X, then $A = K\langle X\rangle = F$ is free associative. So using Theorem 1 of [7] we obtain:

Theorem 5. *Let* L *be a free Lie algebra on the set* X, L', L'' *Lie algebras,* $l \to l'$, $l \to l''$ *maps of* L *into* L' *and* L'' *with kernels* U, V *respectively,* R', R'' *the universal enveloping algebras of* L', L'' *and* T *a free* (R', R'')*-module generated by* $\{\delta(x); x \in X\}$. *Then the kernel of the map*

$$L \to \begin{pmatrix} L'' & 0 \\ T & L' \end{pmatrix} \text{ defined by } x \to \begin{pmatrix} x'' & 0 \\ \delta(x) & x' \end{pmatrix}$$

is $[U \cap V, U \cap V]$.

Proof. By Theorem 1 of [7],

$$x \to \begin{pmatrix} x'' & 0 \\ \delta(x) & x' \end{pmatrix} \text{ induces a map } K\langle X\rangle \to \begin{pmatrix} R'' & 0 \\ T & R' \end{pmatrix}$$

whose kernel is $U_F V_F$. The result follows immediately. □

In particular with $L'' = 0$, we find from the corollary that the map

$$x \to \begin{pmatrix} 0 & 0 \\ \delta(x) & x' \end{pmatrix} \text{ defines an embedding } L/[U, U] \to \begin{pmatrix} 0 & 0 \\ T & L' \end{pmatrix}.$$

If U is an ideal of L, and R' is the universal enveloping algebra of L/U, then the adjoint representation of L induces an R'-module structure on $U/[U, U]$. If J is the augmentation ideal of the free algebra $F = K\langle X\rangle$, then the injection $U \to J$ induces an R'-module homomorphism $U/[U, U] \to J/U_F J$. The corollary shows that this map is injective. (This was shown by Labute [6] if U is generated by a single element.)

If L is finitely generated and R' is left Noetherian, then $J/U_F J$ is left Noetherian and hence so is $U/[U, U]$. This can be used to give an alternate proof of a recent theorem of Amayo and Stewart [1]:

Let L *be a finitely generated Lie algebra having an Abelian ideal* U *such that the universal enveloping algebra of* L/U *is left Noetherian. Then* L *satisfies the maximal condition for ideals.*

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DEPARTMENT OF MATHEMATICS, SYRACUSE UNIVERSITY, SYRACUSE, NEW YORK 13210