A Method for Accurate Transmission Line Impedance Parameter Estimation

Deborah Ritzmann, *Student Member, IEEE,* Paul S. Wright, William Holderbaum, *Member, IEEE,*
and Ben Potter, *Member, IEEE*

Abstract—Real-time estimation of power transmission line impedance parameters has become possible with the availability of synchronized phasor measurements of voltage and current. If sufficiently accurate, the estimated parameter values are a powerful tool for improving the performance of a range of power system monitoring, protection and control applications, including fault location and dynamic thermal line rating. The accuracy of the parameter estimates can be reduced by unknown errors in the synchronized phasors that are introduced in the measurement process. In this paper, a method is proposed with the aim of obtaining accurate estimates of potentially variable impedance parameters, in the presence of systematic errors in voltage and current measurements. The method is based on optimization to identify correction constants for the phasors. A case study of a simulated transmission line is presented to demonstrate the effectiveness of the new method, which is better in comparison with a previously proposed method. The results as well as limits and potential extensions of the new method are discussed.

Index Terms—Accuracy, admittance measurement, impedance measurement, optimization methods, parameter estimation, phasor measurement unit, transmission line measurements

I. INTRODUCTION

CONTINUOUS electricity supply has become one of the backbones of many economies worldwide. For this reason, reliable and efficient operation of power networks is a crucial challenge that needs to keep pace with their increasingly complex nature. Reliability and efficiency are ensured through careful monitoring, protection and control of power systems, which requires a range of electrical measurements as inputs. One of these inputs are the impedance parameters of transmission lines; for example in current differential protection [1] and fault location [2].

Traditionally, parameters were calculated off-line using handbook formulae based on tower geometry and conductor properties [3], [4] or through fault record analysis [5]. These methods are not able to track short-term changes in impedance parameters, which may occur due to Joule heating and ambient temperature variations. Nowadays it is possible to calculate the impedance parameters of transmission lines on-line and in real-time from synchronized phasor (synchrophasor) measurements of voltage and current at both line ends. The

synchrophasors are usually reported by Phasor Measurement Units (PMUs) that are installed in substations [6].

Synchrophasor-based transmission line impedance determination has been investigated by many researchers since the 1990s. Early studies demonstrated the feasibility of the concept and advantages over traditional methods [7]–[9]. The determined parameter values are only useful if they satisfy accuracy requirements, which depend on the specific applications. For fault location [2] and dynamic thermal line rating [10], it is desirable to detect thermally induced variation of the line resistance, which ranges from 1 % to 20 % [11].

Parameter accuracy may be expressed in terms of minimum and maximum limits that are derived from the accuracy of the synchrophasor measurements [1]. The accuracy of the reported synchrophasors is influenced by the entire measurement chain.

PMUs themselves often exceed the requirements of 1 % Total Vector Error and 1 µs time-tagging to UTC, given in IEEE Standard C37.118.1-2011 [12]; for instance, PMUs with accuracies of ± 0.03 % in phasor magnitude and $\pm 0.01°$ in phase angle (± 0.6 µs at 50 Hz) have been manufactured [13]. Hence, if only the accuracy of PMUs is considered, uncertainties in impedance parameter estimates of less than 2 % are possible [1].

It is important to recognize that additional systematic errors of up to 1 % in the magnitude and 1° in the phase angle of the synchrophasors may be introduced by the remaining measurement chain, as is recognized in IEEE Standard C37.242-2013 [14]. The remaining measurement chain includes instrument transformers, cables, burdens and external time synchronization equipment such as GPS antennae and connection cables.

Ideally, these errors should be characterized and corrected before the impedance parameter estimation process. For example, in addition to the nominal transformer ratios, transformer correction factors should be applied, and time-tagging adjusted for delays in the synchronization signal. However, the actual correction factors may differ from their values at the time of characterization due to ageing or modification of the instrumentation channel. Consequently, the measured synchrophasors can be subject to unknown errors, which can have an adverse impact on parameter estimation accuracy.

A number of approaches have been proposed to reduce the impact of random errors in synchrophasor measurements on impedance parameter determination: unbiased linear least squares estimation [15], [16], non-linear least squares algorithms [17]–[19], total least squares estimation [20] as well as optimization procedures [7], [21]. On the other hand, the impact and reduction of systematic errors in the synchrophasor
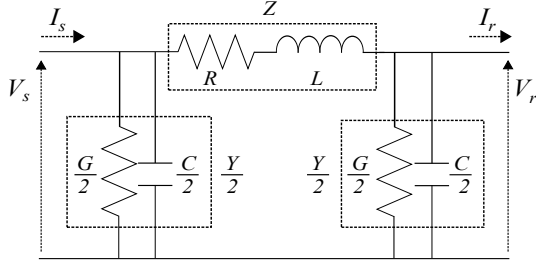
Fig. 1. Nominal pi circuit diagram for a medium length transmission line. It shows the lumped impedance and admittance components as well as the sending and receiving end voltages and currents.

measurements with regards to impedance parameter estimation has received less attention.

One solution is to estimate individual correction factors for both magnitude and phase angle of voltage and current along with the parameters in an optimization procedure [21]. However, this approach makes use of a wide range of measurements and assumes time-invariant impedance parameters. Hence, there is a need to develop effective methods for the reduction of the impact of systematic errors in synchrophasor measurements on real-time impedance parameter estimation.

In a previous paper by the authors, a potential solution for this problem has been proposed [22]. It consists of a method that assumes linear variation of the impedance parameters over short periods of time. Correction constants for the synchrophasors are identified by minimising the residuals of a least squares fit of the calculated parameters to a linear model. The method was shown to effectively reduce the impact of systematic errors in voltage measurements of a short transmission line, neglecting shunt admittance. But for longer lines shunt admittance is significant as it causes the current to vary along the line, and thus needs to be taken into account in parameter calculations. Furthermore, systematic errors can also occur in synchrophasor measurements of current. The aim of this paper is to propose an extension of the method such that it can effectively reduce the impact of systematic errors in all synchrophasor measurements for the general transmission line modelled by the lumped pi circuit, which has both series impedance and shunt admittance.

The rest of the paper is structured as follows: in the next section, models for the transmission line and systematic errors are defined and the proposed method is presented. Thereafter, a case study of a simulated line is considered and a comparison with an existing linear least squares-based method is made. The fourth section is a discussion of the case study results, as well as strengths and limits of the proposed method. The final section concludes the paper.

II. METHODS

A. Transmission Line Model

The nominal pi circuit, as shown in Fig. 1, is the standard model for the electrical parameters of a medium length transmission line (80 km to 240 km) [3]. For medium length lines, the effects of shunt admittance cannot be ignored, but

lumped components are still a good approximation for the actual, distributed parameters [3].

The circuit consists of a series impedance component Z, as well as shunt admittance Y, which is split into two equal components at either end of the line, as shown in Fig. 1. The series impedance Z has resistance R and inductance L, while the shunt admittance Y consists of conductance G and capacitance C. Conductance G is normally considered negligible and omitted from the model; however, it is useful for the consideration and correction of systematic errors as will be shown in Section II-C. The measured currents and voltages at either end of the line are modelled by V_s, I_s, V_r, I_r, which are assumed to be phasors at the nominal power system frequency f; subscript s refers to sending and subscript r to receiving end. By Kirchhoff's Voltage and Current Laws, the circuit equations are

$$V_s = (I_s - \frac{Y}{2}V_s)Z + V_r \tag{1}$$

$$I_s = (V_s + V_r)\frac{Y}{2} + I_r, \tag{2}$$

where $V_s, I_s, V_r, I_r, Z, Y \in \mathbb{C}$, $Z = R + jX$, $X = 2\pi fL$, $Y = G + jB$, $B = 2\pi fC$ and $R, X, G, B, L, C, f \in \mathbb{R}^{>0}$. X is the inductive reactance and B is the capacitive susceptance.

Substitution of (2) into (1) leads to the following formulae for impedance Z and admittance Y:

$$Z = \frac{V_s^2 - V_r^2}{V_sI_r + V_rI_s} \tag{3}$$

$$Y = 2\frac{I_s - I_r}{V_s + V_r}. \tag{4}$$

If a single set of synchronized measurements V_s, I_s, V_r, I_r is available from a PMU or equivalent device, values of Z and Y can be calculated; parameters R, X, G and B are obtained from the real and imaginary parts of Z and Y, respectively.

B. Systematic Errors in the Synchrophasor Measurements

In this paper, systematic errors in the form of a proportional error in the phasor magnitude and additive offset in the phase angle are considered. Let $\tilde{V}_s$ be a synchrophasor measurement of the sending end voltage V_s with systematic errors a_s in magnitude and ϕ_s in phase angle. V_s and $\tilde{V}_s$ are related by

$$V_s = \tilde{V}_s(1 + a_s)\exp(j\phi_s), \tag{5}$$

where $\tilde{V}_s \in \mathbb{C}, a_s, \phi_s \in \mathbb{R}$. This structure is chosen in line with transformer correction factors, which are the general model for expressing errors caused by instrument transformers [23]. The systematic errors are assumed to be constant, since in real-time applications, the utilized voltage and current measurements span only a limited part of the instrument ranges and instrumentation channels are designed for long-term stability.

On the basis of accuracy classes of instrument transformers and previous characterization of instrumentation channels, the errors are assumed to be less than 1% in magnitude and less than $0.01\,\mathrm{rad}$ in phase angle [14]. Thus it is assumed that $|a_s|, |\phi_s| < 0.01$ and the following small angle approximation is made:

$$\exp(j\phi_s) \approx 1 + j\phi_s. \tag{6}$$

Substituting (6) into (5) gives

$$V_s = \tilde{V}_s(1 + a_s)(1 + j\phi_s) = \tilde{V}_s(1 + a_s + j\phi_s + ja_s\phi_s). \quad (7)$$

The lower order term $ja_s\phi_s$ will be omitted. Hence,

$$V_s = \tilde{V}_s(1 + a_s + j\phi_s). \quad (8)$$

Define the overall error $\delta V_s \in \mathbb{C}$ in the synchrophasor measurement $\tilde{V}_s$ as

$$\delta V_s = V_s - \tilde{V}_s = (a_s + j\phi_s)\tilde{V}_s. \quad (9)$$

Similarly, $\tilde{I}_s, \tilde{V}_r, \tilde{I}_r \in \mathbb{C}$ are defined as synchrophasor measurements that have systematic errors $a_r, \phi_r, b_s, \theta_s, b_r, \theta_r$ such that $V_r = \tilde{V}_r(1 + a_r + j\phi_r), I_s = \tilde{I}_s(1 + b_s + j\theta_s), I_r = \tilde{I}_r(1 + b_r + j\theta_r)$, and $\delta I_s, \delta V_r, \delta I_r \in \mathbb{C}$ are overall errors defined as $\delta V_r = (a_r + j\phi_r)\tilde{V}_r, \delta I_s = (b_s + j\theta_s)\tilde{I}_s, \delta I_r = (b_r + j\theta_r)\tilde{I}_r$.

Suppose the values of $a_s, \phi_s, a_r, \phi_r, b_s, \theta_s, b_r, \theta_r$ are unknown. Then the impedance and admittance estimates from synchrophasors with systematic errors are given by

$$\tilde{Z} = \frac{\tilde{V}_s^2 - \tilde{V}_r^2}{\tilde{V}_s\tilde{I}_r + \tilde{V}_r\tilde{I}_s} \quad (10)$$

$$\tilde{Y} = 2\frac{\tilde{I}_s - \tilde{I}_r}{\tilde{V}_s + \tilde{V}_r}, \quad (11)$$

where $\tilde{V}_s, \tilde{I}_s, \tilde{V}_r, \tilde{I}_r$ have been substituted into (3) and (4). $\tilde{Z}$ and $\tilde{Y}$ deviate from Z and Y, respectively, and thus the estimated parameters are in error. This loss of accuracy can be reduced by estimating values of $a_s, \phi_s, a_r, \phi_r, b_s, \theta_s, b_r, \theta_r$ to correct the phasor measurements. The following observations are used to simplify the problem:

- Since Z is proportional to $(V_s^2 - V_r^2)$, it is more sensitive to a_s, ϕ_s, a_r, ϕ_r than to $b_s, \theta_s, b_r, \theta_r$ and the error in Z caused by a_s, ϕ_s is approximately equal and opposite to the error caused by a_r, ϕ_r (see Appendix C).
- Since Y is proportional to $(I_s - I_r)$, it is more sensitive to $b_s, \theta_s, b_r, \theta_r$ than to a_s, ϕ_s, a_r, ϕ_r and the error in Y caused by b_s, θ_s is approximately equal and opposite to the error caused by b_r, θ_r (see Appendix C).

Therefore it is assumed that error constants a_s, a_r, ϕ_s, ϕ_r can be combined into 'net' errors a, ϕ in $\tilde{V}_r$, where $a = a_r - a_s, \phi = \phi_r - \phi_s, |a|, |\phi| < 0.02$. Similarly $b_s, \theta_s, b_r, \theta_r$ are combined into errors b, θ in $\tilde{I}_r$, where $b = b_r - b_s, \theta = \theta_r - \theta_s, |b|, |\theta| < 0.02$.

In the next section a method for estimating values of a, ϕ, b and θ is presented.

C. Proposed Method for Identification of Correction Constants

To reduce the deviations in estimated parameters $\tilde{Z}$ and $\tilde{Y}$ due to systematic errors that were described in the previous subsection, synchrophasor measurements should be corrected before parameters estimates are calculated. A method has been designed to identify such correction constants and is presented in the following paragraphs.

The method assumes no knowledge of the true values of impedance and admittance parameters. Instead, it is assumed

that the behaviour of the resistance and reactance is approximately linear over short periods relative to the thermal time constant of overhead line conductors (5 min to 20 min according to IEEE Standard 738-2012 [24]) because of slow variation in the rate of change of resistance and reactance. Conductance and susceptance are assumed to be constant. Therefore the calculated parameters are fitted to linear models with respect to time. Let the models for R, X, G, B be $f_R, f_X, f_G, f_B : \mathbb{R}^+ \to \mathbb{R}$, respectively, where

$$f_R(t_i) = q_R t_i + r_R \quad (12)$$

$$f_X(t_i) = q_X t_i + r_X \quad (13)$$

$$f_G(t_i) = r_G \quad (14)$$

$$f_B(t_i) = r_B \quad (15)$$

and $q_R, r_R, q_X, r_X, r_G, r_B \in \mathbb{R}$ are constants, which are estimated in a least squares sense from a set of $N \in \mathbb{N}$ parameter values $R_i, X_i, G_i, B_i \in \mathbb{R}$, calculated at time instants $t_i, i = 1, \ldots, N$, with $t_i = i\Delta t$ and $\Delta t \in \mathbb{R}$ the constant time interval between synchrophasor measurements. The time interval $t_N - t_1$ is a moving window that is chosen to be less than the thermal time constant of the line. The details of the estimation of $q_R, r_R, q_X, r_X, r_G, r_B$ are given in Appendix A.

Suppose that R_i, X_i, G_i, B_i are calculated from synchrophasor measurements with systematic errors, then the goodness of fit of f_R, f_X, f_G, f_B is reduced. To measure the goodness of fit, the sum of the squared residual is calculated as

$$S_R = \sum_{i=1}^{N}(R_i - f_R(t_i))^2, \quad (16)$$

where $S_R \in \mathbb{R}^+$. Equivalent expressions are assumed for $S_X, S_B, S_G \in \mathbb{R}^+$, in terms of $X_i, f_X, G_i, f_G, B_i, f_B$, respectively. By minimizing S_R, S_X, S_G and S_B, correction constants can be found that maximize the goodness of fit of f_R, f_X, f_G and f_B, and thus result in impedance and admittance parameter estimates that are more consistent with the expected physical behaviour of the line over time.

S_R and S_X are sensitive to errors in $\tilde{V}_s$ and $\tilde{V}_r$ and can be minimized by finding optimal values of correction constants a, ϕ for $\tilde{V}_r$. Hence, optimization problem 1 is formulated:

$$\begin{aligned} \underset{a,\phi}{\text{minimize}} \quad & g_Z(a, \phi) = S_R + S_X \\ \text{subject to} \quad & |a| < 0.02, |\phi| < 0.02, \end{aligned} \quad (17)$$

with initial values: $a = 0, \phi = 0$. The objective function $g_Z : \mathbb{R}^2 \to \mathbb{R}^+$ is evaluated using correction constants a, ϕ to recalculate the impedance parameters as follows:

$$Z_i = R_i + jX_i = \tilde{Z}_i + \frac{\partial Z}{\partial V_r}\Big|_{V_r = \tilde{V}_{r_i}}\delta V_{r_i}, \quad (18)$$

where $\delta V_{r_i} = (a + j\phi)\tilde{V}_{r_i}$, and $\tilde{Z}_i$ is calculated using (10) from synchrophasor measurements $\tilde{V}_{s_i}, \tilde{I}_{s_i}, \tilde{V}_{r_i}, \tilde{I}_{r_i}$ taken at time t_i. The first order Taylor approximation of Z is taken because δV_{r_i} is small and in this way Z_i remains linear in a, ϕ. The partial derivative $\frac{\partial Z}{\partial V_r}$ is given in Appendix B. Once R_i and X_i have been recalculated using (18), new values for

$q_R, r_R, q_X, r_X, r_G, r_B$ are estimated and S_R as well as S_X are updated to give a new value of g_Z.

Similarly, S_G and S_B are sensitive to errors in $\tilde{I}_s$ and $\tilde{I}_r$. Optimization problem 2 is defined to identify optimal values of correction constants $b, \theta \in \mathbb{R}$ for $\tilde{I}_r$ that minimize S_G and S_B:

$$\underset{b,\theta}{\text{minimize}} \quad g_Y(b, \theta) = \mu(S_G + S_B)$$
$$\text{subject to} \quad |b| < 0.02, |\theta| < 0.02, \tag{19}$$

with initial values: $b = 0, \theta = 0$. Since G and B are of the order of 10^{-6} and 10^{-4}, respectively, S_G and S_B can become very small and factor μ is introduced to avoid bad scaling. The objective function $g_Y : \mathbb{R}^2 \to \mathbb{R}^+$ is evaluated using correction constants b, θ to recalculate the admittance parameters as follows:

$$Y_i = G_i + jB_i = \tilde{Y}_i + \frac{\partial Y}{\partial I_r}\bigg|_{I_r = \tilde{I}_{r_i}} \delta I_{r_i}, \tag{20}$$

where $\delta I_{r_i} = (b + j\theta)\tilde{I}_{r_i}$, and $\tilde{Y}_i$ is calculated using (11) from synchrophasor measurements $\tilde{V}_{s_i}, \tilde{I}_{s_i}, \tilde{V}_{r_i}, \tilde{I}_{r_i}$ taken at time t_i. The first order Taylor approximation of Y is taken because δI_r is small and Y_i remains linear in b, θ. The partial derivative $\frac{\partial Y}{\partial I_r}$ is given in Appendix B.

Both (17) and (19) are nonlinear constrained optimization problems, for which minima can be obtained with a range of algorithms. In this instance the interior-point method was chosen [25]. g_Z and g_Y are convex and thus the local minima are global in the feasible regions. The reason is that Z_i and Y_i are linear in the respective correction constants and estimation of $q_R, r_R, q_X, r_X, r_G, r_B$ (see Appendix A) as well as evaluation of g_Z and g_Y preserve convexity.

Fig. 2 shows a flow chart that summarizes the processes of identifying correction constants and estimating values of the line parameters. The final parameter estimates at a given time t_N are obtained by fitting functions f_R, f_X, f_B to the parameter values calculated from corrected measurements and evaluating $f_R(t_N), f_X(t_N), f_B(t_N)$. The aim of this step is to give parameter estimates with reduced random variation, which occurs in the individually calculated parameter values.

In the next section, the effectiveness of the proposed method is demonstrated in a case study.

III. CASE STUDY

In this section, the specifications of the transmission line simulation are given, and results of the application of the proposed method as well as an existing linear least squares-based method are presented.

A. Transmission Line Simulation

A single phase of the $400\,\text{kV}$, $102\,\text{km}$ long transmission line located between substations Grendon and Staythorpe, East Midlands, England [26], was simulated in *Matlab*. The nominal parameter values are $R_0 = 2.96\,\Omega$, $X_0 = 32.4\,\Omega$ and $B_0 = 3.69 \times 10^{-4}\,\text{S}$. The resistance was assumed to vary sinusoidally within $\pm 4\,\%$ of the nominal value, which corresponds to a change in line temperature of approximately $\pm 10\,°\text{C}$ over the period of the simulation.
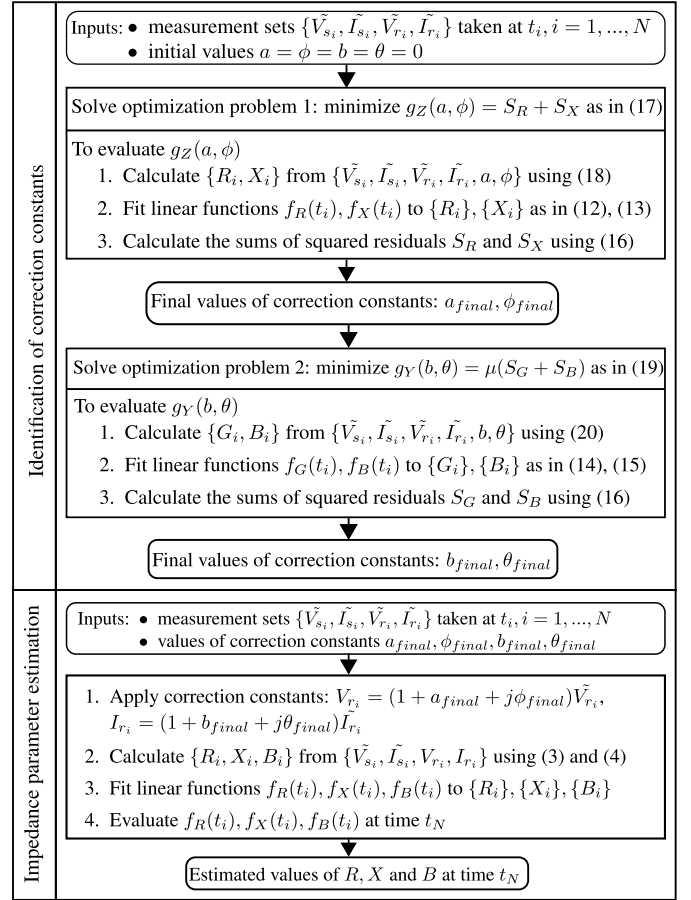
Fig. 2. This flow chart illustrates how values for correction constants and impedance parameters are estimated by the proposed method.
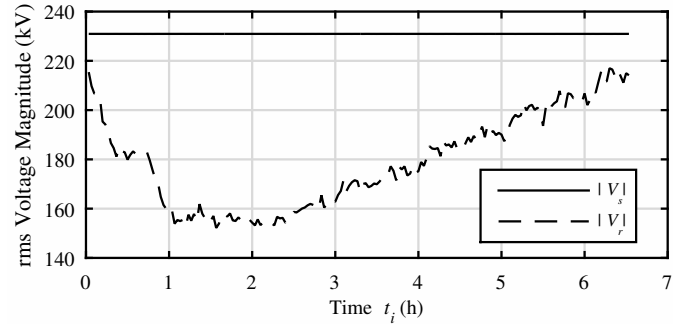
Fig. 3. This graph shows the magnitude of the sending and receiving end voltages over the period of the simulation.

The network at either end of the line was modelled by an equivalent voltage source; Fig. 3 shows the root-mean-square (rms) magnitude of the sending and receiving end voltages. A variable load profile ranging from $15\,\%$ to $100\,\%$ of rated current was assumed to occur over a seven hour period; rms values of current magnitude are shown in Fig. 4. Synchronized measurements of steady-state current and voltage phasors at each line end were taken at time intervals of $\Delta t = 2\,\text{min}$ for blocks of $10\,\text{s}$.
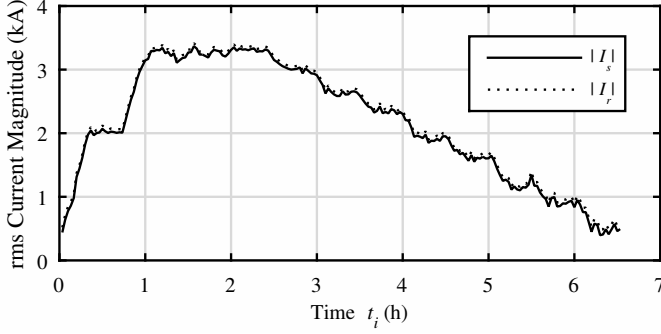
Fig. 4. This graph shows the magnitude of the sending and receiving end currents over the period of the simulation. Their difference is very small compared to the absolute magnitudes.

In order to reflect the measurement uncertainty that would be present in practice, the measurements were contaminated with Gaussian noise of mean zero and standard deviations of 0.03% and 0.04% in magnitudes of voltage and current, respectively, and $0.3\,\mathrm{mrad}$ in all phase angles.

Systematic errors in both sending and receiving end voltages and currents as modelled in Section II-B were applied to all synchrophasor measurements. The mean of the synchrophasors was taken over each $10\,\mathrm{s}$ block to generate an individual set of measurements every two minutes; in total there were 203 measurement sets. A moving window of $N = 8$ measurement points, spanning $16\,\mathrm{min}$, was used to estimate the impedance and admittance parameters of the line in real-time. Thus, 196 estimated values were computed for each of R, X and B.

In order to test the effectiveness of the method, different sets of systematic errors were applied to the measurements. In each case, the magnitude and phase errors were selected randomly from a uniform distribution in the interval $[-0.01, 0.01]$. In total, $100\,000$ cases were studied, giving sufficiently small confidence intervals on the relevant metrics, which will be defined in section III-C.

B. Existing Linear Least Squares Method

The proposed method was applied to identify correction constants a, ϕ, b, θ to improve the accuracy of the calculated Z and Y values. For comparison, an existing linear least squares (LS) method was also applied to each window to obtain parameter estimates [15]. This method models the transmission line as a general two-port network, in which voltages and currents are related by

$$V_s = A * V_r + B * I_r \tag{21}$$

$$I_s = C * V_r + D * I_r, \tag{22}$$

where $A, B, C, D \in \mathbb{C}$ are constants. For the eight measurement sets in a given window, (21) was expanded into two equations by taking the real and imaginary parts to give 16 real equations in total. The real and imaginary parts of A and B were computed through unbiased linear least squares estimation.

TABLE I
SYSTEMATIC ERRORS IN THE SYNCHROPHASOR MEASUREMENTS

	Magnitude		Phase Angle		TVE
$\tilde{V}_s$	$a_s =$	0.0008	$\phi_s =$	0.0059	0.60%
$\tilde{V}_r$	$a_r =$	-0.0021	$\phi_r =$	-0.0076	0.78%
$\tilde{I}_s$	$b_s =$	-0.0016	$\theta_s =$	0.0095	1.02%
$\tilde{I}_r$	$b_r =$	0.0037	$\theta_r =$	-0.0034	0.38%

By assuming a pi circuit (Fig. 1) inside the two-port network, constants A, B, C, D can be expressed in terms of impedance Z and admittance Y:

$$A = 1 + YZ/2 \tag{23}$$

$$B = Z \tag{24}$$

$$C = Y(1 + YZ/4) \tag{25}$$

$$D = 1 + YZ/2. \tag{26}$$

Z and Y are calculated from least squares estimates of A and B using (23) and (24):

$$Z = B \tag{27}$$

$$Y = 2(A - 1)/B. \tag{28}$$

C. Metrics for Evaluation of Method Performance

Two metrics are used to evaluate the accuracy of the impedance and admittance parameter estimates over the simulation period. The first is the rms error $E_{\Delta P}$ calculated over all parameter estimates; it indicates how far the estimates are from the true values.

Let the errors in the individual parameter estimates be $\Delta P_i = P_i - P_0$, where P_i refers to the parameter estimates R_i, X_i, B_i at each time instant $t_i, i = [1 \dots 196]$ and P_0 to the nominal parameter values R_0, X_0, B_0. Then

$$E_{\Delta P} = \frac{1}{P_0}\sqrt{\frac{1}{196}\sum_{i=1}^{196}\Delta P_i^2}. \tag{29}$$

The second metric is $\Sigma_{\Delta P}$, the standard deviation of the parameter errors as a fraction of the nominal values. This metric indicates the variability of the parameter error over the simulation period. $\Sigma_{\Delta P}$ is given by

$$\Sigma_{\Delta P} = \frac{1}{P_0}\sqrt{\frac{1}{195}\sum_{i=1}^{196}(\Delta P_i - \mu_{\Delta P})^2}, \tag{30}$$

where $\mu_{\Delta P} = 1/196 * \sum_{i=1}^{196}\Delta P_i$ is the mean parameter error.

$E_{\Delta P}$ and $\Sigma_{\Delta P}$ are not defined for conductance G as its nominal value is zero.

D. Results

The results of the case study are presented in two parts: first, one individual case with a specific set of systematic errors is considered; then the aggregated results from $100\,000$ cases of systematic errors are presented.
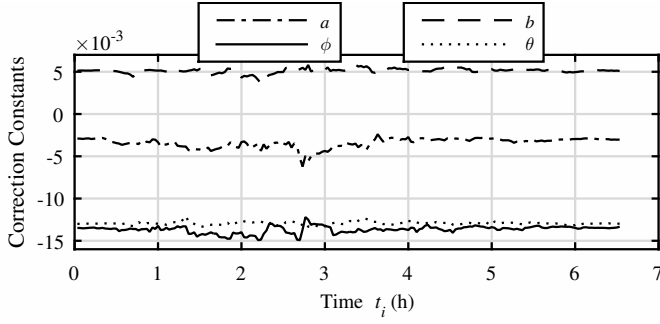
Fig. 5. This plot shows the values of the identified correction constants over time for the individual simulation case.
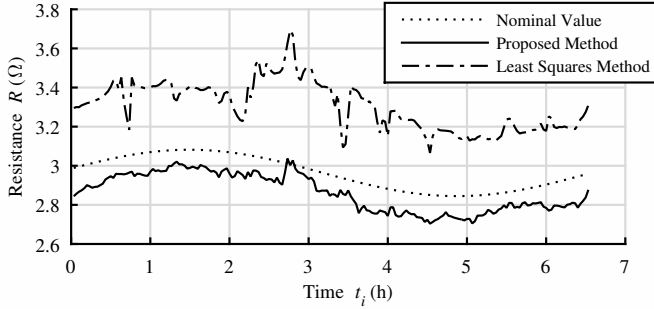

Fig. 6. This plot shows the nominal and estimated values of resistance R over time for the individual simulation case.
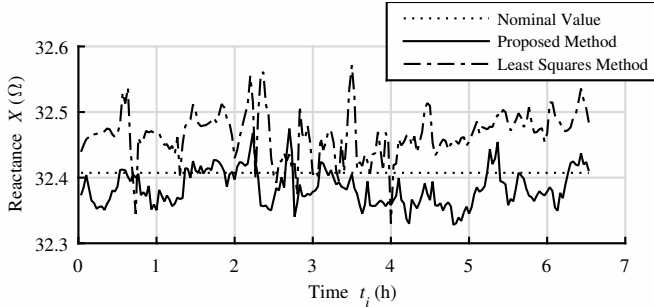

Fig. 7. This plot shows the nominal and estimated values of reactance X over time for the individual simulation case.

1) Individual Case: Table I lists the values of one set of systematic errors that was applied to the voltage and current phasors as well as the resulting Total Vector Errors (TVE). The plot in Fig. 5 shows the values of the correction constants that were identified using a moving window of $N = 8$ measurements as described in section II-C. It can be observed that $a \approx -0.003 \approx a_r - a_s$ (from Table I), which is consistent with the assumption that a corrects the net error. Similar observations can be made for ϕ, b, θ. Fig. 6 to Fig. 8 show the final parameter estimates over the simulation period.

In Table II the root-mean-square and standard deviation of the parameter errors are given. For R, X, B the rms error of the proposed method is significantly smaller than for the existing estimator. Similarly, the standard deviation of the error
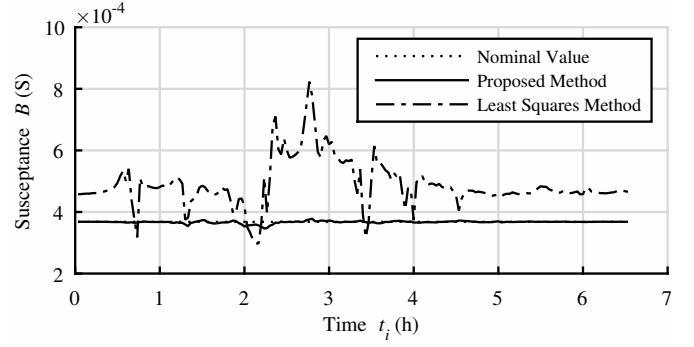

Fig. 8. This plot shows the nominal and estimated values of susceptance B over time for the individual simulation case.

TABLE II
PARAMETER ERRORS FOR ONE INDIVIDUAL CASE

		R	X	B
E_Δ (%)	PM[1]	3.51	0.111	1.11
	LS[2]	11.9	0.212	38.7
Σ_Δ (%)	PM[1]	0.974	0.0855	1.09
	LS[2]	2.85	0.111	20.8

[1] Proposed Method
[2] Least Squares Method

TABLE III
ERRORS IN RESISTANCE R FOR $100\,000$ CASES

		Percentile		
		50th	75th	95th
$E_{\Delta R}$ (%)	PM[1]	6.35(10)	10.8(1)	16.7(1)
	LS[2]	7.07(10)	11.4(1)	17.4(1)
$\Sigma_{\Delta R}$ (%)	PM[1]	0.925(1)	0.998(1)	1.20(1)
	LS[2]	2.85(1)	2.86(1)	2.89(1)

[1] Proposed Method
[2] Least Squares Method

is lower, indicating less variability in the parameter estimates.

2) Large Number of Cases: Tables III to V summarize the results from the simulation of $100\,000$ different cases of systematic error sets. The 50th, 75th and 95th percentile of the distributions of the root-mean-square and standard deviation of parameter errors are listed to give an indication of the level of accuracy and consistency of the applied methods. For each percentile the 95 % confidence interval is given in brackets. For resistance R, the distributions of rms error occupy a similar range for both the proposed and the existing method, with the 95th percentile at 17 %. However, the proposed method yields significantly lower standard deviations of error at around 1 %, whereas the existing method yields 2.9 %. Both methods produce lower errors in reactance X, with rms errors of the order of 1 % and standard deviation of error of approximately 0.1 %. In contrast, for susceptance B the level of error differs greatly between the methods. While the proposed method gives rms errors of 1 % to 2 %, the existing method results in rms errors of over 100 %. The standard deviation of errors is also an order of magnitude larger for the existing method.

TABLE IV
ERRORS IN REACTANCE X FOR 100 000 CASES

		Percentile		
		50th	75th	95th
$E_{\Delta X}$ (%)	PM[1]	0.581(10)	0.984(10)	1.52(1)
	LS[2]	0.597(10)	1.01(1)	1.56(1)
$\Sigma_{\Delta X}$ (%)	PM[1]	0.0818(10)	0.0890(10)	0.108(1)
	LS[2]	0.110(1)	0.111(1)	0.112(1)

[1] Proposed Method
[2] Least Squares Method

TABLE V
ERRORS IN SUSCEPTANCE B FOR 100 000 CASES

		Percentile		
		50th	75th	95th
$E_{\Delta B}$ (%)	PM[1]	1.18(1)	1.39(1)	1.82(1)
	LS[2]	99.8(10)	168(1)	261(1)
$\Sigma_{\Delta B}$ (%)	PM[1]	1.05(1)	1.06(1)	1.08(1)
	LS[2]	21.6(1)	21.9(1)	22.4(1)

[1] Proposed Method
[2] Least Squares Method

IV. DISCUSSION

A. Comparison of Methods

Based on the results presented in the previous section, the proposed method demonstrated equal or better performance compared to the existing linear least squares-based method.

The linear least squares method finds an optimal estimate for the parameters of a two-port network; these are then used to calculate impedance and admittance parameters of the pi line. The advantage of this approach over estimating the pi line parameters directly, is that it makes use of redundant measurements such that constant systematic errors, as modelled in this paper, either cancel or only cause a constant offset in the estimated parameters. However, the linear least squares method also assumes constant parameters in time, and even small variations over a moving window lead to variable parameter errors. This robustness to systematic errors, yet weak accuracy for variable parameters, explains the relatively similar results in the accuracy of the resistance and reactance parameters in the case study for both methods. Therefore it may appear that there is no significant advantage in the suggested method. However, one of the crucial differences is that the proposed method has demonstrated approximately 50% less variability in the errors of resistance values. The resistance is the parameter with the highest temperature sensitivity, hence, it is desirable to monitor changes in its value. This can be done to good accuracy even if there is a constant error in the estimated values, however, the accuracy of the estimated changes deteriorates quickly with increasing error variability.

In field applications, the true value of the impedance and admittance parameters can never be known; thus, the accuracy of estimated parameters has to be assessed on their repeatability and consistency with expected physical variations. Based on these criteria, the proposed method has clear advantages over other estimation techniques.

B. Requirements of the Proposed Method

While the proposed method has strong potential to improve the accuracy of impedance parameter estimation, it also has some limitations. The method relies on increased residuals that are caused by systematic measurement errors to identify correction constants. In the case where the errors have the same size at both line ends ($a_s = a_r, \phi_s = \phi_r, b_s = b_r, \theta_s = \theta_r$) there would be no increase in residuals and hence the method would not yield any improvement. However, in these cases the error in the estimated parameters is constant and only of the order of the systematic errors; therefore the overall effect is small. Increased residuals only occur if there is variation in the load of the transmission line, which is thus a requirement for the method to identify correction constants. The required level of load variation depends on the magnitude of the systematic errors as well as the random noise in the synchrophasor measurements. Measurement noise is in turn related to the overall load level, as the noise increases towards the lower end of instrument scales. A conservative estimate of the minimum load variation would be 10% of maximum line loading. Furthermore, the method assumes that over the time window that spans the utilized measurements the parameters are either constant or varying linearly. This implies that the minimum load variation has to occur within this time, which is limited by the thermal time constant. Depending on the load profile of the transmission line, not all time windows may satisfy these requirements; one possibility of overcoming this issue is to reuse correction constants from previous time windows with higher load variation. To summarize, the applicability and effectiveness of the proposed method depends on the specific circumstances of the transmission line operation and measurement instruments as well as the accuracy requirement for the estimated parameter values. Further work is needed to better understand and predict the relationship between these factors.

C. Limiting Assumptions

In presenting the new method in this paper, some assumptions have been made. Firstly, the method has been defined on a single-phase transmission line model. Most transmission lines in power networks have three phases, which couple and thus require more complex models. In the case of identical conductors and symmetric geometry, the method may be applied to the positive sequence components, provided that the behaviour of the systematic error can be modelled as a proportional error in amplitude and additive in phase angle. Further research is required to confirm whether the method can be effective for various three-phase transmission line systems.

The systematic errors were assumed to be constant, directly proportional in magnitude and additive in the phase angle. The systematic errors may follow different, non-linear models. Over small ranges, these variations may still be approximated well by the error model in this paper. More work is required to investigate if and how the method can be adapted to identify correction constants for other models and if it can be used to select the most appropriate error model.

V. Conclusion

The contribution of this paper is in the field of accurate, real-time synchrophasor-based transmission line impedance parameter estimation.

A method was proposed for estimating the impedance parameters of medium-length transmission lines in the presence of systematic errors in the utilized synchrophasor measurements of voltage and current. The method assumes constant or linearly changing parameters over short periods of time and identifies correction constants through optimization.

The effectiveness of the proposed method was compared with that of an existing linear least squares method in a case study of a simulated transmission line. The results are promising and suggest that the method has significant potential to improve parameter estimation accuracy in practical field applications. Limits of the method and future work have been discussed.

Accurate, real-time synchrophasor-based transmission line impedance parameter estimation is a powerful factor in improving the performance of power system monitoring, protection and control applications and thus in creating more reliable and resilient electricity networks.

Appendix A
Estimation of Constants in Linear Parameter Functions

To estimate q_R, r_R from $R_i, i = [1, \ldots, N]$, vectors $\mathbf{R} \in \mathbb{R}^N, \mathbf{Q_R} \in \mathbb{R}^2$ and matrix $\mathbf{H_Z} \in \mathbb{R}^{N \times 2}$ are defined, where

$$\mathbf{R} = \begin{bmatrix} R_1 & \ldots & R_N \end{bmatrix}^{\mathrm{T}}, \mathbf{Q_R} = \begin{bmatrix} q_R \\ r_R \end{bmatrix}, \mathbf{H_Z} = \begin{bmatrix} t_1 & \ldots & t_N \\ 1 & 1 & 1 \end{bmatrix}^{\mathrm{T}}.$$

The N-dimensional model $\mathbf{R}$, based on the theoretical model $f_R(t_i) = q_R t_i + r_R$, is given by the matrix equation

$$\mathbf{R} = \mathbf{H_Z Q_R} + \varepsilon, \tag{31}$$

where $\varepsilon = [\varepsilon_1, \ldots, \varepsilon_N]^{\mathrm{T}}$ are error terms. To satisfy the least squares criterion, $\min \sum_{i=1}^N \varepsilon_i^2$, $\mathbf{Q_R}$ is computed using

$$\mathbf{Q_R} = (\mathbf{H_Z^T H_Z})^{-1} \mathbf{H_Z^T R}. \tag{32}$$

In the same manner, q_X, r_X are calculated using vectors $\mathbf{X} \in \mathbb{R}^N, \mathbf{X} = \begin{bmatrix} X_1 & \ldots & X_N \end{bmatrix}^{\mathrm{T}}$ and $\mathbf{Q_X} \in \mathbb{R}^2, \mathbf{Q_X} = \begin{bmatrix} q_X & r_X \end{bmatrix}^{\mathrm{T}}$.

To estimate q_G from $G_i, i = [1, \ldots, N]$ vectors $\mathbf{G}, \mathbf{H_Y} \in \mathbb{R}^N$ and $\mathbf{Q_G} \in \mathbb{R}$ are defined, where

$$\mathbf{G} = \begin{bmatrix} G_1 & \ldots & G_N \end{bmatrix}^{\mathrm{T}}, \mathbf{H_Y} = \begin{bmatrix} 1 & \ldots & 1 \end{bmatrix}^{\mathrm{T}}, \mathbf{Q_G} = \begin{bmatrix} r_G \end{bmatrix}.$$

The N-dimensional model $\mathbf{G}$, based on the theoretical model $f_G(t_i) = r_G$, is given by

$$\mathbf{G} = \mathbf{H_Y Q_G} + \varepsilon, \tag{33}$$

where $\varepsilon = [\varepsilon_1, \ldots, \varepsilon_N]^{\mathrm{T}}$ are error terms. To satisfy the least squares criterion, $\min \sum_{i=1}^N \varepsilon_i^2$, $\mathbf{Q_G}$ is computed by

$$\mathbf{Q_G} = (\mathbf{H_Y^T H_Y})^{-1} \mathbf{H_Y^T G}. \tag{34}$$

In the same manner, r_B is calculated using vector $\mathbf{B} \in \mathbb{R}^N, \mathbf{B} = \begin{bmatrix} B_1 & \ldots & B_N \end{bmatrix}^{\mathrm{T}}$ and $\mathbf{Q_B} \in \mathbb{R}, \mathbf{Q_B} = \begin{bmatrix} r_B \end{bmatrix}$.

Appendix B
Partial Derivatives of Z and Y

Let $V_s, I_s, V_r, I_r \in \mathbb{C}, \Omega = \mathbb{C}^4 \setminus \{V_s I_r + V_r I_s = 0\}, \Gamma = \mathbb{C}^4 \setminus \{V_s + V_r = 0\}$. Define complex functions $Z : \Omega \to \mathbb{C}, Y : \Gamma \to \mathbb{C}$, where

$$Z = (V_s^2 - V_r^2)/(V_s I_r + V_r I_s) \tag{35}$$

$$Y = 2(I_s - I_r)(V_s + V_r). \tag{36}$$

Rewrite Z as $Z = h_1/h_2$ and Y as $Y = h_3/h_4$, where $h_1 : \mathbb{C}^2 \to \mathbb{C}, h_2 : \Omega \to \mathbb{C}, h_3 : \mathbb{C}^2 \to \mathbb{C}, h_4 : \mathbb{C}^2 \setminus \{V_s + V_r = 0\} \to \mathbb{C}$,

$$h_1 = V_s^2 - V_r^2, h_2 = V_s I_r + V_r I_s \tag{37}$$

$$h_3 = 2(I_s - I_r), h_4 = V_s + V_r. \tag{38}$$

Since h_1, h_2, h_3, h_4 are complex polynomials, Z and Y are rational functions. By the differentiability of complex polynomials and the quotient rule, Z and Y are differentiable at all points in Ω and Γ, respectively. The partial derivatives of Z with respect to V_s and V_r are

$$\begin{aligned} \frac{\partial Z}{\partial V_s} &= \frac{2V_s}{V_s I_r + V_r I_s} - \frac{(V_s^2 - V_r^2)I_r}{(V_s I_r + V_r I_s)^2} \\ \frac{\partial Z}{\partial V_r} &= \frac{-2V_r}{V_s I_r + V_r I_s} - \frac{(V_s^2 - V_r^2)I_s}{(V_s I_r + V_r I_s)^2}. \end{aligned} \tag{39}$$

The partial derivatives of Y with respect to I_s and I_r are

$$\frac{\partial Y}{\partial I_s} = \frac{2}{V_s + V_r}, \frac{\partial Y}{\partial I_r} = -\frac{2}{V_s + V_r}. \tag{40}$$

Appendix C
Approximation: Errors at One Line End

To a first order linear approximation, the change in Z caused by changes in V_s and V_r is given by

$$\begin{aligned} \delta Z &= \frac{\partial Z}{\partial V_s} \delta V_s + \frac{\partial Z}{\partial V_r} \delta V_r \\ &= \frac{2(V_s \delta V_s - V_r \delta V_r)}{V_s I_r + V_r I_s} - \frac{(V_s^2 - V_r^2)(I_r \delta V_s + I_s \delta V_r)}{(V_s I_r + V_r I_s)^2}, \end{aligned} \tag{41}$$

where results from Appendix B have been used. Let the relative change in Z be

$$\Delta Z = \frac{\delta Z}{Z} = \frac{2(V_s \delta V_s - V_r \delta V_r)}{V_s^2 - V_r^2} - \frac{I_r \delta V_s + I_s \delta V_r}{V_s I_r + V_r I_s}. \tag{42}$$

Suppose errors are modelled at both line ends by $\delta V_s = (a_s + j\phi_s)\tilde{V}_s$ and $\delta V_r = (a_r + j\phi_r)\tilde{V}_r$. Then the relative change around $\tilde{V}_s, \tilde{V}_r$ is

$$\Delta Z_{exact} = \frac{2((a_s + j\phi_s)\tilde{V}_s^2 - (a_r + j\phi_r)\tilde{V}_r^2)}{\tilde{V}_s^2 - \tilde{V}_r^2}, \tag{43}$$

where only the first, dominant term is considered. Now suppose all errors are modelled to be in $\tilde{V}_r$, such that $\delta V_s = 0, \delta V_r = (a + j\phi)\tilde{V}_r$ where $a = a_r - a_s, \phi = \phi_r - \phi_s$. Then the relative error becomes

$$\Delta Z_{app} = \frac{-2(a_r - a_s + j\phi_r - j\phi_s)\tilde{V}_r^2}{\tilde{V}_s^2 - \tilde{V}_r^2}. \tag{44}$$

The difference between the exact and approximate relative error is

$$\Delta Z_{exact} - \Delta Z_{app} = \frac{2(a_s + j\phi_s)(\tilde{V_s}^2 - \tilde{V_r}^2)}{\tilde{V_s}^2 - \tilde{V_r}^2} = 2(a_s + j\phi_s).$$
(45)

Hence, by modelling all error to be in $\tilde{V_r}$, an approximation of $2(a_s + j\phi_s)$ is made in the relative error of the impedance, which is constant and of a lower order than the overall error ΔZ_{exact}. Using an equivalent expression for ΔY, a similar argument can be produced for modelling all errors in current in $\tilde{I_r}$.

REFERENCES

[1] G. Sivanagaraju, S. Chakrabarti, and S. Srivastava, "Uncertainty in transmission line parameters: estimation and impact on line current differential protection," *IEEE Trans. Instrum. Meas.*, vol. 63, no. 6, pp. 1496–1504, Jun 2014.

[2] J.-a. Jiang, J.-z. Yang, Y.-h. Lin, C.-w. Liu, and J.-c. Ma, "An adaptive PMU based fault detection/location technique for transmission lines. I. Theory and algorithms," *IEEE Trans. Power Deliv.*, vol. 15, no. 2, pp. 486–493, Apr 2000.

[3] J. Zaborszky and J. W. Rittenhouse, *Electric Power Transmission: the Power System in the Steady State*. New York, NY, USA: The Ronald Press Company, 1954.

[4] H. Dommel, "Overhead line parameters from handbook formulas and computer programs," *IEEE Trans. Power Appar. Syst.*, vol. PAS-104, no. 2, pp. 366–372, Feb 1985.

[5] R. Schulze, P. Schegner, and R. Zivanovic, "Parameter identification of unsymmetrical transmission lines using fault records obtained from protective relays," *IEEE Trans. Power Deliv.*, vol. 26, no. 2, pp. 1265–1272, Apr 2011.

[6] J. De La Ree, V. Centeno, J. S. Thorp, and A. G. Phadke, "Synchronized phasor measurement applications in power systems," *IEEE Trans. Smart Grid*, vol. 1, no. 1, pp. 20–27, Jun 2010.

[7] L. Philippot and J.-C. Maun, "An application of synchronous phasor measurement to the estimation of the parameters of an overhead transmission line," in *Fault Disturb. Anal. Precise Meas. Power Syst.*, Arlington, VA, 1995.

[8] R. E. Wilson, G. A. Zevenbergen, D. L. Mah, and A. J. Murphy, "Calculation of transmission line parameters from synchronized measurements," *Electr. Mach. Power Syst.*, vol. 27, no. 12, pp. 1269–1278, Nov 1999.

[9] I.-D. Kim and R. K. Aggarwal, "A study on the on-line measurement of transmission line impedances for improved relaying protection," *Int. J. Electr. Power Energy Syst.*, vol. 28, no. 6, pp. 359–366, Jul 2006.

[10] R. Mai, L. Fu, and Xu HaiBo, "Dynamic line rating estimator with synchronized phasor measurement," in *2011 Int. Conf. Adv. Power Syst. Autom. Prot.*, Oct 2011, pp. 940–945.

[11] M. Bockarjova and G. Andersson, "Transmission line conductor temperature impact on state estimation accuracy," in *2007 IEEE Lausanne Power Tech*, 2007, pp. 701–706.

[12] *IEEE Standard for Synchrophasor Measurements for Power Systems*, IEEE Standard C37.118.1, 2011.

[13] "Model 1133a power sentinel GPS-synchronized power quality revenue standard operation manual," Arbiter Systems, Paso Robles, CA, USA, Tech. Rep., 2009.

[14] *IEEE Guide for Synchronization, Calibration, Testing, and Installation of Phasor Measurement Units (PMUs) for Power System Protection and Control*, IEEE Standard C37.242, 2013.

[15] D. Shi, D. J. Tylavsky, N. Logic, and K. M. Koellner, "Identification of short transmission-line parameters from synchrophasor measurements," in *40th North Am. Power Symp.*, Sep 2008, pp. 1–8.

[16] D. Shi, D. J. Tylavsky, K. M. Koellner, N. Logic, and D. E. Wheeler, "Transmission line parameter identification using PMU measurements," *Eur. Trans. Electr. Power*, vol. 21, no. 4, pp. 1574–1588, 2011.

[17] C. Indulkar and K. Ramalingam, "Estimation of transmission line parameters from measurements," *Int. J. Electr. Power Energy Syst.*, vol. 30, no. 5, pp. 337–342, Jun 2008.

[18] Y. Liao and M. Kezunovic, "Online optimal transmission line parameter estimation for relaying applications," *IEEE Trans. Power Deliv.*, vol. 24, no. 1, pp. 96–102, Jan 2009.

[19] C. Borda, A. Olarte, and H. Diaz, "PMU-based line and transformer parameter estimation," in *2009 IEEE/PES Power Syst. Conf. Expo.*, Mar 2009, pp. 1–8.

[20] K. Dasgupta and S. A. Soman, "Line parameter estimation using phasor measurements by the total least squares approach," in *2013 IEEE Power Energy Soc. Gen. Meet.*, 2013, pp. 1–5.

[21] A. M. Dan and D. Raisz, "Estimation of transmission line parameters using wide-area measurement method," in *2011 IEEE Trondheim PowerTech*, Jun 2011, pp. 1–6.

[22] D. Ritzmann, W. Holderbaum, B. Potter, and P. S. Wright, "Improving the accuracy of synchrophasor-based overhead line impedance measurement," in *IEEE Int. Work. Appl. Meas. Power Syst.*, 2015, pp. 132–137.

[23] *IEEE Standard Requirements for Instrument Transformers*, IEEE Standard C57.13-2008, 2008.

[24] *IEEE Standard for Calculating the Current-Temperature Relationship of Bare Overhead Conductors*, IEEE Standard 738-2012, 2013.

[25] R. H. Byrd, M. E. Hribar, and J. Nocedal, "An Interior Point Algorithm for Large-Scale Nonlinear Programming," *SIAM J. Optim.*, vol. 9, no. 4, pp. 877–900, Jan 1999.

[26] "Electricity Ten Year Statement, Appendix B - System Data," National Grid, Warwick, UK, Tech. Rep., 2014.

Deborah Ritzmann (S'15) was born in Germany. She received the B.Sc. degree in mathematics and physics from University College London, London, UK, in 2012 and is currently pursuing the Ph.D. degree at the University of Reading, Reading, UK, in collaboration with the National Physical Laboratory, Teddington, UK.

The focus of her research is synchrophasor-based overhead line impedance measurement for dynamic line rating applications.

Paul S. Wright received the B.Sc. and Ph.D. degrees in electrical and electronic engineering from the University of Surrey, Surrey, UK, in 1987 and 2002.

He spent three years as a research fellow at the University of Surrey working in the field of spacecraft sensors and attitude control. This was followed by three years at the Central Electricity Research Laboratory where he worked on advanced control systems. In 1992 he joined the National Physical Laboratory, Teddington, UK, where he is a Principle Research Scientist specializing in a.c. measurements and waveform analysis. His present research interests include a.c. power standards, a.c./d.c. transfer measurements, digital sampling systems and the analysis of non-sinusoidal / non-stationary waveforms applied to power-quality measurements and smart grid development.

Dr. Wright is a Chartered Engineer and member of the Institution of Engineering and Technology.

William Holderbaum (M'01) received the Ph.D. degree in automatic control from the University of Lille, Lille, France, in 1999.

He was a Research Assistant at the University of Glasgow, Glasgow, UK, from 1999 to 2001. Currently, he is a Professor in the School of Systems Engineering at the University of Reading, Reading, UK. His research interests are in control theory and its applications. These are mainly focused on geometric control theory in particular Hamiltonian systems and optimization problems. This research uses the mathematical engineering skills in order to minimize the energy consumption. Further research relates to control theory in the definition of multiple agent systems for distributed network composed of storages, loads and generators.

Ben Potter (S'97–M'01) received the M.Eng. degree in engineering science from the University of Oxford, Oxford, UK, in 2001 and the Ph.D. degree in the modelling of induction machines from the University of Reading, Reading, UK, in 2005.

He subsequently managed research and development activity for several years at Moog Components Group Ltd., including development work on wireless power transfer. He joined the University of Reading in 2009 and is currently Associate Professor of Energy Systems Engineering. His academic research has been focused on energy systems and power electronics, of various flavours, for over twelve years, with applications including energy storage, electric machines, wind turbines and wireless power transfer. In 2010, he founded the Energy Research Lab within the School of Systems Engineering, and this lab focuses on the development of control methods for the new generation of energy networks the smart grid to ensure that the integration of active elements such as renewable energy resources, electric vehicles and energy storage devices will have positive impacts for both network operators and end users.