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A measurable angular distribution for $\bar{B} \to D^{*}\tau^{-}\bar{\nu}_{\tau}$ decays

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ABSTRACT: At present, the measurements of $R_{D^{(*)}}$ and $R_{J/\psi}$ hint at new physics (NP) in $b \to c\tau^{-}\bar{\nu}$ decays. The angular distribution of $\bar{B} \to D^{*}(\to D\pi)\,\tau^{-}\bar{\nu}_{\tau}$ would be useful for getting information about the NP, but it cannot be measured. The reason is that the three-momentum $\vec{p}_{\tau}$ cannot be determined precisely since the decay products of the τ^{-} include an undetected ν_{τ}. In this paper, we construct a measurable angular distribution by considering the additional decay $\tau^{-} \to \pi^{-}\nu_{\tau}$. The full process is $\bar{B} \to D^{*}(\to D\pi')\,\tau^{-}(\to \pi^{-}\nu_{\tau})\bar{\nu}_{\tau}$, which includes three final-state particles whose three-momenta can be measured: D, π', π^{-}. The magnitudes and relative phases of all the NP parameters can be extracted from a fit to this angular distribution. One can measure CP-violating angular asymmetries. If one integrates over some of the five kinematic parameters parametrizing the angular distribution, one obtains (i) familiar observables such as the q^{2} distribution and the D^{*} polarization, and (ii) new observables associated with the π^{-} emitted in the τ decay: the forward-backward asymmetry of the π^{-} and the CP-violating triple-product asymmetry.

KEYWORDS: Beyond Standard Model, CP violation

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1 Introduction

At the present time, there are discrepancies with the predictions of the standard model (SM) in the measurements of some observables in a number of B decays. These include $R_{D^{(*)}} \equiv \mathcal{B}(\bar{B} \to D^{(*)}\tau^-\bar{\nu}_\tau)/\mathcal{B}(\bar{B} \to D^{(*)}\ell^-\bar{\nu}_\ell)$ $(\ell = e, \mu)$ [1–10] and $R_{J/\psi} \equiv \mathcal{B}(B_c^+ \to J/\psi\tau^+\nu_\tau)/\mathcal{B}(B_c^+ \to J/\psi\mu^+\nu_\mu)$ [11]. The experimental results are shown in table 1. The values of the SM predictions for R_D and R_{D^*}, as well as their experimental measurements, are the average values used by the Heavy Flavor Averaging Group (HFLAV) [12]. They find that the deviation from the SM in R_D and R_{D^*} (combined) is 3.1σ.[1] For $R_{J/\psi}$, the discrepancy with the SM is 1.7σ [14]. These measurements suggest the presence of new physics (NP) in $b \to c\tau^-\bar{\nu}$ decays.

A great many papers have examined the question of what type of NP is required to explain the above anomalies. These include both model-independent [14, 16–32] and model-dependent analyses [33–77]. Clearly there are many possibilities for the NP. In order

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[1] However, we note that this is not completely settled: for example, a more recent analysis finds $(R_{D^*}^{\tau/\ell})_{\rm SM} = 0.250 \pm 0.003$ [13]. With this value, not included in the HFLAV average, the deviation from the SM prediction is larger than 3.1σ.

Observable	SM Prediction	Measurement
$R_{D^*}^{\tau/\ell}$	0.258 ± 0.005 [12]	$0.295 \pm 0.011 \pm 0.008$ [12]
$R_{D}^{\tau/\ell}$	0.299 ± 0.003 [12]	$0.340 \pm 0.027 \pm 0.013$ [12]
$R_{J/\psi}^{\tau/\mu}$	0.283 ± 0.048 [14]	$0.71 \pm 0.17 \pm 0.18$ [11]
$R_{D^*}^{\mu/e}$	~ 1.0	$1.04 \pm 0.05 \pm 0.01$ [15]

Table 1. Measured values of observables that suggest NP in $b \to c\tau^-\bar{\nu}$.

to distinguish the various NP explanations, a variety of observables have been considered. These include the q^2 distribution, the D^* polarization, the τ polarization, etc. [18, 78–98].

The above observables are all CP-conserving. But one can also consider CP-violating observales in $\bar{B} \to D^*\tau^-\bar{\nu}_\tau$ [99–102]. All CP-violating effects require the interference of two amplitudes with different weak (CP-odd) phases. Since the SM has only one amplitude, the observation of CP violation in this decay would be a smoking-gun signal of NP.

In ref. [103], we began to explore the prospects for measuring CP-violating effects in $\bar{B} \to D^*\tau^-\bar{\nu}_\tau$. There, we noted that, since $\bar{B} \to D^*$ is the only hadronic transition in this decay, all amplitudes will have the same strong (CP-even) phase. As a result, the direct CP asymmetry is expected to be very small. The main CP-violating effects appear as CP-violating asymmetries in the angular distribution. These are kinematical observables, and require that the two interfering amplitudes have different Lorentz structures. This fact allows us to distinguish different NP explanations. We demonstrated this by constructing the angular distribution for the decay $\bar{B} \to D^*\mu^-\bar{\nu}_\mu$, and showing that one could extract the different NP contributions from an analysis of the CP-violating angular asymmetries.

The reason we did not apply this to $\bar{B} \to D^*\tau^-\bar{\nu}_\tau$ is that the construction of the angular distribution requires the knowledge of the three-momentum $\vec{p}_\tau$. But since the τ decays to final-state particles that include ν_τ, which is undetected, $\vec{p}_\tau$ cannot be determined with any precision. As a result, the full angular distribution in $\bar{B} \to D^*(\to D\pi)\,\tau^-\bar{\nu}_\tau$ cannot be measured.[2]

In this paper, we construct a *measurable* angular distribution in $\bar{B} \to D^*(\to D\pi)\,\tau^-\bar{\nu}_\tau$. This is obtained by considering the additional decay[3] $\tau^- \to \pi^-\nu_\tau$. Now there are three final-state particles whose three-momenta *can* be measured: the D and π (from D^* decay), and the π^- (from τ decay). The new angular distribution is given in terms of five kinematic parameters: q^2, θ^* (describing $D^* \to D\pi$), and three quantities describing the π^-, E_π, θ_π and χ_π. It includes CP-violating angular asymmetries, which can be measured and used to extract information about the NP.

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[2] In fact, methods do exist that use all available experimental information to reconstruct the angular distribution. For example, ref. [94] uses the topology of decay vertices to perform a kinematic reconstruction. Still, in all of these methods, the angular distribution is obtained with limited precision (due to the uncertainty in the measurement of $\vec{p}_\tau$) and/or ambiguities.

[3] We note in passing that the decay $\tau^- \to \pi^-\nu_\tau$ has been used in the context of a proposed method for measuring the τ polarization in $\bar{B} \to D\tau^-(\to \pi^-\nu_\tau)\bar{\nu}_\tau$ [104, 105].

But the angular distribution yields even more information. All the NP parameters can be extracted from a fit to the full distribution. Thus, even if the NP is CP-conserving, so that no CP-violating angular asymmetries appear, its presence can still be detected. It is also possible to integrate over one or more of the five parameters. When one does this, all the familiar observables that have been proposed to distinguish NP models, such as the q^2 distribution and the D^* polarization, are reproduced. But there are also new observables that depend on the kinematic angles associated with the π^- emitted in the τ decay, θ_π and χ_π. These include the forward-backward asymmetry of the π^-, and the CP-violating triple-product asymmetry.

It should be noted that, in order to use this method, the momentum of the decaying B must be known. Thus, the technique described here is more suited to the experiments at e^+e^- machines such as Belle II.

We begin in section 2 with the derivation of the angular distribution of $B \to D^*(\to D\pi')\tau(\to \pi\nu_\tau)\bar{\nu}_\tau$. Here, some information is given in the appendices. In section 3, we discuss the NP signals, both CP-conserving and CP-violating, in the angular distribution. Observables obtained by integrating this rate over one or more of the kinematical variables are described in section 4. We conclude in section 5.

2 Angular distribution

We begin by describing our method of calculating the angular distribution of $B \to D^*(\to D\pi')\tau(\to \pi\nu_\tau)\bar{\nu}_\tau$. (Note that this section is somewhat technical. The reader wishing to simply see the results may skip to the next section.)

2.1 Structure of the new angular distribution

Consider first the angular distribution of the decay $\bar{B} \to D^*(\to D\pi)\ell^-\bar{\nu}_\ell$. This is obtained as follows. Assuming only left-handed (LH) neutrinos, the decay is parametrized as $\bar{B} \to D^* N^{*-}(\to \ell^-\bar{\nu}_\ell)$, where $N = S-P$, $V-A$, T represent LH scalar, vector and tensor interactions, respectively. For $\ell = \mu, e$, there is no NP, so that $N = W$ and the coupling is $V-A$. But for $\ell = \tau$, all couplings are allowed. The full amplitude is then squared, and can be expressed as a function of the final-state momenta. These momenta are defined in terms of the three helicity angles of figure 1, θ_ℓ, θ^* and χ. In this way, one produces a set of angular functions whose coefficients are different combinations of the helicity amplitudes. This is the angular distribution [103].

We now consider the case where the final-state lepton is $\ell = \tau$. The τ is not directly detected in experiments; instead, it is detected through its decay products. We choose to study the simplest possible hadronic decay of the τ, $\tau \to \pi\nu_\tau$. While NP in the τ decay is a possibility, in this analysis we restrict ourselves to NP only in the B decay. As we will show, even using this simple two-body decay of the τ, one can extract a great deal of information about this NP.

Once we let the τ decay, the process $B \to D^*(\to D\pi')\tau(\to \pi\nu_\tau)\bar{\nu}_\tau$ has five particles in the final state. This decay can be broken down into four successive quasi-two-body decays of the B meson and three intermediate states. The five-body phase space for the decay of a

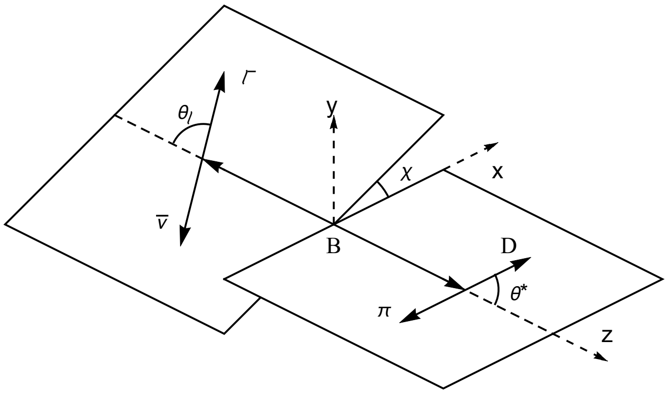

Figure 1. Definition of the angles in the $\bar{B} \to D^*(\to D\pi)\, \ell^- \bar{\nu}_\ell$ distribution.

massive spinless particle, such as the B meson, depends on 8 independent parameters: five helicity angles and the invariant squares of the masses of the three intermediate particles. Since two of these intermediates — the D^* and the τ — can go on shell, two of the three invariant mass parameters are given by m_{D^*} and m_τ. Thus, this decay depends on six independent parameters: five helicity angles and q^2, the invariant mass-squared of the $\tau\bar{\nu}_\tau$ pair. In the following, given that it could be NP that couples to $\tau\bar{\nu}_\tau$, we will refer to the center-of-momentum frame of the $\tau\bar{\nu}_\tau$ pair as the N^* rest frame.

Now, the helicity angles are typically defined in the rest frames of the corresponding intermediate states. Following this procedure, we define (i) θ^* as the polar angle of the D-meson three-momentum in the rest frame of its parent D^* meson, (ii) θ_τ and χ_τ as the polar and azimuthal angles, respectively, of the τ three-momentum in the N^* rest frame, and finally (iii) θ and χ as the polar and azimuthal angles, respectively, of the π three momentum in its parent τ rest frame.

However, this leads to a problem. Although one can in principle theoretically define all five helicity angles, most of them are of no practical use. To be specific, since the τ lepton is not directly observed in experiments, the angles either associated with its three momentum or defined in its rest frame are not measurable. Thus, four of the five helicity angles $(\theta_{(\tau)}, \chi_{(\tau)})$ are of no use to us. This problem can be remedied (at least partially) through a convenient change of variables.

2.2 New parameters

Since we do not have experimental access to the τ rest frame, in our analysis we choose to express the $\tau \to \pi\nu_\tau$ phase space in the N^* rest frame (this frame can be easily determined from information about the hadronic side of the B decay). Since the pion three-momentum can be precisely measured in this frame, we consider three new variables. E_π, θ_π and χ_π represent the pion energy, polar and azimuthal angles, respectively, defined in this frame. (The new helicity angles are shown in figure 2.) These three variables replace three of the unmeasurable helicity angles. The fourth unmeasurable angle is an azimuthal angle and is easily integrated over. We describe below the mathematical method for this transformation.

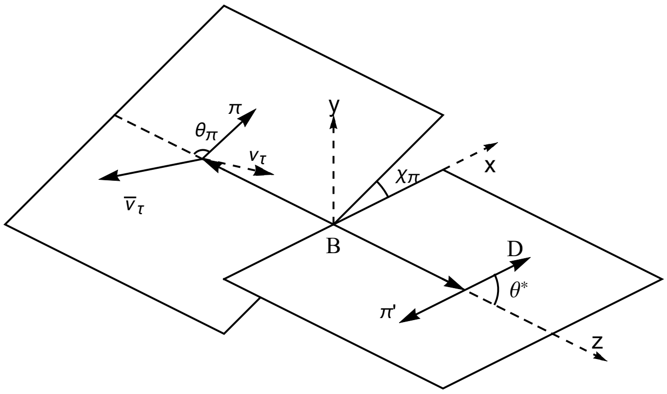

Figure 2. Definition of the angles in the $\bar{B} \to D^*(\to D\pi)\,\tau^-(\to \pi^-\nu_\tau)\bar{\nu}_\tau$ distribution.

Let us consider the product, d^4I, of the quasi-two-body phase spaces for $N^* \to \tau\bar{\nu}_\tau$ (ϕ_{N^*}) and $\tau \to \pi\nu_\tau$ (ϕ_τ). (The d^4 serves as a reminder that this phase-space factor ultimately depends on four independent kinematic variables.) Each phase-space factor is evaluated in the corresponding parent rest frame, and is expressed in terms of the four unmeasurable helicity angles. However, since each individual phase-space factor is Lorentz invariant, we can write this entire product in the measurable N^* rest frame:

$$\begin{aligned} d^4I &= \int d\phi_{N^*}(p_\tau, p_{\bar{\nu}_\tau}) \int d\phi_\tau(p_\pi, p_{\nu_\tau})\,, \\ &= \frac{1}{(4\pi)^4} \int \frac{d^3p_\tau d^3p_{\bar{\nu}_\tau}}{E_\tau E_{\bar{\nu}_\tau}}\, \delta^4(q - p_\tau - p_{\bar{\nu}_\tau}) \int \frac{d^3p_\pi d^3p_{\nu_\tau}}{E_\pi E_{\nu_\tau}} \delta^4(p_\tau - p_\pi - p_{\nu_\tau})\,, \\ &= \frac{1}{(4\pi)^4} \int \frac{d^3p_\tau d^3p_{\bar{\nu}_\tau}}{E_\tau E_{\bar{\nu}_\tau}}\, \delta\left(\sqrt{q^2} - E_\tau - E_{\bar{\nu}_\tau}\right) \delta^3\left(\vec{p}_\tau + \vec{p}_{\bar{\nu}_\tau}\right) \\ &\qquad\qquad \int \frac{d^3p_\pi d^3p_{\nu_\tau}}{E_\pi E_{\nu_\tau}}\, \delta\left(E_\tau - E_\pi - E_{\nu_\tau}\right) \delta^3\left(\vec{p}_\tau - \vec{p}_\pi - \vec{p}_{\nu_\tau}\right)\,, \end{aligned} \tag{2.1}$$

where, in the final line, E_x and $\vec{p}_x$ respectively represent the energy and three-momentum of the particle x in the N^* rest frame. Performing the integrals over the ν_τ and $\bar{\nu}_\tau$ three-momenta, and neglecting neutrino masses, we find

$$d^4I = \frac{1}{(4\pi)^4} \int \frac{d^3p_\tau}{E_\tau |\vec{p}_\tau|}\, \delta\left(\sqrt{q^2} - E_\tau - |\vec{p}_\tau|\right) \int \frac{d^3p_\pi}{E_\pi |\vec{p}_\tau - \vec{p}_\pi|}\, \delta(E_\tau - E_\pi - |\vec{p}_\tau - \vec{p}_\pi|)\,. \tag{2.2}$$

Without loss of generality, we now choose to write the τ and π three-momentum integral measures such that the associated polar angle can be determined, at least theoretically. In the case of d^3p_π, clearly the polar and azimuthal angles of the pion three-momentum relative to the N^* direction, θ_π and χ_π respectively, are measurable. Here, θ_π is defined using three-momenta evaluated in the N^* rest frame,

$$\cos\theta_\pi = -\frac{\vec{p}_{D^*} \cdot \vec{p}_\pi}{|\vec{p}_{D^*}||\vec{p}_\pi|}\,, \tag{2.3}$$

while χ_π is defined using three-momenta evaluated in the B rest frame,

$$\sin\chi_\pi = \frac{[(\vec{p}_{\pi'}\times\vec{p}_D)\times(\vec{p}_{D^*}\times\vec{p}_\pi)]\cdot\vec{p}_{D^*}}{|\vec{p}_{\pi'}\times\vec{p}_D||\vec{p}_{D^*}\times\vec{p}_\pi||\vec{p}_{D^*}|}\,. \tag{2.4}$$

Since $\vec{p}_{D^*} = \vec{p}_D + \vec{p}_{\pi'}$, one can easily verify that $\sin\chi_\pi$ is proportional to the scalar triple product $(\vec{p}_{\pi'}\times\vec{p}_D)\cdot\vec{p}_\pi$.

In the case of d^3p_τ, the polar angle of the τ direction relative to the pion direction, $\theta_{\tau\pi}$, can be theoretically determined. The fourth angle — the corresponding azimuthal angle $\chi_{\tau\pi}$ — cannot be determined. However, at a later stage we will eliminate this angle by integrating over it. After appropriately transforming the delta functions, and writing the phase space in terms of the above new variables (θ_π, χ_π, $\theta_{\tau\pi}$ and $\chi_{\tau\pi}$), we find

$$\begin{aligned} d^4I = \frac{1}{(4\pi)^4}\int \frac{d|\vec{p}_\tau|}{\sqrt{q^2}}\, d\cos\theta_{\tau\pi}\, d\chi_{\tau\pi}\, dE_\pi\, d\cos\theta_\pi\, d\chi_\pi \\ \delta\left(|\vec{p}_\tau| - \frac{q^2-m_\tau^2}{2\sqrt{q^2}}\right)\,\delta\left(\cos\theta_{\tau\pi} - \frac{2E_\tau E_\pi - m_\tau^2 - m_\pi^2}{2|\vec{p}_\tau||\vec{p}_\pi|}\right)\,. \end{aligned} \tag{2.5}$$

Expressed in the above form, it is clear that the remaining two delta functions can be used to remove the two variables $|\vec{p}_\tau|$ and $\cos\theta_{\tau\pi}$. We are thus left with a phase-space factor that depends only on four variables ($\chi_{\tau\pi}$, E_π, θ_π and χ_π), as expressed below:

$$d^4I = \frac{1}{(4\pi)^4}\frac{1}{\sqrt{q^2}}\, d\chi_{\tau\pi}\, dE_\pi\, d\cos\theta_\pi\, d\chi_\pi\,, \tag{2.6}$$

where the following replacements in the squared invariant amplitude of the decay ($|\mathcal{M}|^2$) are understood:

$$E_\tau \to \frac{q^2+m_\tau^2}{2\sqrt{q^2}}\,, \quad |\vec{p}_\tau| \to \frac{q^2-m_\tau^2}{2\sqrt{q^2}}\,, \quad \cos\theta_{\tau\pi} \to \frac{2E_\tau E_\pi - m_\tau^2 - m_\pi^2}{2|\vec{p}_\tau||\vec{p}_\pi|}\,. \tag{2.7}$$

Using the above choice of kinematic parameters we may now express the differential decay rate for the full process as follows:

$$\frac{d^5\Gamma}{dq^2\, d\cos\theta^*\, dE_\pi\, d\cos\theta_\pi\, d\chi_\pi} = \frac{|\vec{p}_{D^*}|\,|\vec{p}_D|}{2^{15}\,\pi^7\, m_B^2\, m_{D^*}\,\sqrt{q^2}}\int d\chi_{\tau\pi}\,\frac{dp_{D^*}^2}{2\pi}\,\frac{dp_\tau^2}{2\pi}|\mathcal{M}|^2\,. \tag{2.8}$$

Here $|\vec{p}_{D^*}| = \sqrt{\lambda(m_B^2; q^2, m_{D^*}^2)}/(2m_B)$ and $|\vec{p}_D| = \sqrt{\lambda(m_{D^*}^2; m_D^2, m_\pi^2)}/(2m_{D^*})$, where

$$\lambda(a;b,c) = a^2 + b^2 + c^2 - 2ab - 2bc - 2ca\,. \tag{2.9}$$

The right-hand side of eq. (2.8) contains integrals over three independent variables (out of the eight variables discussed in the previous subsection). We will see in the following subsection that these integrals can be performed quite simply once we express $|\mathcal{M}|^2$ as an explicit function of these variables.

2.3 Calculating $|\mathcal{M}|^2$

The next step is to calculate $|\mathcal{M}|^2$, appropriately summed over spins and polarizations. In ref. [103], we derived the angular distribution for $B \to D^* \mu \bar{\nu}_\mu$. In the presence of NP, the relevant two-body processes are $\bar{B} \to D^* N^{*-} (\to \mu^- \bar{\nu}_\mu)$, where $N = S-P, V-A, T$ represent left-handed scalar, vector and tensor interactions, respectively. These are labeled SP, VA and T. (The VA contribution includes that of the SM.) For each of the leptonic SP, VA and T Lorentz structures, the hadronic piece (the $b \to c$ transition) also has a NP contribution. The effective Hamiltonian is

$$\begin{aligned}\mathcal{H}_{\text{eff}} = \frac{G_F V_{cb}}{\sqrt{2}} \Big\{ & \left[(1+g_L)\, \bar{c}\gamma_\alpha (1-\gamma_5) b + g_R\, \bar{c}\gamma_\alpha (1+\gamma_5) b \right] \bar{\mu}\gamma^\alpha (1-\gamma_5) \nu_\mu \\ & + \left[g_S\, \bar{c} b + g_P\, \bar{c}\gamma_5 b \right] \bar{\mu}(1-\gamma_5)\nu_\mu + g_T\, \bar{c}\sigma^{\alpha\beta}(1-\gamma_5) b \bar{\mu}\sigma_{\alpha\beta}(1-\gamma_5)\nu_\mu \Big\} + h.c. \end{aligned} \tag{2.10}$$

The decay amplitude is then written as the product of a hadronic piece $\mathcal{H}_{D^*}$, a leptonic piece $\mathcal{L}^{N^*}$, and a helicity amplitude piece $\mathcal{M}^{N^*}$, appropriately summed over helicities labeled by m, n, and p.

This all applies to the decay $B \to D^* \tau \bar{\nu}_\tau$, except that now one must also include the decay $\tau \to \pi \nu_\tau$. In addition to numerical factors and factors of $f_\pi |V_{ud}|$ coming from the $\tau \to \pi \nu_\tau$ transition, the leptonic piece changes. Representing the new leptonic pieces by $\tilde{\mathcal{L}}^{N^*}$, the spin-summed squared invariant amplitude for the full 5-body decay can now be expressed as

$$\begin{aligned}|\mathcal{M}|^2 = & \frac{96\pi\, G_F^2\, |V_{cb}|^2\, m_{D^*}}{|\vec{p}_D|^3\, (m_\tau^2 - m_\pi^2)^2} \, \frac{m_{D^*}\, \Gamma_{D^*}\, \mathcal{B}(D^* \to D\pi')}{(p_{D^*}^2 - m_{D^*}^2)^2 + m_{D^*}^2 \Gamma_{D^*}^2} \, \frac{m_\tau\, \Gamma_\tau\, \mathcal{B}(\tau \to \pi\nu_\tau)}{(p_\tau^2 - m_\tau^2)^2 + m_\tau^2 \Gamma_\tau^2} \\ & \left| \sum_{m=\pm,0} \mathcal{H}_{D^*}(m) \left(\mathcal{M}^{SP}_{(m)}\, \tilde{\mathcal{L}}^{SP} + \sum_{n=t,\pm,0} g_{nn} \mathcal{M}^{VA}_{(m;n)} \tilde{\mathcal{L}}^{VA}(n) \right.\right. \\ & \left.\left. \qquad + \sum_{n,p=t,\pm,0} g_{nn} g_{pp} \mathcal{M}^{T}_{(m;n,p)} \tilde{\mathcal{L}}^{T}(n,p) \right) \right|^2 . \end{aligned} \tag{2.11}$$

In the above, the new leptonic pieces are of the form

$$\begin{aligned} \tilde{\mathcal{L}}^{SP} &= m_\tau \bar{u}(\nu_\tau) \not{p}_\pi (1-\gamma_5) v(\bar{\nu}_\tau) , \\ \tilde{\mathcal{L}}^{VA}(n) &= \epsilon^\beta_{VA}(n) \left[\bar{u}(\nu_\tau) \not{p}_\pi \not{p}_\tau \gamma_\beta (1-\gamma_5) v(\bar{\nu}_\tau) \right] , \\ \tilde{\mathcal{L}}^{T}(n,p) &= -i\, m_\tau\, \epsilon^\beta_T(n) \epsilon^\delta_T(p) \left[\bar{u}(\nu_\tau) \not{p}_\pi \sigma_{\beta\delta} (1-\gamma_5) v(\bar{\nu}_\tau) \right] , \end{aligned} \tag{2.12}$$

and we have used the SM expressions for the branching fractions $\mathcal{B}(D^* \to D\pi')$, and $\mathcal{B}(\tau \to \pi\nu_\tau)$:

$$\mathcal{B}(\tau \to \pi\nu_\tau) = \frac{G_F^2\, |V_{ud}|^2\, f_\pi^2}{16\pi\, m_\tau\, \Gamma_\tau} (m_\tau^2 - m_\pi^2)^2 , \quad \mathcal{B}(D^* \to D\pi') = \frac{|\vec{p}_D|^3}{6\pi\, m_{D^*}^2\, \Gamma_{D^*}} . \tag{2.13}$$

The hadronic pieces, $\mathcal{H}_{D^*}$, and the helicity amplitude pieces $\mathcal{M}^{N^*}$ are the same as those obtained in our earlier work, ref. [103]. For completeness, we have provided this information in appendix A.

We now see that the dependence of $|\mathcal{M}|^2$ on the variables $p^2_{D^*}$ and p^2_τ appears only through the propagators of the corresponding intermediate particles. Since both of these particles — the D^* and the τ — go on shell, we can apply the narrow-width approximation to replace these propagators with delta functions, making the corresponding integrals simple. Under the narrow-width approximation, one can show that

$$\int \frac{dp^2}{2\pi} \frac{m_X \Gamma_X \mathcal{B}}{(p^2 - m_X^2)^2 + m_X^2 \Gamma_X^2} \to \frac{\mathcal{B}}{2} \,. \tag{2.14}$$

Furthermore, the dependence of $|\mathcal{M}|^2$ on the unmeasurable azimuthal angle $\chi_{\tau\pi}$ is a result of fermionic traces over products of the leptonic pieces ($\tilde{\mathcal{L}}^{N^*}$). This dependence turns out to be combinations of simple trigonometric functions, such as $\sin\chi_{\tau\pi}$ and $\cos\chi_{\tau\pi}$. It is therefore straightforward to integrate over $\chi_{\tau\pi}$.

After integrating over the three variables $p^2_{D^*}, p^2_\tau$ and $\chi_{\tau\pi}$, the full five-body differential decay rate is given by

$$\begin{aligned} \frac{d^5\Gamma}{dq^2\, dE_\pi\, d\cos\theta^*\, d\cos\theta_\pi\, d\chi_\pi} &= \frac{3|V_{cb}|^2 G_F^2\, |\vec{p}_{D^*}|\, (q^2)^{3/2}\, m_\tau^2}{2^{11}\pi^4 m_B^2 (m_\tau^2 - m_\pi^2)^2} \mathcal{B}(D^* \to D\pi')\mathcal{B}(\tau \to \pi\nu_\tau) \\ &\quad \times \sum_{i,j} \left(\mathcal{N}_i^S\, |\mathcal{A}_i|^2 + \mathcal{N}_{i,j}^R\, \mathrm{Re}[\mathcal{A}_i\mathcal{A}_j^*] + \mathcal{N}_{i,j}^I\, \mathrm{Im}[\mathcal{A}_i\mathcal{A}_j^*]\right) , \end{aligned} \tag{2.15}$$

where $i,j = t, 0, \perp, \|, SP, (0,T), (\perp,T), (\|,T)$. Here the $\mathcal{A}_i$ represent the helicity amplitudes that contain the crucial physics information that can be extracted from this analysis, while the $\mathcal{N}_{i(,j)}^{(S,R,I)}$ are functions of the five independent kinematic variables of interest to us ($q^2, \theta^*, E_\pi, \theta_\pi$, and χ_π). We present the information relevant for the $\mathcal{N}_i^S\, |\mathcal{A}_i|^2$ pieces of eq. (2.15) in table 5 of appendix B. The first column contains the various $|\mathcal{A}_i|^2$ helicities, while the second column contains the associated $\mathcal{N}_i^S$ terms. In these terms, we have separated out the parts that depend on q^2 and E_π, and put them into the S_i factors. The expressions for the S_i are also given in appendix B. The information relevant for the $\mathcal{N}_{i,j}^R\, \mathrm{Re}[\mathcal{A}_i\mathcal{A}_j^*]$ and $\mathcal{N}_{i,j}^I\, \mathrm{Im}[\mathcal{A}_i\mathcal{A}_j^*]$ pieces is given in tables 6 and 7 of appendix B, respectively. The expressions for the R_i and I_i are also given in appendix B.

It is standard to express the differential decay rate as an angular distribution, written as a sum over a product of angular functions and functions of non-angular variables including the helicity amplitudes. In order to write eq. (2.15) as an angular distribution, it is necessary to separate the $\mathcal{N}_{i(,j)}^{(S,R,I)}$ into angular functions and functions of q^2 and E_π. Following this separation, eq. (2.15) can be rewritten as a sum over a product of 12 angular functions and their respective coefficients:

$$\begin{aligned} \frac{d^5\Gamma}{dq^2\, dE_\pi\, d\cos\theta^*\, d\cos\theta_\pi\, d\chi_\pi} &= \frac{3|V_{cb}|^2 G_F^2\, |\vec{p}_{D^*}|\, (q^2)^{3/2}\, m_\tau^2}{2^{11}\pi^4 m_B^2 (m_\tau^2 - m_\pi^2)^2} \mathcal{B}(D^* \to D\pi')\mathcal{B}(\tau \to \pi\nu_\tau) \\ &\quad \times \left[\sum_{i=1}^{9} f_i^R(q^2, E_\pi)\Omega_i^R(\theta^*, \theta_\pi, \chi_\pi) + \sum_{i=1}^{3} f_i^I(q^2, E_\pi)\Omega_i^I(\theta^*, \theta_\pi, \chi_\pi)\right] . \end{aligned} \tag{2.16}$$

The first nine angular functions above, denoted by $\Omega^R_{1,\ldots 9}$, arise from a rearrangement of the $\mathcal{N}_i^S\, |\mathcal{A}_i|^2 + \mathcal{N}_{i,j}^R\, \mathrm{Re}[\mathcal{A}_i\mathcal{A}_j^*]$ terms. These are presented in table 2. The $\mathcal{N}_{i,j}^I\, \mathrm{Im}[\mathcal{A}_i\mathcal{A}_j^*]$ terms

Coefficient $f_i^R(q^2, E_\pi)$	Angular Function $\Omega_i^R(\theta^*, \theta_\pi, \chi_\pi)$
$S_t \lvert\mathcal{A}_t\rvert^2 + S_{0,1} \lvert\mathcal{A}_0\rvert^2 + S_{SP} \lvert\mathcal{A}_{SP}\rvert^2 + S_{0T,1} \lvert\mathcal{A}_{0,T}\rvert^2$ $+R_{0T01}\,\mathrm{Re}[\mathcal{A}_{0,T}\mathcal{A}_0^*] + R_{SPt}\,\mathrm{Re}[\mathcal{A}_{SP}\mathcal{A}_t^*]$	$\cos^2\theta^*$
$S_{\perp,1} \lvert\mathcal{A}_\perp\rvert^2 + S_{\parallel,1} \lvert\mathcal{A}_\parallel\rvert^2 + S_{\perp T,1} \lvert\mathcal{A}_{\perp,T}\rvert^2 + S_{\parallel T,1} \lvert\mathcal{A}_{\parallel,T}\rvert^2$ $+R_{\parallel T\parallel,1}\,\mathrm{Re}[\mathcal{A}_{\parallel,T}\mathcal{A}_\parallel^*] + R_{\perp T\perp,1}\,\mathrm{Re}[\mathcal{A}_{\perp,T}\mathcal{A}_\perp^*]$	$\sin^2\theta^*$
$R_{SP0}\,\mathrm{Re}[\mathcal{A}_{SP}\mathcal{A}_0^*] + R_{t0}\,\mathrm{Re}[\mathcal{A}_t\mathcal{A}_0^*]$ $+R_{SP0T}\,\mathrm{Re}[\mathcal{A}_{SP}\mathcal{A}_{0,T}^*] + R_{0Tt}\,\mathrm{Re}[\mathcal{A}_{0,T}\mathcal{A}_t^*]$	$\cos^2\theta^*\ \cos\theta_\pi$
$S_{0,2} \lvert\mathcal{A}_0\rvert^2 + S_{0T,2} \lvert\mathcal{A}_{0,T}\rvert^2 + R_{0T0,2}\,\mathrm{Re}[\mathcal{A}_{0,T}\mathcal{A}_0^*]$	$\cos^2\theta^*\ \cos 2\theta_\pi$
$R_{\perp T\parallel}\,\mathrm{Re}[\mathcal{A}_{\perp,T}\mathcal{A}_\parallel^*] + R_{\perp T\parallel T}\,\mathrm{Re}[\mathcal{A}_{\perp,T}\mathcal{A}_{\parallel,T}^*]$ $+R_{\parallel\perp}\,\mathrm{Re}[\mathcal{A}_\parallel\mathcal{A}_\perp^*] + R_{\parallel T\perp}\,\mathrm{Re}[\mathcal{A}_{\parallel,T}\mathcal{A}_\perp^*]$	$\sin^2\theta^*\ \cos\theta_\pi$
$S_{\parallel,2} \lvert\mathcal{A}_\parallel\rvert^2 + S_{\perp,2} \lvert\mathcal{A}_\perp\rvert^2 + S_{\parallel T,2} \lvert\mathcal{A}_{\parallel,T}\rvert^2 + S_{\perp T,2} \lvert\mathcal{A}_{\perp,T}\rvert^2$ $+R_{\parallel T\parallel,2}\,\mathrm{Re}[\mathcal{A}_{\parallel,T}\mathcal{A}_\parallel^*] + R_{\perp T\perp,2}\,\mathrm{Re}[\mathcal{A}_{\perp,T}\mathcal{A}_\perp^*]$	$\sin^2\theta^*\ \cos 2\theta_\pi$
$2(S_{\perp,2} \lvert\mathcal{A}_\perp\rvert^2 - S_{\parallel,2} \lvert\mathcal{A}_\parallel\rvert^2) + 2(S_{\perp T,2} \lvert\mathcal{A}_{\perp,T}\rvert^2 - S_{\parallel T,2} \lvert\mathcal{A}_{\parallel,T}\rvert^2)$ $+2(R_{\perp T\perp,2}\,\mathrm{Re}[\mathcal{A}_{\perp,T}\mathcal{A}_\perp^*] - R_{\parallel T\parallel,2}\,\mathrm{Re}[\mathcal{A}_{\parallel,T}\mathcal{A}_\parallel^*])$	$\sin^2\theta^*\ \sin^2\theta_\pi \cos 2\chi_\pi$
$R_{\perp T0}\,\mathrm{Re}[\mathcal{A}_{\perp,T}\mathcal{A}_0^*] + R_{SP\parallel}\,\mathrm{Re}[\mathcal{A}_{SP}\mathcal{A}_\parallel^*] + R_{t\parallel}\,\mathrm{Re}[\mathcal{A}_t\mathcal{A}_\parallel^*]$ $+R_{SP\parallel T}\,\mathrm{Re}[\mathcal{A}_{SP}\mathcal{A}_{\parallel,T}^*] + R_{0\perp}\,\mathrm{Re}[\mathcal{A}_0\mathcal{A}_\perp^*] + R_{0T\perp}\,\mathrm{Re}[\mathcal{A}_{0,T}\mathcal{A}_\perp^*]$ $+R_{0T\perp T}\,\mathrm{Re}[\mathcal{A}_{0,T}\mathcal{A}_{\perp,T}^*] + R_{\parallel Tt}\,\mathrm{Re}[\mathcal{A}_{\parallel,T}\mathcal{A}_t^*]$	$\sin 2\theta^*\ \sin\theta_\pi\ \cos\chi_\pi$
$R_{\parallel T0}\,\mathrm{Re}[\mathcal{A}_{\parallel,T}\mathcal{A}_0^*] + R_{0\parallel}\,\mathrm{Re}[\mathcal{A}_0\mathcal{A}_\parallel^*]$ $+R_{0T\parallel}\,\mathrm{Re}[\mathcal{A}_{0,T}\mathcal{A}_\parallel^*] + R_{0T\parallel T}\,\mathrm{Re}[\mathcal{A}_{0,T}\mathcal{A}_{\parallel,T}^*]$	$\sin 2\theta^*\ \sin 2\theta_\pi\ \cos\chi_\pi$

Table 2. Contributions of $\mathcal{N}_i^S\,\lvert\mathcal{A}_i\rvert^2$ and $\mathcal{N}_{i,j}^R\,\mathrm{Re}[\mathcal{A}_i\mathcal{A}_j^*]$ [eq. (2.15)] to the angular distribution. These terms are CP-conserving.

involve an additional three angular functions, denoted by $\Omega^I_{1,2,3}$; these contributions are given in table 3. Together, these constitute the $\bar{B} \to D^*(\to D\pi')\,\tau^-(\to \pi^-\nu_\tau)\bar{\nu}_\tau$ angular distribution.

3 Angular distribution: new-physics signals

Tables 2 and 3 describe the angular distribution of the decay $\bar{B} \to D^*(\to D\pi')\tau^-(\to \pi^-\nu_\tau)\bar{\nu}_\tau$. The question now is: how can we use it to obtain information about NP? This is discussed in the present section.

In decays such as $B \to K^*(\to K\pi)\mu^+\mu^-$, where the only non-angular parameter is q^2, the data is separated into q^2 bins before an angular analysis is performed. In the present case, the situation is similar, except that there are two non-angular variables, q^2 and E_π.

Coefficient $f_i^I(q^2, E_\pi)$	Angular Function $\Omega_i^I(\theta^*, \theta_\pi, \chi_\pi)$
$I_{t\perp}\,\mathrm{Im}[\mathcal{A}_t\mathcal{A}_\perp^*] + I_{\parallel T0}\,\mathrm{Im}[\mathcal{A}_{\parallel,T}\mathcal{A}_0^*] + I_{SP\perp}\,\mathrm{Im}[\mathcal{A}_{SP}\mathcal{A}_\perp^*]$ $+I_{SP\perp T}\,\mathrm{Im}[\mathcal{A}_{SP}\mathcal{A}_{\perp,T}^*] + I_{0T\parallel}\,\mathrm{Im}[\mathcal{A}_{0,T}\mathcal{A}_\parallel^*] + I_{\perp Tt}\,\mathrm{Im}[\mathcal{A}_{\perp,T}\mathcal{A}_t^*]$	$\sin 2\theta^* \sin\theta_\pi \sin\chi_\pi$
$I_{0\perp}\,\mathrm{Im}[\mathcal{A}_0\mathcal{A}_\perp^*] + I_{0T\perp}\,\mathrm{Im}[\mathcal{A}_{0,T}\mathcal{A}_\perp^*] + I_{\perp T0}\,\mathrm{Im}[\mathcal{A}_{\perp,T}\mathcal{A}_0^*]$	$\sin 2\theta^* \sin 2\theta_\pi \sin\chi_\pi$
$I_{\parallel\perp}\,\mathrm{Im}[\mathcal{A}_\parallel\mathcal{A}_\perp^*] + I_{\perp T\parallel}\,\mathrm{Im}[\mathcal{A}_{\perp,T}\mathcal{A}_\parallel^*] + I_{\parallel T\perp}\,\mathrm{Im}[\mathcal{A}_{\parallel,T}\mathcal{A}_\perp^*]$	$\sin^2\theta^* \sin^2\theta_\pi \sin 2\chi_\pi$

Table 3. Contributions of $\mathcal{N}_{i,j}^I\,\mathrm{Im}[\mathcal{A}_i\mathcal{A}_j^*]$ [eq. (2.15)] to the angular distribution. These terms are CP-violating.

Helicity Amplitude	Coupling
$\mathcal{A}_0$, $\mathcal{A}_\parallel$, $\mathcal{A}_t$	$1 + g_L - g_R$
$\mathcal{A}_\perp$	$1 + g_L + g_R$
$\mathcal{A}_{SP}$	g_P
$\mathcal{A}_{0,T}$, $\mathcal{A}_{\parallel,T}$, $\mathcal{A}_{\perp,T}$	g_T

Table 4. Contributions of the NP couplings to the various helicity amplitudes.

Therefore, the data has to be separated into both q^2 and E_π bins. An angular fit to the data can then be performed, permitting the extraction of the coefficients in tables 2 and 3.

These coefficients involve the eight helicity amplitudes $\mathcal{A}_0$, $\mathcal{A}_\parallel$, $\mathcal{A}_\perp$, $\mathcal{A}_t$, $\mathcal{A}_{SP}$, $\mathcal{A}_{0,T}$, $\mathcal{A}_{\parallel,T}$ and $\mathcal{A}_{\perp,T}$. In eq. (2.10), there are five NP parameters: g_L, g_R, g_S, g_P and g_T. Of these, g_S does not contribute to this decay. The eight helicity amplitudes are generated, at least in part, by the remaining four NP parameters. The dependence of the helicity amplitudes on the NP parameters is shown in table 4. Note that the Lorentz structure associated with g_L is $(V-A)\times(V-A)$, as in the SM. For this reason, it is the quantity $1+g_L$ that appears in the table, where the 1 is due to the SM. Thus, $\mathcal{A}_0$, $\mathcal{A}_\parallel$, $\mathcal{A}_\perp$ and $\mathcal{A}_t$ are present in the SM — they are associated with W exchange — while $\mathcal{A}_{SP}$, $\mathcal{A}_{0,T}$, $\mathcal{A}_{\parallel,T}$ and $\mathcal{A}_{\perp,T}$ are purely NP helicity amplitudes.

The couplings g_L, g_R, g_P and g_T are complex quantities, i.e. each coupling has an independent magnitude and a weak (CP-odd) phase. The amplitudes $\mathcal{A}_i$ are constructed by taking a product of a coupling with a corresponding QCD matrix element, and a numerical factor that appears in the effective Hamiltonian of eq. (2.10). In principle, each QCD matrix element has a strong (CP-even) phase. This means that, in principle, amplitudes may have both weak (CP-odd) and strong (CP-even) phases. However, as argued in refs. [103, 106, 107] (and summarized in the introduction), we expect all amplitudes to have the same strong phase as that of the SM.

The coefficients in tables 2 and 3 involve products of the helicity amplitudes: $|\mathcal{A}_i|^2$, $\mathrm{Re}[\mathcal{A}_i\mathcal{A}_j^*]$, $\mathrm{Im}[\mathcal{A}_i\mathcal{A}_j^*]$. With the above assumption, these products can all be written in terms of seven NP parameters: the four magnitudes of $1+g_L$, g_R, g_P and g_T, and their three relative weak phases. Thus, the measurement of the angular distribution allows us to probe these NP parameters.

If all NP quantities have the same weak phase, then $\mathrm{Im}[\mathcal{A}_i \mathcal{A}_j^*] = 0$, so that all the entries of table 3 vanish. On the other hand, those of table 2 do not. For this reason, we refer to table 2 as CP-conserving, and table 3 as CP-violating.

Note that, if the strong-phase differences are nonzero, this is not completely accurate. With nonzero strong-phase differences, the entries of tables 5 and 6 can differ between $\bar{B} \to D^* \tau^- \bar{\nu}_\tau$ and its CP-conjugate process. That is, there can be direct CP violation. However, if an untagged data sample is used to measure the angular distribution, i.e., both process and CP-conjugate process are combined, then table 2 is indeed CP-conserving. As for table 3, its entries are CP-violating and can be nonzero even in the untagged data sample (details are given below).

In the following subsections, we examine how to obtain NP information from the measurement of tables 3 and 2. As we will see, table 3 provides smoking-gun signals of NP, while more work is require to identify NP in table 2.

3.1 CP-violating angular terms

Above, we argued that the strong-phase differences between the various amplitudes are expected to be very small. This implies that all direct CP-violating effects are also expected to be tiny. Even so, CP-violating effects can be present in the angular distribution. To be specific, the coefficients of certain angular terms are related to triple products (TPs) of the form $\vec{p}_1 \cdot (\vec{p}_2 \times \vec{p}_3)$, where the $\vec{p}_i$ are the final-state momenta. As we will see below, TP asymmetries do not require a strong-phase difference between the interfering amplitudes. Indeed, they are maximal when this strong-phase difference vanishes. In the decay $B \to D^*(\to D\pi')\tau(\to \pi\nu_\tau)\bar{\nu}_\tau$, the 3-momenta of the final-state particles D, π and π' can be measured. From these, a TP can be constructed; all the entries of table 3 involve this TP.

Now, all entries are proportional to $\mathrm{Im}[A_i A_j^*]$, where A_i and A_j are the two interfering helicity amplitudes. Writing

$$A_i = |A_i| e^{i\phi_i} e^{i\delta_i} \,, \quad A_j = |A_j| e^{i\phi_j} e^{i\delta_j} \,, \tag{3.1}$$

where $\phi_{i,j}$ ($\delta_{i,j}$) are the weak (strong) phases, we see that

$$\mathrm{Im}[A_i A_j^*] = |A_i||A_j| \sin(\phi_i - \phi_j + \delta_i - \delta_j) \,. \tag{3.2}$$

If, as we have assumed, the strong-phase difference is negligible, the TP is proportional to $\sin(\phi_i - \phi_j)$. This is a CP-violating quantity. On the other hand, if the strong-phase difference is not negligible, the TP can be nonzero even if the weak-phase difference vanishes. That is, this is not CP-violating (it is known as a “fake TP”). To obtain a true CP-violating term, this must be compared to the TP in the CP-conjugate process. In the CP-conjugate process, the weak phases change sign, but the strong phases do not. But there is an additional change. Each angular function in table 3 is proportional to $\sin\chi_\pi$, so that these functions are parity odd. This means that, in going from process to CP-conjugate process, there is an additional minus sign [108, 109], so that the CP-conjugate TP is proportional to

$$-\,\mathrm{Im}[\bar{A}_i \bar{A}_j^*] = |A_i||A_j| \sin(\phi_i - \phi_j - \delta_i + \delta_j) \,. \tag{3.3}$$

The true, CP-violating effect is then found by *adding* the TPs in process and CP-conjugate process [108], so that it remains even in an untagged data sample.[4]

The key point is that, in the SM, CP-violating effects are absent. Thus, the observation of a nonzero entry in table 3 would be a smoking-gun signal of NP.

3.2 CP-conserving angular terms

NP signals are not as easy to obtain from the measurement of table 2. In each of the nine entries, the coefficient contains at least one term involving the helicity amplitudes $\mathcal{A}_0$, $\mathcal{A}_\parallel$, $\mathcal{A}_\perp$ and $\mathcal{A}_t$, all of which are present in the SM. That is, even if there is no NP, all the angular functions of table 2 will be found in the angular distribution.

On the other hand, in the presence of NP, the coefficients will be modified from their SM predictions. Thus, the way to detect NP is to measure the coefficients in as many q^2-E_π bins as possible, and then perform a combined fit to all measurements and extract the best-fit values of the magnitudes and relative weak phases of $1+g_L$, g_R, g_P and g_T.

If a smoking-gun signal of NP has already been observed in the measurement of table 3, the values of the NP parameters responsible for it can be determined in this way. And even if no such signal has been seen, the presence of CP-conserving NP can be detected through the measurement of the angular distribution of table 2 in a sufficient number of different q^2-E_π bins.

4 Integrated observables

The full differential decay rate for $B \to D^*(\to D\pi')\tau(\to \pi\nu_\tau)\bar{\nu}_\tau$ depends on the five kinematic parameters q^2, E_π, θ^*, θ_π and χ_π. While a complete study of the decay distribution as a function of all five parameters can reveal NP effects, a full experimental analysis may be statistics limited. Effects of NP can still be studied through "integrated observables," obtained by integrating the differential decay rate over one or more of the kinematic parameters.

We separate the integrated observables into two types. The first type is found by integrating over all three of the lepton-side parameters (E_π, θ_π, χ_π). Such observables are functions of q^2, and are independent of the dynamics of the lepton decay. They can, therefore, be used to study lepton-flavor universality. Observables such as the longitudinal and transverse D^* polarizations ($F^{D^*}_{L,T}$) fall in this category. The second type of observables are constructed by integrating over the hadron-side parameter, θ^*, and either of the parameters θ_π, and χ_π. These observables explicitly depend on the effects from the $\tau \to \pi\nu_\tau$ decay. Since lighter leptons cannot decay to a pion, this second type of observables appears only when the intermediate lepton is a τ.

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[4] Whether to add or subtract individual angular terms for the construction of a true CP-violating effect depends on the sign convention used to define the azimuthal angle. Theory sign conventions for the decay $B \to K^*\mu^-\mu^+$, which our discussion follows for the $B \to D^*\tau^-\bar{\nu}_\tau$, can be found in ref. [110]. Ref. [111] presents detailed comparisons between sign conventions used in $B \to K^*\mu^+\mu^-$ theory versus experiment.

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4.1 Lepton flavor universality

Here we consider observables constructed from the differential decay distribution by integrating over E_π, θ_π, χ_π. The resulting distribution in q^2 and θ^* can be expressed as

$$\begin{aligned}\frac{d^2\Gamma}{dq^2\,d\cos\theta^*} &= \frac{3}{2}\frac{d\Gamma}{dq^2}\frac{a(q^2)+c(q^2)\cos^2\theta^*}{3a(q^2)+c(q^2)}\\ &= \frac{3}{4}\frac{d\Gamma}{dq^2}\left[2F_L^{D*}(q^2)\cos^2\theta^* + F_T^{D*}(q^2)\sin^2\theta^*\right]\,, \end{aligned}\tag{4.1}$$

where $F_L^{D*}(q^2)$ and $F_T^{D*}(q^2) = 1 - F_L^{D*}(q^2)$ are the longitudinal and transverse polarization fractions of the D^*. The functions $a(q^2)$ and $c(q^2)$ are given by

$$\begin{aligned} a(q^2) &= 2\left(1+\frac{m_\tau^2}{2q^2}\right)\left(|\mathcal{A}_\parallel|^2+|\mathcal{A}_\perp|^2\right) + 16\left(1+\frac{2m_\tau^2}{q^2}\right)\left(|\mathcal{A}_{\parallel,T}|^2+|\mathcal{A}_{\perp,T}|^2\right)\\ &\quad - \frac{24m_\tau}{\sqrt{q^2}}\left(\mathrm{Re}[\mathcal{A}_\parallel\mathcal{A}_{\parallel,T}^*] + \mathrm{Re}[\mathcal{A}_\perp\mathcal{A}_{\perp,T}^*]\right)\,, \end{aligned}\tag{4.2}$$

$$\begin{aligned} c(q^2) &= 2\left(1+\frac{m_\tau^2}{2q^2}\right)\left(2|\mathcal{A}_0|^2-|\mathcal{A}_\parallel|^2-|\mathcal{A}_\perp|^2\right) + 6\left|\frac{m_\tau}{\sqrt{q^2}}\mathcal{A}_t + \mathcal{A}_{SP}\right|^2\\ &\quad + 16\left(1+\frac{2m_\tau^2}{q^2}\right)\left(2|\mathcal{A}_{0,T}|^2-|\mathcal{A}_{\parallel,T}|^2-|\mathcal{A}_{\perp,T}|^2\right)\\ &\quad - \frac{24m_\tau}{\sqrt{q^2}}\left(2\mathrm{Re}\left[\mathcal{A}_0\mathcal{A}_{0,T}^*\right] - \mathrm{Re}[\mathcal{A}_\parallel\mathcal{A}_{\parallel,T}^*] - \mathrm{Re}[\mathcal{A}_\perp\mathcal{A}_{\perp,T}^*]\right)\,. \end{aligned}\tag{4.3}$$

The longitudinal and transverse polarization fractions $F_{L,T}^{D^*}$ can be obtained from eq. (4.1):

$$F_L^{D^*} = \frac{a(q^2)+c(q^2)}{3a(q^2)+c(q^2)}\,,\qquad F_T^{D^*} = \frac{2a(q^2)}{3a(q^2)+c(q^2)}\,.\tag{4.4}$$

Further integration over $\cos\theta^*$ gives us the decay distribution as a function of q^2:

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2|V_{cb}|^2|\vec{p}_{D^*}|q^2}{128m_B^2\pi^3}\left(1-\frac{m_\tau^2}{q^2}\right)^2\mathcal{B}(D^*\to D\pi')\mathcal{B}(\tau\to\pi\nu_\tau)\left(a(q^2)+\frac{c(q^2)}{3}\right)\,.\tag{4.5}$$

The integrated observables constructed above are not affected by the dynamics of the τ decay, since the relevant kinematic parameters have been integrated over. Indeed, the expressions for these observables agree with those found elsewhere in the literature (apart from the factor $\mathcal{B}(\tau\to\pi\nu_\tau)$ in eq. (4.5)). The comparison of the measured values of these observables with those found in decays involving the light leptons, taking into account the larger τ mass and the associated kinematic differences, provides a test of lepton flavor universality.

4.2 Lepton-side observables

Here we discuss observables obtained by integrating the full differential distribution over θ^* and either (or both) of θ_π and χ_π. These observables depend on at least one kinematic parameter associated with the decay of the τ, E_π. Therefore, these observables can only be constructed in the τ lepton case, and specifically for the decay $\tau\to\pi\nu_\tau$.

The first step is to integrate the full differential decay rate of eq. (2.15) over θ^*. The θ^* dependence of the full angular distribution can be retrieved from tables 2 and 3. The angular functions in these tables are proportional to one of three forms — $\cos^2\theta^*$, $\sin^2\theta^*$ and $\sin 2\theta^*$. The integral over θ^* eliminates all helicity-amplitude combinations proportional to $\sin 2\theta^*$, but keeps the other two. Thus, terms in the angular distribution proportional to $f^R_{1,\dots,7}$ and f^I_3 survive. The remaining expression is long and may not carry any more insight than the full angular distribution itself. We therefore proceed one step further and integrate over χ_π.

Once again, the χ_π dependence can be retrieved from tables 2 and 3. Only terms that are independent of χ_π at this stage still survive after we integrate over χ_π. These terms appear in table 2 as those proportional to $f^R_{1,\dots,6}$. The remaining differential decay rate is a function of q^2, E_π and θ_π, and can be expressed in terms of the functions $f_1, \dots, 6$ as

$$\frac{d^3\Gamma}{dq^2 dE_\pi d\cos\theta_\pi} = \frac{3}{2}\,\frac{d^2\Gamma}{dq^2 dE_\pi}\,\frac{a_\pi + b_\pi\cos\theta_\pi + c_\pi\cos^2\theta_\pi}{3a_\pi + c_\pi}\,, \tag{4.6}$$

where the coefficients a_π, b_π and c_π are functions of q^2 and E_π (the $f^R_i(q^2, E_\pi)$ are defined in table 2):

$$\begin{aligned}
a_\pi &= f^R_1(q^2,E_\pi) + 2f^R_2(q^2,E_\pi) - f^R_4(q^2,E_\pi) - 2f^R_6(q^2,E_\pi)\\
&= (S_{0,1} - S_{0,2})|\mathcal{A}_0|^2 + (S_{0T,1} - S_{0T,2})|\mathcal{A}_{0,T}|^2 + S_{SP}|\mathcal{A}_{SP}|^2 + S_t|\mathcal{A}_t|^2\\
&\quad + 2(S_{\parallel,1} - S_{\parallel,2})|\mathcal{A}_\parallel|^2 + 2(S_{\parallel T,1} - S_{\parallel T,2})|\mathcal{A}_{\parallel,T}|^2 + 2(S_{\perp,1} - S_{\perp,2})|\mathcal{A}_\perp|^2\\
&\quad + 2(S_{\perp T,1} - S_{\perp T,2})|\mathcal{A}_{\perp,T}|^2 + (R_{0T0,1} - R_{0T0,2})\,\mathrm{Re}[\mathcal{A}_{0,T}\mathcal{A}^*_0] + R_{SPt}\,\mathrm{Re}[\mathcal{A}_{SP}\mathcal{A}^*_t]\\
&\quad + 2(R_{\parallel T\parallel,1} - R_{\parallel T\parallel,2})\,\mathrm{Re}[\mathcal{A}_{\parallel,T}\mathcal{A}^*_\parallel] + 2(R_{\perp T\perp,1} - R_{\perp T\perp,2})\,\mathrm{Re}[\mathcal{A}_{\perp,T}\mathcal{A}^*_\perp]\,,
\end{aligned} \tag{4.7}$$

$$\begin{aligned}
b_\pi &= f^R_3(q^2,E_\pi) + 2f^R_5(q^2,E_\pi)\\
&= R_{0Tt}\,\mathrm{Re}[\mathcal{A}_{0,T}\mathcal{A}^*_t] + R_{SP0}\,\mathrm{Re}[\mathcal{A}_{SP}\mathcal{A}^*_0] + R_{SP0T}\,\mathrm{Re}[\mathcal{A}_{SP}\mathcal{A}^*_{0,T}] + R_{t0}\,\mathrm{Re}[\mathcal{A}_t\mathcal{A}^*_0]\\
&\quad + 2R_{\parallel T\perp}\,\mathrm{Re}[\mathcal{A}_{\parallel,T}\mathcal{A}^*_\perp] + 2R_{\parallel\perp}\,\mathrm{Re}[\mathcal{A}_\parallel\mathcal{A}^*_\perp] + 2R_{\perp T\parallel}\,\mathrm{Re}[\mathcal{A}_{\perp,T}\mathcal{A}^*_\parallel]\\
&\quad + 2R_{\perp T\parallel T}\,\mathrm{Re}[\mathcal{A}_{\perp,T}\mathcal{A}^*_{\parallel,T}]\,,
\end{aligned} \tag{4.8}$$

$$\begin{aligned}
c_\pi &= 2f^R_4(q^2,E_\pi) + 4f^R_6(q^2,E_\pi)\\
&= 2S_{0,2}|\mathcal{A}_0|^2 + 2S_{0T,2}|\mathcal{A}_{0,T}|^2 + 4S_{\parallel,2}|\mathcal{A}_\parallel|^2\\
&\quad + 4S_{\parallel T,2}|\mathcal{A}_{\parallel,T}|^2 + 4S_{\perp,2}|\mathcal{A}_\perp|^2 + 4S_{\perp T,2}|\mathcal{A}_{\perp,T}|^2\\
&\quad + 2R_{0\mathrm{T}0,2}\,\mathrm{Re}[\mathcal{A}_{0,T}\mathcal{A}^*_0] + 4R_{\parallel T\parallel,2}\,\mathrm{Re}[\mathcal{A}_{\parallel,T}\mathcal{A}^*_\parallel] + 4R_{\perp T\perp,2}\,\mathrm{Re}[\mathcal{A}_{\perp,T}\mathcal{A}^*_\perp]\,.
\end{aligned} \tag{4.9}$$

Further integrating over θ_π gives us the decay distribution as a function of q^2 and E_π:

$$\frac{d^2\Gamma}{dq^2 dE_\pi} = \frac{m^2_\tau\,(q^2)^{5/2}}{2\,(m^2_\tau - m^2_\pi)^2\,(q^2 - m^2_\tau)^2}\,\frac{3\,a_\pi + c_\pi}{3\,a(q^2) + c(q^2)}\,\frac{d\Gamma}{dq^2}\,. \tag{4.10}$$

At this stage, we can perform an asymmetric integral over $\cos\theta_\pi$, to find the forward-backward asymmetry in the distribution of the π coming from the τ decay. This is done by integrating the differential decay rate with a uniform negative weight for the positive values of $\cos\theta_\pi$, and subtracting this from a similar integral with a uniform positive weight for the negative values of $\cos\theta_\pi$. Appropriately normalizing this function, we can define the forward-backward asymmetry (A_{FB}) as follows:

$$\begin{aligned} A_{FB}(q^2, E_\pi) &\equiv \frac{\int\limits_{-1}^{0} \frac{d^3\Gamma}{dq^2 dE_\pi d\cos\theta_\pi} d\cos\theta_\pi - \int\limits_{0}^{1} \frac{d^3\Gamma}{dq^2 dE_\pi d\cos\theta_\pi} d\cos\theta_\pi}{\frac{d^2\Gamma}{dq^2 dE_\pi}}\,, \\ &= -\frac{3}{2}\,\frac{b_\pi}{3\,a_\pi + c_\pi}\,. \end{aligned} \tag{4.11}$$

As can be seen from the form of b_π [eq. (4.8)], A_{FB} is nonzero in the SM. In order to see if NP is present, one must combine this measurement with that of other observables, or of other terms in the angular distribution. With enough independent measurements of functions of the helicity amplitudes, it is possible to determine if some NP amplitudes must be nonzero.

Changing the order of integrals over χ_π and θ_π can yield valuable complementary information. In the preceding discussion we obtained observables by first integrating over χ_π and then over θ_π. If instead the integral over θ_π is performed first, the helicity-amplitude combinations proportional to $\cos\theta_\pi$ and $\sin\theta_\pi$ are removed. The terms in the angular distribution proportional to $f^R_{1,2,4,6,7}$ and f^I_3 survive. The remaining differential decay rate, a function of q^2, E_π and θ_π, is found to be

$$\frac{d^3\Gamma}{dq^2 dE_\pi d\chi_\pi} = \frac{d^2\Gamma}{dq^2 dE_\pi}\,\frac{B_1 + B_2\cos 2\chi_\pi + B_3 \sin 2\chi_\pi}{2\pi B_1}\,, \tag{4.12}$$

where the coefficients B_i are functions of q^2 and E_π (the $f^R_i(q^2, E_\pi)$ are defined in table 2):

$$\begin{aligned} B_1 &= 3f^R_1(q^2,E_\pi) + 6f^R_2(q^2,E_\pi) - f^R_4(q^2,E_\pi) - 2f^R_6(q^2,E_\pi) \\ &= (3S_{0,1} - S_{0,2})|\mathcal{A}_0|^2 + (3S_{0T,1} - S_{0T,2})|\mathcal{A}_{0,T}|^2 + 3S_{SP}|\mathcal{A}_{SP}|^2 + 3S_t|\mathcal{A}_t|^2 \\ &\quad + 2(3S_{\parallel,1} - S_{\parallel,2})|\mathcal{A}_\parallel|^2 + 2(3S_{\parallel T,1} - S_{\parallel T,2})|\mathcal{A}_{\parallel,T}|^2 + 2(3S_{\perp,1} - S_{\perp,2})|\mathcal{A}_\perp|^2 \\ &\quad + 2(3S_{\perp T,1} - S_{\perp T,2})|\mathcal{A}_{\perp,T}|^2 + (3R_{0T0,1} - R_{0T0,2})\mathrm{Re}[\mathcal{A}_{0,T}\mathcal{A}_0^*] + 3R_{SPt}\mathrm{Re}[\mathcal{A}_{SP}\mathcal{A}_t^*] \\ &\quad + 2(3R_{\parallel T\parallel,1} - R_{\parallel T\parallel,2})\mathrm{Re}[\mathcal{A}_{\parallel,T}\mathcal{A}_\parallel^*] + 2(3R_{\perp T\perp,1} - R_{\perp T\perp,2})\mathrm{Re}[\mathcal{A}_{\perp,T}\mathcal{A}_\perp^*]\,, \end{aligned} \tag{4.13}$$

$$\begin{aligned} B_2 &= 4f^R_7(q^2,E_\pi) \\ &= 8\left(S_{\perp,2}\,|\mathcal{A}_\perp|^2 - S_{\parallel,2}\,|\mathcal{A}_\parallel|^2\right) + 8\left(S_{\perp T,2}\,|\mathcal{A}_{\perp,T}|^2 - S_{\parallel T,2}\,|\mathcal{A}_{\parallel,T}|^2\right) \\ &\quad + 8\left(R_{\perp T\perp,2}\,\mathrm{Re}[\mathcal{A}_{\perp,T}\mathcal{A}_\perp^*] - R_{\parallel T\parallel,2}\,\mathrm{Re}[\mathcal{A}_{\parallel,T}\mathcal{A}_\parallel^*]\right)\,, \end{aligned} \tag{4.14}$$

$$\begin{aligned} B_3 &= 4f^I_3(q^2,E_\pi) \\ &= 4\left(I_{\parallel\perp}\mathrm{Im}[\mathcal{A}_\parallel\mathcal{A}_\perp^*] + I_{\parallel T\perp}\mathrm{Im}[\mathcal{A}_{\parallel,T}\mathcal{A}_\perp^*] + I_{\perp T\parallel}\mathrm{Im}[\mathcal{A}_{\perp,T}\mathcal{A}_\parallel^*]\right)\,. \end{aligned} \tag{4.15}$$

Note that the coefficient B_1 is related to the coefficients a_π and c_π:

$$B_1 = 3\,a_\pi + c_\pi\,. \tag{4.16}$$

An asymmetric integral over χ_π can now isolate an observable that is nonzero only if true CP-violating TP asymmetries are present. This new observable, A_{TP}, can be defined as

$$A_{TP}(q^2, E_\pi) = \left(\frac{d^2\Gamma}{dq^2 dE_\pi}\right)^{-1} \left(\int_0^{\pi/2} - \int_{\pi/2}^{\pi} + \int_{\pi}^{3\pi/2} - \int_{3\pi/2}^{2\pi}\right) \frac{d^3\Gamma}{dq^2 dE_\pi d\chi_\pi} d\chi_\pi = \frac{B_3}{2\pi B_1} \,. \quad (4.17)$$

From eq. (4.15), we see that B_3 vanishes in the absence of weak-phase differences. This shows that A_{TP} is a CP-violating observable.

Above, A_{TP} is defined as a function of q^2 and E_π. However, one can further integrate this function over both of these variables. The resulting integrated observable can be directly compared to an experimental event analysis in which one obtains the asymmetry between the number of events with $\sin 2\chi_\pi > 0$ and $\sin 2\chi_\pi < 0$.

We note that A_{TP} involves interferences of vector-vector and vector-tensor type. The only way to generate a nonzero value of A_{TP} is if there is a nonzero weak phase in at least one of g_R and g_T. Thus, if A_{TP} (and/or its form integrated over q^2 and E_π) is found to be nonzero, this will be an unmistakeable sign of CP-violating NP.

5 Conclusions

At the present time, the measurements of $R_{D^{(*)}} \equiv \mathcal{B}(\bar{B} \to D^{(*)}\tau^-\bar{\nu}_\tau)/\mathcal{B}(\bar{B} \to D^{(*)}\ell^-\bar{\nu}_\ell)$ ($\ell = e, \mu$) and $R_{J/\psi} \equiv \mathcal{B}(B_c^+ \to J/\psi\tau^+\nu_\tau)/\mathcal{B}(B_c^+ \to J/\psi\mu^+\nu_\mu)$ disagree with the predictions of the SM, hinting at NP in $b \to c\tau^-\bar{\nu}$ decays. There are many possibilities for this NP. A variety of observables have been proposed to distinguish the various NP explanations: the q^2 distribution, the D^* polarization, the τ polarization, etc.

Another potential way of distinguishing the NP explanations involves CP violation. Within the SM, there are no CP-violating effects in $\bar{B} \to D^*\tau^-\bar{\nu}_\tau$, so that any observation of CP violation in this decay would be a smoking-gun signal of NP. Here, the main CP-violating effects appear as CP-violating asymmetries in the angular distribution. However, this is problematic. The construction of the angular distribution requires the knowledge of the three-momentum $\vec{p}_\tau$. But this cannot be measured precisely, since the τ decays to final-state particles that include ν_τ, which is undetected. The result is that the full angular distribution in $\bar{B} \to D^*(\to D\pi)\,\tau^-\bar{\nu}_\tau$ cannot be measured.

In this paper, we construct a *measurable* angular distribution by considering the additional decay $\tau^- \to \pi^-\nu_\tau$. The full process then is $\bar{B} \to D^*(\to D\pi')\,\tau^-(\to \pi^-\nu_\tau)\bar{\nu}_\tau$. Here there are three final-state particles whose three-momenta *can* be measured: the D and π' (from D^* decay), and the π^- (from τ decay). The new angular distribution is given in terms of five kinematic parameters: q^2, θ^* (describing $D^* \to D\pi$), and three quantities describing the π^-, E_π, θ_π and χ_π. It includes CP-violating angular asymmetries, which can be measured and used to extract information about the NP.

But much more information can be extracted from the angular distribution. In the most general case, the angular distribution involves the couplings $1 + g_L$, g_R, g_P and g_T, where g_L, g_R, g_P and g_T are the NP parameters. The magnitudes and relative phases of all four couplings can be extracted from a fit to the full distribution. This will go a long way towards identifying the NP.

It is also possible to integrate over one or more of the five kinematic parameters. If one integrates over the lepton-side parameters E_π, θ_π and χ_π, all the familiar observables that have been proposed to distinguish NP models are reproduced. These include the q^2 distribution and the D^* polarization. And if one integrates over the hadron-side quantities, one obtains new observables that depend on the kinematic angles associated with the π^- emitted in the τ decay, θ_π and χ_π. These include the forward-backward asymmetry of the π^-, and the CP-violating triple-product asymmetry.

In principle, one can construct angular distributions using other τ decays. The analysis of $\tau \to 3\pi\nu_\tau$ is similar to that of $\tau \to \pi\nu_\tau$, treating the 3π system as a single "particle," except that one must allow for it to have spin 0, 1, 2, etc. And $\tau \to \mu\bar{\nu}_\mu\nu_\tau$ is more complicated, since one must also integrate over the kinematic angles of the $\bar{\nu}_\mu$.

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A Hadronic and helicity amplitude pieces

The differential decay rate for the process $B \to D^*(\to D\pi')\tau(\to \pi\nu_\tau)\bar{\nu}_\tau$ has been written in terms of a collection of hadronic pieces $\mathcal{H}_{D^*}$, helicity amplitude pieces $\mathcal{M}^{N^*}$, and leptonic pieces $\tilde{\mathcal{L}}^{N^*}$ in section 2.3. While the leptonic pieces are new in this analysis, the hadronic and helicity amplitudes were presented in ref. [103]. For convenience, below we summarize these relationships.

The hadronic pieces, H_{D^*}, can be expressed as

$$\mathcal{H}_{D^*}(m) = \epsilon_{D^*}(m) \cdot p_D \,, \quad m = 0, \pm \,, \tag{A.1}$$

where p_D represents the four-momentum of the D meson, and $\epsilon_{D^*}(m)$ represents the polarization of the D^* meson. We follow the convention of expressing the four-momentum and polarizations of the D^* meson in the B-meson rest frame as follows:

$$p^\mu_{D^*} = (k_0, 0, 0, k_z) \,, \quad \epsilon^\mu_{D^*}(\pm) = (0, 1, \pm i, 0)/\sqrt{2} \,, \quad \epsilon^\mu_{D^*}(0) = (k_z, 0, 0, k_0)/m_{D^*} \,. \tag{A.2}$$

In addition to the hadronic pieces corresponding to the three well-defined helicities of the on-shell D^* meson, we make use of a fourth timelike helicity for an off-shell particle, defined such that $\mathcal{H}_{D^*}(t) = \mathcal{H}_{D^*}(0)$.

The helicity amplitude pieces, $\mathcal{M}^{N^*}$, also depend on the helicities of the intermediate particles. These are of the scalar-pseudoscalar (SP), the vector-axialvector (VA), and the tensor (T) types. Furthermore, since the decaying B meson is spinless, only certain helicity combinations survive. The list of non-zero components of the helicity amplitude pieces are listed below:

$$
\begin{aligned}
\mathcal{M}^{SP}_{(0)}(B\to D^*SP^*) &= \mathcal{A}_{SP}\,,\\
\mathcal{M}^{VA}_{(+;+)}(B\to D^*VA^*) &= \mathcal{A}_{+}\,,\\
\mathcal{M}^{VA}_{(-;-)}(B\to D^*VA^*) &= \mathcal{A}_{-}\,,\\
\mathcal{M}^{VA}_{(0;0)}(B\to D^*VA^*) &= \mathcal{A}_{0}\,,\\
\mathcal{M}^{VA}_{(0;t)}(B\to D^*VA^*) &= \mathcal{A}_{t}\,,\\
\mathcal{M}^{T}_{(+;+,0)}(B\to D^*T^*) &= \mathcal{M}^{T}_{(+;+,t)}(B\to D^*T^*) = \mathcal{A}_{+,T}\,,\\
\mathcal{M}^{T}_{(0;-,+)}(B\to D^*T^*) &= \mathcal{M}^{T}_{(0;0,t)}(B\to D^*T^*) = \mathcal{A}_{0,T}\,,\\
\mathcal{M}^{T}_{(-;0,-)}(B\to D^*T^*) &= \mathcal{M}^{T}_{(-;-,t)}(B\to D^*T^*) = \mathcal{A}_{-,T}\,.
\end{aligned}
\tag{A.3}
$$

As seen in the above, there are a total of 8 independent helicity amplitudes: one of type SP, four of type VA, and three independent amplitudes of type T. Using the definitions for the $B\to D^*$ form factors $A_i(q^2)$, $V(q^2)$ and $T_i(q^2)$ given in refs. [38, 112], we can further represent each helicity amplitude as follows:

$$
\begin{aligned}
\mathcal{A}_{SP} &= -g_P\,\frac{\sqrt{\lambda(m_B^2,m_{D^*}^2,q^2)}}{m_b+m_c}A_0(q^2)\,,\\
\mathcal{A}_0 &= -\frac{(1+g_L-g_R)\,(m_B+m_{D^*})}{2m_{D^*}\sqrt{q^2}}\left((m_B^2-m_{D^*}^2-q^2)A_1(q^2)+\frac{\lambda(m_B^2,m_{D^*}^2,q^2)}{(m_B+m_{D^*})^2}A_2(q^2)\right),\\
\mathcal{A}_t &= -(1+g_L-g_R)\,\frac{\sqrt{\lambda(m_B^2,m_{D^*}^2,q^2)}}{\sqrt{q^2}}A_0(q^2)\,,\\
\mathcal{A}_\pm &= (1+g_L-g_R)\,(m_B+m_{D^*})A_1(q^2)\mp(1+g_L+g_R)\frac{\sqrt{\lambda(m_B^2,m_{D^*}^2,q^2)}}{m_B+m_{D^*}}V(q^2)\,,\\
\mathcal{A}_{0,T} &= \frac{g_T}{2m_{D^*}}\left((m_B^2+3m_{D^*}^2-q^2)T_2(q^2)-\frac{\lambda(m_B^2,m_{D^*}^2,q^2)T_3(q^2)}{m_B^2-m_{D^*}^2}\right),\\
\mathcal{A}_{\pm,T} &= g_T\,\frac{\sqrt{\lambda(m_B^2,m_{D^*}^2,q^2)}T_1(q^2)\pm(m_B^2-m_{D^*}^2)T_2(q^2)}{\sqrt{q^2}}\,,
\end{aligned}
\tag{A.4}
$$

where $\lambda(a,b,c) = a^2+b^2+c^2-2ab-2bc-2ca\,.$

Finally, the amplitudes for the vector and tensor types can be expressed in the transversity basis (using $\perp,\parallel$) instead of the helicity basis (using $\pm$), using the following relationships,

$$
\begin{aligned}
\mathcal{A}_{\parallel(,T)} &= (\mathcal{A}_{+(,T)}+\mathcal{A}_{-(,T)})/\sqrt{2}\,,\\
\mathcal{A}_{\perp(,T)} &= (\mathcal{A}_{+(,T)}-\mathcal{A}_{-(,T)})/\sqrt{2}\,.
\end{aligned}
\tag{A.5}
$$

B $\mathcal{N}_i^S$, $\mathcal{N}_{i,j}^R$ and $\mathcal{N}_{i,j}^I$ contributions

The information relevant for the $\mathcal{N}_i^S\,|\mathcal{A}_i|^2$, $\mathcal{N}_{i,j}^R\,\mathrm{Re}[\mathcal{A}_i\mathcal{A}_j^*]$ and $\mathcal{N}_{i,j}^I\,\mathrm{Im}[\mathcal{A}_i\mathcal{A}_j^*]$ pieces of eq. (2.15) are found in tables 5, 6 and 7, respectively. The dependence on q^2 and E_pi is contained in the S_i, R_i, and I_i functions, respectively, whose expressions are given below.

The kinematics of the five-body decay restricts the range of values that the parameters q^2 and E_π can take.

$$m_\tau^2 \le q^2 \le (m_B - m_{D^*})^2\,, \qquad \frac{m_\tau^4 + m_\pi^2 q^2}{2m_\tau^2\sqrt{q^2}} \le E_\pi \le \frac{q^2 + m_\pi^2}{2\sqrt{q^2}}\,. \tag{B.1}$$

We define the normalized parameters $\rho_\tau \equiv m_\tau/\sqrt{q^2}$, $\rho_\pi \equiv m_\pi/\sqrt{q^2}$, and $\mathcal{E}_\pi \equiv E_\pi/\sqrt{q^2}$. Based on the above limits, the normalized parameters are limited to values between 0 and 1. Below we express S_i, R_i, and I_i in terms of these normalized parameters.

The expressions for the S_i factors are

$$\begin{aligned}
S_t = \rho_\tau^2 S_{SP} = 16\rho_\tau^2\left(2\mathcal{E}_\pi\rho_\tau^2-\rho_\tau^4-\rho_\pi^2\right),& \\
S_{0,1} = \frac{1}{\mathcal{E}_\pi^2-\rho_\pi^2}\left(4\rho_\tau^2\right)\Big(&2\mathcal{E}_\pi\left(\rho_\tau^2\left(1-\rho_\pi^2\right)+\rho_\tau^4+3\rho_\pi^2\right) \\
&+2\mathcal{E}_\pi^2\left(1-2\rho_\tau^2-\rho_\tau^4\right)-4\mathcal{E}_\pi^3\left(1-\rho_\tau^2\right) \\
&+\rho_\tau^4\left(-1+\rho_\pi^2\right)-3\rho_\pi^2-\rho_\pi^4\Big), \\
S_{0,2} = -4S_{\perp,2} = \frac{1}{\mathcal{E}_\pi^2-\rho_\pi^2}\left(4\rho_\tau^2\right)\Big(&2\mathcal{E}_\pi\left(1+\rho_\tau^2\right)\left(3\rho_\tau^2+\rho_\pi^2\right) \\
&-2\mathcal{E}_\pi^2\left(1+6\rho_\tau^2+\rho_\tau^4+2\rho_\pi^2\right)+4\mathcal{E}_\pi^3\left(1+\rho_\tau^2\right) \\
&-\rho_\tau^4\left(3+\rho_\pi^2\right)-\rho_\pi^2+\rho_\pi^4\Big), \\
S_{\perp,1} = \frac{1}{\mathcal{E}_\pi^2-\rho_\pi^2}\left(-\rho_\tau^2\right)\Big(&2\mathcal{E}_\pi\left(\rho_\tau^2\left(1+3\rho_\pi^2\right)+\rho_\tau^4-5\rho_\pi^2\right) \\
&-2\mathcal{E}_\pi^2\left(3+2\rho_\tau^2-\rho_\tau^4+2\rho_\pi^2\right)+4\mathcal{E}_\pi^3\left(3-\rho_\tau^2\right) \\
&-\rho_\tau^4\left(1+3\rho_\pi^2\right)+5\rho_\pi^2+3\rho_\pi^4\Big), \\
S_{\parallel,1} = \frac{\rho_\tau^2}{\mathcal{E}_\pi^2-\rho_\pi^2}&\left(-2\mathcal{E}_\pi\left(\rho_\tau^2+\rho_\tau^4+\rho_\pi^2\left(-5+3\rho_\tau^2\right)\right)+\mathcal{E}_\pi^2\left(6+4\rho_\pi^2+4\rho_\tau^2-2\rho_\tau^4\right)\right. \\
&\left.+\,4\mathcal{E}_\pi^3\left(-3+\rho_\tau^2\right)-3\rho_\pi^4+\rho_\tau^4+\rho_\pi^2\left(-5+3\rho_\tau^4\right)\right) \\
S_{\parallel,2} = \frac{\rho_\tau^2}{\mathcal{E}_\pi^2-\rho_\pi^2}&\left(-2\mathcal{E}_\pi\left(1+\rho_\tau^2\right)\left(\rho_\pi^2+3\rho_\tau^2\right)+2\mathcal{E}_\pi^2\left(1+2\rho_\pi^2+6\rho_\tau^2+\rho_\tau^4\right)-4\mathcal{E}_\pi^3\left(1+\rho_\tau^2\right)\right. \\
&\left.+\rho_\pi^2-\rho_\pi^4+\left(3+\rho_\pi^2\right)\rho_\tau^4\right) \\
S_{0T,1} = \frac{1}{\mathcal{E}_\pi^2-\rho_\pi^2}(-64)\Big(&2\mathcal{E}_\pi\rho_\pi^2\left(\rho_\tau^2\left(1-\rho_\pi^2\right)-3\rho_\tau^4-\rho_\pi^2\right) \\
&+2\mathcal{E}_\pi^2\rho_\pi^2\left(1+2\rho_\tau^2-\rho_\tau^4\right)-4\mathcal{E}_\pi^3\rho_\pi^2\left(1-\rho_\tau^2\right) \\
&+\rho_\tau^4\rho_\pi^2\left(1+3\rho_\pi^2\right)-\rho_\pi^4+\rho_\pi^6\Big),
\end{aligned}$$

$$
\begin{aligned}
S_{0T,2} &= -4S_{\perp T,2} = -4S_{\parallel T,2} \\
&= \frac{1}{\mathcal{E}_\pi^2-\rho_\pi^2}64\Big(2\mathcal{E}_\pi\rho_\pi^2\left(1+\rho_\tau^2\right)\left(\rho_\tau^2+3\rho_\pi^2\right) \\
&\qquad -2\mathcal{E}_\pi^2\left(6\rho_\tau^2\rho_\pi^2+\rho_\tau^4\left(2+\rho_\pi^2\right)+\rho_\pi^2\right)+4\mathcal{E}_\pi^3\rho_\tau^2\left(1+\rho_\tau^2\right) \\
&\qquad +\rho_\tau^4\rho_\pi^2\left(1-\rho_\pi^2\right)-\rho_\pi^4-3\rho_\pi^6\Big)\,, \\
S_{\perp T,1} = S_{\parallel T,1} &= \frac{1}{\mathcal{E}_\pi^2-\rho_\pi^2}(-16)\Big(2\mathcal{E}_\pi\rho_\pi^2\left(\rho_\tau^2\left(3+\rho_\pi^2\right)-5\rho_\tau^4+\rho_\pi^2\right) \\
&\qquad -\mathcal{E}_\pi^2\left(4\rho_\tau^2\rho_\pi^2+\rho_\tau^4\left(4+6\rho_\pi^2\right)-2\rho_\pi^2\right)-4\mathcal{E}_\pi^3\rho_\tau^2\left(1-3\rho_\tau^2\right) \\
&\qquad +\rho_\tau^4\rho_\pi^2\left(3+5\rho_\pi^2\right)-3\rho_\pi^4-\rho_\pi^6\Big)\,.
\end{aligned}
\tag{B.2}
$$

The expressions for the R_i factors are

$$
\begin{aligned}
R_{t0} = 2\sqrt{2}R_{t\parallel} = \rho_\tau R_{SP0} = 2\sqrt{2}\rho_\tau R_{SP\parallel} &= \frac{-32\rho_\tau^2\left(1-\mathcal{E}_\pi\right)\left(2\mathcal{E}_\pi\rho_\tau^2-\rho_\tau^4-\rho_\pi^2\right)}{\sqrt{\mathcal{E}_\pi^2-\rho_\pi^2}}\,, \\
R_{0\parallel} &= \frac{2\sqrt{2}\rho_\tau^2}{\mathcal{E}_\pi^2-\rho_\pi^2}\Big(2\mathcal{E}_\pi\left(1+\rho_\tau^2\right)\left(3\rho_\tau^2+\rho_\pi^2\right)-2\mathcal{E}_\pi^2\left(1+6\rho_\tau^2+\rho_\tau^4+2\rho_\pi^2\right) \\
&\qquad +4\mathcal{E}_\pi^3\left(1+\rho_\tau^2\right)-\rho_\tau^4\left(3+\rho_\pi^2\right)-\rho_\pi^2+\rho_\pi^4\Big)\,, \\
R_{\parallel\perp} = -\sqrt{2}R_{0\perp} &= \frac{16\rho_\tau^2\left(1-2\mathcal{E}_\pi+\rho_\pi^2\right)\left(\rho_\tau^2-\mathcal{E}_\pi\right)}{\sqrt{\mathcal{E}_\pi^2-\rho_\pi^2}}\,, \\
R_{SPt} &= 32\rho_\tau\left(+2\mathcal{E}_\pi\rho_\tau^2-\rho_\tau^4-\rho_\pi^2\right)\,, \\
R_{SP0T} = 2\sqrt{2}R_{SP\parallel T} = \frac{1}{\rho_\tau}R_{0Tt} = \frac{2\sqrt{2}}{\rho_\tau}R_{\parallel Tt} &= \frac{128\left(\mathcal{E}_\pi-\rho_\pi^2\right)\left(2\mathcal{E}_\pi\rho_\tau^2-\rho_\tau^4-\rho_\pi^2\right)}{\sqrt{\mathcal{E}_\pi^2-\rho_\pi^2}}\,, \\
R_{0T0,1} &= \frac{32\rho_\tau\Big(\mathcal{E}_\pi\left(1+\rho_\pi^2\right)\left(\rho_\tau^4+\rho_\pi^2\right)-4\mathcal{E}_\pi^2\rho_\tau^2\left(1+\rho_\pi^2\right)+4\mathcal{E}_\pi^3\rho_\tau^2+2\rho_\pi^2\left(\rho_\tau^2-\rho_\pi^2\right)\left(1-\rho_\tau^2\right)\Big)}{\mathcal{E}_\pi^2-\rho_\pi^2}\,, \\
R_{0T0,2} = 2\sqrt{2}R_{0T\parallel} &= \frac{32\rho_\tau}{\mathcal{E}_\pi^2-\rho_\pi^2}\Big(\mathcal{E}_\pi\left(8\rho_\tau^2\rho_\pi^2+3\rho_\tau^4\left(1+\rho_\pi^2\right)+3\rho_\pi^2+3\rho_\pi^4\right)-4\mathcal{E}_\pi^2\left(1+\rho_\tau^2\right)\left(\rho_\tau^2+\rho_\pi^2\right) \\
&\qquad +4\mathcal{E}_\pi^3\rho_\tau^2-2\rho_\pi^2\left(1+\rho_\tau^2\right)\left(\rho_\tau^2+\rho_\pi^2\right)\Big)\,, \\
R_{0T\parallel T} &= \frac{32\sqrt{2}}{\mathcal{E}_\pi^2-\rho_\pi^2}\Big(2\mathcal{E}_\pi\rho_\pi^2\left(1+\rho_\tau^2\right)\left(3\rho_\pi^2+\rho_\tau^2\right)-2\mathcal{E}_\pi^2\left(6\rho_\tau^2\rho_\pi^2+\rho_\tau^4\left(2+\rho_\pi^2\right)+\rho_\pi^2\right) \\
&\qquad +4\mathcal{E}_\pi^3\rho_\tau^2\left(1+\rho_\tau^2\right)+\rho_\tau^4\rho_\pi^2\left(1-\rho_\pi^2\right)-\rho_\pi^4-3\rho_\pi^6\Big)\,, \\
R_{\perp T\parallel} = -\sqrt{2}R_{0T\perp} = -\sqrt{2}R_{\perp T0} = R_{\parallel T\perp} &= \frac{-32\rho_\tau\left(1-2\mathcal{E}_\pi+\rho_\pi^2\right)\left(\rho_\tau^4-\rho_\pi^2\right)}{\sqrt{\mathcal{E}_\pi^2-\rho_\pi^2}}\,, \\
R_{\perp T\parallel T} = -\sqrt{2}R_{0T\perp T} &= \frac{256\rho_\tau^2\left(1-2\mathcal{E}_\pi+\rho_\pi^2\right)\left(\mathcal{E}_\pi\rho_\tau^2-\rho_\pi^2\right)}{\sqrt{\mathcal{E}_\pi^2-\rho_\pi^2}}\,, \\
R_{\parallel T0} &= -\sqrt{2}R_{\perp T\perp,2} = -\sqrt{2}R_{\parallel T\parallel,2} \\
&= \frac{8\sqrt{2}\rho_\tau}{\mathcal{E}_\pi^2-\rho_\pi^2}\Big(\mathcal{E}_\pi\left(8\rho_\tau^2\rho_\pi^2+3\rho_\tau^4\left(1+\rho_\pi^2\right)+3\rho_\pi^2+3\rho_\pi^4\right)-4\mathcal{E}_\pi^2\left(1+\rho_\tau^2\right)\left(\rho_\pi^2+\rho_\tau^2\right) \\
&\qquad +4\mathcal{E}_\pi^3\rho_\tau^2-2\rho_\pi^2\left(1+\rho_\tau^2\right)\left(\rho_\pi^2+\rho_\tau^2\right)\Big)\,,
\end{aligned}
$$

Helicity info	$\mathcal{N}^S$
$\lvert\mathcal{A}_t\rvert^2$	$S_t\,\cos^2\theta^*$
$\lvert\mathcal{A}_0\rvert^2$	$[S_{0,1}+S_{0,2}\,\cos 2\theta_\pi]\cos^2\theta^*$
$\lvert\mathcal{A}_\perp\rvert^2$	$\left[S_{\perp,1}+S_{\perp,2}\left(\cos 2\chi_\pi+2\cos 2\theta_\pi\sin^2\chi_\pi\right)\right]\sin^2\theta^*$
$\lvert\mathcal{A}_\parallel\rvert^2$	$\left[S_{\parallel,1}+S_{\parallel,2}\left(\cos 2\theta_\pi-2\sin^2\theta_\pi\cos 2\chi_\pi\right)\right]\sin^2\theta^*$
$\lvert\mathcal{A}_{SP}\rvert^2$	$S_{SP}\cos^2\theta^*$
$\lvert\mathcal{A}_{0,T}\rvert^2$	$[S_{0T,1}+S_{0T,2}\cos 2\theta_\pi]\cos^2\theta^*$
$\lvert\mathcal{A}_{\perp,T}\rvert^2$	$\left[S_{\perp T,1}+S_{\perp T,2}\left(\cos 2\theta_\pi+2\cos 2\chi_\pi\sin^2\theta_\pi\right)\right]\sin^2\theta^*$
$\lvert\mathcal{A}_{\parallel,T}\rvert^2$	$\left[S_{\parallel T,1}+S_{\parallel T,2}\left(\cos 2\theta_\pi-2\cos 2\chi_\pi\sin^2\theta_\pi\right)\right]\sin^2\theta^*$

Table 5. Contributions to the $\mathcal{N}_i^S\,|\mathcal{A}_i|^2$ pieces of eq. (2.15). The coefficients S_i depend on the kinematic parameters q^2 and E_π, and are listed in eq. (B.2).

$$\begin{aligned}
R_{\parallel T\parallel,1}=R_{\perp T\perp,1}=\frac{-8\rho_\tau}{\mathcal{E}_\pi^2-\rho_\pi^2}\Big(&\mathcal{E}_\pi\left(8\rho_\tau^2\rho_\pi^2+\rho_\tau^4\left(1+\rho_\pi^2\right)+\rho_\pi^2+\rho_\pi^4\right)-4\mathcal{E}_\pi^2\left(1-\rho_\tau^2\right)\left(\rho_\pi^2-\rho_\tau^2\right)\\
&-4\mathcal{E}_\pi^3\rho_\tau^2-2\rho_\pi^2\left(3\rho_\tau^2\left(1+\rho_\pi^2\right)-\rho_\tau^4-\rho_\pi^2\right)\Big)\,. \qquad \text{(B.3)}
\end{aligned}$$

And finally the expressions for the I_i factors are

$$\begin{aligned}
I_{t\perp}=\rho_\tau I_{SP\perp}&=\frac{8\sqrt{2}\rho_\tau^2\left(1-\mathcal{E}_\pi\right)\left(2\mathcal{E}_\pi\rho_\tau^2-\rho_\tau^4-\rho_\pi^2\right)}{\sqrt{\mathcal{E}_\pi^2-\rho_\pi^2}}\,,\\
I_{\parallel\perp}=\sqrt{2}I_{0\perp}&=\frac{-4\rho_\tau^2}{\mathcal{E}_\pi^2-\rho_\pi^2}\Big(2\mathcal{E}_\pi\left(1+\rho_\tau^2\right)\left(\rho_\pi^2+3\rho_\tau^2\right)-2\mathcal{E}_\pi^2\left(1+6\rho_\tau^2+\rho_\tau^4+2\rho_\pi^2\right)\\
&\qquad+4\mathcal{E}_\pi^3\left(1+\rho_\tau^2\right)-\left(3+\rho_\pi^2\right)\rho_\tau^4-\rho_\pi^2+\rho_\pi^4\Big)\,,\\
I_{0T\parallel}=-I_{\parallel T0}&=\frac{-16\sqrt{2}\rho_\tau\left(1-2\mathcal{E}_\pi+\rho_\pi^2\right)\left(\rho_\tau^4-\rho_\pi^2\right)}{\sqrt{\mathcal{E}_\pi^2-\rho_\pi^2}}\,,\\
I_{\perp Tt}=-\rho_\tau I_{SP\perp T}&=\frac{32\sqrt{2}\rho_\tau\left(\mathcal{E}_\pi-\rho_\pi^2\right)\left(2\mathcal{E}_\pi\rho_\tau^2-\rho_\tau^4-\rho_\pi^2\right)}{\sqrt{\mathcal{E}_\pi^2-\rho_\pi^2}}\,,\\
I_{\perp T0}=-I_{0T\perp}=\frac{1}{\sqrt{2}}I_{\perp T\parallel}&=-\frac{1}{\sqrt{2}}I_{\parallel T\perp}\\
&=\frac{8\sqrt{2}\rho_\tau}{\mathcal{E}_\pi^2-\rho_\pi^2}\Big(\mathcal{E}_\pi\left(8\rho_\pi^2\rho_\tau^2+3\rho_\tau^4\left(1+\rho_\pi^2\right)+3\rho_\pi^2+3\rho_\pi^4\right)\\
&\qquad-4\mathcal{E}_\pi^2\left(1+\rho_\tau^2\right)\left(\rho_\pi^2+\rho_\tau^2\right)+4\mathcal{E}_\pi^3\rho_\tau^2-2\rho_\pi^2\left(1+\rho_\tau^2\right)\left(\rho_\tau^2+\rho_\pi^2\right)\Big)\,,\\
I_{\parallel T\perp}=\sqrt{2}I_{0T\perp}&=\frac{-16\rho_\tau}{\mathcal{E}_\pi^2-\rho_\pi^2}\Big(\mathcal{E}_\pi\left(8\rho_\tau^2\rho_\pi^2+3\rho_\tau^4\left(1+\rho_\pi^2\right)+3\rho_\pi^2+3\rho_\pi^4\right)\\
&\qquad-4\mathcal{E}_\pi^2\left(1+\rho_\tau^2\right)\left(\rho_\pi^2+\rho_\tau^2\right)+4\mathcal{E}_\pi^3\rho_\tau^2\\
&\qquad-2\rho_\pi^2\left(1+\rho_\tau^2\right)\left(\rho_\tau^2+\rho_\pi^2\right)\Big)\,. \qquad \text{(B.4)}
\end{aligned}$$

Helicity info	$\mathcal{N}^R$
$\mathrm{Re}[\mathcal{A}_t\mathcal{A}_0^*]$	$R_{t0}\ \cos\theta_\pi \cos^2\theta^*$
$\mathrm{Re}[\mathcal{A}_t\mathcal{A}_\parallel^*]$	$R_{t\parallel}\ \cos\chi_\pi \sin\theta_\pi \sin 2\theta^*$
$\mathrm{Re}[\mathcal{A}_0\mathcal{A}_\parallel^*]$	$R_{0\parallel}\ \cos\chi_\pi \sin 2\theta_\pi \sin 2\theta^*$
$\mathrm{Re}[\mathcal{A}_0\mathcal{A}_\perp^*]$	$R_{0\perp}\ \cos\chi_\pi \sin\theta_\pi \sin 2\theta^*$
$\mathrm{Re}[\mathcal{A}_\parallel\mathcal{A}_\perp^*]$	$R_{\parallel\perp}\ \cos\theta_\pi \sin^2\theta^*$
$\mathrm{Re}[\mathcal{A}_{SP}\mathcal{A}_t^*]$	$R_{SPt}\ \cos^2\theta^*$
$\mathrm{Re}[\mathcal{A}_{SP}\mathcal{A}_0^*]$	$R_{SP0}\ \cos\theta_\pi \cos^2\theta^*$
$\mathrm{Re}[\mathcal{A}_{SP}\mathcal{A}_\parallel^*]$	$R_{SP\parallel}\ \cos\chi_\pi \sin\theta_\pi \sin 2\theta^*$
$\mathrm{Re}[\mathcal{A}_{SP}\mathcal{A}_{0,T}^*]$	$R_{SP0T}\ \cos\theta_\pi \cos^2\theta^*$
$\mathrm{Re}[\mathcal{A}_{SP}\mathcal{A}_{\parallel,T}^*]$	$R_{SP\parallel T}\ \cos\chi_\pi \sin\theta_\pi \sin 2\theta^*$
$\mathrm{Re}[\mathcal{A}_{0,T}\mathcal{A}_t^*]$	$R_{0Tt}\ \cos\theta_\pi \cos^2\theta^*$
$\mathrm{Re}[\mathcal{A}_{0,T}\mathcal{A}_0^*]$	$[R_{0T0,1} + R_{0T0,2}\ \cos 2\theta_\pi] \cos^2\theta^*$
$\mathrm{Re}[\mathcal{A}_{0,T}\mathcal{A}_\perp^*]$	$R_{0T\perp}\ \cos\chi_\pi \sin\theta_\pi \sin 2\theta^*$
$\mathrm{Re}[\mathcal{A}_{0,T}\mathcal{A}_\parallel^*]$	$R_{0T\parallel}\ \cos\chi_\pi \sin 2\theta_\pi \sin 2\theta^*$
$\mathrm{Re}[\mathcal{A}_{0,T}\mathcal{A}_{\perp,T}^*]$	$R_{0T\perp T}\ \cos\chi_\pi \sin\theta_\pi \sin 2\theta^*$
$\mathrm{Re}[\mathcal{A}_{0,T}\mathcal{A}_{\parallel,T}^*]$	$R_{0T\parallel T}\ \cos\chi_\pi \sin 2\theta_\pi \sin 2\theta^*$
$\mathrm{Re}[\mathcal{A}_{\perp,T}\mathcal{A}_0^*]$	$R_{\perp T0}\ \cos\chi_\pi \sin\theta_\pi \sin 2\theta^*$
$\mathrm{Re}[\mathcal{A}_{\perp,T}\mathcal{A}_\perp^*]$	$[R_{\perp T\perp,1} + R_{\perp T\perp,2}\,(\cos 2\theta_\pi + 2\cos 2\chi_\pi \sin^2\theta_\pi)] \sin^2\theta^*$
$\mathrm{Re}[\mathcal{A}_{\perp,T}\mathcal{A}_\parallel^*]$	$R_{\perp T\parallel}\ \cos\theta_\pi \sin^2\theta^*$
$\mathrm{Re}[\mathcal{A}_{\perp,T}\mathcal{A}_{\parallel,T}^*]$	$R_{\perp T\parallel T}\ \cos\theta_\pi \sin^2\theta^*$
$\mathrm{Re}[\mathcal{A}_{\parallel,T}\mathcal{A}_t^*]$	$R_{\parallel Tt}\ \cos\chi_\pi \sin\theta_\pi \sin 2\theta^*$
$\mathrm{Re}[\mathcal{A}_{\parallel,T}\mathcal{A}_0^*]$	$R_{\parallel T0}\ \cos\chi_\pi \sin 2\theta_\pi \sin 2\theta^*$
$\mathrm{Re}[\mathcal{A}_{\parallel,T}\mathcal{A}_\perp^*]$	$R_{\parallel T\perp}\ \cos\theta_\pi \sin^2\theta^*$
$\mathrm{Re}[\mathcal{A}_{\parallel,T}\mathcal{A}_\parallel^*]$	$\big[R_{\parallel T\parallel,1} + R_{\parallel T\parallel,2}\,(\cos 2\theta_\pi - 2\cos 2\chi_\pi \sin^2\theta_\pi)\big] \sin^2\theta^*$

Table 6. Contributions to the $\mathcal{N}^R_{i,j}\,\mathrm{Re}[\mathcal{A}_i\mathcal{A}_j^*]$ pieces of eq. (2.15). The coefficients R_i depend on the kinematic parameters q^2 and E_π, and are listed in eq. (B.3).

Helicity info	$\mathcal{N}^I$
$\mathrm{Im}[\mathcal{A}_t\mathcal{A}_\perp^*]$	$I_{t\perp}\sin 2\theta^*\sin\theta_\pi\sin\chi_\pi$
$\mathrm{Im}[\mathcal{A}_{\parallel,T}\mathcal{A}_0^*]$	$I_{\parallel T0}\sin 2\theta^*\sin\theta_\pi\sin\chi_\pi$
$\mathrm{Im}[\mathcal{A}_{SP}\mathcal{A}_\perp^*]$	$I_{SP\perp}\sin 2\theta^*\sin\theta_\pi\sin\chi_\pi$
$\mathrm{Im}[\mathcal{A}_{SP}\mathcal{A}_{\perp,T}^*]$	$I_{SP\perp T}\sin 2\theta^*\sin\theta_\pi\sin\chi_\pi$
$\mathrm{Im}[\mathcal{A}_{0,T}\mathcal{A}_\parallel^*]$	$I_{0T\parallel}\sin 2\theta^*\sin\theta_\pi\sin\chi_\pi$
$\mathrm{Im}[\mathcal{A}_{\perp,T}\mathcal{A}_t^*]$	$I_{\perp Tt}\sin 2\theta^*\sin\theta_\pi\sin\chi_\pi$
$\mathrm{Im}[\mathcal{A}_0\mathcal{A}_\perp^*]$	$I_{0\perp}\sin 2\theta^*\sin 2\theta_\pi\sin\chi_\pi$
$\mathrm{Im}[\mathcal{A}_{0,T}\mathcal{A}_\perp^*]$	$I_{0T\perp}\sin 2\theta^*\sin 2\theta_\pi\sin\chi_\pi$
$\mathrm{Im}[\mathcal{A}_{\perp,T}\mathcal{A}_0^*]$	$I_{\perp T0}\sin 2\theta^*\sin 2\theta_\pi\sin\chi_\pi$
$\mathrm{Im}[\mathcal{A}_\parallel\mathcal{A}_\perp^*]$	$I_{\parallel\perp}\sin^2\theta^*\sin^2\theta_\pi\sin 2\chi_\pi$
$\mathrm{Im}[\mathcal{A}_{\perp,T}\mathcal{A}_\parallel^*]$	$I_{\perp T\parallel}\sin^2\theta^*\sin^2\theta_\pi\sin 2\chi_\pi$
$\mathrm{Im}[\mathcal{A}_{\parallel,T}\mathcal{A}_\perp^*]$	$I_{\parallel T\perp}\sin^2\theta^*\sin^2\theta_\pi\sin 2\chi_\pi$

Table 7. Contributions to the $\mathcal{N}^I_{i,j}\,\mathrm{Im}[\mathcal{A}_i\mathcal{A}_j^*]$ pieces of eq. (2.15). The coefficients I_i depend on the kinematic parameters q^2 and E_π, and are listed in eq. (B.4).

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