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A METHOD FOR DETERMINING THE DISTRIBUTION OF LOADS IN ROLLING PAIRS IN CYCLOIDAL PLANETARY GEAR

METODA WYZNACZANIA ROZKŁADÓW OBCIĄŻEŃ W WĘZŁACH TOCZNYCH OBIEGOWEJ PRZEKŁADNI CYKLOIDALNEJ

Key words: cycloidal planetary gear, rolling pairs, distribution of load.

Abstract: The power transmission system in the cycloidal planetary gear is created by a serial connection of three rolling pairs: central cylindrical roller bearings, a set of rolling pins in the straight-line mechanism, and cycloidal meshing. The paper presents the numerical method for determining the distribution of forces acting on each rolling pair of this gear. Unlike analytical methods, numerical methods allow one to find that distribution in corrected meshing. Geometrical dimensions used in the equations of balance for the planetary gear transmission Palmgren`s dependences for the deformation line contact were used to calculate forces between co-operating elements. Once the distribution of load is known, one can predict the fatigue life of Cyclo's gears in rolling pairs. The fatigue of rolling pairs is a very good criterion to optimize geometrical parameters of the power transmission system.

Słowa kluczowe: obiegowa przekładnia cykloidalna, węzły toczne, rozkład obciążenia.

Streszczenie: Układ przeniesienia mocy obiegowej przekładni cykloidalnej tworzy szeregowe połączenie trzech węzłów tocznych – walcowych łożysk centralnych, zestawu tocznych sworzni w mechanizmie równowodowym oraz zazębienia cykloidalnego. W pracy przedstawiono numeryczną metodę wyznaczania rozkładów sił występujących w poszczególnych węzłach tocznych obiegowej przekładni cykloidalnej. W przeciwieństwie do metod analitycznych metoda numeryczna pozwala również na znalezienie rozkładów obciążenia dla przekładni cykloidalnej z korygowanym zazębieniem. Zostały wyznaczone wielkości geometryczne, które wykorzystano w przedstawionych w artykule równaniach równowagi koła obiegowego przekładni. Do obliczenia sił w stykach współpracujących elementów zastosowano zależność Palmgrena na odkształcenie w styku liniowym. Znajomość rozkładów obciążenia umożliwia prognozowanie trwałości zmęczeniowej węzłów tocznych obiegowej przekładni cykloidalnej. Trwałość zmęczeniowa węzłów tocznych stanowi bardzo dobre kryterium optymalizacji parametrów geometrycznych przekładni.

INTRODUCTION

Planetary gears are more and more frequently used in driving systems. They have a number of advantages. The cycloidal gear, also known as the Cyclo gear, has the most compact design of all types of gears. The rolling friction necessary in power transmission is provided by applying internal eccentric meshing gear [**L. 16**]. The Cyclo gear power transmission system consists of a serial connection of three rolling pairs: central cylindrical roller bearings (1), a set of rolling pins in the straight-line mechanism (2), and cycloidal meshing (3) (**Fig. 1**).

Meshing is formed by planetary wheels (usually two, staggered by an angle of 180°) co-operating with a fixed set of z_k rollers. Each planetary gear has external teeth in the form of the equidistant of shortened epicycloid. Each planetary wheel has the number of teeth $| z_s = | i_c |$ (where i_c is the gear ratio) [**L. 3, 4**]. The input torque M_h is transmitted via central bearings mounted on the input

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eccentric shaft to the planetary gears. To output torque, $M_1 = 2\ M_c$, from the planetary gears to the output shaft, a straight-line mechanism is used which is formed by the holes in the planetary gears in which pivotal sleeves are rolling. The pivots are rigidly connected to the disc of the output shaft. The third torque M_2 loads a set of pivotal sleeves on stationary pins.

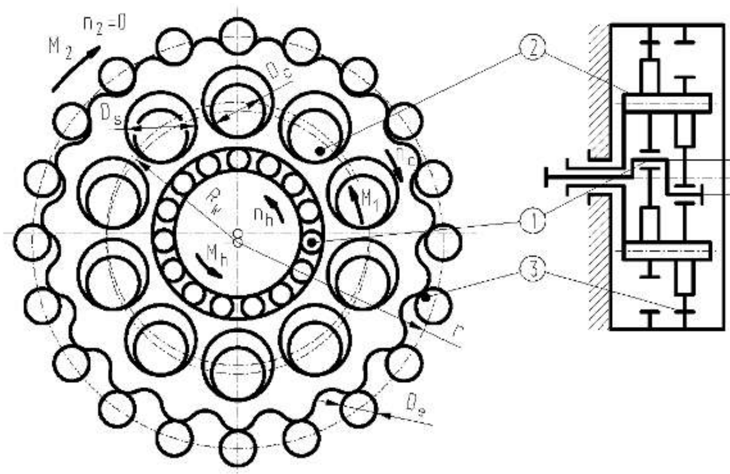

Fig. 1. Cycloidal planetary gear operation principle [L. 4, 5]
Rys. 1. Zasada działania obiegowej przekładni cykloidalnej [**L. 4, 5**]

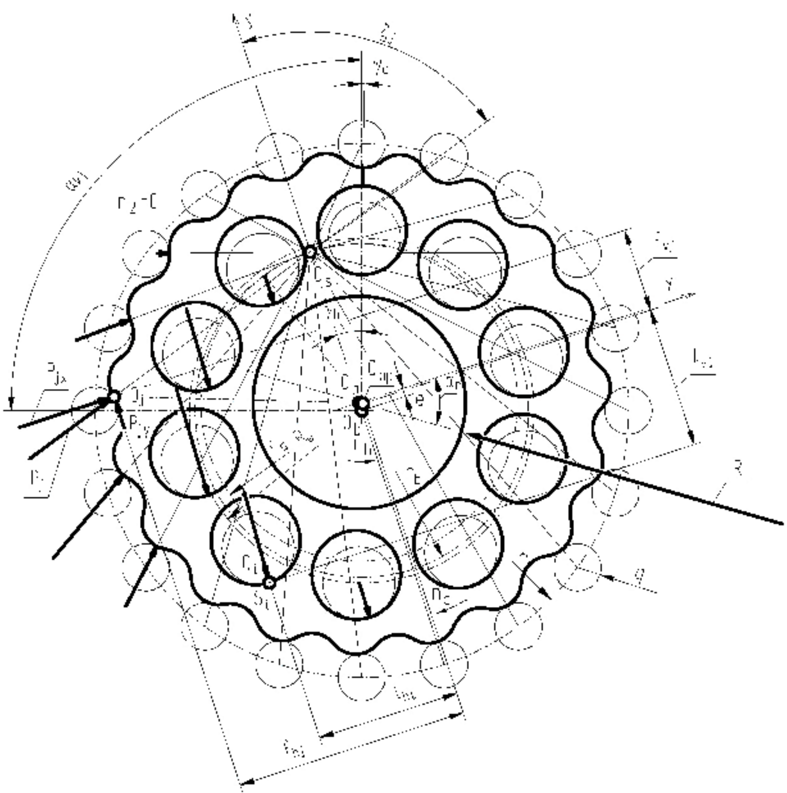

Fig. 2. The geometric parameters used in determining the force distribution between the teeth
Rys. 2. Parametry geometryczne wykorzystywane do wyznaczenia rozkładu sił międzyzębnych

Acting of torques M_h, M_1 and M_2 causes that three unknown loads appear in the contact surfaces of rolling elements (**Fig. 2**):

- The distribution of forces P_j,
- The load distribution on the pins of the straight-line mechanism Q_i, and
- The load distribution on the central bearing rollers caused by impact force *R* in the eccentric input shaft.

Loads *Pj* and *Qi* are the reactions of the bonds resulting from the torque *Mh* load is transmitted to the planetary gear through force *R*. Forces *Pj* and *Qi* are a function of displacements that arise from the actions of these forces. These forces, after the adoption of the relevant simplifications, can be determined by analytical methods. The analytical method allows one to specify the load distribution only for nominal meshing, which is characterized by a lack of inter-meshing clearance. Because of the unavoidable inaccuracy, the nominal gearing could not work, because interference would exist in the meshing. The proper cooperation of the teeth of the planetary gears and rollers of the stationary disc is provided by suitable modification (correction) of the meshing consists in changing the parameters describing the outline of equidistant teeth of the planetary gears [**L. 4, 5**]. For corrected meshing, the analytical method ceases to be useful. To determine the distribution of forces *Pj* and *Qi,* it is necessary to use numerical methods.

Up to now, the most accurate information of the Cyclo load distribution in the transmission is obtained by using the finite element method [**L. 4, 79**]. This method allows the modelling of the actual characteristics of the transmission, including the flexibility of planetary gears, and the actual shape of the teeth. However, it is very time-consuming. Therefore, analytical methods and simplified numerical methods are still used to determine load distributions [**L. 10**]. This paper presents the equations determining load distributions in the rolling pairs of the Cyclo gear using numerical methods. This methodology, which is a development on the method described in [**L. 5**], takes into account the calculation of the actual geometry of the transmission while allowing the execution of the calculation for a number of data variants in a relatively short time.

FORCE DISTRIBUTION BETWEEN THE TEETH

To determine the force distribution between the teeth in the Cyclo gear, the following simplifying assumptions were adopted:

- The deformations of the planetary gear disk do not exist (except for local deformations in the contacts).
- The load is evenly distributed on the planetary wheels.
- The directions of the forces P_j acting on the individual teeth form a bundle of lines intersecting at the rolling point of the meshing O_s (**Fig. 2**).
- The force in contact of the planetary gear tooth and the co-operating roller *j*-th of the stationary disc is a function of deflection δ_{sj} existing there, resulting from the rotation angle of the planetary gear of angle β_s.

Due to the small angle ${}_s$, the deformation δ_{sj} can be defined as the difference in the length of the segment $O_{spo}O_{jkpo}$, plus an amount resulting from the displacement of the planetary gear, and the length of the segment O_sO_j, which determines the distance of the rolling point of the meshing O_s from the point O_j, which is the point of contact the nominal equidistant (without correction) with the co-operating wheel roller of a radius q. $O_{spo}O_{jkpo}$ length of the segment is equal to the length of the segment O_sO_{jk}, which describes the distance from the point O_s to the point O_{jk}, which is the tangent point of the circle with q_k radius and the corrected equidistant. The deformation δ_{sj} is described by the following relation (**Fig. 3**):

$$\delta_{sj} = O_{spo}O_{jkpo} + \delta_{s0}\cos\kappa_j - O_sO_j \tag{1}$$

Length of the segments O_sO_j and $O_{spo}O_{jkpo} = O_sO_{jk}$ can be determined from the following formulas:

$$O_sO_j = \frac{r\sin\left(\alpha_{kj} - \gamma_h\right)}{\sin\zeta_j} - q, \qquad O_{spo}O_{jkpo} = \frac{r_k\sin\left(\alpha_{kj} - \gamma_h\right)}{\sin\zeta_{jk}} - q_k, \tag{2}$$

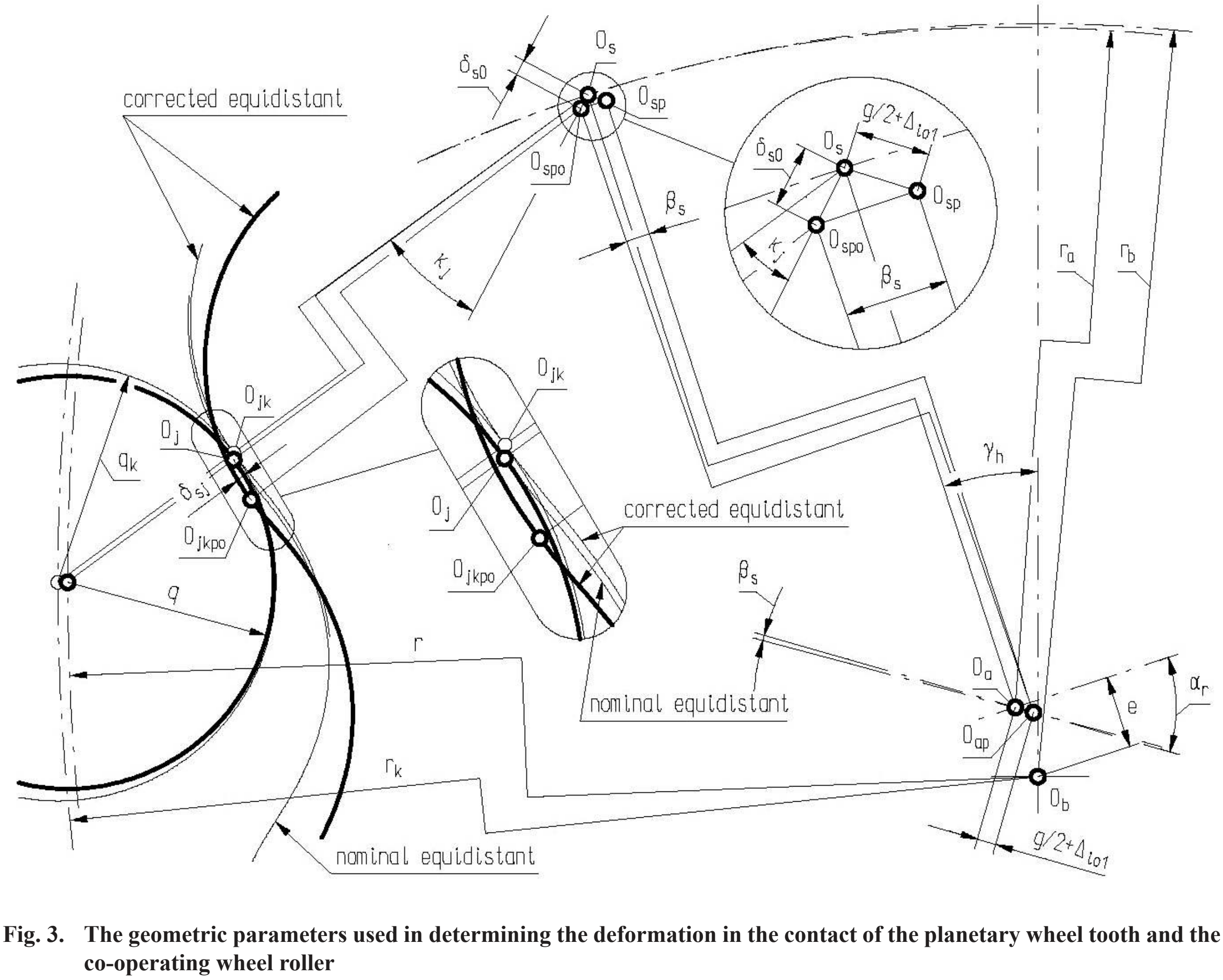

Fig. 3. The geometric parameters used in determining the deformation in the contact of the planetary wheel tooth and the co-operating wheel roller

Rys. 3. Parametry geometryczne wykorzystywane do wyznaczenia odkształceń w styku zęba koła obiegowego z rolką koła współpracującego where

$$\zeta_j = arctan\left[\frac{r\sin\left(\alpha_{kj}-\gamma_h\right)}{r_a+e-r\cos\left(\alpha_{kj}-\gamma_h\right)}\right],$$
$$\zeta_{jk} = arctan\left[\frac{r_k\sin\left(\alpha_{kj}-\gamma_h\right)}{r_a+e-r_k\cos\left(\alpha_{kj}-\gamma_h\right)}\right], \tag{3}$$

$$\alpha_{kj} = 2\pi\left(j-1\right)/z_k, \quad j=1\ 2,\dots z_k\ . \tag{4}$$

While the lengths of O_sO_j and $O_{spo}O_{jkpo}$ are a function of the geometrical parameters of the transmission, displacement δ_{s0} and angle κ_j depend on two variables:

- The planetary gear rotation angle β_s; and,
- The displacement of the planetary gear centre from point O_a to point O_{ap} arising from the action of the resultant force R in the central bearing:

$$O_aO_{ap} = O_sO_{sp} = g/2+\Delta_{io1}, \tag{5}$$

where g is the radial clearance in the bearing and Δ_{io1} is the sum of the deformation of the most loaded roller contacted with the raceways of the centre bearing.

The force R, which determines the size of Δ_{io1}, is a function of the eccentricity e, the action angle of the resultant force in the central bearing α_r and torque M_h. For the gear of two planetary wheels, the drive torque is represented by the following formula:

$$M_h = 2R\cdot e\cos\alpha_r. \tag{6}$$

Deformations δ_{s0} and the angle κ_j take the following form:

$$\kappa_j = \zeta_j - arctan\left[\frac{r_a\sin\beta_s - O_aO_{ap}\cos\alpha_r}{r_a\left(1-\cos\beta_s\right)+O_aO_{ap}\sin\alpha_r}\right], \tag{7}$$

$$\delta_{s0} = \left\{\left[r_a\left(1-\cos\beta_s\right)+O_aO_{ap}\sin\alpha_r\right]^2 + \left[r_a\sin\beta_s - O_aO_{ap}\cos\alpha_r\right]^2\right\}^{1/2}. \tag{8}$$

Assuming that the roller-tooth contact is Hertzian linear, the deformation that is the result of planetary disc angular displacement causes the contact reaction force equal [**L. 11, 12**]:

$$P_j = 78000\,\delta_{sj}^{10/9} l_e^{8/9}, \tag{9}$$

where l_e is a the width of a tooth in the planetary wheel.

To determine the moment resulting from the forces P_j, with respect to O_{ap}, it is necessary to know the arms of the forces P_{jx}, and P_{jy} (**Fig. 2**). These values are described in the following relations:

$$f_{hj} = r\sin\left(\alpha_{kj}-\gamma_h\right) - q\sin\zeta_j + O_aO_{ap}\cos\alpha_r, \tag{10}$$

$$f_{vj} = r\cos\left(\alpha_{kj}-\gamma_h\right) + q\cos\zeta_j - e + O_aO_{ap}\sin\alpha_r. \tag{11}$$

DISTRIBUTION OF FORCES IN STRAIGHT-LINE MECHANISM

The distribution of forces in the straight-line mechanism was determined on the assumption that the force Q_i in the contact between *i*-roller and the proper hole is the function of the deformation δ_{ci}, as a consequence of both the rotation angle of the planetary gear β_s and the rotation angle of the output shaft β_c. Similarly, as in the case of the gear teeth contact of the planetary wheel with the co-operating wheel roller, the force Q_i is equal to the following:

$$Q_i = 78000\,\delta_{ci}^{10/9} l_e^{8/9}. \tag{12}$$

The deformation δ_{cj} can be determined from the relation (**Fig. 4**):

$$\delta_{ci} = h_i + D_c/2 - D_s/2. \tag{13}$$

The distance between the geometric centre of the hole in the wheel and the geometric centre of the straight-line mechanism roller is described by the following equation:

$$h_i = \left\{\left[R_w\left(\sin\xi_{ci}-\sin\xi_{si}\right)+O_aO_{ap}\cos\alpha_r\right]^2 + \left[R_w\left(\cos\xi_{si}-\cos\xi_{ci}\right)+e-O_aO_{ap}\sin\alpha_r\right]^2\right\}^{1/2}, \tag{14}$$

$$\varepsilon_i = arctan\frac{R_w\left(\sin\xi_{ci}-\sin\xi_{si}\right)+O_aO_{ap}\cos\alpha_r}{R_w\left(\cos\xi_{si}-\cos\xi_{ci}\right)+e-O_aO_{ap}\sin\alpha_r}, \tag{15}$$

$$\xi_{si} = \alpha_{ci} + \beta_s - \left(\gamma_c+\gamma_h\right), \tag{16}$$

$$\xi_{ci} = \alpha_{ci} + \beta_c - \left(\gamma_c+\gamma_h\right), \tag{17}$$

$$\alpha_{ci} = 2\pi\left(i-1\right)/z_c, \quad i=1,\ 2,\dots z_c\ . \tag{18}$$

Component action force arms Q_{ix}, Q_{iy} with respect to O_{ap} are described in the following relations:

$$l_{hi} = \left(D_c/2+h_i\right)\sin\varepsilon_i + R_w\sin\xi_{si}, \tag{19}$$

$$l_{vi} = \left(D_c/2+h_i\right)\cos\varepsilon_i + R_w\cos\xi_{si}. \tag{20}$$

PLANETARY WHEEL EQUILIBRIUM EQUATIONS

Equilibrium equations of cycloidal planetary gear wheel are as follows:

$$\Sigma M_{O_{ap}} = \sum_{j=1}^{j=z_k} f_{vj}P_j\sin\zeta_j + \sum_{j=1}^{j=z_k} f_{hj}P_j\cos\zeta_j + \sum_{i=1}^{i=z_c} l_{vi}Q_i\sin\varepsilon_i - \sum_{i=1}^{i=z_c} l_{hi}Q_i\cos\varepsilon_i = 0, \tag{21}$$

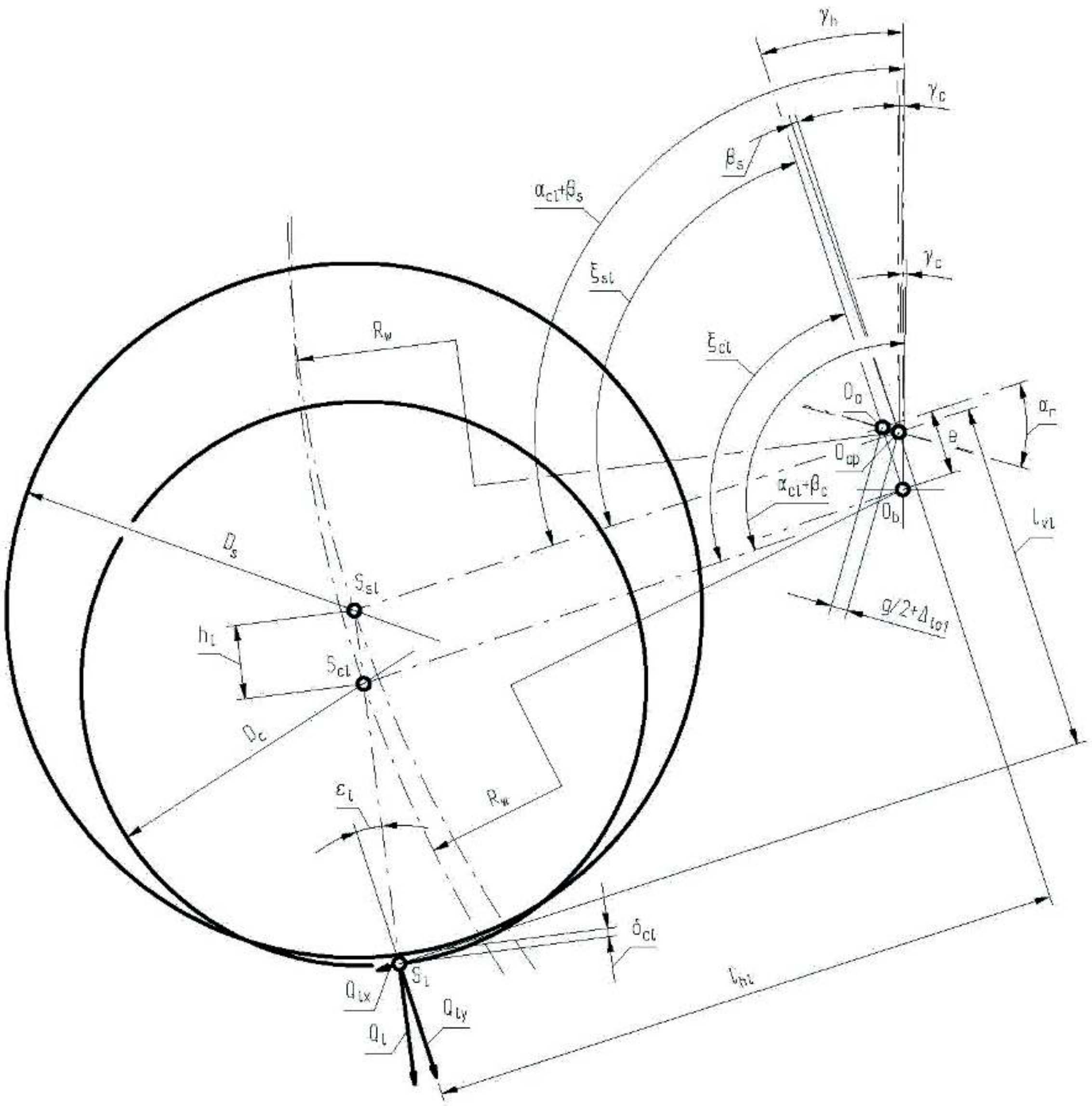

Fig. 4. The geometric parameters used in determining the deformation in the contact between planetary wheel hole surface and straight-line mechanism roller

Rys. 4. Parametry geometryczne wykorzystywane do wyznaczenia odkształceń w styku rolki mechanizmu równowodowego z otworem w kole obiegowym

$$\Sigma F_x = \sum_{j=1}^{j=z_k} P_j \sin\zeta_j - \sum_{i=1}^{i=z_c} Q_i \sin\varepsilon_i - R\cos\alpha_r = 0, \quad (22)$$

$$\Sigma F_y = \sum_{j=1}^{j=z_k} P_j \cos\zeta_j - \sum_{i=1}^{i=z_c} Q_i \cos\varepsilon_i + R\sin\alpha_r = 0. \quad (23)$$

The solution of equilibrium equations of the planetary wheels is only possible by numeric methods. The Hook and Jeeves method has been applied. As the initial values in the iterative process, and at the same time variables, the following have been adopted:

- The angle of the resultant force in the central bearing α_r,
- Planetary wheel angle β_s, and
- The rotation angle of the output shaft β_c.

These quantities, for a given geometry of the gear and the load torque M_l of the output shaft, allow to one calculate other parameters necessary to solve the equilibrium equations of the planetary wheel, including the forces between the teeth P_j, forces Q_i in the straight-line mechanism and the loading force R of the central bearing.

The target function of the iterative method used is the sum of the squares of the equilibrium equations of the planetary wheel (21), (22), and (23). An additional criterion for the convergence of the computer program is the condition that the calculated value of the output shaft torque of the transmission was equal to, with a certain accuracy, the command value.

Apart from the distribution of forces between the teeth and the forces in the straight-line mechanism, it is necessary to know the distribution of forces on the rolling elements of central bearing. The method of determining this distribution is described in [**L. 5**]. These three distributions will enable the determination of the predicted fatigue life of all three rolling pairs of the Cyclo gear [**L. 13**]. Rolling pair fatigue is a very good criterion for optimization of geometrical parameters of the transmission, in particular, cycloidal gearing correction parameters.

CALCULATION RESULTS FOR THE CHOSEN DATA VARIANTS

The subject of the analysis was the Cyclo gear described in [**L. 5**] with parameters presented in **Table 1**. The distributions of forces in the rolling pairs of the gear are given in **Tables 2, 3,** and **4**.

Table 1. Parameters of the examined Cyclo gear
Tabela 1. Parametry badanej obiegowej przekładni cykloidalnej

Eccentricity	$e = 3$ mm
The number of teeth of the planetary wheel	$z_s = 19$
The number of rollers of the co-operating wheel	$z_k = 20$
Diameter of roller of the co-operating wheel	$q = 8.5$ mm
Spacing radius of rollers of the co-operating wheel	$r = 96$ mm
Corrected diameter of roller of the co-operating wheel	$q_k = 9$ mm
Corrected spacing radius of rollers of the co-operating wheel	$r_k = 96.406$ mm
Width of the planetary wheel	$l_e = 14$ mm
Spacing radius of rollers of the straight-line mechanism	$R_w = 62$ mm
Side hole diameter	$D_s = 32$ mm
Diameter of roller of the straight-line mechanism	$D_c = 26$ mm
The number of rollers of the straight-line mechanism	$z_c = 10$
Diameter of the central hole	$d_{bo} = 76.5$ mm
Diameter of roller of the central bearing	$D_r = 11$ mm
Length of roller of the central bearing	$L_r = 12$ mm
Roller chamfer	$r_c = 0.5$ mm
The number of rollers of the central bearing	$Z_r = 15$
Input torque	$M_h = 46.37$ Nm

The calculations were carried out with the software called CYCLOAD for four values of radial clearance in the central bearing (**Tables 2-4**) and the drive shaft angle $\gamma_h = 18°$.

Table 2. The distribution of forces between the teeth
Tabela 2. Rozkład sił międzyzębnych

j	P_j [N]			
	g = -0.02 mm	g = 0	g = 0.02 mm	g = 0.05 mm
1	0	0	0	0
2	0	0	0	0
3	0	0	0	0
4	1389.8	580.1	0	0
5	1906.8	1592.7	1052.0	253.7
6	2658.3	2810.8	2828.4	2617.8
7	2360.9	2919.6	3465.0	3903.0
8	0	654.7	1553.2	2510.6
9	0	0	0	0
10	0	0	0	0
11	0	0	0	0
12	0	0	0	0
13	0	0	0	0
14	0	0	0	0
15	0	0	0	0
16	0	0	0	0
17	0	0	0	0
18	0	0	0	0
19	0	0	0	0
20	0	0	0	0

Table 3. The distribution of forces in the straight-line mechanism
Tabela 3. Rozkład sił w mechanizmie równowodowym

i	Q_i [N]			
	g = -0.02 mm	g = 0	g = 0.02 mm	g = 0.05 mm
1	0	0	0	0
2	0	0	0	0
3	2262.4	2120.8	1971.6	1831.7
4	3414.7	3641.1	3926.6	4265.6
5	2338.8	2202.6	2033.2	1825.6
6	0	0	0	0
7	0	0	0	0
8	0	0	0	0
9	0	0	0	0
10	0	0	0	0

Table 4. The distribution of radial forces Q_{rr} in central bearing, resultant force R and angle of the resultant force α_r.

Tabela 4. Rozkład sił promieniowych Q_{rr} w łożysku centralnym, siła wypadkowa R i kąt działania siły α_r

r	Q_{rr} [N]			
	g = -0.02 mm	g = 0	g = 0.02 mm	g = 0.05 mm
1	3201.6	2527.6	2828.7	3239.7
2	3083.7	2288.8	2380.5	2477.0
3	2741.8	1619.5	1154.6	468.4
4	2245.9	686.4	0	0
5	1689.9	0	0	0
6	1174.9	0	0	0
7	787.5	0	0	0
8	582.9	0	0	0
R [N]	9950.4	9301.0	8723.1	8392.2
α_r [°]	39.04	33.81	27.63	22.94

The results show the number of teeth subjected to load. This number is primarily determined by the parameters of the correction of the meshing and by the gear load [**L. 4, 5**]. The influence can also be noticed of the radial clearance in the central bearing on the magnitude of forces in the tooth-roller contacts as well as the fact of which teeth (and rollers) are loaded. The character of load distribution determines the value of the resultant force R in the central bearing and the angle α_r. The increase in radial clearance reduces the resultant force and the angle of its operation.

SUMMARY

A method for determining the distribution of the forces occurring in the rolling pairs of the Cyclo gear is described. Thanks to the adopted simplifying assumptions the execution of numerical calculations for a number of variants can be accomplished in a short time. This will allow one to carry out the optimization of geometric parameters of the transmission, including the outline of the teeth while taking into account the longest fatigue life. This will be the subject of further work.

In contrast to the analytical method, the proposed method allows one to find the distributions for corrected gear meshing in any position of the planet wheel and the determination of the number of teeth simultaneously subjected to load. It is also possible to take into account the effect of the radial clearance in the central bearing on the load of the rolling pairs. The calculation results showed that a radial clearance in the central bearing significantly affect the character of the distribution of forces in the Cyclo gear meshing and thus the value of the resultant force and the angle of its action.

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