A Method for Estimating the Parameters of the K Distribution

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***Abstract*—A method that combines the maximum likelihood and the method of moments for estimating the parameters of the K distribution is proposed. The method results in the lowest variance of parameter estimates when compared with existing non-ML techniques.**

***Index Terms*—GBK distribution, K distribution, maximum likelihood estimation, method of moments.**

I. Introduction

The K distribution has been successfully used in many signal processing applications such as modeling the radar clutter envelope in radar systems. The statistics of a K-distributed random variable X are described by the probability density function [1]

$$f_X(x) = \frac{2}{a\Gamma(\nu+1)} \left(\frac{x}{2a}\right)^{\nu+1} K_\nu\left(\frac{x}{a}\right) \tag{1}$$

where $x > 0, \Gamma(\cdot)$ is the standard Gamma function [2, p. 255, eq. (6.1.1)], $K_\nu(\cdot)$ is the modified Bessel function of order $\nu > -1$ [2, p. 375, eq. 9.6.2], and a is a positive constant.

The K distribution is completely specified by the shape parameter ν and the scale parameter a. Maximum likelihood (ML)-based methods can be used to estimate a and ν [3]. They yield asymptotically efficient estimates but are computationally too expensive to be implemented in real-time systems. Recently, estimation techniques based on moments have been proposed [1], [4]. They lead to accurate estimates but still involve computationally expensive numerical methods to solve nonlinear equations.

The simplest approach for estimating the parameters of the K distribution is based on the fourth- and second-order moments [1], [3]. This method performs well when the number of samples is large (usually greater than 1000) [3]. However, when the amount of data available is small, a good performance cannot be achieved. The small sample size case is important in radar applications because only then, the assumption of local stationarity can be made. Therefore, methods that are computationally implementable in real-time and at the same time lead to accurate parameter estimates for a small sample size are sought.

In this correspondence, an approach that combines the maximum likelihood principle with the method of moments (MOM) is presented. The proposed method leads to the lowest variance of the parameter estimates when compared with existing non-ML methods.

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II. Parameter Estimation

Most of the existing techniques for estimating the parameters of the K distribution are based on the MOM. The principle is as follows. Estimate the kth-order moment of the K distribution [1]

$$\mu_k = \mathrm{E}[X^k] = \frac{\Gamma\left(\frac{k}{2}+1\right)\Gamma\left(\nu+1+\frac{k}{2}\right)}{\Gamma(\nu+1)} (2a)^k \tag{2}$$

by its sample counterpart $\hat{\mu}_k = (1/N)\ \Sigma_{i=1}^N\ x_i^k$, where $\{x_i; i = 1, \cdots, N\}$ is a set of realizations of N statistically independent random variables $\{X_i; i = 1, \cdots, N\}$ from f_X. It is readily seen that we can estimate the parameters a and ν using *any two* estimates of the moments given in (2).

The simplest choice for the two moments, as suggested in [4], is the sample mean and the sample variance. However, the derivation of ν and a requires solving a tedious nonlinear equation, which needs to be performed numerically.

Another estimation scheme has been proposed by Raghavan [1], where the arithmetic and the geometric sample means $\hat{\mu}_a$ and $\hat{\mu}_g$, respectively, were used to estimate the parameters of the Gamma distribution with probability density function $f_X(x) = (x^{\rho-1}/b^\rho\Gamma(\rho))\ \exp(-(x/b))$. The ML estimates of the parameters b and ρ can be obtained as $\hat{\rho}\ \exp[-\Psi(\hat{\rho})] = \hat{\mu}_a/\hat{\mu}_g$ and $\hat{b} = \hat{\mu}_a/\hat{\rho}$, where $\Psi(\cdot)$ is the digamma function [2, p. 258, eq. 6.3.1]. To find the estimates of the K distribution, the method takes into account the fact that the maximum likelihood estimates of the Gamma distribution are related to the ones of the K distribution. This approach yields good approximations for values of ν in the range $-0.8 < \nu < 1$. This range corresponds to the important case when both the tail of the distribution is higher (and the standard deviation-to-mean ratio is larger) than that predicted by the Rayleigh distribution (for example, when the clutter in radar applications is spiky). To find the parameter ν, a solution of a nonlinear equation is required, involving the digamma function.

Another moment-based technique was proposed in [5]. It is based on the moment ratio

$$r_{p,q} = \frac{\mu_{p+2q}}{\mu_p\mu_{2q}}, \qquad p > 0, \quad q = 1, 2, \cdots \tag{3}$$

which is independent of the shape parameter a. Considering that estimates based on higher order moments show large variability, it is of interest to estimate the parameters from lower order moments. Setting $q = 1$ in (3), the ratio

$$R_p = r_{p,1} = \frac{\mu_{p+2}}{\mu_p\mu_2} = \frac{[(p+2)/2]^2 + \nu(p+2)/2}{\nu+1} \tag{4}$$

was considered, where $p > 0$. The parameter ν is then given by

$$\nu = f(R_p) = \frac{[(p+2)/2]^2 - R_p}{R_p - (p+2)/2}, \qquad p > 0. \tag{5}$$

For $p = 2$, the ratio R_2 reduces to the one based on the second- and fourth-order moments [1]. It has been shown that it is convenient to choose the range $0 < p \leq 2$ in order to reduce the variance of the moment estimates. This range corresponds to *fractional moments*, i.e., moments of order other than a positive integer. It was shown in [5] that fractional moments lead to better parameter estimates than traditional methods based on the second- and fourth-order moments. However, the method of Raghavan, although computationally more intensive, still provides better estimates.

III. Estimation Based on ML and MOM

Let $X_1, X_2, \cdots, X_N$, be N independent variables, where each is distributed according to (1). The log-likelihood function of the K distribution based on $X_1, X_2, \cdots, X_N$ is given by

$$L = N \log \left(\frac{2^{-\nu}}{\Gamma(\nu+1)a^{\nu+2}}\right) + (\nu+1) \sum_{i=1}^{N} \log(x_i) + \sum_{i=1}^{N} \log \left[K_\nu\left(\frac{x_i}{a}\right)\right] \tag{6}$$

where $\{x_i, i=1,\cdots,N\}$ are realizations of $\{X_i, i=1,\cdots,N\}$. The partial derivatives of the log-likelihood function (6) are given by

$$\frac{\partial L}{\partial a} = -\frac{(\nu+2)N}{a} - \frac{1}{2a} \cdot \sum_{i=1}^{N} \frac{K_{\nu-1}\left(\frac{x_i}{a}\right) + K_{\nu+1}\left(\frac{x_i}{a}\right)}{K_\nu\left(\frac{x_i}{a}\right)} \left(\frac{x_i}{a}\right) \tag{7}$$

$$\frac{\partial L}{\partial \nu} = -N[\log(2a) + \Psi(\nu+1)] + \sum_{i=1}^{N} \log(x_i) + \sum_{i=1}^{N} \frac{\frac{\partial}{\partial \nu} K_\nu\left(\frac{x_i}{a}\right)}{K_\nu\left(\frac{x_i}{a}\right)} \tag{8}$$

where $\Psi(\cdot)$ is the digamma function. The ML estimates of the parameters ν and a of the K distribution can be found by calculating the log-likelihood function and equating its two partial derivatives (7) and (8) to zero. This, however, does not lead to closed-form expressions, even in the case where one of the parameters is known.

In order to overcome this problem, partial derivatives of the newly developed generalized Bessel function K (GBK) distribution are used [6]. The GBK distribution is a four-parameter distribution with density function

$$f_X(x) = \frac{2c\left(\frac{x}{\beta}\right)^{(c/2)(\alpha_1+\alpha_2)-1}}{\beta\Gamma(\alpha_1)\Gamma(\alpha_2)} K_{(\alpha_2-\alpha_1)}\left[2\left(\frac{x}{\beta}\right)^{c/2}\right] \tag{9}$$

that includes the K distribution as a special case for the set of the parameters $(\alpha_1, \alpha_2, \beta, c) = (1, \nu+1, 2a, 2)$. The ML equations for the GBK distribution are derived by equating to zero the partial derivatives of the log-likelihood function for the GBK distribution (see Appendix A). From (19)–(22) and noting that [2]

$$\frac{\partial}{\partial \alpha_1} K_{(\alpha_2-\alpha_1)}\left[2\left(\frac{x}{\beta}\right)^{c/2}\right] = -\frac{\partial}{\partial \alpha_2} K_{(\alpha_2-\alpha_1)}\left[2\left(\frac{x}{\beta}\right)^{c/2}\right]$$

we have

$$c = \frac{\Psi(\alpha_1) + \Psi(\alpha_2)}{\frac{1}{N}\sum_{i=1}^{N} \log\left(\frac{x_i}{\beta}\right)} \tag{10}$$

$$\frac{1}{N}\sum_{i=1}^{N} \mathcal{K}(x_i) = \alpha_1 + \alpha_2 \tag{11}$$

$$\frac{1}{N}\sum_{i=1}^{N} \mathcal{K}(x_i) \log\left(\frac{x_i}{\beta}\right) = \frac{2}{c} + (\alpha_1+\alpha_2)\frac{1}{N} \cdot \sum_{i=1}^{N} \log\left(\frac{x_i}{\beta}\right). \tag{12}$$

To find the maximum likelihood estimates of the K distribution, we set in (10)–(12) $\alpha_1 = 1, \alpha_2 = \nu+1, \beta = 2a$, and $c = 2$ and solve for the unknowns so that

$$\hat{a} = \frac{1}{2} \exp\left(\frac{\gamma - \Psi(\hat{\nu}+1)}{2} + \frac{1}{N}\sum_{i=1}^{N} \log(x_i)\right) \tag{13}$$

and

$$\sum_{i=1}^{N} \mathcal{K}(x_i) \log\left(\frac{x_i}{2\hat{a}}\right) = \frac{1}{N}\sum_{i=1}^{N} \mathcal{K}(x_i) \sum_{i=1}^{N} \log\left(\frac{x_i}{2\hat{a}}\right) + N \tag{14}$$

where $\gamma = 0.5772\cdots$ is Euler's constant, and

$$\mathcal{K}(x_i) = \frac{K_{\hat{\nu}-1}\left(\frac{x_i}{\hat{a}}\right) + K_{\hat{\nu}+1}\left(\frac{x_i}{\hat{a}}\right)}{K_{\hat{\nu}}\left(\frac{x_i}{\hat{a}}\right)} \left(\frac{x_i}{\hat{a}}\right). \tag{15}$$

Substituting $\hat{a}$ from (13) into (14), we can obtain an estimate for ν. Such a procedure needs to be performed numerically and is computationally intensive, even for small data sizes. On the other hand, the result given in (13) alone may be useful in some radar applications where the values of ν can be expressed as functions of the grazing angle, cross-range resolution, polarization, and a suitable aspect angle. This is in contrast to the fact that no analytical result, such as (13), can be obtained when directly applying the ML technique to the K distribution.

A. The ML/MOM Approach

To provide an explicit formula for an estimate of the shape parameter ν, kth-order moments of the K distribution, given in (2), are used. Replacing these moments by its sample counterpart, we can obtain a relation between the estimate $\hat{\nu}$ of the shape parameter ν and the estimate $\hat{a}$ of the scale parameter a as

$$\hat{a} = \frac{1}{2}\left[\frac{\Gamma(\hat{\nu}+1)}{\Gamma(0.5k+1)\Gamma(\hat{\nu}+1+0.5k)} \frac{1}{N}\sum_{i=1}^{N} x_i^k\right]^{1/k}. \tag{16}$$

By combining (13) and (16), we obtain a function of $\hat{\nu}$ with parameter k

$$\begin{aligned} g_k(\hat{\nu}) &= \log\left[\frac{\Gamma(\hat{\nu}+1)}{\Gamma\left(\hat{\nu}+1+\frac{k}{2}\right)}\right] + \frac{k\Psi(\hat{\nu}+1)}{2} \\ &= \frac{1}{N}\sum_{i=1}^{N} \log(x_i^k) - \log\left[\frac{1}{N}\sum_{i=1}^{N} x_i^k\right] + \frac{k\gamma}{2} \\ &\quad + \log\left[\Gamma\left(1+\frac{k}{2}\right)\right]. \end{aligned} \tag{17}$$

Note that the right-hand side of the equation does not depend on the estimate of the shape parameter ν and can be evaluated from the data sample when k is given. The function $g_k(\hat{\nu})$ in (17) is strictly monotonically increasing, as shown in Fig. 1 for $k = 0.5$ (solid line) $k = 1$ (dashed line), $k = 1.5$ (dashed-dotted line), and $k = 2$ (dotted line). The proof for monotonicity is given in Appendix B.

Therefore, we can estimate the shape parameter ν of the K distribution using

$$\hat{\nu} = g_k^{-1}\left(\frac{1}{N}\sum_{i=1}^{N} \log(x_i^k) - \log\left[\frac{1}{N}\sum_{i=1}^{N} x_i^k\right] + \frac{k\gamma}{2} + \log\left[\Gamma\left(1+\frac{k}{2}\right)\right]\right) \tag{18}$$

where $g_k^{-1}(\cdot)$ is the inverse function of $g_k(\cdot)$. Note that in (18), the order is not restricted to a positive integer, and fractional moments can be used as well.

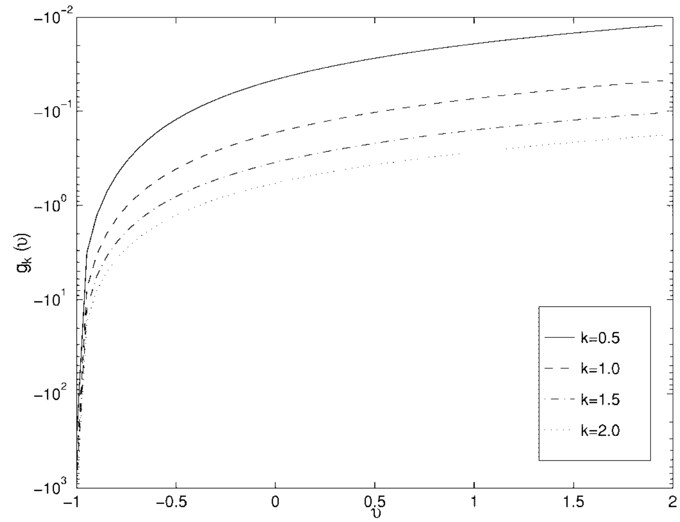

Fig. 1. Plot of the function $g_{1/2}(\nu)$ (solid line), $g_1(\nu)$ (dashed line), $g_{3/2}(\nu)$ (dashed-dotted line), and the function $g_2(\nu)$ (dotted line).

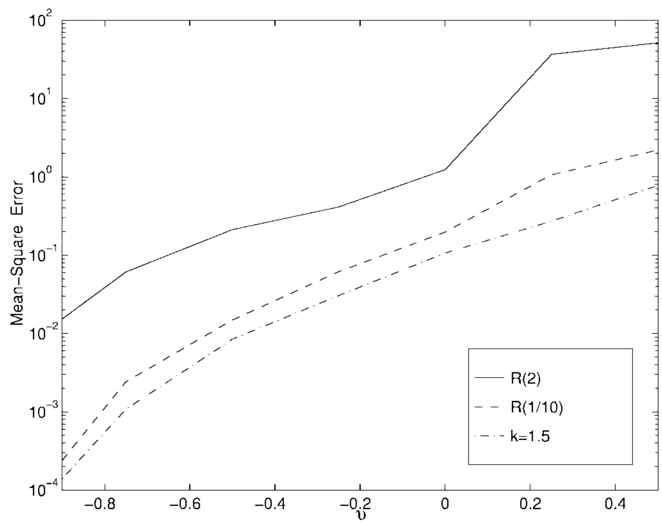

Fig. 2. Mean-square error of the estimates of ν for $N = 100$ using R_2 (solid line), $R_{1/10}$ (dashed line), and $g_{3/2}(v)$ (dashed-dotted line).

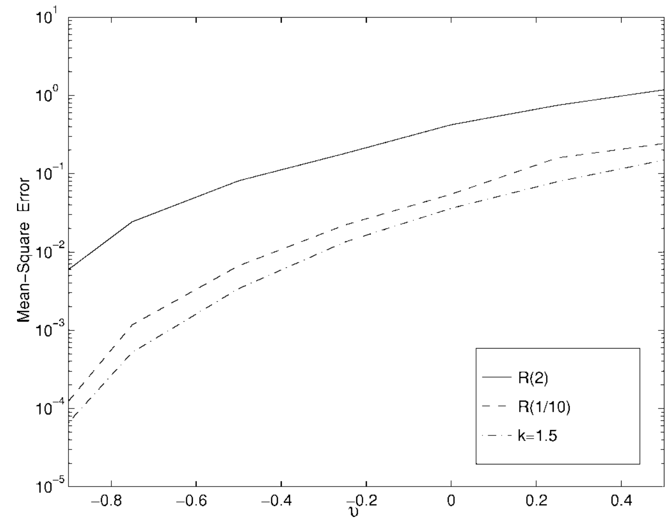

Fig. 3. Mean-square error of the estimates of ν for $N = 200$ using R_2 (solid line), $R_{1/10}$ (dashed line), and $g_{3/2}(v)$ (dashed-dotted line).

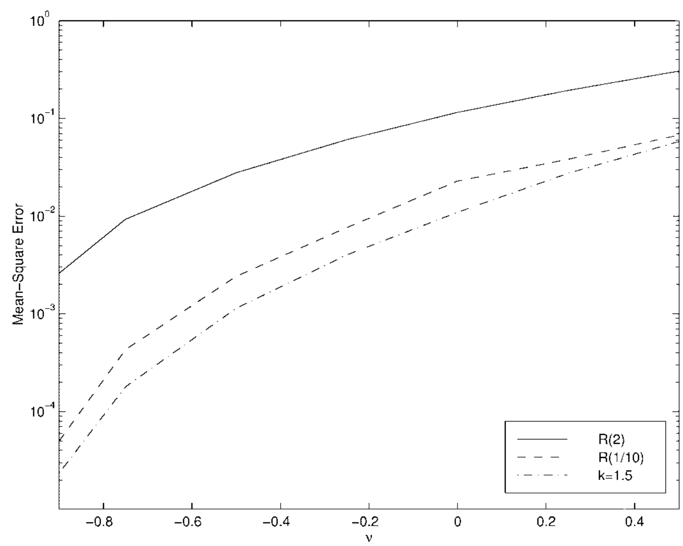

Fig. 4. Mean-square error of the estimates of ν for $N = 500$ using R_2 (solid line), $R_{1/10}$ (dashed line), and $g_{3/2}(v)$ (dashed-dotted line).

Although an analytical expression of the inverse function $g_k^{-1}(\cdot)$ has not been found, the numerical solution is easy to implement. First, we evaluate the function $g_k(\nu)$ for a given k and a range of ν. From the estimate $g_k(\hat{\nu})$ and function $g_k(\nu)$, we can obtain the estimate of ν using, for example, cubic spline interpolation, e.g., $\hat{\nu} = \texttt{spline}(g_k(\nu), \nu, g_k(\hat{\nu}))$ in Matlab©. Once the parameter $\hat{\nu}$ is calculated from (18), the estimate of the scale parameter a can be simply obtained from the ML estimator given in (13). Note that in [1] and [4], the scale parameter a was derived from the first-order sample moment because no ML estimator for the parameter a was available in closed form.

B. Simulation Results

In the following simulations, the power of the data was normalized so that the second-order moment of the process is unity. The normalization procedure leads to the relation $a = (1/2\sqrt{\nu + 1})$. K-distributed data was generated using seven different values of the shape parameter ν in the range $[-0.9, 0.5]$. Several estimators $g_k(\nu)$ were used with k ranging from 0.5–2. The number of data samples was chosen to be $N = 100, N = 200$, and $N = 500$. Estimation was performed over 1000 independent trials in each case, and averages were obtained.

Discussion of the Results: For $N = 100$, it was difficult to decide which k results in the best estimator (in the mean-square sense). On the other hand, for $N = 200$ and $N = 500$, the best estimates of the parameter ν were obtained for $k = 1.5$. In Figs. 2–4, the mean-square error of the estimates of ν for $N = 100, N = 200$, and $N = 500$ are shown, respectively. For comparison, the results based on the moment ratio R_2 and $R_{1/10}$ (see [5]) are also included. Further analysis indicates that for $N > 500$, the best estimator for the shape parameter ν is reached using $1 < k < 2$.

Comparison with Raghavan's Method: In order to compare the ML/MOM-based procedure with the method proposed by Raghavan [1], a similar performance analysis as for the ML/MOM was conducted for Raghavan's method.

In Figs. 5–7, the mean-square errors of the estimates of ν using Raghavan's method (solid line) and the one based on $g_{3/2}(v)$ (dashed

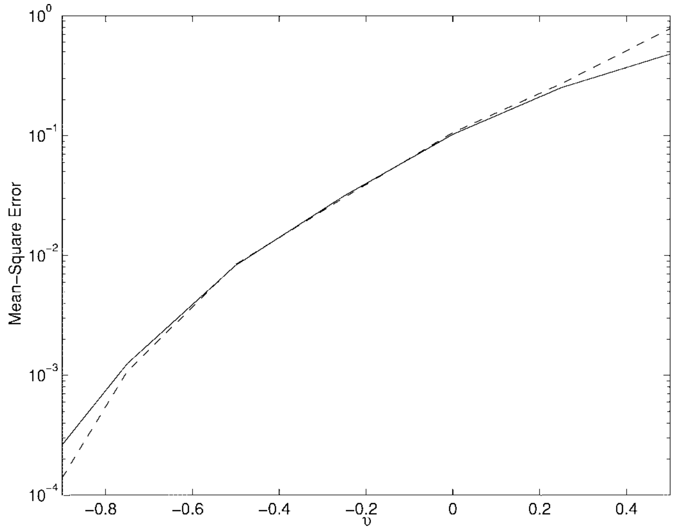

Fig. 5. Mean-square error of the estimates of ν for $N = 100$ using Raghavan's method (solid line) and $g_{3/2}(v)$ (dashed line).

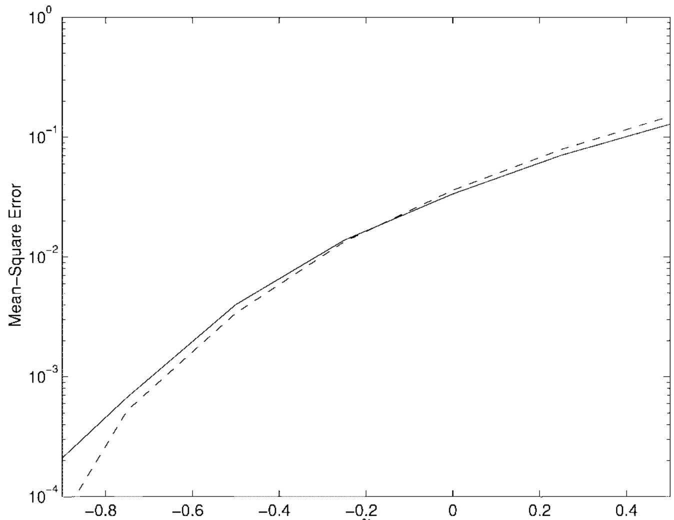

Fig. 6. Mean-square error of the estimates of ν for $N = 200$ using Raghavan's method (solid line) and $g_{3/2}(v)$ (dashed line).

line) are depicted for $N = 100, N = 200$, and $N = 500$, respectively. It is readily seen that for $N = 100$, the performance of the ML/MOM-based estimator is similar to the performance of Raghavan's estimator. For very small values of the parameter ν (close to -1), which is of great interest in radar applications, our technique performs better. On the other hand, the performance slightly degrades (with respect to Raghavan's method) for $\nu > 0.3$. However for larger sample size, e.g. $N = 500$, this effect diminishes, and both methods become equivalent for larger ν, as depicted in Fig. 7.

Computational Requirements: For $N = 100$, the method based on fractional moments (with $k = 1/10$) requires 620 flops, Raghavan's method requires 165 000 flops, while the proposed ML/MOM method (with $k = 1.5$) requires 150 600 flops. The numbers of flops for the proposed method and the method of Raghavan are for the case where the splines were used for calculating the inverse functions in the methods. The proposed method is computationally more efficient than Raghavan's method. On the other hand, the computational complexity of the method based on lower order and fractional moments [5] is much lower.

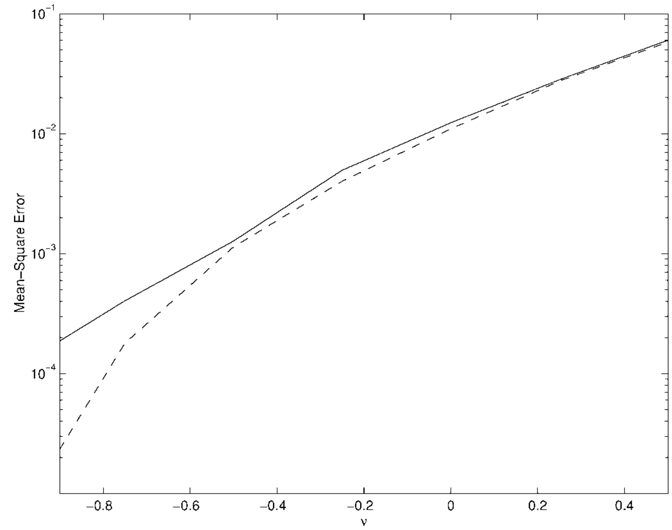

Fig. 7. Mean-square error of the estimates of ν for $N = 500$ using Raghavan's method (solid line) and $g_{3/2}(v)$ (dashed line).

IV. Conclusions

The maximum likelihood estimates of the parameters of the K distribution are generally difficult to obtain and require computationally expensive numerical methods. This correspondence has presented an alternative method that combines the method of maximum likelihood and the method of moments (ML/MOM) to estimate the parameters of the K distribution. This approach leads to parameter estimates with lower mean-square error when compared with standard methods such as the one based on the second- and fourth-order moments. In addition, the computational burden of the proposed ML/MOM-based method is smaller than other techniques such as Raghavan's method, whereas its performance is comparable. An explicit and mathematically tractable maximum likelihood relationship between the shape parameter ν and the scale parameter a is derived using log-likelihood function derivatives of the recently developed GBK distribution. This relationship is useful in radar applications where the values of the shape parameter can be expressed as functions of the grazing angle, cross-range resolution, polarization, and a suitable aspect angle.

Appendix A

The partial derivatives of the log-likelihood function for the GBK distribution are

$$\frac{\partial L}{\partial \alpha_1} = -\frac{N\,c\log(\beta)}{2} - N\,\Psi(\alpha_1) + \frac{c}{2}\sum_{i=1}^{N}\log(x_i) + \sum_{i=1}^{N}\frac{\frac{\partial}{\partial \alpha_1}K_{(\alpha_2-\alpha_1)}\left[2\left(\frac{x_i}{\beta}\right)^{c/2}\right]}{K_{(\alpha_2-\alpha_1)}\left[2\left(\frac{x_i}{\beta}\right)^{c/2}\right]} \tag{19}$$

$$\frac{\partial L}{\partial \alpha_2} = -\frac{N\,c\log(\beta)}{2} - N\,\Psi(\alpha_2) + \frac{c}{2}\sum_{i=1}^{N}\log(x_i) + \sum_{i=1}^{N}\frac{\frac{\partial}{\partial \alpha_2}K_{(\alpha_2-\alpha_1)}\left[2\left(\frac{x_i}{\beta}\right)^{c/2}\right]}{K_{(\alpha_2-\alpha_1)}\left[2\left(\frac{x_i}{\beta}\right)^{c/2}\right]} \tag{20}$$

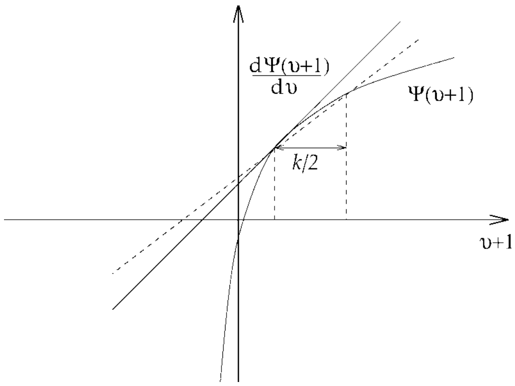

Fig. 8. Graphical interpretation of the proof in Appendix 2.

$$\frac{\partial L}{\partial \beta} = \frac{c}{2\beta}\left[-(\alpha_1+\alpha_2)N + \sum_{i=1}^{N} \mathcal{K}(x_i)\right] \quad (21)$$

$$\frac{\partial L}{\partial c} = \frac{N}{c} + \frac{\alpha_1+\alpha_2}{2} \sum_{i=1}^{N} \log\left(\frac{x_i}{\beta}\right) - \frac{1}{2}\sum_{i=1}^{N} \mathcal{K}(x_i) \log\left(\frac{x_i}{\beta}\right) \quad (22)$$

where $\mathcal{K}(x_i)$ is given in (15).

APPENDIX B

For $g_k(\nu)$ to be a strictly monotonically increasing function

$$\frac{dg_k(\nu)}{d\nu} = \Psi(\nu+1) - \Psi\left(\nu+1+\frac{k}{2}\right) + \frac{k}{2}\,\frac{d\Psi(\nu+1)}{d\nu} > 0 \quad (23)$$

must hold. Rearranging the expression in (23), we obtain

$$\frac{d\Psi(\nu+1)}{d\nu} > \frac{\Psi\left(\nu+1+\frac{k}{2}\right) - \Psi(\nu+1)}{\frac{k}{2}}, \qquad k > 0. \quad (24)$$

We recognize that in the limit $k \to 0$, the inequality in (24) becomes equality. Since the digamma function $\Psi(\nu+1)$ is a *strictly monotonically increasing function* and its first derivative (the trigamma function) is a *positive strictly monotonically decreasing function* [2, ch. 6], the inequality (24) holds (see Fig. 8).

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A Fast CFAR Detection Space-Time Adaptive Processing Algorithm

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***Abstract*— All of the conventional CFAR detection algorithms that use space-time processing involve a time-consuming matrix-inversion operation. Based on today's technology, this computational complexity sometimes makes the full-rank solution difficult to realize. In this correspondence, a CFAR detection algorithm, which does not need a matrix inversion, is developed by an adaptation and extension of Hotelling's principal-component method studied recently by Kirsteins and Tufts. Finally, the performance of the new CFAR test statistic is analyzed, and the effect of the rank reduction on performance is evaluated for an example scenario.**

***Index Terms*—CFAR detection, reduced-rank, STAP radar.**

I. INTRODUCTION

An all-encompassing generalized likelihood ratio test (GLRT) on space-time adaptive processing (STAP) for radar detection was derived by Kelly [1]. Later, Chen and Reed [2] and, independently, Fuhrmann *et al.* developed a simpler CFAR detection test to the GLRT obtained originally by Kelly. However, all of the previous STAP algorithms involved a time-consuming matrix inversion operation.

The principal-components (PC) technique was developed originally by Hotelling [4]. Hotelling showed that the dimension of the problem often could be reduced without sacrificing too much of the information contained in the covariance matrix. A new unnormalized GLRT, which uses the PC technique, was developed by Kirsteins and Tufts in [5], but they did not obtain the probability density functions (PDF) needed to evaluate the performance of the test statistic.

In this study, the normalized LRT in [2] and [3] is modified by a use of the PC technique. This test exhibits the very desirable property that no matrix inversions are needed and that the computational complexity can be reduced by incorporating the HT method [7] or other methods, e.g., the fast PASTd algorithm [8].

The PDF of this new detection statistic is derived here for both the noise-alone case and the signal-plus-noise case when the noise covariance is unknown but with dominating clutter-basis vectors. Under this situation, the principal eigenvectors of the clutter can be estimated accurately. In addition, the false-alarm probability is shown not to depend on the signal and noise power. Thus, this new PC test is a CFAR criterion. Finally, these results are validated by a computer simulation.

II. RANK REDUCTION OF THE CLUTTER-PLUS-NOISE COVARIANCE

Let $\mathbf{x}$ be a $N \times 1$ snapshot of a given range gate, i.e., the observation vector $\mathbf{x} = [x_1, x_2, \cdots, x_N]^T$. Two hypotheses are

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