NASA TECHNICAL MEMORANDUM

NASA TM X-64633

A METHOD FOR NONLINEAR EXPONENTIAL REGRESSION ANALYSIS

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October 29, 1971

NASA

George C. Marshall Space Flight Center
Marshall Space Flight Center, Alabama

TECHNICAL REPORT STANDARD TITLE PAGE

1. REPORT NO. TM X-64633	2. GOVERNMENT ACCESSION NO.	3. RECIPIENT'S CATALOG NO.
4. TITLE AND SUBTITLE A Method for Nonlinear Exponential Regression Analysis		5. REPORT DATE October 29, 1971
		6. PERFORMING ORGANIZATION CODE
7. AUTHOR (S) Bobby G. Junkin		8. PERFORMING ORGANIZATION REPORT #
9. PERFORMING ORGANIZATION NAME AND ADDRESS George C. Marshall Space Flight Center Marshall Space Flight Center, Alabama 35812		10. WORK UNIT NO.
		11. CONTRACT OR GRANT NO.
		13. TYPE OF REPORT & PERIOD COVERED Technical Memorandum
12. SPONSORING AGENCY NAME AND ADDRESS National Aeronautics and Space Administration Washington, D.C. 20546		14. SPONSORING AGENCY CODE
15. SUPPLEMENTARY NOTES Prepared by Computation Laboratory, Science and Engineering		

16. ABSTRACT

This report presents a computer-oriented technique for performing a nonlinear exponential regression analysis on decay-type experimental data. The technique involves the least squares procedure wherein the nonlinear problem is linearized by expansion in a Taylor series. A linear curve-fit procedure for determining the initial nominal estimates for the unknown exponential model parameters is included as an integral part of this technique. A correction matrix is derived and then applied to the nominal estimate to produce an improved set of model parameters. The solution cycle is repeated until some predetermined criterion is satisfied.

17. KEY WORDS mathematical model regression analysis exponential iterative method	initial estimate nonlinear least squares computer program	18. DISTRIBUTION STATEMENT Unclassified-unlimited Bobby G. Junkin

19. SECURITY CLASSIF. (of this report)	20. SECURITY CLASSIF. (of this page)	21. NO. OF PAGES	22. PRICE
Unclassified	Unclassified	94	$3.00

MSFC - Form 3292 (May 1969)

ACKNOWLEDGMENT

The author acknowledges the assistance of Mr. M. C. Davidson of the Space Sciences Laboratory who provided the physical process data used in the analysis herein.

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DEFINITION OF SYMBOLS

Symbol	Definition
f^o	Observed response variable
f^c	Computed response variable
t	Independent variable
A_i , B_i	Exponential regression coefficients
$\tilde{A}_i$, $\tilde{B}_i$	Initial approximations for A_i , B_i
$\bar{\Delta}$	Parameter correction matrix
V_{fi}	Residuals associated with response variable
S	Minimizing function
$\bar{W}$	Parameter weight matrix
$\bar{B}$	Partial derivative matrix
σ_o^2	Unit variance
σ_f	Standard deviation of response variable
$\tilde{F}$	Sum of squares of deviations
n	Number of data points
$( )\|_o$	Partial derivative evaluation at nominal estimate
m	Number of exponential terms ($n \geq 2m + 1$)

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A METHOD FOR NONLINEAR EXPONENTIAL REGRESSION ANALYSIS

INTRODUCTION

The investigation of physical processes frequently requires the use of models that simulate or describe the processes. A model is often chosen so that certain variables interact in the model according to physical theories associated with the particular process. Formulation of a model often results in the form referred to as mathematical models. This is the familiar representation of the physical process by one or more equations that encompass the physical theory. A model equation contains identified independent variables and unknown parameters. Regression analysis is the statistical tool used to determine these unknown parameters, thereby providing an analytical representation of the experimental data.

The general procedure in regression analysis is to take partial derivatives of a specific model-dependent minimizing function. These partial derivatives are taken with respect to each of the unknown model parameters. If the set of equations obtained by setting these partial derivatives equal to zero can be solved by the usual algebraic methods, the fitting or analytical representation is accomplished. However, if these equations are transcendental in one or more of the unknown parameters, they cannot be solved by the usual algebraic methods.

The processes of particular interest in this report are those that can be described by decaying exponential forms. A mathematical model that contains more than one exponential term results in a set of transcendental normal equations if conventional forms of regression analysis are used. Thus, one usually resorts to iterative methods that require initial estimates for the parameters. The method described herein involves the least squares procedure, whereby the nonlinear problem is linearized by expanding in a Taylor series. In this iterative method, we first develop a starting nominal guess for the model parameters. A correction matrix is derived and then applied to the nominal guess to produce an improved set of model parameters. This procedure is continued until some predetermined criterion is satisfied. The number of iterations necessary for convergence is closely related to this criterion, the initial estimates, and the form of the exponential model.

Additional information on the various methods of curve-fitting decay-type data to a sum of exponentials is given in References 1 through 5. Procedures for obtaining the initial parameter estimates are discussed in References 6 through 8. It is noted that the initial estimate procedure herein is not restricted to equally spaced data.

Application of the procedure is illustrated with data obtained from a particular process concerning the anodic oxidation of metals. In this process one expects an exponential or logarithmic behavior. From an analysis of the results, it is concluded that an adequate two-term exponential representation of the data is obtained. Thus, the analytical representation of the physical process data is accomplished using an exponential decay-type model.

MATHEMATICAL THEORY FOR EXPONENTIAL REGRESSION ANALYSIS

In general, we are given the set of observed values $\left\{ (t_1, f_1^o), (t_2, f_2^o), \ldots, (t_n, f_n^o) \right\}$. We assume that the function to be fitted to these data is of the following form:

$$f^c = A_1 e^{-B_1 t} + A_2 e^{-B_2 t} + \ldots + A_m e^{-B_m t} + K \equiv f(A, B, K, t) \quad , \qquad (1)$$

where f^c represents the calculated value of the response and K, A_i, $B_i (i = 1, 2, \ldots, m)$ are the $2m + 1$ parameters to be estimated. The independent variable is time t. We first consider the simple case $m = 1$ and $K = 0$ which results in a conventional least squares solution for the unknowns A_1 and B_1. We have the following model:

$$f^c = A_1 e^{-B_1 t} \quad . \qquad (2)$$

By taking the natural log of both sides,

$$\ln f^c = \ln A_1 - B_1 t \quad . \qquad (3)$$

Let

$$\left.\begin{aligned} C &= \ln A_1 \\ &\text{and} \\ Y &= \ln f^o \end{aligned}\right\} \qquad (4)$$

Then,

$$Y = C - B_1 t \qquad (5)$$

The least squares solutions for C and B_1 then become

$$B_1 = -\left[\frac{m \sum_{i=1}^{n} t_i Y_i - \sum_{i=1}^{n} t_i \sum_{i=1}^{n} Y_i}{m \sum_{i=1}^{n} t_i^2 - \left(\sum_{i=1}^{n} t_i\right)^2}\right] \qquad (6)$$

and

$$C = \left[\frac{\sum_{i=1}^{n} t_i^2 \sum_{i=1}^{n} Y_i - \sum_{i=1}^{n} t_i \sum_{i=1}^{n} t_i Y_i}{m \sum_{i=1}^{n} t_i^2 - \left(\sum_{i=1}^{n} t_i\right)^2}\right] \qquad (7)$$

For $m > 1$ and $K \neq 0$ we proceed as follows. We write the parameters as initial approximations or nominal values plus unknown corrections; that is,

$$\left.\begin{aligned}
A_1 &= \tilde{A}_1 + \Delta A_1 \\
B_1 &= \tilde{B}_1 + \Delta B_1 \\
A_2 &= \tilde{A}_2 + \Delta A_2 \\
B_2 &= \tilde{B}_2 + \Delta B_2 \\
&\;\vdots \\
A_m &= \tilde{A}_m + \Delta A_m \\
B_m &= \tilde{B}_m + \Delta B_m \\
K &= \tilde{K} + \Delta K
\end{aligned}\right\} \quad . \tag{8}$$

For any assumed functional form, the following condition equation can be written:

$$f_i^o - f_i^c - V_{fi} = 0 \quad , \tag{9}$$

where

f_i^o = observed values of the response variable,

f_i^c = calculated values of the response variable,

and

V_{fi} = residuals associated with the response variable.

If we substitute equation (8) into equation (1) and the result into equation (9), we obtain

$$f_i^o - f_i\Big(\tilde{A}_1 + \Delta A_1 , \tilde{B}_1 + \Delta B_1 , \ldots , \tilde{A}_m + \Delta A_m , \tilde{B}_m + \Delta B_m ,$$

$$\tilde{K} + \Delta K, t\Big) - V_{fi} = 0 \quad , \tag{10}$$

or

$$f_i\left(\widetilde{A}_1 + \Delta A_1 , \widetilde{B}_1 + \Delta B_1 , \ldots , \widetilde{A}_m + \Delta A_m , \widetilde{B}_m + \Delta B_m ,\right.$$

$$\left.\widetilde{K} + \Delta K, t\right) = f_i^o - V_{fi} \quad . \tag{11}$$

Expanding the left side of equation (11) in a Taylor series about the estimates $\widetilde{A}_1 , \widetilde{B}_1 , \ldots , \widetilde{A}_m , \widetilde{B}_m , \widetilde{K}$ and neglecting higher order terms than the first, we have $(i = 1, 2, \ldots , n)$

$$V_{fi} = N_i - F_{1i}\,\Delta A_1 - F_{2i}\,\Delta B_1 - F_{3i}\,\Delta A_2 - F_{4i}\,\Delta B_2 - \ldots$$

$$- F_{2m-1,i}\,\Delta A_m - F_{2m,i}\,\Delta B_m - F_{2m+1,i}\,\Delta K \quad , \tag{12}$$

where

$$\left.\begin{aligned} F_{1i} &= \left.\frac{\partial f_i}{\partial A_1}\right|_o & F_{4i} &= \left.\frac{\partial f_i}{\partial B_2}\right|_o \\ & & &\vdots \\ F_{2i} &= \left.\frac{\partial f_i}{\partial B_1}\right|_o & F_{2m-1,i} &= \left.\frac{\partial f_i}{\partial A_m}\right|_o \\ F_{3i} &= \left.\frac{\partial f_i}{\partial A_2}\right|_o & F_{2m,i} &= \left.\frac{\partial f_i}{\partial B_m}\right|_o \\ & & F_{2m+1,i} &= \left.\frac{\partial f_i}{\partial K}\right|_o \end{aligned}\right\} \tag{13}$$

and

$$N_i = f_i^o - f_i\,(\tilde{A},\ \tilde{B},\ \tilde{K},\ t) \quad . \tag{14}$$

The n equations given by equation (12) are the linearized condition or residual equations. According to the Gauss least squares principle, the best representation of the data is that which makes the weighted sum of the squares of the residuals a minimum. Thus, the minimizing function is

$$\begin{aligned} S &= f\left(\Delta A_1 , \Delta B_1 , \ldots , \Delta A_m , \Delta B_m , \Delta K\right) \\ &= W_1 V_{f1}^{\,2} + W_2 V_{f2}^{\,2} + \ldots + W_n V_{fn}^{\,2} \quad . \end{aligned} \tag{15}$$

The $2m + 1$ linear algebraic equations for determining the Δ increments to the initial estimates are now obtained by taking the partial derivative of S with respect to each of the unknown corrections and setting the result equal to zero. These equations, frequently referred to as the normal equations, are given by

$$\left.\begin{aligned} \frac{\partial S}{\partial \Delta A_1} &= 2W_1 V_{f1} \frac{\partial V_{f1}}{\partial \Delta A_1} + 2W_2 V_{f2} \frac{\partial V_{f2}}{\partial \Delta A_1} + \ldots + 2W_n \frac{\partial V_{fn}}{\partial \Delta A_1} = 0 \\ \frac{\partial S}{\partial \Delta B_1} &= 2W_1 V_{f1} \frac{\partial V_{f1}}{\partial \Delta B_1} + 2W_2 V_{f2} \frac{\partial V_{f2}}{\partial \Delta B_1} + \ldots + 2W_n \frac{\partial V_{fn}}{\partial \Delta B_1} = 0 \\ &\ \ \vdots \\ \frac{\partial S}{\partial \Delta K} &= 2W_1 V_{f1} \frac{\partial V_{f1}}{\partial \Delta K} + 2W_2 V_{f2} \frac{\partial V_{f2}}{\partial \Delta K} + \ldots + 2W_n \frac{\partial V_{fn}}{\partial \Delta K} = 0 \end{aligned}\right\} \tag{16}$$

or

$$\left.\begin{aligned}
&-W_1V_{f1}F_{11} - W_2V_{f2}F_{12} - \cdots - W_nV_{fn}F_{1n} = 0 \\
&-W_1V_{f1}F_{21} - W_2V_{f2}F_{22} - \cdots - W_nV_{fn}F_{2n} = 0 \\
&\qquad\vdots \\
&-W_1V_{f1}F_{2m+1,1} - W_2V_{f2}F_{2m+1,1} - \cdots - W_nV_{fn}F_{2m+1,n} = 0
\end{aligned}\right\} . \tag{17}$$

We now express equation (12) in the following more convenient matrix expression

$$\overline{V} + \overline{B}\,\overline{\Delta} - \overline{N} = 0 \quad , \tag{18}$$

where

$$\underset{[n\times 1]}{\overline{V}} = \begin{bmatrix} V_{f1} \\ V_{f2} \\ \vdots \\ V_{fn} \end{bmatrix} , \tag{19}$$

$$\underset{[n\times(2m+1)]}{\overline{B}} = \begin{bmatrix} F_{11} & F_{21} & \cdots & F_{2m+1,1} \\ F_{12} & F_{22} & \cdots & F_{2m+1,2} \\ \vdots & \vdots & & \vdots \\ F_{1n} & F_{2n} & \cdots & F_{2m+1,n} \end{bmatrix} , \tag{20}$$

$$\underset{[(2m+1)\times 1]}{\overline{\Delta}} = \begin{bmatrix} \Delta A_1 \\ \Delta B_1 \\ \Delta A_2 \\ \Delta B_2 \\ \vdots \\ \Delta A_m \\ \Delta B_m \\ \Delta K \end{bmatrix} \quad , \tag{21}$$

and

$$\underset{[n\times 1]}{\overline{N}} = \begin{bmatrix} N_1 \\ N_2 \\ \vdots \\ N_n \end{bmatrix} \quad . \tag{22}$$

By using the denotations for $\overline{V}$ and $\overline{B}$ as given by equations (19) and (20), respectively, we can also rewrite equation (17) as

$$\overline{B}^T \overline{W} \overline{V} = 0 \quad , \tag{23}$$

where

$$\underset{[n\times n]}{\overline{W}} = \begin{bmatrix} \sigma_o^2/\sigma_{f1}^2 & 0 & \cdots & 0 \\ 0 & \sigma_o^2/\sigma_{f2}^2 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & \sigma_o^2/\sigma_{fn}^2 \end{bmatrix} \quad . \tag{24}$$

Solving equation (18) for $\overline{V}$,

$$\overline{V} = \overline{N} - \overline{B}\,\overline{\Delta} \quad . \tag{25}$$

Substituting equation (25) into equation (23),

$$\overline{B}^T \overline{W} (\overline{N} - \overline{B}\,\overline{\Delta}) = 0 \quad . \tag{26}$$

Solving for $\overline{\Delta}$,

$$\overline{B}^T \overline{W}\,\overline{N} - \overline{B}^T \overline{W}\,\overline{B}\,\overline{\Delta} = 0 \tag{27}$$

or

$$\overline{\Delta} = (\overline{B}^T \overline{W}\,\overline{B})^{-1} (\overline{B}^T \overline{W}\,\overline{N}) \quad . \tag{28}$$

An improved set of values for the parameters is then given by equation (8). The process or cycle is repeated to produce the corrections resulting from the second cycle. These corrections are then added to the estimates from the first cycle:

$$\left.\begin{aligned}
A_1^1 &= \Delta A_1^1 + A_1 \\
B_1^1 &= \Delta B_1^1 + B_1 \\
A_2^1 &= \Delta A_2^1 + A_2 \\
B_2^1 &= \Delta B_2^1 + B_2 \\
&\vdots \\
A_m^1 &= \Delta A_m^1 + A_m \\
B_m^1 &= \Delta B_m^1 + B_m \\
K^1 &= \Delta K^1 + K
\end{aligned}\right\} \quad . \tag{29}$$

These values represent an improved set of estimates to use for the third cycle. An iterative procedure is thus set up for improving the parameter estimates to any prescribed degree of accuracy consistent with the accuracy of the observed data.

The standard algorithm is based on obtaining a nominal solution that, hopefully, converges to the correct solution. The algorithm is summarized as follows:

1. Let $\tilde{A}^k$ denote the kth nominal; linearize about $\tilde{A}^k$.

2. Solve the resulting linear least squares problem.

3. Use the new solution as the new nominal.

4. Check for convergence. If convergence has not occurred, repeat steps 1 through 3.

The standard deviation of each of the converged parameters is calculated from

$$\bar{\sigma} = \sigma_f \, \bar{c} \quad , \tag{30}$$

where

$$\sigma_f = \left[\frac{\sum_{i=1}^{n} \left(f_i^o - f_i^c \right)^2}{n - 2m - 1} \right]^{1/2} \tag{31}$$

and

$$\underset{[(2m+1)\times 1]}{\bar{c}} = \begin{bmatrix} \sqrt{c_{11}} \\ \sqrt{c_{22}} \\ \vdots \\ \sqrt{c_{2m+1,\,2m+1}} \end{bmatrix} . \tag{32}$$

The c elements in equation (32) refer to diagonal elements in the inverse matrix $(\overline{B}^T \overline{W} \overline{B})^{-1}$.

PROCEDURE FOR OBTAINING THE INITIAL PARAMETER ESTIMATES

"Peeling-Off" Approach

An iterative method for nonlinear exponential regression analysis was developed in the previous section. Inherent in this method is a requirement for initial estimates of the parameters. This section presents a least squares "peeling-off" procedure for arriving at these initial estimates.

Our assumed exponential model is of the form given by equation (1). Generally speaking, if we plot decay-type data in the form $\ln f^o$ against t where f^o is the observed response and t is the independent variable, then for large t the curve is approximately a straight line. Consider the following Figure 1.

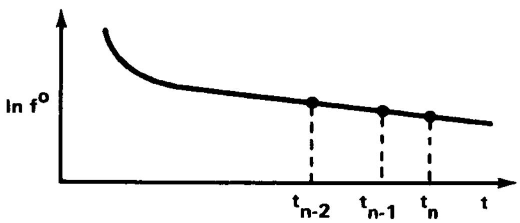


Figure 1. Logarithmic time decay illustration.

If we fit a straight line to the last three data points by the method of least squares, the assumed form is

$$\ln f^o = -\tilde{B}_m t + D \quad . \tag{33}$$

However, this is equivalent to the equation

$$f^o = \tilde{A}_m e^{-\tilde{B}_m t} \quad ,$$

where

$$D = \ln \tilde{A}_m \quad . \tag{34}$$

The coefficients $\tilde{B}_m$ and D are given by

$$D = \frac{\sum_{i=n}^{n-2} t_i^2 \sum_{i=n}^{n-2} \ln f_i^o - \sum_{i=n}^{n-2} t_i \sum_{i=n}^{n-2} t_i \ln f_i^o}{3 \sum_{i=n}^{n-2} t_i^2 - \left(\sum_{i=n}^{n-2} t_i \right)^2} \tag{35}$$

and

$$\tilde{B}_m = - \left[\frac{3 \sum_{i=n}^{n-2} t_i \ln f_i^o - \sum_{i=n}^{n-2} t_i \sum_{i=n}^{n-2} \ln f_i^o}{3 \sum_{i=n}^{n-2} t_i^2 - \left(\sum_{i=n}^{n-2} t_i \right)^2} \right] \quad . \tag{36}$$

Thus, we have determined the least squares values for $\tilde{A}_m$ and $\tilde{B}_m$. We can then obtain values for the residuals from $(i = 0, 1, 2)$

$$R_{n-i} = f_{n-i}^o - \tilde{A}_m e^{-\tilde{B}_m t_{n-i}} \quad . \tag{37}$$

We now take the next three data points at t_{n-3}, t_{n-4}, and t_{n-5} and subtract the corresponding term $\tilde{A}_m e^{-\tilde{B}_m t}$ and the arbitrary constant $\tilde{K}$ from f^o to obtain the following residuals

$$\left.\begin{aligned}
R_{n-3} &= f^o_{n-3} - \tilde{A}_m e^{-\tilde{B}_m t_{n-3}} - \tilde{K} \\
R_{n-4} &= f^o_{n-4} - \tilde{A}_m e^{-\tilde{B}_m t_{n-4}} - \tilde{K} \\
R_{n-5} &= f^o_{n-5} - \tilde{A}_m e^{-\tilde{B}_m t_{n-5}} - \tilde{K}
\end{aligned}\right\} . \qquad (38)$$

Next, a straight line is fitted to the data for $\ln |R_{n-i}|$ against t_{n-i}, $i = 3, 4, 5$. This determines $\tilde{A}_{m-1}$ and $\tilde{B}_{m-1}$. The residuals for the next three data points are determined from

$$\left.\begin{aligned}
R_{n-6} &= f^o_{n-6} - \tilde{A}_{m-1} e^{-\tilde{B}_{m-1} t_{n-6}} - \tilde{A}_m e^{-\tilde{B}_m t_{n-6}} - \tilde{K} \\
R_{n-7} &= f^o_{n-7} - \tilde{A}_{m-1} e^{-\tilde{B}_{m-1} t_{n-7}} - \tilde{A}_m e^{-\tilde{B}_m t_{n-7}} - \tilde{K} \\
R_{n-8} &= f^o_{n-8} - \tilde{A}_{m-1} e^{-\tilde{B}_{m-1} t_{n-8}} - \tilde{A}_m e^{-\tilde{B}_m t_{n-8}} - \tilde{K}
\end{aligned}\right\} . \qquad (39)$$

We now fit a straight line to $\ln |R_{n-i}|$ against t_{n-i}, $i = 6, 7, 8$. This determines $\tilde{A}_{m-2}$ and $\tilde{B}_{m-2}$. We continue this process until all the $\tilde{A}$'s and $\tilde{B}$'s are determined. In general, $\tilde{A}_1$ and $\tilde{B}_1$ are determined from a set that contains more than three data points. That is, the points remaining after $\tilde{A}_2$ and $\tilde{B}_2$ have been determined are used for determining $\tilde{A}_1$ and $\tilde{B}_1$. It is noted that since three points are chosen as a minimum for calculating a particular $\tilde{A}$ and $\tilde{B}$, we must have $n \geq 3m$ where n is the number of data points. At this point we calculate the weighted sum of squares of the deviations using the initial parameter estimates

$$\tilde{F}_1 = \sum_{i=1}^{n} W_i \left(f_i^o - f_i^c\right)^2 \quad , \qquad (40)$$

where

$$W_i = 1/\sigma_{fi}^2 \quad ,$$

$$f_i^o = \text{observed response} \quad ,$$

and

$$f_i^c = \text{fitted response.}$$

Iteration Philosophy

The iteration logic can be summarized by the following steps:

1. Repeat the process, but use the last four data points to obtain the initial estimates of $\tilde{A}_m$ and $\tilde{B}_m$. The next three data points are used to obtain $\tilde{A}_{m-1}$, $\tilde{B}_{m-1}$, etc., for other $\tilde{A}$'s and $\tilde{B}$'s. This yields a second set of parameter estimates from which we can calculate another $\tilde{F}$; call it $\tilde{F}_2$.

2. Use the last five data points and obtain a third set of parameters which yield $\tilde{F}_3$.

3. We continue this, always keeping three points as a minimum in determining $\tilde{A}_i$ and $\tilde{B}_i$.

4. Increase the constant to $\tilde{K} + 0.05\,\tilde{K}$ and repeat steps 1 through 3. Terminate when the constant reaches a predetermined value.

5. Parameter estimates yielding the minimum $\tilde{F}$ are chosen as the initial estimates.

COMPUTER PROGRAM DEVELOPMENT

The computer programs to implement the previously developed theory for exponential regression analysis were organized and developed according to two general types of exponential models. One concerns a single exponential and the sum of exponentials without a constant, and the other concerns the sum of exponentials with a constant included. Two highly flexible computer programs were thus developed for the MSFC UNIVAC 1108 digital computer. Each program contains double-precision capability and SC-4020 plotting procedures. In addition, the "peeling-off" procedure for obtaining initial parameter estimates is an integral part of each program segment.

The logic flow for Programs I and II is depicted in Figures 2 and 3. The parameter NCASES is the number of cases of data processed in each program. The models that can be investigated in Program I are

Model I: $$f^c = A_1e^{-B_1t} + A_2e^{-B_2t} + A_3e^{-B_3t} + K$$

and

Model II: $$f^c = A_1e^{-B_1t} + A_2e^{-B_2t} + K \quad .$$

Those models that can be investigated in Program II are

Model III: $$f^c = A_1e^{-B_1t} + A_2e^{-B_2t} + A_3e^{-B_3t} \quad ,$$

Model IV: $$f^c = A_1e^{-B_1t} + A_2e^{-B_2t} \quad ,$$

and

Model V: $$f^c = A_1e^{-B_1t} \quad .$$

The characteristics of both programs are summarized in Table 1. It should be noted that both programs can be easily extended to include additional exponential terms if desired.

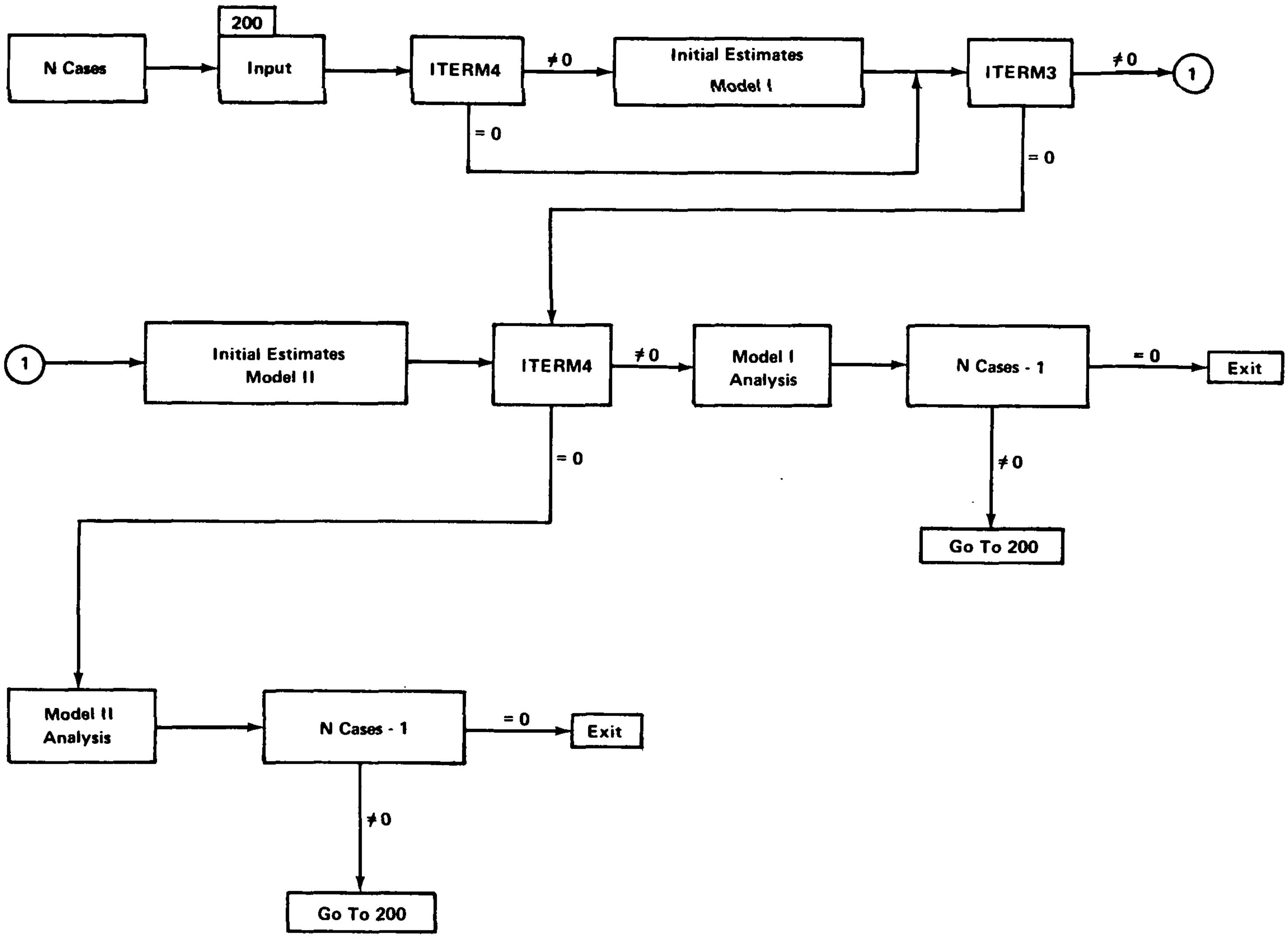


Figure 2. Program I block diagram summary.

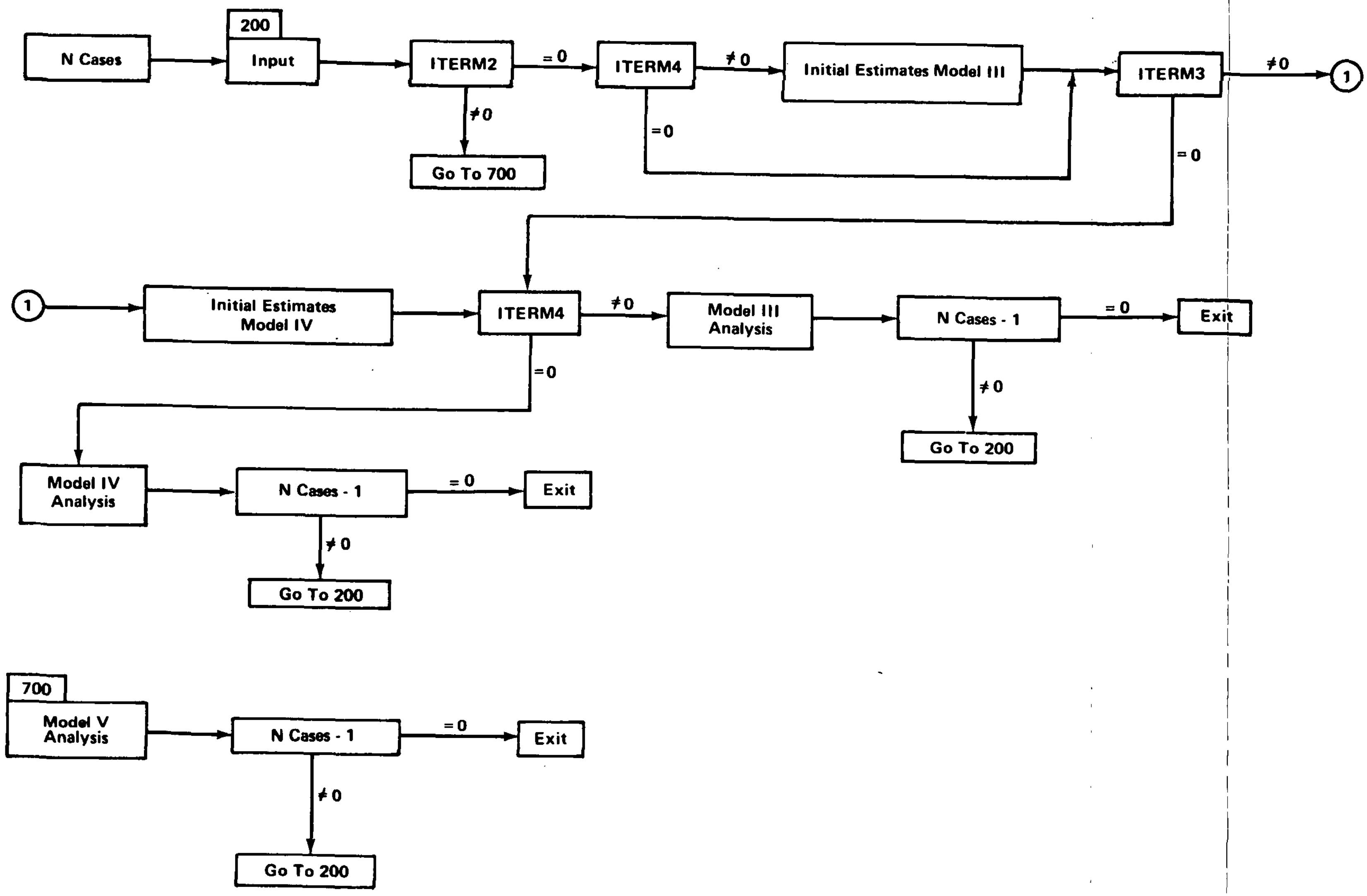


Figure 3. Program II block diagram summary.

TABLE 1. COMPUTER PROGRAM SUMMARY FOR EXPONENTIAL CURVE FITTING

Program Number[a]	Exponential Model Fitted
I	(1) A_i , B_i , K : i = 1, 2, 3
	(2) A_i , B_i , K : i = 1, 2
II	(1) A_i , B_i : i = 1, 2, 3
	(2) A_i , B_i : i = 1, 2
	(3) A_1 , B_1

a. Each program has a built-in capability for obtaining initial parameter estimates.

The parameter ITERM is used to determine control transfer in each program. For example, if ITERM has the value one, parameters for the particular model associated with it are determined. A value of zero indicates that parameters for the associated model are not determined. A double-precision matrix inverse routine using the Gaussian elimination procedure is used in each program.

RESULTS AND CONCLUSIONS

Discussion of the Physical Process

In an open circuit transient analysis of the anodic oxidation of metals, one expects an exponential or logarithmic behavior. This fact is evident from the experimental data when the layer is passive; that is, normal growth is taking place. In some cases, however, growth is truncated by the onset of oxygen evolution. In this region we have electronic conduction in addition to a small amount of anodic or growth conduction. During the open circuit break, therefore, we expect the conduction to be initially dominated by electrons and as the voltage decreases, the conduction becomes primarily anodic conduction. Thus, the transient analysis has to have the capability to consider two or more conduction mechanisms, each with different relaxation times.

Basically, the voltage data from the anodic oxidation process are classified into four sets. Each set of data represents a different time held in oxygen evolution. When plotted as a function of time, these voltage data exhibit an exponential or decay-type behavior. The response data are represented as an observed voltage, which is designated as f^o.

Discussion of Results

The observed decay data were processed through both Programs I and II to assess the validity of the various exponential models. A summary of the initial estimates for the various assumed models is presented in Tables 2, 3, and 4. The minimum value for the $\tilde{F}$ quantity associated with the selected initial estimates is given in these tables.

TABLE 2. INITIAL ESTIMATE SUMMARY FOR TWO-TERM EXPONENTIAL MODEL PLUS A CONSTANT

	Set 1	Set 2	Set 3	Set 4
Groups of Initial Estimates	441	441	441	441
Group Number with Smallest $\tilde{F}$	49	44	61	63
Smallest $\tilde{F}$ Value	0.2083	0.3113	2.9021	5.986

TABLE 3. INITIAL ESTIMATE SUMMARY FOR TWO-TERM EXPONENTIAL MODEL

	Set 1	Set 2	Set 3	Set 4
Groups of Initial Estimates	63	63	63	63
Group Number with Smallest $\tilde{F}$	52	63	61	59
Smallest $\tilde{F}$ Value	0.0827	0.0455	0.01185	0.02952

TABLE 4. INITIAL ESTIMATE SUMMARY FOR THREE-TERM EXPONENTIAL MODEL PLUS A CONSTANT

	Set 1	Set 2	Set 3	Set 4
Groups of Initial Estimates	420	420	420	—
Group Number with Smallest $\tilde{F}$	59	47	14	—
Smallest $\tilde{F}$ Value	0.2132	0.3532	57.45	—

Numerical problems were encountered for both the three-term model and the three-term plus a constant model. As indicated in Table 5, divergence occurred for the set 2 and set 3 data. It is noted that the determinant of the coefficients for solving for the correction matrix was 0.911×10^{-18} in one case and 0.162×10^{-2} in the other case. Convergence failed to occur for the set 2 data even with relatively good initial estimates.

TABLE 5. RESULTS FOR THREE-TERM EXPONENTIAL MODEL PLUS A CONSTANT

Data Set Number	Set 1		Set 2		Set 3		Set 4[b]	
Parameter and Error Estimate	Initial Estimate	Improved Estimate	Initial Estimate	Improved Estimate[a]	Initial Estimate	Improved Estimate[a]	Initial Estimate	Improved Estimate
A_1	0.506027	0.550958	0.524024		0.289545			
σ	—	0.0319	—		—			
B_1	1.144510	1.003510	0.121633		0.006907			
σ	—	0.3064	—		—			
A_2	0.525248	0.571808	0.036133		0.237985			
σ	—	0.0200	—		—			
B_2	0.117757	0.063911	0.065537		0.000905			
σ	—	0.0070	—		—			
A_3	0.231458	0.108318	0.473872		0.777985			
σ	—	0.0066	—		—			
B_3	0.010038	0.012974	0.011831		0.006935			
σ	—	0.0005	—		—			
K	0.0525	0.0689	0.0770		0.21350			
σ	—	0.0039	—		—			
σ_f	0.0591	0.0321	0.0761	—	0.9705			
Determinant Value	—	7.90	—	0.0016	—	0.911×10^{-18}		

a. Diverged.

b. Numerical problems for assumed model.

The data in Table 6 summarize the results for the two-term plus a constant model. As shown in this table convergence failed to occur for the set 3 and set 4 data. The initial estimates for the set 1 and set 2 data showed some disagreement with the cycle 1 estimates. The σ_f initial estimate values for set 1 and set 2 are significantly smaller than the σ_f values at the end of cycle 1. Rather high error estimates for the A_1 and B_1 parameters are also evident in this table.

Results obtained for the two-term exponential model are perhaps the most encouraging from the standpoint of adequately describing the data. As indicated in Table 7, highly accurate parameter estimates were obtained. The converged estimates represent an improvement over the initial estimates with the exception of the set 4 data. Here it is noted that $\sigma_f = 0.0215$ for the initial estimates as compared to $\sigma_f = 0.0600$ for the improved estimates. The initial estimates were thus chosen as the representation for the set 4 data. The models that appear to adequately describe the observation data are

$$\text{Set 1: } f^c = 0.924798e^{-0.160783t} + 0.230238e^{-0.010751t} \quad ,$$

$$\text{Set 2: } f^c = 0.629470e^{-0.23023t} + 0.676240e^{-0.01642t} \quad ,$$

$$\text{Set 3: } f^c = 0.598702e^{-0.126326t} + 0.707297e^{-0.006132t} \quad ,$$

and

$$\text{Set 4: } f^c = 0.424970e^{-0.109368t} + 0.897037e^{-0.005876t} \quad .$$

The observation data and the models evaluated at the corresponding time point are presented in graphical form in Figures 4 through 7. Residual data are presented in Figures 8 through 11. These figures indicate an adequate model representation of the data. It is concluded that changing the exponential functional form for these data to one other than a two-term model is not warranted in view of the problems encountered with the other models.

The coefficients for each model are plotted in Figure 12 as a function of the time to oxygen evolution associated with each set. These data enable one to simulate the physical process using a two-term exponential model at conditions other than those tested.

TABLE 6. RESULTS FOR TWO-TERM EXPONENTIAL MODEL PLUS A CONSTANT

Data Set Number	Set 1		Set 2		Set 3		Set 4	
Parameter and Error Estimate	Initial Estimate	Cycle 1 Estimate	Initial Estimate	Cycle 1 Estimate	Initial Estimate	Cycle 1 Estimate[a]	Initial Estimate	Cycle 1 Estimate[a]
A_1	0.912635	0.826217	0.673531	0.315643	0.332689		0.023109	
σ	—	0.4746	—	0.2146	—		—	
B_1	0.158482	0.206468	0.113233	0.218638	0.287924		0.065336	
σ	—	0.1355	—	0.0569	—		—	
A_2	0.169134	0.472633	0.437571	0.414676	0.746388		0.948405	
σ	—	0.1237	—	0.0759	—		—	
B_2	0.006810	0.067656	0.011059	0.049739	0.006686		0.006476	
σ	—	0.0123	—	0.0035	—		—	
K	0.05250	0.834446	0.0770	0.55022	0.21350		0.29750	
σ	—	0.0840	—	0.0431	—		—	
σ_f	0.0575	0.6926	0.0703	0.3556	0.21463	—	0.3059	—
Determinant Value	—	$0.9687 \cdot 10^4$	—	$0.7801 \cdot 10^5$	—	$0.8811 \cdot 10^4$	—	129.94

a. Diverged.

TABLE 7. RESULTS FOR TWO-TERM EXPONENTIAL MODEL

Data Set Number	Set 1		Set 2		Set 3		Set 4	
Parameter and Error Estimate	Initial Estimate	Improved Estimate	Initial Estimate	Improved Estimate	Initial Estimate	Improved Estimate	Initial Estimate	Improved Estimate
A_1	0.951721	0.924798	0.696720	0.629470	0.593509	0.598702	0.424970	0.488538
σ	—	0.0224	—	0.0161	—	0.0065	—	0.0357
B_1	0.140500	0.160783	0.256310	0.230230	0.182970	0.126326	0.109368	0.036581
σ	—	0.0054	—	0.0109	—	0.0033	—	0.0144
A_2	0.181307	0.230238	0.629470	0.676240	0.746388	0.707297	0.897037	0.804932
σ	—	0.0063	—	0.0048	—	0.0016	—	0.0102
B_2	0.007505	0.010751	0.014747	0.01642	0.006686	0.006132	0.005876	0.004847
σ	—	0.0006	—	0.0002	—	0.00004	—	0.0002
σ_f	0.0359	0.0342	0.0267	0.0201	0.0136	0.0089	0.0215	0.0600
Determinant Value	—	$0.5229 \cdot 10^6$	—	$0.1292 \cdot 10^6$	—	$0.2446 \cdot 10^7$	—	$0.6845 \cdot 10^7$

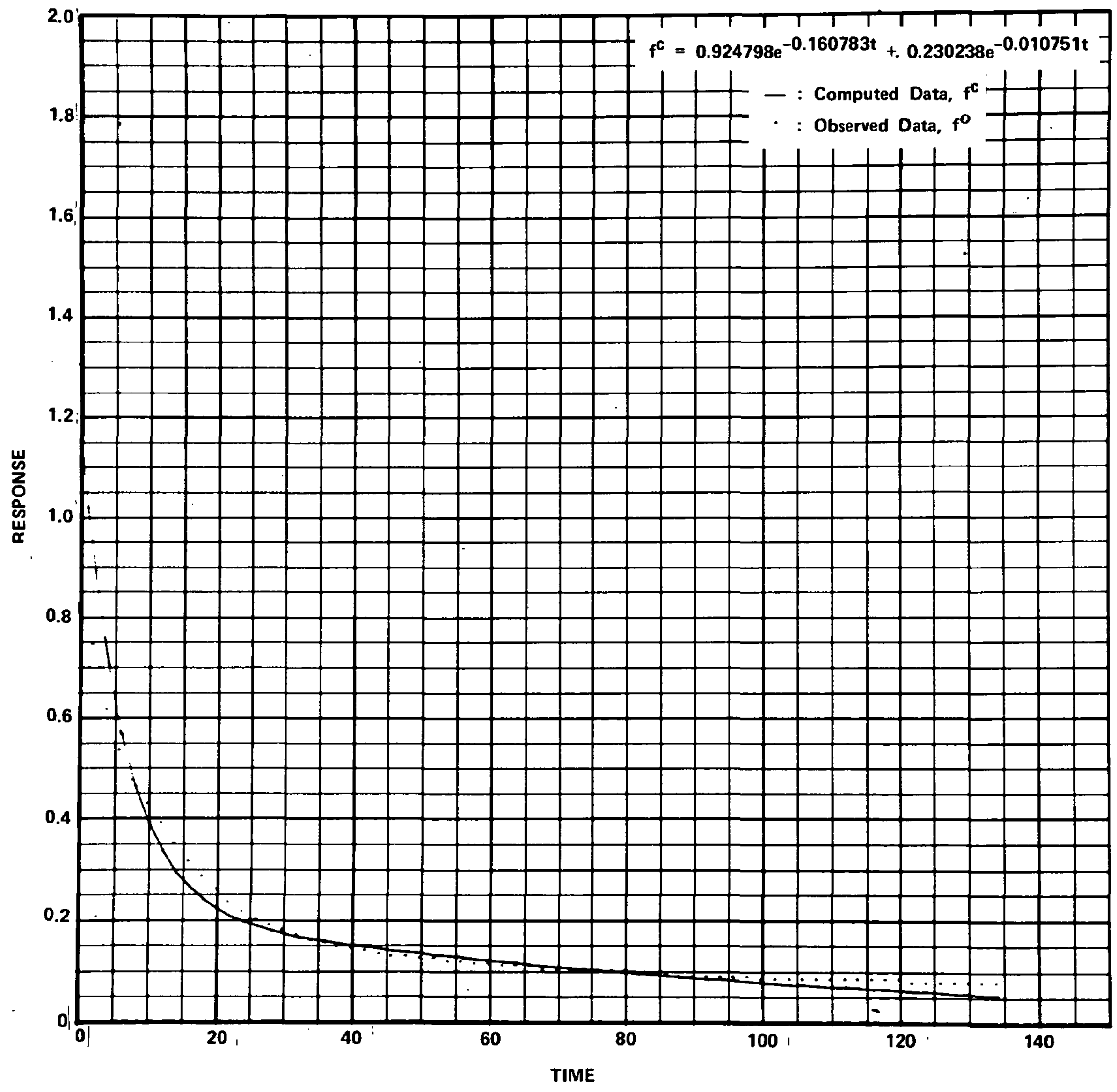


Figure 4. Observed and computed response for Model IV analysis on set 1 data.

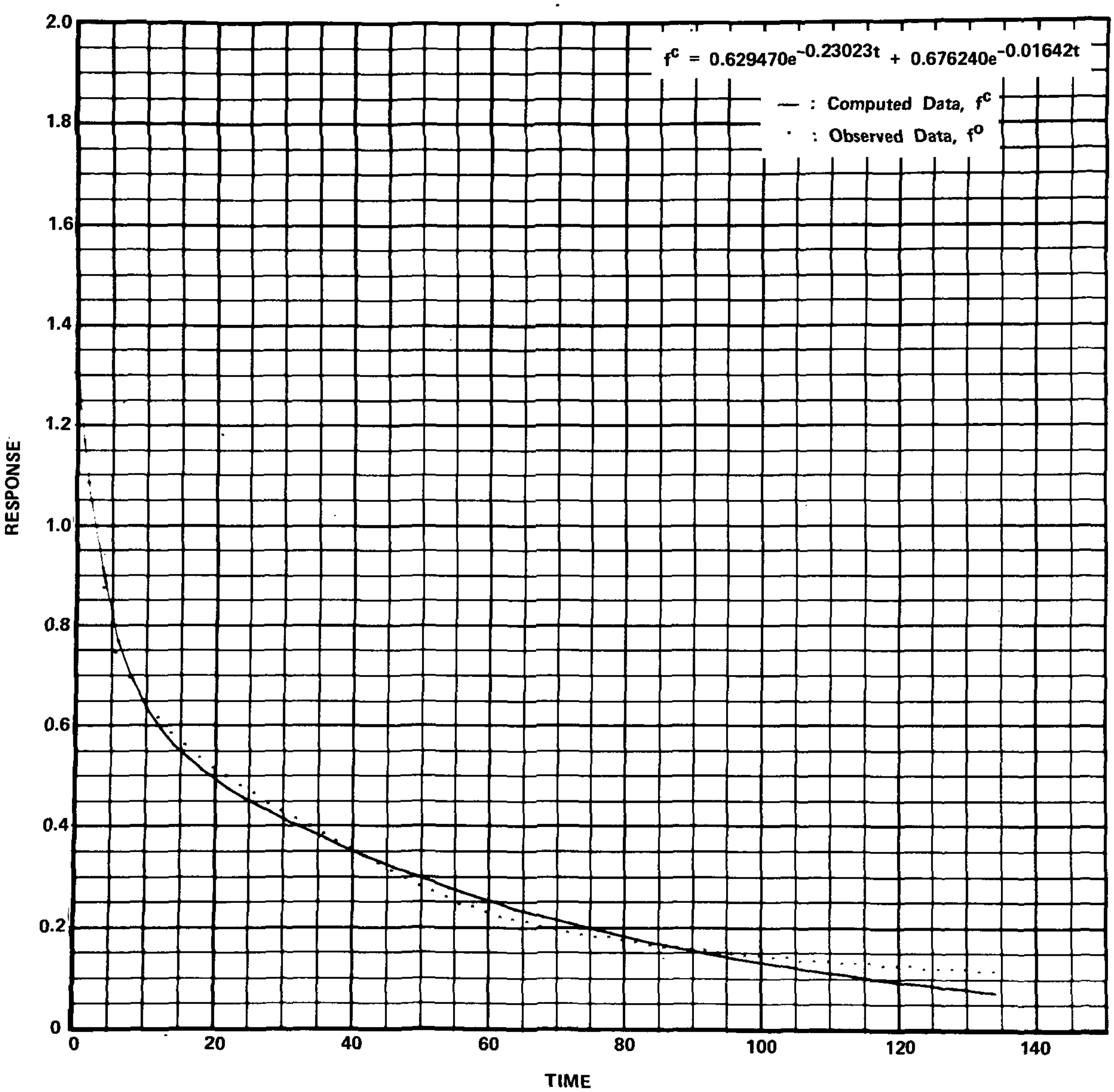


Figure 5. Observed and computed response for Model IV analysis on set 2 data.

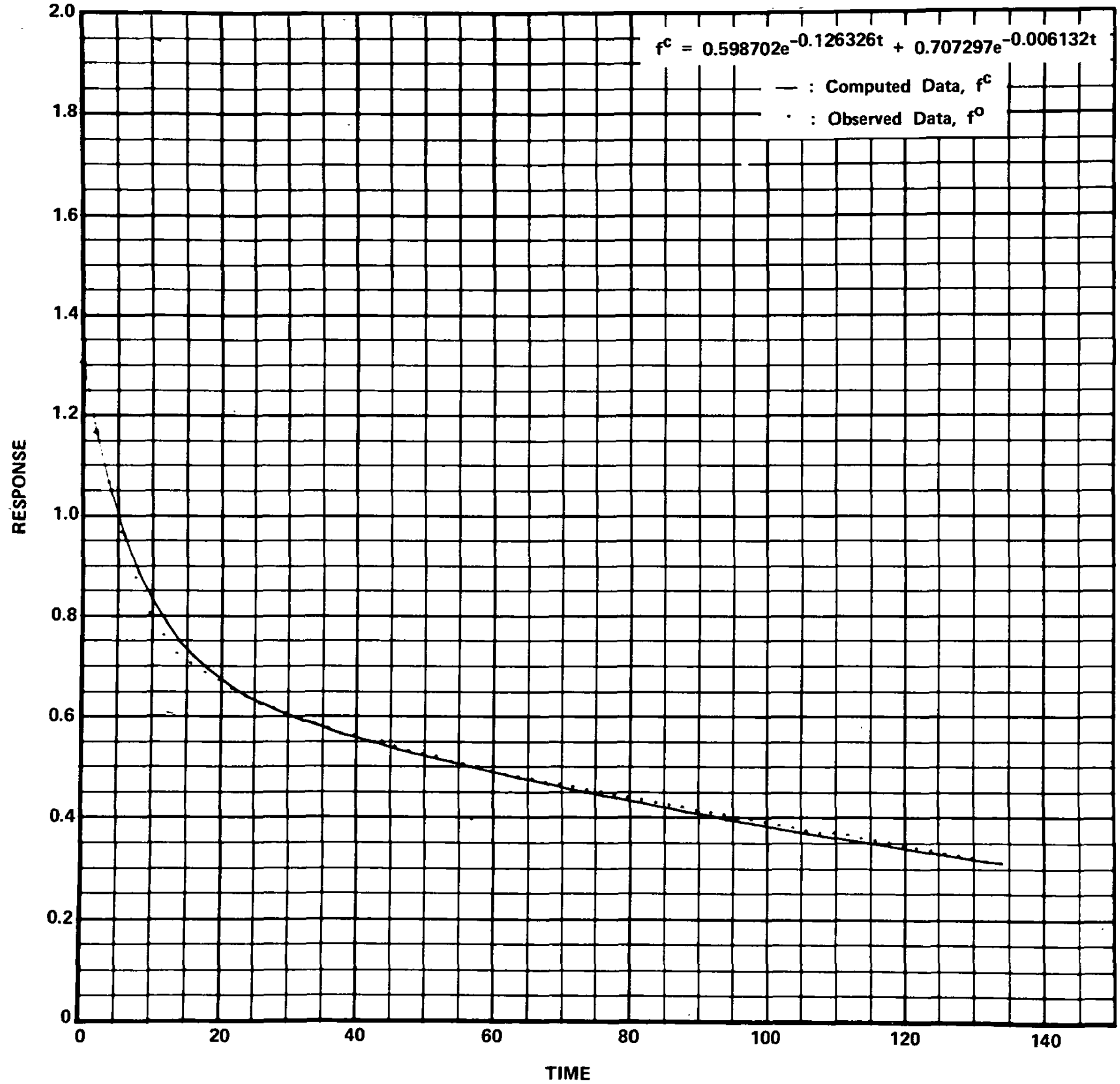


Figure 6. Observed and computed response for Model IV analysis on set 3 data.

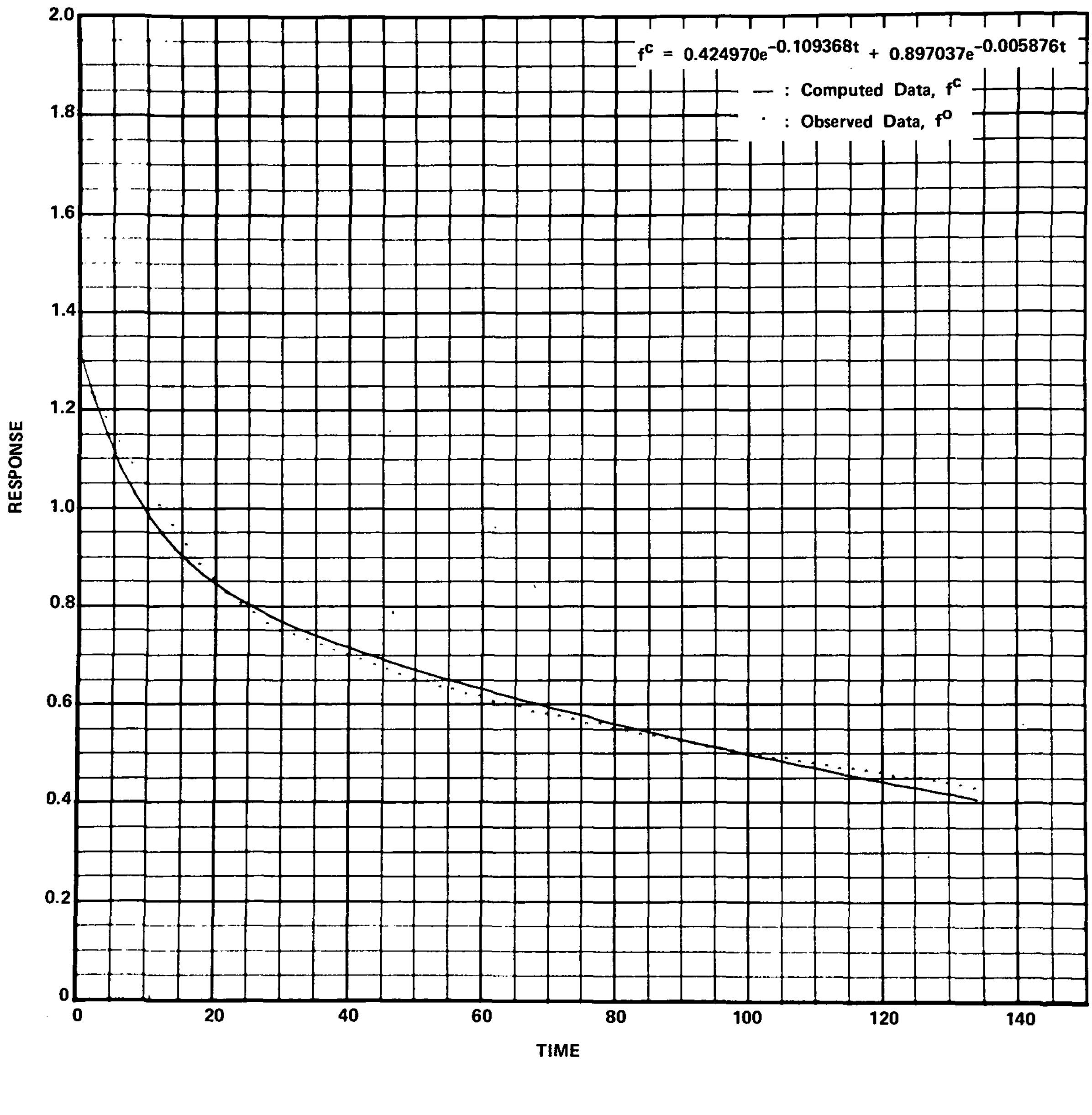


Figure 7. Observed and computed response for Model IV analysis on set 4 data.

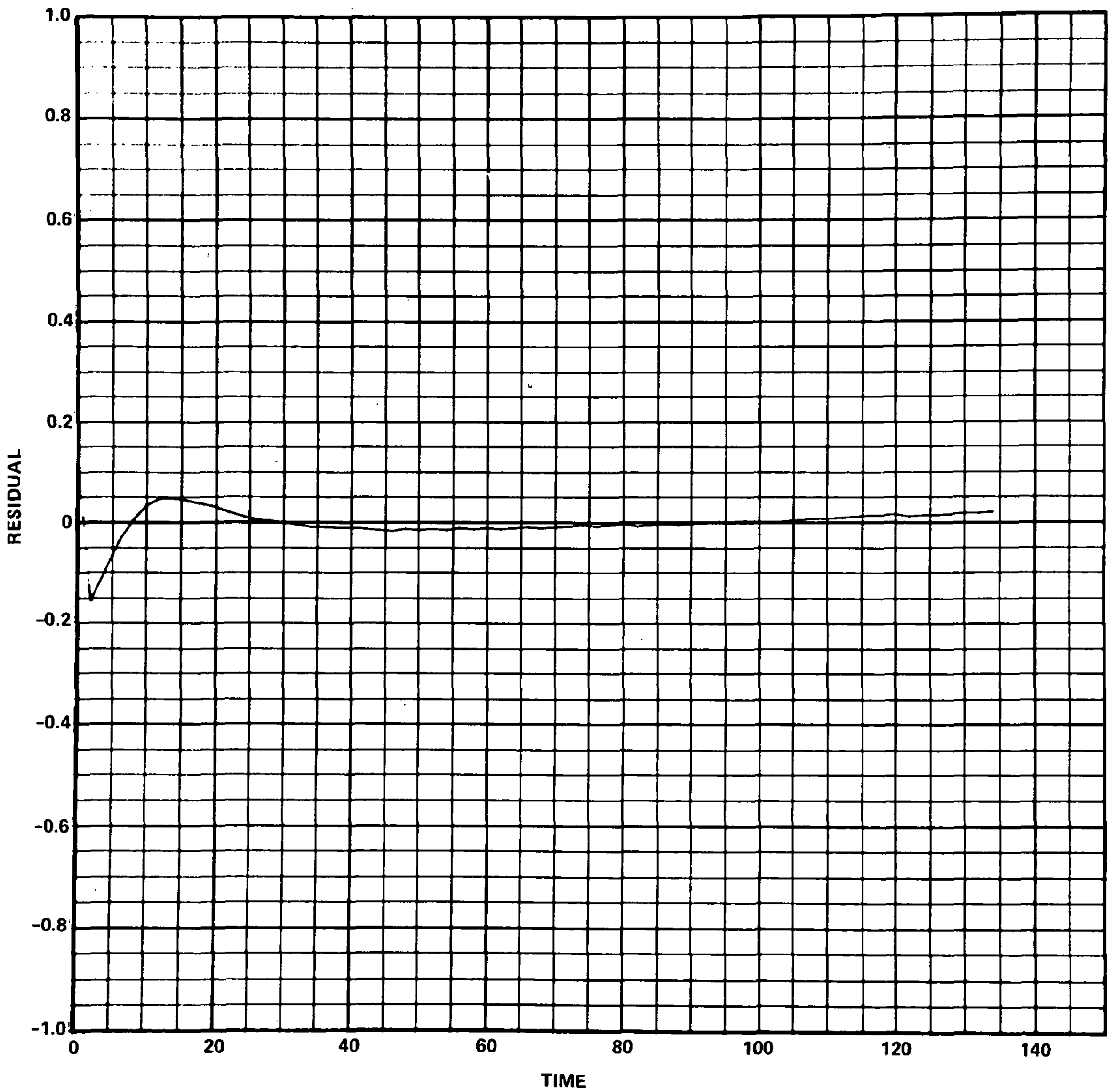


Figure 8. Residuals for Model IV analysis on set 1 data.

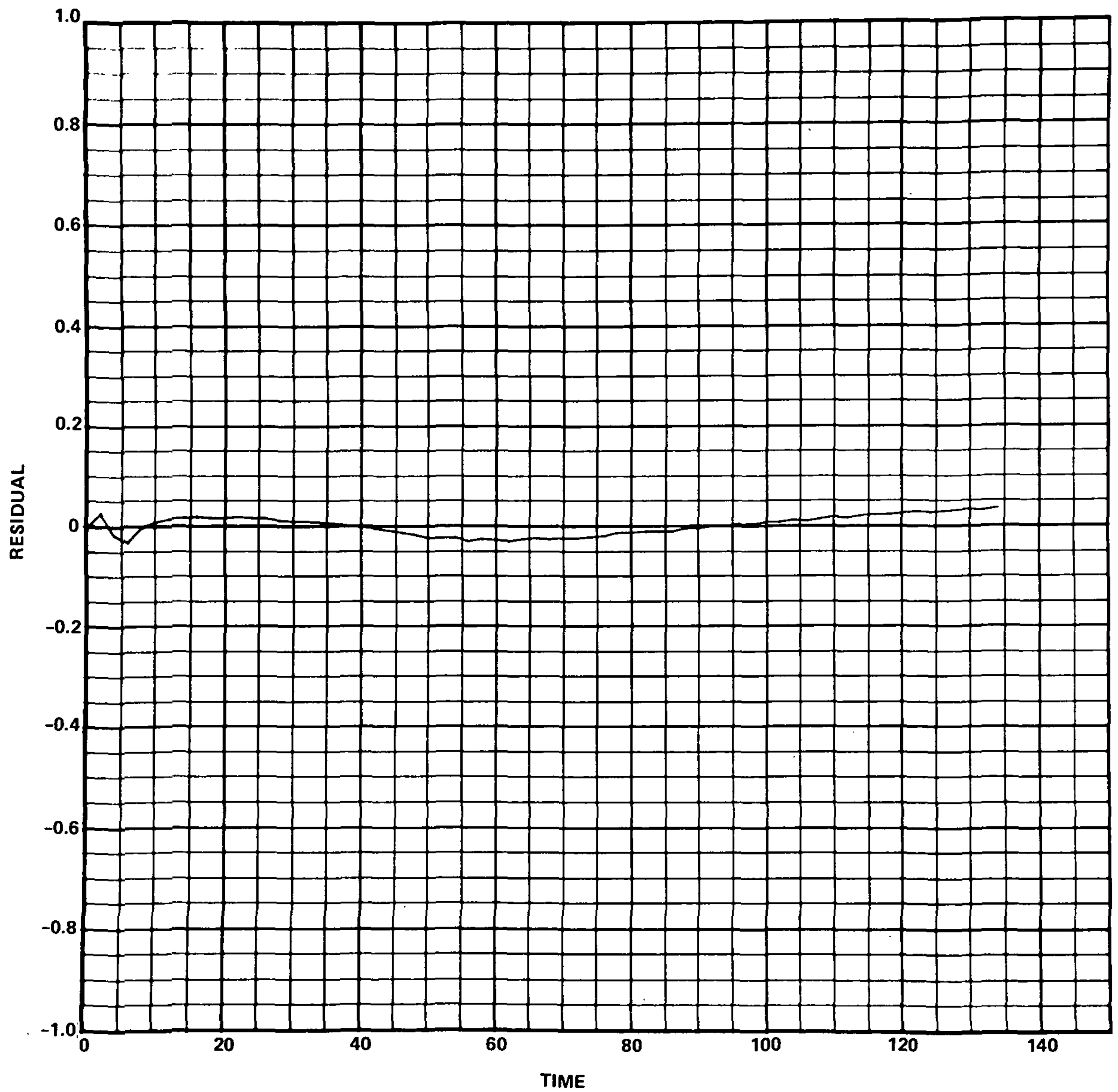


Figure 9. Residuals for Model IV analysis on set 2 data.

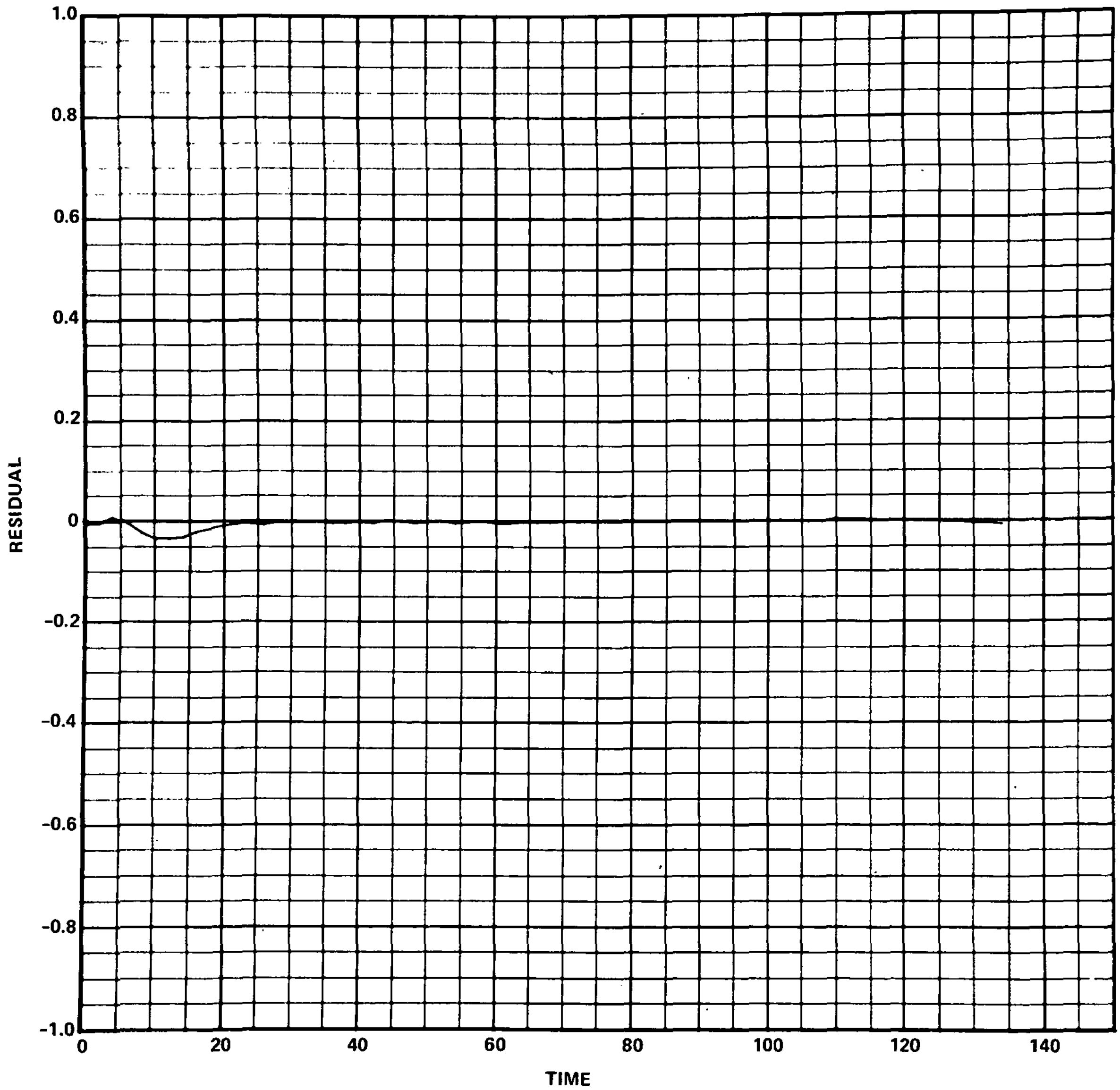


Figure 10. Residuals for Model IV analysis on set 3 data.

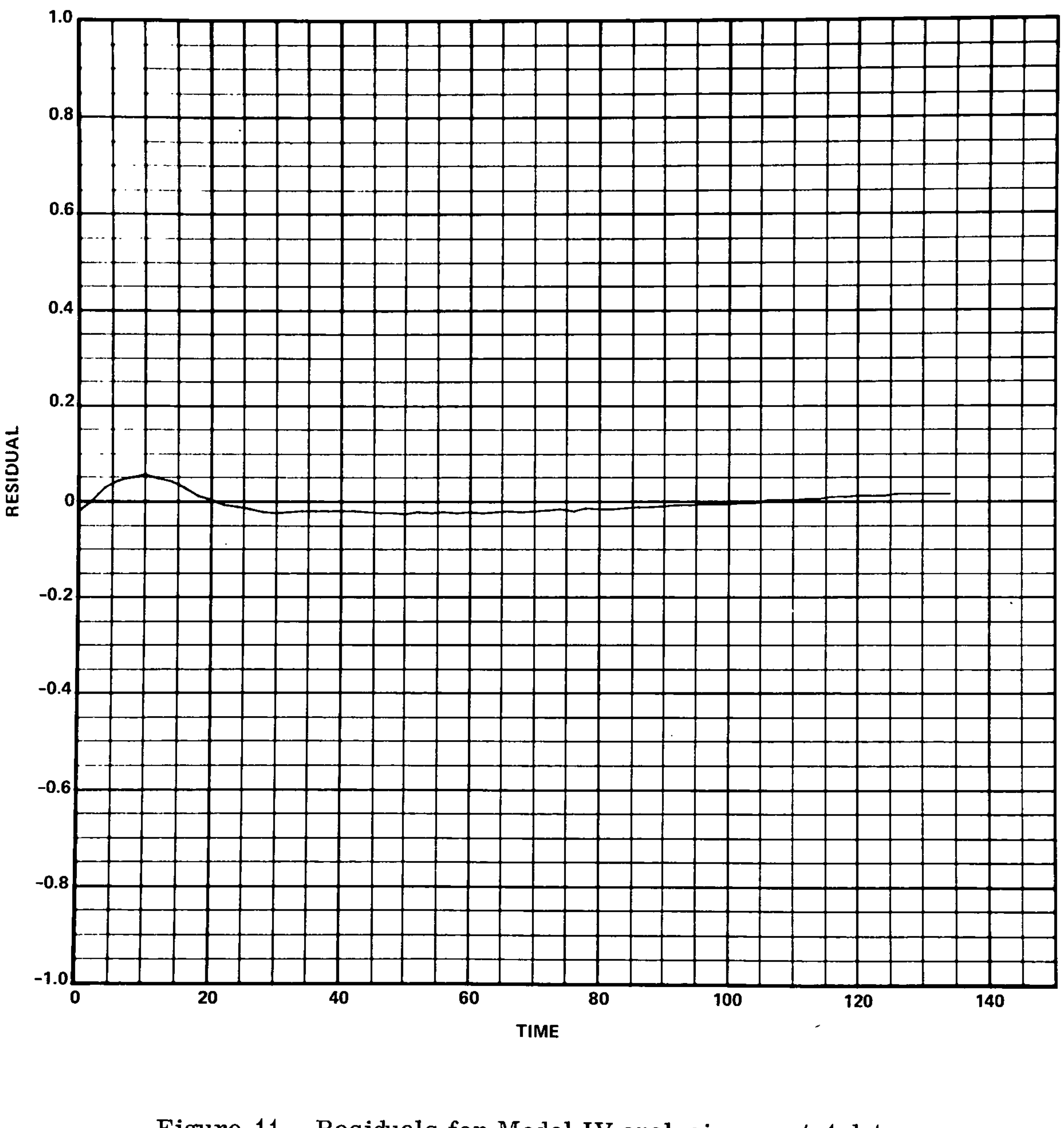


Figure 11. Residuals for Model IV analysis on set 4 data.

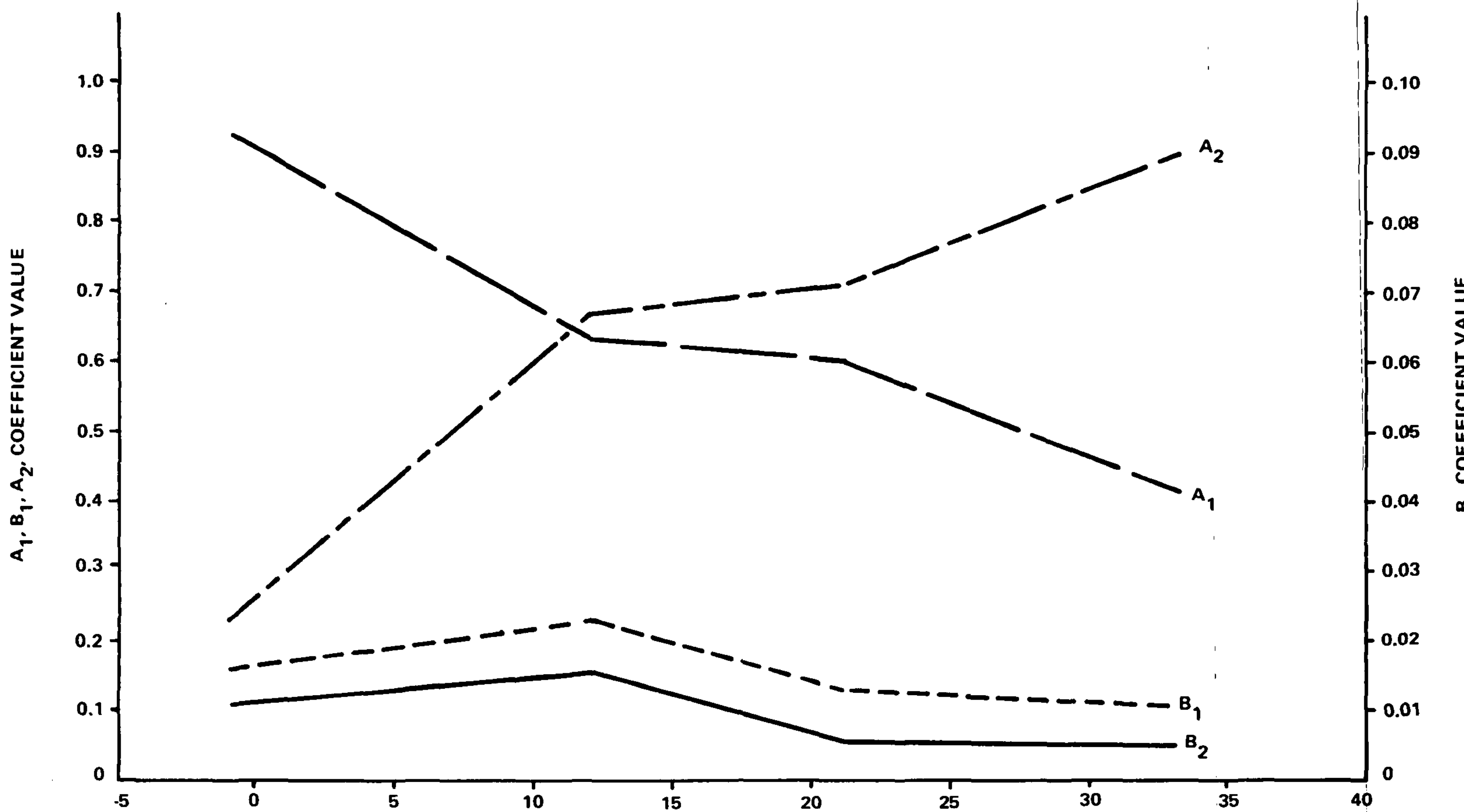


Figure 12. Model IV coefficients versus oxygen evolution time.

APPENDIX

COMPUTER PROGRAM DOCUMENTATION

This appendix presents operational information on the UNIVAC 1108 computer programs concerning the regression analysis application of exponential models to decay-type data. The general organization of the operational version of the two developed programs is depicted in Figure A-1. Since both programs are basically similar, only information concerning Program II is presented. A complete program listing, job card example (Fig. A-2), input preparation, and sample output are included.

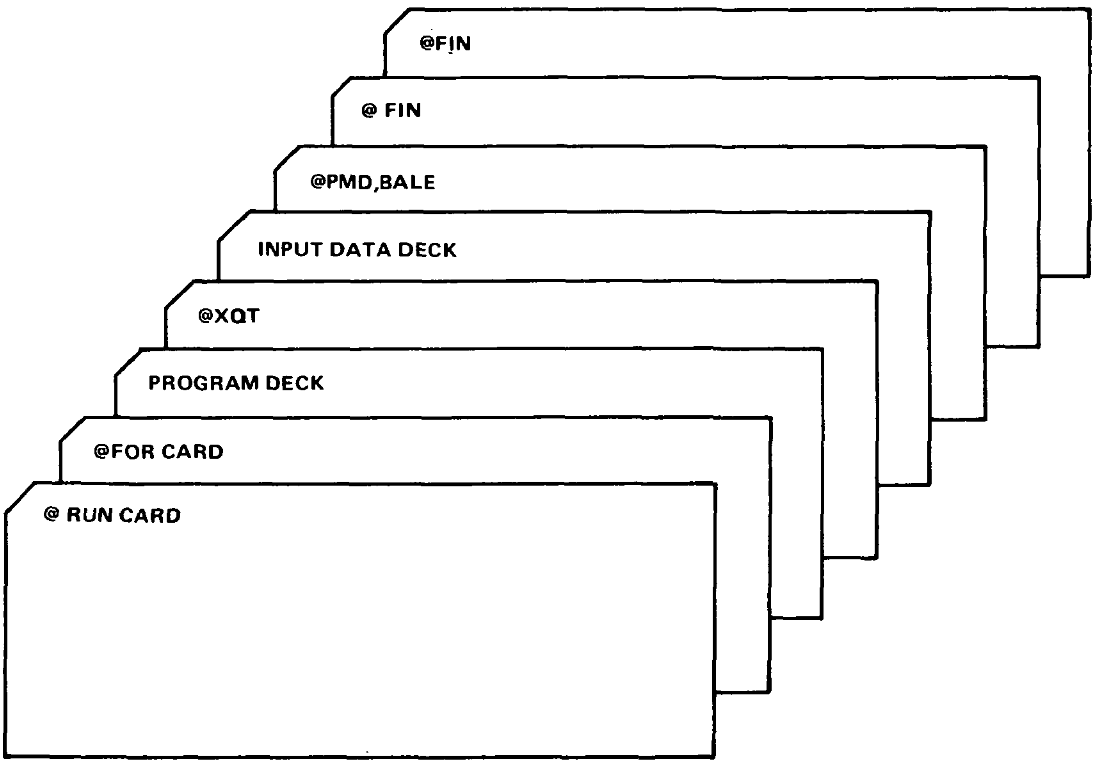


Figure A-1. Program organization.

1108 RUN REQUEST & INSTRUCTIONS

NAME (LAST & INITIAL)	OP CODE	JOB#	PROD #
JUNKIN, B.	12	320600	

BIN#	BLDG#	RM#	RUN-ID	RUN
313	4663	A-217	EXPREG	1 OF 1

EST. CPU RUN TIME	CORE SIZE	PUNCH	
0 HRS. 3 MINS.	24K	N/A	☒ COMPILE ☒ EXECUTE ☐ SORT

	LANGUAGE	MAX. PAGES	☐ SPECIAL FORMS
☐ ☒ EXEC VIII	FOR4	1,500	TYPE COPIES

DOES THIS JOB HAVE A RESTART PROCEDURE? ☐ YES ☐ NO

INPUT TAPES			OUTPUT TAPES ONLY				
REEL NO.	FILE NAME	UNIT	REEL NO.	FILE NAME	LOGIC	UNIT	SAVE

PROGRAMMER COMMENTS:

Set 1 Data: 2 exponentials

☐ OVER

MICRO FILM		COPIES		COPY FLO		OPER. INIT.
#FILES	#FRAMES	P	F	P	F	SEQ.#

OPERATOR COMMENTS: ☐ SEE TECH. ☐ SEE OPER.

MSFC - Form 3019 (Rev August 1969) ☐ OVER

Figure A-2. Job card example.

Description of Data Deck Input Parameters

The card immediately following the @XQT card is the first card of the input data deck. This card specifies the number of cases (NCASES) of data that are to be processed. The format is of the form +XX(I3) and appears in columns 1 through 3. The information between the $INPUT card and the $ card is associated with a specific set of data and is input under the non-executable NAMELIST statement. For example, the input statement in the program is

NAMELIST/INPUT/T, Y, NN, TL, TR, YB, YT, YB1, YT1, VARY, TOLER, ITERM4, ITERM3, ITERM2.

The forms that the input data take include variable name and subscripted variable. In the usage above, T and Y are subscripted arrays and the remaining variables are simple variable names. The specific format of the data can be either integer constants (i. e., +218) or real constants (i. e., 1.85921E+00, with or without the E notation). The description of the variables in the NAMELIST statement follows.

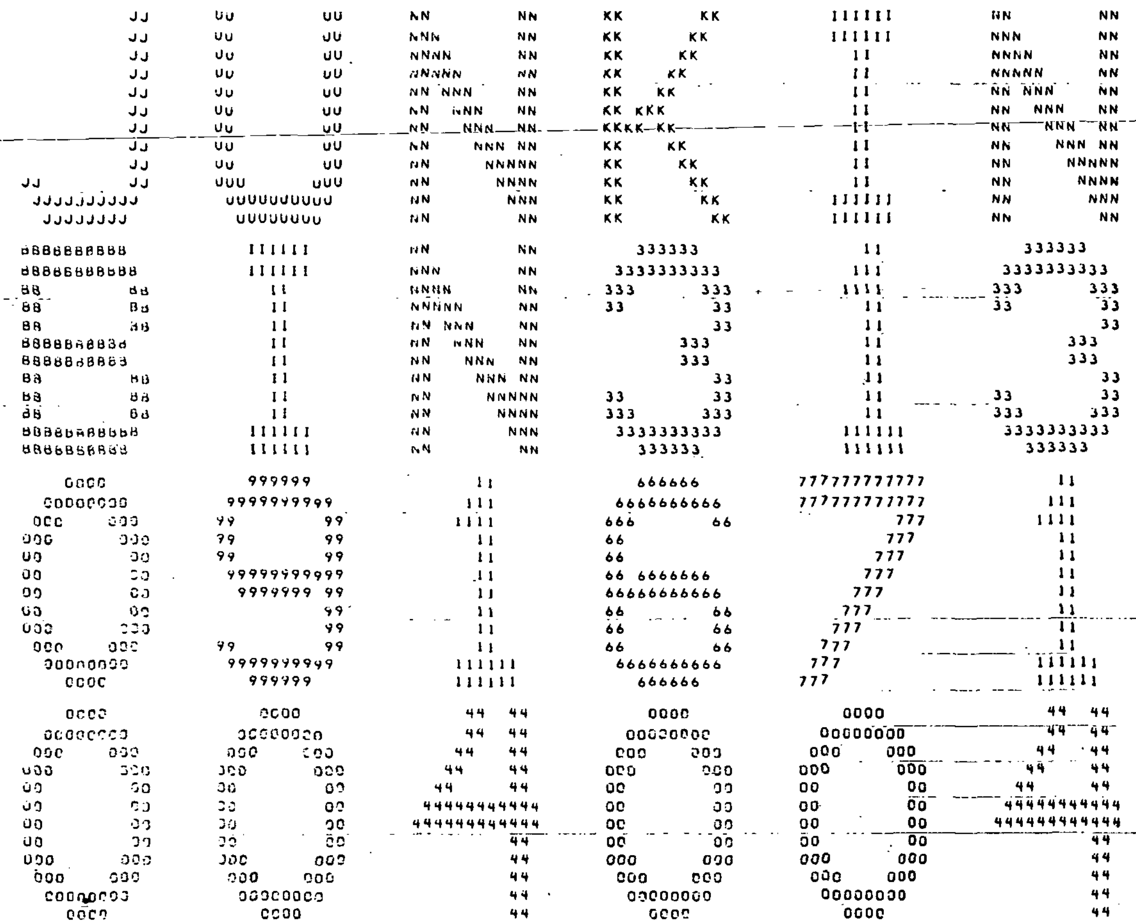
T	—	array containing the values for the independent variable
Y	—	array containing the values for the dependent variable
NN	—	number of data points
TL	—	left plot limit for the horizontal T axis
TR	—	right plot limit for the horizontal T axis
YB	—	bottom plot limit for the vertical Y axis
YT	—	top plot limit for the vertical Y axis
YB1	—	bottom plot limit for vertical residual axis
YT1	—	top plot limit for vertical residual axis
VARY	—	σ_f^2, variance for dependent variable
TOLER	—	iteration parameter

ITERM4 — control parameter for Model III

ITERM3 — control parameter for Model IV

ITERM2 — control parameter for Model V

Computer Listing of Program II


JUNKIN
BIN313
091671
004004

```
@FOR,IS LGCEXP,LWCEXP
HVI 009-09/16-13:31 (,0)
```

MAIN PROGRAM

STORAGE USED: CODE(1) 005617; DATA(0) 023777; BLANK COMMON(2) 000000

EXTERNAL REFERENCES (BLOCK, NAME)

```
0003   TRACSF
0004   IDENT
0005   QUIK3L
0006   INVRT
0007   ENDJOB
0010   NINTRS
0011   NRDUS
0012   NIO2S
0013   NWDUS
0014   NWNLS
0015   NWNLS
0016   DLOG
0017   EXP
0020   DEXP
0021   NIOIS
0022   SQRT
0023   DSQRT
0024   NSTOPS
```

STORAGE ASSIGNMENT (BLOCK, TYPE, RELATIVE LOCATION, NAME)

0000	023210 100F	0000	023211 101F	0001	002077 1012G	0000	023212 102F	0001	002124 1023G
0000	023231 103F	0001	002172 1035G	0000	023240 104F	0001	002206 1043G	0001	002207 1046G
0000	023247 105F	0001	002223 1054G	0000	023266 106F	0001	002241 1061G	0001	002247 1066G
0000	023272 107F	0001	002255 1073G	0000	023277 108F	0000	023317 109F	0000	023326 110F
0001	002263 1100G	0001	002265 1103G	0000	023336 111F	0001	002301 1111G	0001	002302 1114G
0000	023344 112F	0001	002313 1122G	0001	002315 1125G	0000	023353 113F	0001	002327 1133G
0001	002330 1136G	0000	023373 114F	0001	002342 1144G	0001	002344 1147G	0000	023376 115F
0001	002345 1152G	0000	023401 116F	0001	002374 1163G	0001	002401 1167G	0000	023410 117F
0001	002414 1176G	0000	023422 118F	0000	023423 119F	0000	023435 120F	0001	002415 1201G
0000	023451 121F	0000	023455 122F	0001	002467 1223G	0000	023464 123F	0000	023466 124F
0001	002512 1240G	0001	002513 1243G	0000	023477 125F	0001	002526 1252G	0001	002527 1255G
0000	023510 126F	0001	002541 1263G	0001	002543 1266G	0000	023514 127F	0001	002544 1271G
0000	023522 128F	0000	023525 129F	0000	023532 130F	0001	002605 1305G	0000	023541 131F
0000	023553 132F	0001	002647 1327G	0000	023567 133F	0001	002655 1334G	0001	002657 1337G
0000	023601 134F	0000	023614 135F	0001	002715 1353G	0000	023617 136F	0001	002734 1362G
0000	023621 137F	0000	023625 138F	0000	023630 139F	0000	023642 140F	0001	003006 1401G
0000	023652 141F	0000	023657 142F	0001	003050 1420G	0001	003070 1427G	0000	023675 143F
0000	023701 144F	0000	023703 145F	0001	003202 1450G	0000	023712 146F	0000	023717 147F
0001	003601 1616G	0001	003761 1652G	0001	004004 1666G	0001	004025 1676G	0001	004060 1706G
0001	004074 1714G	0001	004075 1717G	0001	004111 1725G	0001	004124 1732G	0001	004132 1737G
0001	004140 1744G	0001	004146 1751G	0001	004150 1754G	0001	004164 1762G	0001	004165 1765G
0001	004176 1773G	0001	004200 1776G	0001	000013 200L	0001	004212 2004G	0001	004213 2007G
0001	004225 2015G	0001	004227 2020G	0001	004230 2023G	0001	004257 2034G	0001	004264 2040G

```
0001   004300 2047G    0001   004321 2052G    0001   004352 2073G    0001   004375 2110G    0001   004376 2113G
0001   004411 2122G    0001   004412 2125G    0001   004424 2133G    0001   004426 2136G    0001   004427 2141G
0001   004470 2155G    0001   004532 2177G    0001   004540 2204G    0001   004542 2207G    0001   004572 2221G
0001   004605 2227G    0001   004651 2245G    0001   004705 2264G    0001   004727 2273G    0001   000072 231G
0001   005024 2312G    0001   005316 2423G    0001   005442 2457G    0001   000177 254G     0001   000041 3000L
0001   000320 301G     0001   000554 305L     0001   000565 306L     0001   000567 307L     0001   000603 308L
0001   000626 309L     0001   000650 310L     0001   000442 321G     0001   000457 331G     0001   000560 360G
0001   000760 4001L    0001   000767 401L     0001   001327 405L     0001   001340 406L     0001   001342 407L
0001   001356 408L     0001   001401 409L     0001   001423 410L     0001   000721 437G     0001   000731 445G
0001   001320 476G     0001   001527 5000L    0001   002454 5140L    0001   002507 5141L    0001   002573 5152L
0001   002622 5153L    0001   001126 521G     0001   003320 524L     0001   003343 525L     0001   001235 540G
0001   001247 547G     0001   001333 573G     0001   003461 600L     0001   004337 6140L    0001   004372 6141L
0001   004456 6152L    0001   004505 6153L    0001   005136 624L     0001   005166 625L     0001   001470 650G
0001   001500 656G     0001   005276 700L     0001   001657 740G     0001   002054 776G     0001   005611 800L
0000 R 014512 AR       0000 R 023067 A1       0000 D 023000 A11      0000 R 023062 A2       0000 D 023004 A22
0000 R 023054 A3       0000 D 023010 A33      0000 D 002114 B        0000 R 022634 BCOR     0000 R 022304 BCOT
0000 R 022020 BCDY     0000 R 023102 BJ       0000 R 023047 BJ1      0000 R 023063 BJ2      0000 D 007474 BN
0000 D 004704 BT       0000 R 023066 B1       0000 D 023002 B11      0000 R 023061 B2       0000 D 023006 B22
0000 R 023053 B3       0000 D 023012 B33      0000 R 023064 CAA1     0000 R 023057 CAA2     0000 R 023051 CAA3
0000 R 023065 CA1      0000 R 023060 CA2      0000 R 023052 CA3      0000 D 012754 D        0000 D 022050 DD
0000 D 013116 DEL      0000 D 023014 DETER    0000 R 023076 DF       0000 D 021642 DIDEN    0000 R 023133 DTOL
0000 D 010054 D1       0000 D 012574 D2       0000 D 012612 D3       0000 D 021500 D33      0000 R 013754 F
0000 I 023043 I        0000 I 023045 IJ       0000   023134 INPUT    0000 I 023041 ITERM2   0000 I 023040 ITERM3
0000 I 023037 ITERM4   0000 I 023117 IW       0000 I 023111 IX       0000 I 023112 I2       0000 I 023074 J
0000 I 023106 JB       0000 I 023050 JJ       0000 I 023073 JJ1      0000 I 023056 JN       0000 I 023100 JSEL2
0000 I 023077 JSEL3    0000 I 023046 J1       0000 I 023055 J2       0000 I 023075 K        0000 I 023101 KCYCLE
0000 I 023105 KK       0000 I 023071 KTER     0000 I 023113 K1       0000 I 023114 K2       0000 I 023115 K3
0000 I 023116 K4       0000 I 023110 M        0000 I 023107 N        0000 I 023044 NCASES   0000 I 023072 NKOT
0000 I 023026 NN       0000 I 023042 P        0000 D 013134 RES      0000 R 022160 RSS      0000 R 023124 SGA1
0000 R 023126 SGA2     0000 R 023130 SGA3     0000 R 023125 SGB1     0000 R 023127 SGB2     0000 R 023131 SGB3
0000 R 023104 SIGYIN   0000 R 023121 SIGY1    0000 R 023123 SIGY2    0000 D 023016 SM1      0000 D 023020 SM2
0000 D 023022 SM3      0000 D 023024 SM4      0000 R 023070 SUMDEV   0000 R 023103 SUMSQ    0000 R 023120 SUM1
0000 R 023122 SUM2     0000 D 000000 T        0000 R 021334 TEMP     0000 R 023027 TL       0000 R 023036 TOLER
0000 R 023132 TOL1     0000 R 023030 TR       0000 R 022324 TS       0000 R 023035 VARY     0000 R 000620 W
0000 D 000764 X1       0000 D 001274 X2       0000 D 001604 X3       0000 D 000310 Y        0000 R 023031 YB
0000 R 023033 YB1      0000 D 013444 YC       0000 R 022634 YCS      0000 R 022470 YS       0000 R 023032 YT
0000 R 023034 YT1

00100     C
00100     C NUMERICAL TECHNIQUE FOR EXPONENTIAL REGRESSION ANALYSIS
00100     C
00101           DIMENSION T(100),Y(100),A(100),X1(100),X2(100),X3(100),B(100,7),
00101          -BT(7,100),BN(100,1),D1(100,7),D2(7,1),D3(7,7),D(49),DEL(7,1),
00101          -RES(100),YC(100),F(350),AR(7,350),TEMP(100),D33(7,7),DIDEN(7,7),
00101          -BCDT(12),BCDY(12),BCDR(12),DD(36)
00103           DIMENSION RSS(100),TS(100),YS(100),YCS(100)
00104           DOUBLE PRECISION X1,X2,X3,YC,RES,A11,B11,A22,B22,A33,B33,DEL
00105           DOUBLE PRECISION B,BT,BN,D1,D2,D3,D,DD,DETER,D33,DIDEN,T,Y
00106           DOUBLE PRECISION SM1,SM2,SM3,SM4
00107           NAMELIST/INPUT/T,Y,NN,TL,TR,YB,YT,YB1,YT1,VARY,TOLER,ITERM4,
00107          -ITERM3,ITERM2
00110           CALL IDENT(935)
00111           INTEGER P
```

```
00112        100   FORMAT (1H1)
00113        101   FORMAT (///)
00114        102   FORMAT (83H NON-LINEAR EXPONENTIAL REGRESSION ANALYSIS USING AN IT
00114             -ERATIVE CORRECTION PROCEDURE)
00115        103   FORMAT (36H ASSUMED MODEL IS THREE EXPONENTIALS)
00116        104   FORMAT (33H INITIAL ESTIMATES FOR PARAMETERS)
00117        105   FORMAT (3X,3HA1=E13.6,2X,3HB1=E13.6,2X,3HA2=E13.6,2X,3HB2=E13.6,2X
00117             -,3HA3=E13.6,2X,3HB3=E13.6)
00120        106   FORMAT (15H CYCLE NUMBER =I3)
00121        107   FORMAT (3X,22H PARAMETER CORRECTIONS)
00122        108   FORMAT (3X,4HDA1=E13.6,1X,4HDB1=E13.6,1X,4HDA2=E13.6,1X,4HDB2=E13.
00122             -6,1X,4HDA3=E13.6,1X,4HDB3=E13.6)
00123        109   FORMAT (3X,29H IMPROVED PARAMETER ESTIMATES)
00124        110   FORMAT (3X,37H RESPONSE VARIABLE STANDARD DEVIATION)
00125        111   FORMAT (3X,6HSIGY1=E13.6,3X,6HSIGY2=E13.6)
00126        112   FORMAT (3X,31H COEFFICIENT STANDARD DEVIATION)
00127        113   FORMAT (3X,4HSA1=E13.6,1X,4HSB1=E13.6,1X,4HSA2=E13.6,1X,4HSB2=E13.
00127             -6,1X,4HSA3=E13.6,1X,4HSB3=E13.6)
00130        114   FORMAT (3X,6HTOLER=E13.6)
00131        115   FORMAT (3X,5HDTOL=E13.6)
00132        116   FORMAT (3X,29HCONVERGENCE HAS BEEN ACHIEVED)
00133        117   FORMAT (3X,48H VALUES IN LAST CYCLE ARE FINAL PARAMETER VALUES)
00134        118   FORMAT (I3)
00135        119   FORMAT (51H INITIAL ESTIMATES FOR THREE-TERM EXPONENTIAL MODEL)
00136        120   FORMAT (5X,1HI,8X,1HF,14X,2HA1,13X,2HB1,13X,2HA2,13X,2HB2,13X,2HA3
00136             -,13X,2HB3)
00137        121   FORMAT (3X,I3,8(E13.6,2X))
00140        122   FORMAT (3X,31H THE INITIAL ESTIMATES USED ARE)
00141        123   FORMAT (3X,2HI=)
00142        124   FORMAT (49H INITIAL ESTIMATES FOR TWO-TERM EXPONENTIAL MODEL)
00143        125   FORMAT (5X,1HI,8X,1HF,14X,2HA1,13X,2HB1,13X,2HA2,13X,2HB2)
00144        126   FORMAT (3X,I3,6(E13.6,2X))
00145        127   FORMAT (30H INVERSE TIMES ORIGINAL MATRIX)
00146        128   FORMAT (3X,7(E13.6,1X))
00147        129   FORMAT (3X,18H DETERMINANT VALUE)
00150        130   FORMAT (34H ASSUMED MODEL IS TWO EXPONENTIALS)
00151        131   FORMAT (3X,3HA1=E13.6,3X,3HB1=E13.6,3X,3HA2=E13.6,3X,3HB2=E13.6)
00152        132   FORMAT (3X,6HDELA1=E13.6,3X,6HDELB1=E13.6,3X,6HDELA2=E13.6,3X,6HDE
00152             -LB2=E13.6)
00153        133   FORMAT (3X,3HA1=E13.6,3X,3HB1=E13.6,3X,3HA2=E13.6,3X,3HB2=E13.6)
00154        134   FORMAT (3X,5HSGA1=E13.6,3X,5HSGB1=E13.6,3X,5HSGA2=E13.6,3X,5HSGB2=
00154             -E13.6)
00155        135   FORMAT (3X,10H RESIDUALS)
00156        136   FORMAT (3X,E13.6)
00157        137   FORMAT (18H INVERSE OF BT*W*B)
00160        138   FORMAT (9H B MATRIX)
00161        139   FORMAT (51H OBSERVED RESPONSE, COMPUTED RESPONSE AND RESIDUALS)
00162        140   FORMAT (6X,8HOBSERVED,6X,8HCOMPUTED,6X,8HRESIDUAL)
00163        141   FORMAT (21H END OF CYCLE NUMBER I3)
00164        142   FORMAT (76H OBSERVED RESPONSE, COMPUTED RESPONSE, AND RESIDUALS US
00164             -ING INITIAL ESTIMATES)
00165        143   FORMAT (3X,7HSIGYIN=E13.6)
00166        144   FORMAT (7H BT*W*B)
00167        145   FORMAT (33H ASSUMED MODEL IS ONE EXPONENTIAL)
00170        146   FORMAT (20H PARAMETER ESTIMATES)
00171        147   FORMAT (3X,3HA1=E13.6,2X,3HB1=E13.6)
```

```
00172              DATA (BCDT(I),I=1,12)/6HTIME  ,11*6H        /
00174              DATA (BCDY(I),I=1,12)/6HRESPON,6HSE    ,10*6H      /
00176              DATA (BCDR(I),I=1,12)/6HRESIDU,6HAL    ,10*6H      /
00200              READ (5,118) NCASES
00203        200   WRITE (6,100)
00205              READ (5,INPUT)
00210              WRITE (6,INPUT)
00213              IF (ITERM2.NE.0) GO TO 700
00215              IF (ITERM4.EQ.0) GO TO 4000
00215        C
00215        C INITIAL ESTIMATES FOR THREE EXPONENTIALS
00215        C     STRAIGHT LINE FIT TO LOG Y VS. TIME
00215        C
00217              IJ=0
00220              J1=2
00221              BJ1=3.
00222        3000  IJ=IJ+1
00223              SM1=0.
00224              SM2=0.
00225              SM3=0.
00226              SM4=0.
00227              JJ=NN-J1
00230              DO 300 I=JJ,NN
00233              SM1=T(I)**2+SM1
00234              SM2=DLOG(Y(I))+SM2
00235              SM3=T(I)+SM3
00236        300   SM4=T(I)*DLOG(Y(I))+SM4
00240              CAA3=BJ1*SM1-SM3**2
00241              CA3=(SM1*SM2-SM3*SM4)/CAA3
00241        C
00241        C ESTIMATES FOR A3,B3
00241        C
00242              B3=-((BJ1*SM4-SM3*SM2)/CAA3)
00243              A3=EXP(CA3)
00244              J2=J1+3
00244        C
00244        C STRAIGHT LINE FIT TO LOG RESIDUAL VS. TIME
00244        C
00245              JJ=NN-J2
00246              JN=NN-J1-1
00247              SM1=0.
00250              SM2=0.
00251              SM3=0.
00252              SM4=0.
00253              DO 301 I=JJ,JN
00256              X3(I)=-B3*T(I)
00257              RES(I)=Y(I)-A3*DEXP(X3(I))
00260              RES(I)=DABS(RES(I))
00261              SM1=T(I)**2+SM1
00262              SM2=DLOG(RES(I))+SM2
00263              SM3=T(I)+SM3
00264        301   SM4=T(I)*DLOG(RES(I))+SM4
00266              CAA2=3.*SM1-SM3**2
00267              CA2=(SM1*SM2-SM3*SM4)/CAA2
00267        C
00267        C ESTIMATES FOR A2,B2
```

```
      C
            B2=-((3.*SM4-SM3*SM2)/CAA2)
            A2=EXP(CA2)
      C
      C STRAIGHT LINE FIT TO LOG RESIDUAL VS. TIME
      C
            JN=NN-J2-1
            BJ2=JN
            SM1=0.
            SM2=0.
            SM3=0.
            SM4=0.
            DO 302 I=1,JN
            X3(I)=-B3*T(I)
            X2(I)=-B2*T(I)
            RES(I)=Y(I)-A2*DEXP(X2(I))-A3*DEXP(X3(I))
            RES(I)=DABS(RES(I))
            SM1=T(I)**2+SM1
            SM2=DLOG(RES(I))+SM2
            SM3=T(I)+SM3
  302       SM4=T(I)*DLOG(RES(I))+SM4
            CAA1=BJ2*SM1-SM3**2
            CA1=(SM1*SM2-SM3*SM4)/CAA1
      C
      C ESTIMATES FOR A1,B1
      C
            B1=-((BJ2*SM4-SM3*SM2)/CAA1)
            A1=EXP(CA1)
      C
      C WEIGHTED SUM OF SQUARES OF DEVIATIONS,W=1.
      C
            DO 303 I=1,NN
            X1(I)=-B1*T(I)
            X2(I)=-B2*T(I)
  303       X3(I)=-B3*T(I)
            SUMDEV=0.
            DO 304 I=1,NN
            B(I,1)=DEXP(X1(I))
            B(I,3)=DEXP(X2(I))
            B(I,5)=DEXP(X3(I))
  304       SUMDEV=(Y(I)-A1*B(I,1)-A2*B(I,3)-A3*B(I,5))**2+SUMDEV
            J1=J1+1
            BJ1=BJ1+1.
            KTER=NN-J1-6
            F(IJ)=SUMDEV
            AR(1,IJ)=A1
            AR(2,IJ)=B1
            AR(3,IJ)=A2
            AR(4,IJ)=B2
            AR(5,IJ)=A3
            AR(6,IJ)=B3
            NKOT=IJ
            IF (KTER.EQ.0) GO TO 305
            GO TO 3000
  305       CONTINUE
      C
```

```
00356         C SELECTION OF SMALLEST F AND CORRESPONDING PARAMETERS
00356         C
00357                DO 307 I=1,IJ
00362                IF (F(I).LT.0.) GO TO 306
00364                GO TO 307
00365         306    F(I)=-F(I)
00366         307    CONTINUE
00370                JJ1=1
00371                NKOT=NKOT-1
00372                J=1
00373                K=2
00374                TEMP(1)=F(1)
00375         308    DF=TEMP(J)-F(K)
00376                IF (DF.GT.0.) GO TO 309
00400                K=K+1
00401                JJ1=K-1
00402                NKOT=NKOT-1
00403                IF (NKOT.EQ.0) GO TO 310
00405                GO TO 308
00406         309    J=JJ1+1
00407                TEMP(J)=F(K)
00410                K=K+1
00411                JJ1=K-1
00412                NKOT=NKOT-1
00413                IF (NKOT.EQ.0) GO TO 310
00415                GO TO 308
00416         310    CONTINUE
00417                A1=AR(1,J)
00420                B1=AR(2,J)
00421                A2=AR(3,J)
00422                B2=AR(4,J)
00423                A3=AR(5,J)
00424                B3=AR(6,J)
00425                JSEL3=J
00426                WRITE (6,100)
00430                WRITE (6,101)
00432                WRITE (6,119)
00434                WRITE (6,120)
00436                DO 311 I=1,IJ
00441         311    WRITE (6,121) (I,F(I),(AR(J,I),J=1,6))
00452                WRITE (6,101)
00454                WRITE (6,122)
00456                WRITE (6,123) JSEL3
00461         4000   CONTINUE
00462                IF (ITERM3.EQ.0) GO TO 5000
00462         C
00462         C INITIAL ESTIMATES FOR TWO EXPONENTIALS
00462         C      STRAIGHT LINE FIT TO LOG Y VS. TIME
00462         C
00464                IJ=0
00465                J1=2
00466                BJ1=3.
00467         401    IJ=IJ+1
00470                SM1=0.
00471                SM2=0.
00472                SM3=0.
```

```
00473          SM4=0.
00474          JJ=NN-J1
00475          DO 400 I=JJ,NN
00500          SM1=T(I)**2+SM1
00501          SM2=DLOG(Y(I))+SM2
00502          SM3=T(I)+SM3
00503    400   SM4=T(I)*DLOG(Y(I))+SM4
00505          CAA2=BJ1*SM1-SM3**2
00506          CA2=(SM1*SM2-SM3*SM4)/CAA2
00506    C
00506    C ESTIMATES FOR A2,B2
00506    C
00507          B2=-((BJ1*SM4-SM3*SM2)/CAA2)
00510          A2=EXP(CA2)
00511          J2=J1+3
00511    C
00511    C STRAIGHT LINE FIT TO LOG RESIDUAL VS. TIME
00511    C
00512          JN=NN-J1-1
00513          BJ2=JN
00514          SM1=0.
00515          SM2=0.
00516          SM3=0.
00517          SM4=0.
00520          DO 402 I=1,JN
00523          X2(I)=-B2*T(I)
00524          RES(I)=Y(I)-A2*DEXP(X3(I))
00525          RES(I)=DABS(RES(I))
00526          SM1=T(I)**2+SM1
00527          SM2=DLOG(RES(I))+SM2
00530          SM3=T(I)+SM3
00531    402   SM4=T(I)*DLOG(RES(I))+SM4
00533          CAA1=BJ2*SM1-SM3**2
00534          CA1=(SM1*SM2-SM3*SM4)/CAA1
00534    C
00534    C ESTIMATES FOR A1,B1
00534    C
00535          B1=-((BJ2*SM4-SM3*SM2)/CAA1)
00536          A1=EXP(CA1)
00536    C
00536    C WEIGHTED SUM OF SQUARES OF DEVIATIONS,W=1.
00536    C
00537          DO 403 I=1,NN
00542          X1(I)=-B1*T(I)
00543    403   X2(I)=-B2*T(I)
00545          SUMDEV=0.
00546          DO 404 I=1,NN
00551          B(I,1)=DEXP(X1(I))
00552          B(I,3)=DEXP(X2(I))
00553    404   SUMDEV=(Y(I)-A1*B(I,1)-A2*B(I,3))**2+SUMDEV
00555          J1=J1+1
00556          BJ1=BJ1+1.
00557          KTER=NN-J1-3
00560          F(IJ)=SUMDEV
00561          AR(1,IJ)=A1
00562          AR(2,IJ)=B1
```

```
00563                  AR(3,IJ)=A2
00564                  AR(4,IJ)=B2
00565                  NKOT=IJ
00566                  IF (KTER.EQ.0) GO TO 405
00570                  GO TO 401
00571          405     CONTINUE
00571          C
00571          C SELECTION OF SMALLEST F AND CORRESPONDING PARAMETERS
00571          C
00572                  DO 407 I=1,IJ
00575                  IF (F(I).LT.0.) GO TO 406
00577                  GO TO 407
00600          406     F(I)=-F(I)
00601          407     CONTINUE
00603                  JJ1=1
00604                  NKOT=NKOT-1
00605                  J=1
00606                  K=2
00607                  TEMP(1)=F(1)
00610          408     DF=TEMP(J)-F(K)
00611                  IF (DF.GT.0.) GO TO 409
00613                  K=K+1
00614                  JJ1=K-1
00615                  NKOT=NKOT-1
00616                  IF (NKOT.EQ.0) GO TO 410
00620                  GO TO 408
00621          409     J=JJ1+1
00622                  TEMP(J)=F(K)
00623                  K=K+1
00624                  JJ1=K-1
00625                  NKOT=NKOT-1
00626                  IF (NKOT.EQ.0) GO TO 410
00630                  GO TO 408
00631          410     CONTINUE
00632                  A1=AR(1,J)
00633                  B1=AR(2,J)
00634                  A2=AR(3,J)
00635                  B2=AR(4,J)
00636                  JSEL2=J
00637                  WRITE (6,100)
00641                  WRITE (6,101)
00643                  WRITE (6,124)
00645                  WRITE (6,125)
00647                  DO 411 I=1,IJ
00652          411     WRITE (6,126) (I,F(I),(AR(J,I),J=1,4))
00663                  WRITE (6,101)
00665                  WRITE (6,122)
00667                  WRITE (6,123) JSEL2
00672          5000    CONTINUE
00672          C
00672          C STATEMENTS NUMBERED 500-599 REFER TO PROGRAM FOR THREE
00672          C       EXPONENTIALS
00672          C
00673                  IF (ITERM4.EQ.0) GO TO 600
00675          500     CONTINUE
00675          C
```

```
00675      C THREE EXPONENTIALS
00675      C       ITERM4=1
00675      C
00676              WRITE (6,100)
00700              WRITE (6,102)
00702              WRITE (6,101)
00704              WRITE (6,103)
00706              WRITE (6,104)
00710              WRITE (6,105) A1,B1,A2,B2,A3,B3
00720              KCYCLE=0
00721              A11=A1
00722              B11=B1
00723              A22=A2
00724              B22=B2
00725              A33=A3
00726              B33=B3
00727              BJ=NN
00730              SUMSQ=0.
00731              WRITE (6,142)
00733              WRITE (6,101)
00735              WRITE (6,140)
00737              DO 5001 I=1,NN
00742              X1(I)=-B11*T(I)
00743              X2(I)=-B22*T(I)
00744              X3(I)=-B33*T(I)
00745              B(I,1)=DEXP(X1(I))
00746              B(I,3)=DEXP(X2(I))
00747              B(I,5)=DEXP(X3(I))
00750              YC(I)=A11*B(I,1)+A22*B(I,3)+A33*B(I,5)
00751              RES(I)=Y(I)-YC(I)
00752              RSS(I)=RES(I)
00753              TS(I)=T(I)
00754              YS(I)=Y(I)
00755              YCS(I)=YC(I)
00756              SUMSQ=RES(I)**2+SUMSQ
00757      5001    WRITE (6,128) Y(I),YC(I),RES(I)
00765              SIGYIN=SQRT(SUMSQ/(BJ-6.))
00766              WRITE (6,143) SIGYIN
00771              KK=NN
00772              CALL QUIK3L (-1,TL,TR,YB,YT,43,BCDT,BCDY,KK,TS,YS)
00773              CALL QUIK3L (0,TL,TR,YB,YT,35,BCDT,BCDY,-KK,TS,YCS)
00774              CALL QUIK3L(-1,TL,TR,YB1,YT1,35,BCDT,BCDR,-KK,TS,RSS)
00774      C
00774      C W MATRIX
00774      C
00775              DO 501 I=1,NN
01000      501     W(I)=1./VARY
01000      C
01000      C B MATRIX
01000      C
01002      5010    CONTINUE
01003              KCYCLE=KCYCLE+1
01004              WRITE (6,101)
01006              WRITE (6,106) KCYCLE
01011              DO 502 I=1,NN
01014              X1(I)=-B11*T(I)
```

```
01015                X2(I)=-B22*T(I)
01016          502   X3(I)=-B33*T(I)
01020                WRITE (6,138)
01022                DO 503 I=1,NN
01025                B(I,1)=DEXP(X1(I))
01026                B(I,2)=-A11*T(I)*B(I,1)
01027                B(I,3)=DEXP(X2(I))
01030                B(I,4)=-A22*T(I)*B(I,3)
01031                B(I,5)=DEXP(X3(I))
01032                B(I,6)=-A33*T(I)*B(I,5)
01033          503   WRITE (6,128) (B(I,JB),JB=1,6)
01033          C
01033          C TRANSPOSE OF B MATRIX
01033          C
01042                DO 504 K=1,6
01045                DO 504 I=1,NN
01050          504   BT(K,I)=B(I,K)
01050          C
01050          C N MATRIX
01050          C
01053                DO 505 I=1,NN
01056          505   BN(I,1)=Y(I)-A11*B(I,1)-A22*B(I,3)-A33*B(I,5)
01056          C
01056          C B-TRANSPOSE*W*N
01056          C
01060                DO 506 I=1,NN
01063          506   D1(I,1)=0.0D0
01065                DO 507 N=1,NN
01070          507   D1(N,1)=W(N)*BN(N,1)
01072                DO 508 I=1,6
01075          508   D2(I,1)=0.0D0
01077                DO 509 M=1,6
01102                DO 509 N=1,NN
01105          509   D2(M,1)=BT(M,N)*D1(N,1)+D2(M,1)
01105          C
01105          C B-TRANSPOSE*W*B
01105          C
01110                DO 510 I=1,NN
01113                DO 510 J=1,6
01116          510   D1(I,J)=0.0D0
01121                DO 511 P=1,6
01124                DO 511 N=1,NN
01127          511   D1(N,P)=W(N)*B(N,P)
01132                DO 512 I=1,6
01135                DO 512 J=1,6
01140          512   D3(I,J)=0.0D0
01143                DO 513 M=1,6
01146                DO 513 P=1,6
01151                DO 513 N=1,NN
01154          513   D3(M,P)=BT(M,N)*D1(N,P)+D3(M,P)
01160                WRITE (6,144)
01162                DO 5130 I=1,6
01165          5130  WRITE (6,128) (D3(I,IX),IX=1,6)
01165          C
01165          C INVERSE OF B-TRANSPOSE*W*B
01165          C
```

```
01174                I2=0
01175                DO 514 I=1,6
01200                DO 514 J=1,6
01203                I2=I2+1
01204                D(I2)=D3(J,I)
01205        514     DD(I2)=D3(J,I)
01210                N=6
01211                M=0
01212                CALL INVRT (D,N,M,DETER)
01212        C
01212        C INVERSE MATRIX
01212        C
01213                K1=1
01214                K2=6
01215                K3=6
01216                K4=6
01217                WRITE (6,137)
01221        5140    WRITE (6,128) (D(I),I=K1,K2)
01227                K1=K2+1
01230                K2=K2+K3
01231                K4=K4-1
01232                IF (K4.EQ.0) GO TO 5141
01234                GO TO 5140
01235        5141    CONTINUE
01235        C
01235        C INVERSE*ORIGINAL MATRIX
01235        C
01236                I2=0
01237                DO 515 I=1,6
01242                DO 515 J=1,6
01245                I2=I2+1
01246        515     D33(J,I)=D(I2)
01251                DO 5150 I=1,6
01254                DO 5150 J=1,6
01257        5150    DIDEN(I,J)=0.0D0
01262                DO 5151 M=1,6
01265                DO 5151 P=1,6
01270                DO 5151 N=1,6
01273        5151    DIDEN(M,P)=D33(M,N)*D3(N,P)+DIDEN(M,P)
01277                IW=6
01300                I=1
01301                WRITE (6,127)
01303        5152    WRITE (6,128) (DIDEN(I,J),J=1,6)
01311                IW=IW-1
01312                IF (IW.EQ.0) GO TO 5153
01314                I=I+1
01315                GO TO 5152
01316        5153    CONTINUE
01317                WRITE (6,101)
01321                WRITE (6,129)
01323                WRITE (6,128) DETER
01323        C
01323        C DELTA MATRIX
01323        C
01326                DO 516 I=1,6
01331        516     DEL(I,I)=0.0D0
```

```
01333          DO 517 M=1,6
01336          DO 517 N=1,6
01341   517    DEL(M,1)=D33(M,N)*D2(N,1)+DEL(M,1)
01341   C
01341   C IMPROVED PARAMETER ESTIMATES
01341   C
01344          A11=A11+DEL(1,1)
01345          B11=B11+DEL(2,1)
01346          A22=A22+DEL(3,1)
01347          B22=B22+DEL(4,1)
01350          A33=A33+DEL(5,1)
01351          B33=B33+DEL(6,1)
01351   C
01351   C RESIDUALS USING IMPROVED PARAMETER ESTIMATES
01351   C
01352          DO 519 I=1,NN
01355          X1(I)=-B11*T(I)
01356          X2(I)=-B22*T(I)
01357   519    X3(I)=-B33*T(I)
01361          DO 520 I=1,NN
01364          B(I,1)=DEXP(X1(I))
01365          B(I,3)=DEXP(X2(I))
01366   520    B(I,5)=DEXP(X3(I))
01370          WRITE (6,100)
01372          WRITE (6,139)
01374          WRITE (6,101)
01376          WRITE (6,140)
01400          DO 521 I=1,NN
01403          YC(I)=A11*B(I,1)+A22*B(I,3)+A33*B(I,5)
01404          RES(I)=Y(I)-YC(I)
01405          RSS(I)=RES(I)
01406          YCS(I)=YC(I)
01407   521    WRITE (6,128) Y(I),YC(I),RES(I)
01407   C
01407   C STANDARD DEVIATION OF RESPONSE VARIABLE USING INITIAL ESTIMATES
01407   C
01415          BJ=NN
01416          SUM1=0.
01417          DO 522 I=1,NN
01422   522    SUM1=BN(I,1)**2+SUM1
01424          SIGY1=SQRT(SUM1/(BJ-6.))
01424   C
01424   C STANDARD DEVIATION OF RESPONSE VARIABLE USING IMPROVED ESTIMATES
01424   C
01425          SUM2=0.
01426          DO 523 I=1,NN
01431          SIGY2=SQRT(SUM2/(BJ-6.))
01432   523    SUM2=RES(I)**2+SUM2
01432   C
01432   C PARAMETER STANDARD DEVIATIONS
01432   C
01434          SGA1=SIGY2/SQRT(D3(1,1))
01435          SGB1=SIGY2/SQRT(D3(2,2))
01436          SGA2=SIGY2/SQRT(D3(3,3))
01437          SGB2=SIGY2/SQRT(D3(4,4))
01440          SGA3=SIGY2/SQRT(D3(5,5))
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01441         SGB3=SIGY2/SQRT(D3(6,6))
01442         WRITE (6,101)
01444         WRITE (6,107)
01446         WRITE (6,108) (DEL(I,I),I=1,6)
01454         WRITE (6,101)
01456         WRITE (6,109)
01460         WRITE (6,105) A11,B11,A22,B22,A33,B33
01470         WRITE (6,101)
01472         WRITE (6,110)
01474         WRITE (6,111) SIGY1,SIGY2
01500         WRITE (6,101)
01502         WRITE (6,112)
01504         WRITE (6,113) SGA1,SGB1,SGA2,SGB2,SGA3,SGB3
01504   C
01504   C ITERATION LOGIC CYCLE
01504   C
01514         TOL1=SIGY2**2-SIGY1**2
01515         DTOL=TOLER-TOL1
01516         DTOL=ABS(DTOL)
01517         IF ((DTOL-.00001).GT.0.) GO TO 524
01521         GO TO 525
01522   524   TOLER=TOL1
01523         WRITE (6,101)
01525         WRITE (6,114) TOLER
01530         WRITE (6,141) KCYCLE
01533   525   WRITE (6,101)
01535         WRITE (6,100)
01537         WRITE (6,115) DTOL
01542         WRITE (6,116)
01544         WRITE (6,117)
01546         KK=NN
01547         CALL QUIK3L (-1,TL,TR,YB,YT,43,BCDT,BCDY,KK,TS,YS)
01550         CALL QUIK3L (0,TL,TR,YB,YT,35,BCDT,BCDY,-KK,TS,YCS)
01551         CALL QUIK3L (-1,TL,TR,YB1,YT1,35,BCDT,BCDR,-KK,TS,RSS)
01552         NCASES=NCASES-1
01553         IF (NCASES.EQ.0) GO TO 800
01553   C
01553   C STATEMENTS NUMBERED 600-699 REFER TO PROGRAM FOR TWO
01553   C     EXPONENTIALS
01553   C
01555         IF (ITERM3.EQ.0) GO TO 200
01557   600   CONTINUE
01557   C
01557   C TWO EXPONENTIALS
01557   C     ITERM3=1
01557   C
01560         WRITE (6,100)
01562         WRITE (6,102)
01564         WRITE (6,101)
01566         WRITE (6,130)
01570         WRITE (6,104)
01572         WRITE (6,131) A1,B1,A2,B2
01600         KCYCLE=0
01601         A11=A1
01602         B11=B1
01603         A22=A2
```

```
01604              B22=B2
01605              BJ=NN
01606              SUMSQ=0.
01607              WRITE (6,142)
01611              WRITE (6,101)
01613              WRITE (6,140)
01615              DO 6001 I=1,NN
01620              X1(I)=-B11*T(I)
01621              X2(I)=-B22*T(I)
01622              B(I,1)=DEXP(X1(I))
01623              B(I,3)=DEXP(X2(I))
01624              YC(I)=A11*B(I,1)+A22*B(I,3)
01625              RES(I)=Y(I)-YC(I)
01626              RSS(I)=RES(I)
01627              TS(I)=T(I)
01630              YS(I)=Y(I)
01631              YCS(I)=YC(I)
01632              SUMSQ=RES(I)**2+SUMSQ
01633        6001  WRITE (6,128) Y(I),YC(I),RES(I)
01641              SIGYIN=SQRT(SUMSQ/(BJ-4.))
01642              WRITE (6,143) SIGYIN
01645              KK=NN
01646              CALL QUIK3L (-1,TL,TR,YB,YT,43,BCDT,BCDY,KK,TS,YS)
01647              CALL QUIK3L (0,TL,TR,YB,YT,35,BCDT,BCDY,-KK,TS,YCS)
01650              CALL QUIK3L(-1,TL,TR,YB1,YT1,35,BCDT,BCDR,-KK,TS,RSS)
01650        C
01650        C W MATRIX
01650        C
01651              DO 601 I=1,NN
01654        601   W(I)=1./VARY
01654        C
01654        C B MATRIX
01654        C
01656        6010  CONTINUE
01657              KCYCLE=KCYCLE+1
01660              WRITE (6,101)
01662              WRITE (6,106) KCYCLE
01665              DO 602 I=1,NN
01670              X1(I)=-B11*T(I)
01671        602   X2(I)=-B22*T(I)
01673              WRITE (6,138)
01675              DO 603 I=1,NN
01700              B(I,1)=DEXP(X1(I))
01701              B(I,2)=-A11*T(I)*B(I,1)
01702              B(I,3)=DEXP(X2(I))
01703              B(I,4)=-A22*T(I)*B(I,3)
01704        603   WRITE (6,128) (B(I,JB),JB=1,4)
01704        C
01704        C TRANSPOSE OF B MATRIX
01704        C
01713              DO 604 K=1,4
01716              DO 604 I=1,NN
01721        604   BT(K,I)=B(I,K)
01721        C
01721        C N MATRIX
01721        C
```

```
01724              DO 605 I=1,NN
01727      605     BN(I,1)=Y(I)-A11*B(I,1)-A22*B(I,3)
01727      C
01727      C B-TRANSPOSE*W*N
01727      C
01731              DO 606 I=1,NN
01734      606     D1(I,1)=0.0D0
01736              DO 607 N=1,NN
01741      607     D1(N,1)=W(N)*BN(N,1)
01743              DO 608 I=1,4
01746      608     D2(I,1)=0.0D0
01750              DO 609 M=1,4
01753              DO 609 N=1,NN
01756      609     D2(M,1)=BT(M,N)*D1(N,1)+D2(M,1)
01756      C
01756      C B-TRANSPOSE*W*B
01756      C
01761              DO 610 I=1,NN
01764              DO 610 J=1,4
01767      610     D1(I,J)=0.0D0
01772              DO 611 P=1,4
01775              DO 611 N=1,NN
02000      611     D1(N,P)=W(N)*B(N,P)
02003              DO 612 I=1,4
02006              DO 612 J=1,4
02011      612     D3(I,J)=0.0D0
02014              DO 613 M=1,4
02017              DO 613 P=1,4
02022              DO 613 N=1,NN
02025      613     D3(M,P)=BT(M,N)*D1(N,P)+D3(M,P)
02031              WRITE (6,144)
02033              DO 6130 I=1,4
02036      6130    WRITE (6,128) (D3(I,IX),IX=1,4)
02036      C
02036      C INVERSE OF B-TRANSPOSE*W*B
02036      C
02045              I2=1
02046              DO 614 I=1,4
02051              DO 614 J=1,4
02054              D(I2)=D3(J,I)
02055      614     I2=I2+1
02060              N=4
02061              M=0
02062              CALL INVRT (D,N,M,DETER)
02062      C
02062      C INVERSE MATRIX
02062      C
02063              K1=1
02064              K2=4
02065              K3=4
02066              K4=4
02067              WRITE (6,137)
02071      6140    WRITE (6,128) (D(I),I=K1,K2)
02077              K1=K2+1
02100              K2=K2+K3
02101              K4=K4-1
```

```
02102                IF (K4.EQ.0) GO TO 6141
02104                GO TO 6140
02105        6141    CONTINUE
02106                I2=0
02107                DO 615 I=1,4
02112                DO 615 J=1,4
02115                I2=I2+1
02116        615     D33(J,I)=D(I2)
02116        C
02116        C INVERSE*ORIGINAL MATRIX
02116        C
02121                DO 6150 I=1,4
02124                DO 6150 J=1,4
02127        6150    DIDEN(I,J)=0.0D0
02132                DO 6151 M=1,4
02135                DO 6151 P=1,4
02140                DO 6151 N=1,4
02143        6151    DIDEN(M,P)=D33(M,N)*D3(N,P)+DIDEN(M,P)
02147                IW=4
02150                I=1
02151                WRITE (6,127)
02153        6152    WRITE (6,128) (DIDEN(I,J),J=1,4)
02161                IW=IW-1
02162                IF (IW.EQ.0) GO TO 6153
02164                I=I+1
02165                GO TO 6152
02166        6153    CONTINUE
02167                WRITE (6,101)
02171                WRITE (6,129)
02173                WRITE (6,128) DETER
02173        C
02173        C DELTA MATRIX
02173        C
02176                DO 616 I=1,4
02201        616     DEL(I,1)=0.0D0
02203                DO 617 M=1,4
02206                DO 617 N=1,4
02211        617     DEL(M,1)=D33(M,N)*D2(N,1)+DEL(M,1)
02211        C
02211        C IMPROVED PARAMETER ESTIMATES
02211        C
02214                A11=A11+DEL(1,1)
02215                B11=B11+DEL(2,1)
02216                A22=A22+DEL(3,1)
02217                B22=B22+DEL(4,1)
02217        C
02217        C RESIDUALS USING IMPROVED PARAMETER ESTIMATES
02217        C
02220                DO 619 I=1,NN
02223                X1(I)=-B11*T(I)
02224        619     X2(I)=-B22*T(I)
02226                DO 620 I=1,NN
02231                B(I,1)=DEXP(X1(I))
02232        620     B(I,3)=DEXP(X2(I))
02234                WRITE (6,100)
02236                WRITE (6,139)
```

```
02240        WRITE (6,101)
02242        WRITE (6,140)
02244        DO 621 I=1,NN
02247        YC(I)=A11*B(I,1)+A22*B(I,3)
02250        RES(I)=Y(I)-YC(I)
02251        RSS(I)=RES(I)
02252        YCS(I)=YC(I)
02253  621   WRITE (6,128) Y(I),YC(I),RES(I)
02253  C
02253  C STANDARD DEVIATION OF RESPONSE VARIABLE USING INITIAL ESTIMATES
02253  C
02261        BJ=NN
02262        SUM1=0.
02263        DO 622 I=1,NN
02266  622   SUM1=BN(I,1)**2+SUM1
02270        SIGY1=SQRT(SUM1/(BJ-4.))
02270  C
02270  C STANDARD DEVIATION OF RESPONSE VARIABLE USING IMPROVED ESTIMATES
02270  C
02271        SUM2=0.
02272        DO 623 I=1,NN
02275  623   SUM2=RES(I)**2+SUM2
02277        SIGY2=SQRT(SUM2/(BJ-5.))
02277  C
02277  C PARAMETER STANDARD DEVIATIONS
02277  C
02300        SGA1=SIGY2/SQRT(D3(1,1))
02301        SGB1=SIGY2/SQRT(D3(2,2))
02302        SGA2=SIGY2/SQRT(D3(3,3))
02303        SGB2=SIGY2/SQRT(D3(4,4))
02304        WRITE (6,101)
02306        WRITE (6,107)
02310        WRITE (6,132) (DEL(I,1),I=1,4)
02316        WRITE (6,101)
02320        WRITE (6,109)
02322        WRITE (6,133) A11,B11,A22,B22
02330        WRITE (6,101)
02332        WRITE (6,110)
02334        WRITE (6,111) SIGY1,SIGY2
02340        WRITE (6,101)
02342        WRITE (6,112)
02344        WRITE (6,134) SGA1,SGB1,SGA2,SGB2
02344  C
02344  C ITERATION LOGIC CYCLE
02344  C
02352        TOL1=SIGY2**2-SIGY1**2
02353        DTOL=TOLER-TOL1
02354        DTOL=ABS(DTOL)
02355        IF ((DTOL-.00001).GT.0.) GO TO 624
02357        GO TO 625
02360  624   TOLER=TOL1
02361        WRITE (6,101)
02363        WRITE (6,114) TOLER
02366        WRITE (6,141) KCYCLE
02371        WRITE (6,100)
02373  625   WRITE (6,101)
```

```
02375          WRITE (6,115) DTOL
02400          WRITE (6,116)
02402          WRITE (6,117)
02404          KK=NN
02405          CALL QUIK3L (-1,TL,TR,YB,YT,43,BCDT,BCDY,KK,TS,YS)
02406          CALL QUIK3L (0,TL,TR,YB,YT,35,BCDT,BCDY,-KK,TS,YCS)
02407          CALL QUIK3L(-1,TL,TR,YB1,YT1,35,BCDT,BCDR,-KK,TS,RSS)
02410          NCASES=NCASES-1
02411          IF (NCASES.EQ.0) GO TO 800
02413          GO TO 200
02413    C
02413    C  STATEMENTS NUMBERED 700-799 REFER TO PROGRAM FOR ONE EXPONENTIAL
02413    C   ITERM2=1
02413    C
02413    C
02414    700   SM1=0.0D0
02415          SM2=0.0D0
02416          SM3=0.0D0
02417          SM4=0.0D0
02420          BJ1=NN
02421          SUMSQ=0.
02422          DO 701 I=1,NN
02425          SM1=T(I)**2+SM1
02426          SM2=DLOG(Y(I))+SM2
02427          SM3=T(I)+SM3
02430    701   SM4=T(I)*DLOG(Y(I))+SM4
02432          CAA1=BJ1*SM1-SM3**2
02433          CA1=(SM1*SM2-SM3*SM4)/CAA1
02434          B11=-((BJ1*SM4-SM3*SM2)/CAA1)
02435          A11=DEXP(CA1)
02436          WRITE (6,100)
02440          WRITE (6,145)
02442          WRITE (6,146)
02444          WRITE (6,147) A11,B11
02450          WRITE (6,139)
02452          WRITE (6,101)
02454          WRITE (6,140)
02456          DO 702 I=1,NN
02461          X1(I)=-B11*T(I)
02462          B(I,1)=DEXP(X1(I))
02463          YC(I)=A11*B(I,1)
02464          RES(I)=Y(I)-YC(I)
02465          RSS(I)=RES(I)
02466          TS(I)=T(I)
02467          YS(I)=Y(I)
02470          YCS(I)=YC(I)
02471          SUMSQ=RES(I)**2+SUMSQ
02472    702   WRITE (6,128) Y(I),YC(I),RES(I)
02500          SIGYIN=SQRT(SUMSQ/(BJ1-2.))
02501          WRITE(6,143) SIGYIN
02504          KK=NN
02505          CALL QUIK3L (-1,TL,TR,YB,YT,43,BCDT,BCDY,KK,TS,YS)
02506          CALL QUIK3L (0,TL,TR,YB,YT,35,BCDT,BCDY,-KK,TS,YCS)
02507          CALL QUIK3L (-1,TL,TR,YB1,YT1,35,BCDT,BCDR,-KK,TS,RSS)
02510          NCASES=NCASES-1
02511          IF (NCASES.EQ.0) GO TO 800
```

```
02513    912*          GO TO 200
02514    913*    800   CALL ENDJOB
02515    914*          STOP
02516    915*          END
```

```
        END OF COMPILATION:        NO  DIAGNOSTICS.
```

```
@FOR,IS INVRT,INVRT
HVI 009-09/16-13:32 (,0)

   SUBROUTINE INVRT      ENTRY POINT 000365

   STORAGE USED: CODE(1) 002413; DATA(0) 000144; BLANK COMMON(2) 000000

   EXTERNAL REFERENCES (BLOCK, NAME)

    0003    TRACSF
    0004    NERR3$

   STORAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

    0001     000016 111G      0001     000146 115L      0001     000027 117G      0001     000034 123G      0001     000042 131G
    0001     000116 157G      0001     000255 161L      0001     000163 201G      0001     000203 207G      0001     000225 220G
    0001     000263 234G      0001     000333 235L      0001     000307 245G      0001     000343 250L      0001     000345 255L
    0001     000071 75L       0001     000073 76L       0000 D 000002 AMAX       0000 I 000103 I          0000 I 000106 IC
    0000 I 000104 IND         0000 I 000030 INDEX      0000 I 000110 IND2       0000   000116 INJPS      0000 I 000004 IPIV
    0000 I 000105 IR          0000 I 000100 J          0000 I 000102 K          0000 I 000107 L          0000 I 000101 NN
    0000 D 000000 SIGN


00101               SUBROUTINE INVRT(A,N,M,DETER)
00103               PARAMETER IDIM=20
00103        C      MATRIX INVERSION AND SIMULTANEOUS EQUATIONS SOLVER
00103        C      A=INPUT MATRIX FOR INVERISON OR AUGUMENTED MATRIX FOR SIME. EQS.  IGS00030
00103        C      N=ORDER OF COEFFICIENT MATRIX                                     IGS00040
00103        C      M=0 FOR INVERISON ONLY
00103        C      M=NUMBER OF CONSTANT VECTORS
00103        C      DETER=DETERMINANT OF COEFFICIENT MATRIX
00104               DOUBLE PRECISION A(1),DETER,SIGN,AMAX
00105               DIMENSION IPIV(IDIM),INDEX(IDIM,2)
00106               DETER=1.0D0
00107               SIGN=1.0D0
00110               DO 20 J=1,N
00113            20 IPIV(J)=0
00115               NN=N+M
00116               DO 182 K=1,N
00121               AMAX=0.0D0
00122            40 DO 76 I=1,N
00125               IF (IPIV(I)-1)50,76,50
00130            50 DO 75 J=1,N
00133               IF (IPIV(J)-1)55,75,250
00136            55 IND=(J-1)*N+I
00137               IF(AMAX-DABS(A(IND))) 60,75,75
00142            60 IR=I
00143               IC=J
00144               AMAX=DABS(A(IND))
00145            75 CONTINUE
```

```
00147          76 CONTINUE
00151             IPIV(IC)=IPIV(IC)+1
00152             IF (IR-IC)90,115,90
00155          90 SIGN=-SIGN
00156             DO 110 L=1,NN
00161             IND=(L-1)*N+IR
00162             IND2=(L-1)*N+IC
00163             AMAX=A(IND)
00164             A(IND)=A(IND2)
00165         110 A(IND2)=AMAX
00167         115 INDEX(K,1)=IR
00170             INDEX(K,2)=IC
00171             IND=(IC-1)*N+IC
00172             AMAX=A(IND)
00173             DETER=DETER*AMAX
00174             IF (DETER)140,255,140
00177         140 A(IND)=1.0D0
00200             DO 150 L=1,NN
00203             IND=(L-1)*N+IC
00204         150 A(IND)=A(IND)/AMAX
00206             DO 181 L=1,N
00211             IF (L-IC)165,181,165
00214         165 IND=(IC-1)*N+L
00215             AMAX=A(IND)
00216             A(IND)=0.0D0
00217             DO 180 I=1,NN
00222             IND=(I-1)*N+L
00223             IND2=(I-1)*N+IC
00224             A(IND)=A(IND)-A(IND2)*AMAX
00225         180 CONTINUE
00227         181 CONTINUE
00231         182 CONTINUE
00233             DO 235 I=1,N
00236             L=N+1-I
00237             IR=INDEX(L,1)
00240             IC=INDEX(L,2)
00241             IF (IR-IC)210,235,210
00244         210 DO 230 K=1,N
00247             IND=(IR-1)*N+K
00250             IND2=(IC-1)*N+K
00251             AMAX=A(IND)
00252             A(IND)=A(IND2)
00253         230 A(IND2)=AMAX
00255         235 CONTINUE
00257             DETER=SIGN*DETER
00260             RETURN
00261         250 M=-1
00262         255 RETURN
00263              END

      END OF COMPILATION:          NO  DIAGNOSTICS.
```

```
@FOR,IS DMATML,DMATML
MVI 009-09/16-13:32 (,0)

   SUBROUTINE DMATML     ENTRY POINT 000160

   STORAGE USED: CODE(1) 000176; DATA(0) 000053; BLANK COMMON(2) 000000

   EXTERNAL REFERENCES (BLOCK, NAME)

    0003   TRACSF
    0004   NERR3$

   STORAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

    0001     000034 1L        0001     000061 130G      0001     000067 135G      0001     000102 143G      0001     000035 2L
    0001     000046 3L        0001     000052 4L        0000 D 000000 CD        0000 I 000005 IA1       0000 I 000006 IA2
    0000 I 000002 IA3       0000 I 000007 IB1       0000 I 000010 IB2       0000 I 000013 IB3       0000 I 000003 IM
    0000 I 000004 IN        0000   000022 INJP$     0000 I 000015 LA        0000 I 000016 LB        0000 I 000012 LC
    0000 I 000011 LM        0000 I 000017 LN        0000 I 000014 LP


00101            SUBROUTINE DMATML(C,A,B,M,N,K)
00101      C*    ABSTRACT
00101      C     GENERAL MATRIX MULTIPLICATION ROUTINE  WITH TRANSPOSE OPTIONS
00101      C      WHERE, M IS THE NUMBER OF ROWS OF (A)
00101      C             N IS THE NUMBER OF ROWS OF (B)
00101      C             K IS THE NUMBER OF COLUMNS OF (B) OR (B)T
00101      C     TRANSPOSE OPTIONS ARE CONTROLLED BY THE SIGNS OF M AND N.
00101      C     THE FOLLOWING PRODUCTS MAY BE OBTAINED
00101      C      (C)=(A)(B)        M AND N POSITIVE
00101      C      (C)=(A)T(B)       M NEGATIVE FOR (A)T
00101      C      (C)=(A)(B)T       N NEGATIVE FOR (B)T
00101      C      (C)=(A)T(B)T      M AND N NEGATIVE
00101      C      WHERE T INDICATES TRANSPOSE
00101      C      IF M IS NEGATIVE, M IS THE NUMBER OF ROWS OF (A)T
00101      C      IF N IS NEGATIVE, N IS THE NUMBER OF ROWS OF (B)T
00101      C
00101      C*    OUTPUT ARGUMENT * C
00103            DIMENSION C(1)
00103      C
00103      C*    INPUT ARGUMENTS * A,B,M,N,K
00104            DIMENSION A(1),B(1)
00104      C
00104      C*****************************************************************
00105            DOUBLE PRECISION CD,C,A,B
00106            IA3=1
00107            IM=IABS(M)
00110            IN=IABS(N)
00111            IF(M .LT. 0)GO TO 1
00113            IA1=IM
```

```
00114             IA2=1
00115             GO TO 2
00116       1     IA1 = 1
00117             IA2=IN
00120       2     IF(N .LT. 0)GO TO 3
00122             IB1=1
00123             IB2=IN
00124             GO TO 4
00125       3     IB1=K
00126             IB2=1
00127       4     DO 7 LM=1,IM
00132             LC=LM
00133             IB3=1
00134             DO 6 LP=1,K
00137             CD = 0.0D0
00140             LA=IA3
00141             LB=IB3
00142             DO 5 LN=1,IN
00145             CD = CD + A(LA)*B(LB)
00146             LA=LA+IA1
00147       5     LB=LB+IB1
00151             C(LC) = CD
00152             LC = LC + IM
00153       6     IB3=IB3+IB2
00155       7     IA3=IA3+IA2
00157             RETURN
00160             END

        END OF COMPILATION:        NO  DIAGNOSTICS.
```

```
@MAP,IS EXPO
MAP 17M1-09/16-13:33 -(1,0)

    1.      LIB SYS$*MSFC$.

    2.      IN LQCEXP

H$MONITOR      ENTRY POINT      TRACE          ALREADY DEFINED

ADDRESS LIMITS      001000 037532      040000 076226
STARTING ADDRESS    031714

WORDS DECIMAL        15707 IBANK        15511 DBANK

                SEGMENT MAIN                001000 037532      040000 076226

NSWTC$/FOR                     1   001000 001021
NRBLK$/MSFC55                  1   001022 001110      0   040000 040001
NRWND$/FOR50                   1   001111 001170      2   040002 040013
NWEF$/MSFC55                   1   001171 001425      2   040014 040034
NEXP6$/MSFC57                  1   001426 001620      2   040035 040106
ALOG$/FOR51                    1   001621 001736      2   040107 040147
CSIG6V/SC4020                  1   001737 002225      0   040150 040206
                                                      2   BLANK$COMMON
CERMRK/SC4020                  1   002226 002255      0   040207 040222
                                                      2   BLANK$COMMON
CLABLV/SC4020                  1   002256 003315      0   040223 040331
                                                      2   BLANK$COMMON
NFTCH$/FOR57                   1   003316 003615      2   040332 040367
NFTV$/FOR                      1   003616 003640
NCLOS$/MSFC57                  1   003641 004007      2   040370 040415
NWBLK$/MSFC57                  1   004010 004177      0   040416 040420
NBSBL$/FOR                     1   004200 004235
NUPDA$/FOR                     1   004236 004271
NBF00$/FOR                                            2   040421 042622
CYMODV/SC4020                  1   004272 004317      0   042623 042631
                                                      2   BLANK$COMMON
CACCBY/SC4020                  1   004320 004341      0   042632 042642
                                                      2   BLANK$COMMON
CXMODV/SC4020                  1   004342 004367      0   042643 042651
                                                      2   BLANK$COMMON
CONCAT/MSFC                    1   004370 004541      0   042652 042673
SETINT/SC4020                  1   004542 004574      0   042674 042701
                                                      2   BLANK$COMMON
CHOLLV/SC4020                  1   004575 004657      0   042702 042716
                                                      2   BLANK$COMMON
```

CNONLN/SC4020	1	004600 005265	0	042717 042770
			2	BLANK$COMMON
CLINRV/SC4020	1	005266 006127	0	042771 043070
	3	GGG	2	BLANK$COMMON
CYSCLV/SC4020	1	006130 006332	0	043071 043117
	3	GGG	2	BLANK$COMMON
CXSCLV/SC4020	1	006333 006535	0	043120 043146
	3	GGG	2	BLANK$COMMON
CERNLV/SC4020	1	006536 006712	0	043147 043161
			2	BLANK$COMMON
CERRLN/SC4020	1	006713 007022	0	043162 043177
			2	BLANK$COMMON
CSETCV/SC4020	1	007023 007063	0	043200 043210
			2	BLANK$COMMON
NEXP5$/FOR57	1	007064 007147	2	043211 043220
CSETMV/SC4020	1	007150 007226	0	043221 043235
			2	BLANK$COMMON
CFRAM/SC4020	1	007227 007446	0	043236 043322
	3	GGG	2	BLANK$COMMON
CXAXIS/SC4020	1	007447 007652	0	043323 043356
			2	BLANK$COMMON
VCHARV/SC4020	1	007653 010116	0	043357 043376
RITE2V/SC4020	1	010117 010347	0	043377 043425
BPLOTK/SC4020	1	010350 010430	0	043426 044005
			2	BLANK$COMMON
CCAMRA/SC4020	1	010431 010502	0	044006 044017
	3	GGG	2	BLANK$COMMON
TABLIV/SC4020			0	044020 044340
NBDCV$/FOR57	1	010503 010636	2	044341 044400
NCNVT$/FOR57	1	010637 011071	2	044401 044470
NOTIN$/MSFC55	1	011072 011422	2	044471 044501
NOUT$/FOR57	1	011423 012377	2	044502 044532
NIOER$/MSFC57	1	012400 012557	2	044533 044655
NININ$/MSFC55	1	012560 013011	2	044656 044677
NINPT$/FOR57	1	013012 013671	2	044700 044722
NFMT$/FOR57	1	013672 014576	2	044723 044741
NFCHK$/MSFC57	1	014577 015415	2	044742 045116
			4	045117 045170
NTAB$/MSFC55			2	045171 045257
CPLOTV/SC4020	1	015416 015567	0	045260 045317
	3	GGG	2	BLANK$COMMON
CLINEV/SC4020	1	015570 016153	0	045320 045376
			2	BLANK$COMMON
YSCLVI/SC4020	1	016154 016272	0	045377 045407
	3	GGG	2	BLANK$COMMON
XSCLVI/SC4020	1	016273 016410	0	045410 045420
	3	GGG	2	BLANK$COMMON
CAPLOT/SC4020	1	016411 016653	0	045421 045474
	3	GGG	2	BLANK$COMMON
CAPRNV/SC4020	1	016654 016736	0	045475 045513
			2	BLANK$COMMON
CPRNTV/SC4020	1	016737 017330	0	045514 045550
	3	GGG	2	BLANK$COMMON
CGRDIV/SC4020	1	017331 020235	0	045551 045664
	3	GGG	2	BLANK$COMMON
CDXDYV/SC4020	1	020236 021030	0	045665 045751

```
                                                         2   BLANK$COMMON
CBRITV/SC4020                  1   021031 021132         0   045752 045765
                               3   GGG                   2   BLANK$COMMON
CMARGN/SC4020                  1   021133 021211         0   045766 046027
                                                         2   BLANK$COMMON
CNBLNK/SC4020                  1   021212 021261         0   046030 046043
                                                         2   BLANK$COMMON
BMOV/MSFC                                                0   046044 046120
CIDENT/SC4020                  1   021262 022431         0   046121 046304
                               3   GGG                   2   BLANK$COMMON
ERU$/MSFC55
DSQRT$/FOR57                   1   022432 022501         2   046305 046323
SQRT$/FOR55                    1   022502 022541         2   046324 046335
DEXP$/FOR54                    1   022542 022710         2   046336 046371
EXP$/FOR57                     1   022711 022777         2   046372 046412
DLOG$/FOR54                    1   023000 023120         2   046413 046517
NLOUT$/MSFC55                  1   023121 024206         2   046520 046563
NLINP$/MSFC57                  1   024207 025753         2   046564 046775
NOBUF$/FOR51                   1   025754 026013
NIER$/FOR52                    1   026014 026076         2   046776 047125
NIBUF$/FOR52                   1   026077 026140
H$MONITOR/MSFC55               1   026141 027250         2   047126 047703
NERR$/FOR57                    1   027251 027605         2   047704 050060
GGG (COMMON BLOCK)                                           050061 050214
CQUIKL/SC4020                  1   027606 030217         0   050215 050277
                               3   GGG                   2   BLANK$COMMON
IDENT/SC4020                   1   030220 031154         0   050300 051772
TRACE                          1   031155 031300         0   051773 052000
                                                         2   052001 052063
INVRT                          1   031301 031713         0   052064 052227
                                                         2   BLANK$COMMON
BLANK$COMMON (COMMON BLOCK)
LQCEXP                         1   031714 037532         0   052230 076226
                                                         2   BLANK$COMMON


SYS$*RLIB$. LEVEL M57-0
END OF COLLECTION - TIME 3.972 SECONDS
```

@XQT EXPO

Typical Output Results for Set 1
Data Using a Two-Term Exponential Model

```
$INPUT
T        =          .000000000000000000D+000,      .200000000000000000D+001,
                    .400000000000000000D+001,      .599999999999999999D+001,
                    .799999999999999999D+001,      .100000000000000000D+002,
                    .120000000000000000D+002,      .140000000000000000D+002,
                    .160000000000000000D+002,      .180000000000000000D+002,
                    .200000000000000000D+002,      .220000000000000000D+002,
                    .240000000000000000D+002,      .260000000000000000D+002,
                    .280000000000000000D+002,      .300000000000000000D+002,
                    .320000000000000000D+002,      .340000000000000000D+002,
                    .360000000000000000D+002,      .380000000000000000D+002,
                    .400000000000000000D+002,      .420000000000000000D+002,
                    .439999999999999999D+002,      .460000000000000000D+002,
                    .480000000000000000D+002,      .500000000000000000D+002,
                    .520000000000000000D+002,      .539999999999999999D+002,
                    .560000000000000000D+002,      .579999999999999999D+002,
                    .599999999999999999D+002,      .619999999999999999D+002,
                    .639999999999999999D+002,      .660000000000000000D+002,
                    .679999999999999999D+002,      .700000000000000000D+002,
                    .719999999999999999D+002,      .740000000000000000D+002,
                    .759999999999999999D+002,      .780000000000000000D+002,
                    .799999999999999999D+002,      .820000000000000000D+002,
                    .839999999999999999D+002,      .860000000000000000D+002,
                    .879999999999999999D+002,      .900000000000000000D+002,
                    .919999999999999999D+002,      .940000000000000000D+002,
                    .959999999999999999D+002,      .980000000000000000D+002,
                    .100000000000000000D+003,      .102000000000000000D+003,
                    .104000000000000000D+003,      .106000000000000000D+003,
                    .108000000000000000D+003,      .110000000000000000D+003,
                    .112000000000000000D+003,      .114000000000000000D+003,
                    .116000000000000000D+003,      .118000000000000000D+003,
                    .120000000000000000D+003,      .122000000000000000D+003,
                    .124000000000000000D+003,      .126000000000000000D+003,
                    .128000000000000000D+003,      .130000000000000000D+003,
                    .132000000000000000D+003,      .134000000000000000D+003,
                    .000000000000000000D+000,      .000000000000000000D+000,
                    .000000000000000000D+000,      .000000000000000000D+000,
                    .000000000000000000D+000,      .000000000000000000D+000,
                    .000000000000000000D+000,      .000000000000000000D+000,
                    .000000000000000000D+000,      .000000000000000000D+000,
                    .000000000000000000D+000,      .000000000000000000D+000,
                    .000000000000000000D+000,      .000000000000000000D+000,
                    .000000000000000000D+000,      .000000000000000000D+000,
                    .000000000000000000D+000,      .000000000000000000D+000,
                    .000000000000000000D+000,      .000000000000000000D+000,
                    .000000000000000000D+000,      .000000000000000000D+000,
                    .000000000000000000D+000,      .000000000000000000D+000,
                    .000000000000000000D+000,      .000000000000000000D+000,
                    .000000000000000000D+000,      .000000000000000000D+000,
                    .000000000000000000D+000,      .000000000000000000D+000,
                    .000000000000000000D+000,      .000000000000000000D+000
         =          .130000000000000000D+001,      .740000000000000000D+000,
                    .613000000000000000D+000,      .530000000000000000D+000,
                    .470000000000000000D+000,      .425000000000000000D+000,
                    .385000000000000000D+000,      .345000000000000000D+000,
```

```
.31000000000000000D+000,          .28000000000000000D+000,
.25500000000000000D+000,          .23000000000000000D+000,
.21000000000000000D+000,          .19500000000000000D+000,
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.13000000000000000D+000,          .12500000000000000D+000,
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.11500000000000000D+000,          .11000000000000000D+000,
.11000000000000000D+000,          .10500000000000000D+000,
.10500000000000000D+000,          .10500000000000000D+000,
.10000000000000000D+000,          .10000000000000000D+000,
.10000000000000000D+000,          .10000000000000000D+000,
.94999999999999999D-001,          .94999999999999999D-001,
.94999999999999999D-001,          .89999999999999999D-001,
.89999999999999999D-001,          .89999999999999999D-001,
.84999999999999999D-001,          .84999999999999999D-001,
.84999999999999999D-001,          .84999999999999999D-001,
.84999999999999999D-001,          .84999999999999999D-001,
.79999999999999999D-001,          .79999999999999999D-001,
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.74999999999999999D-001,          .74999999999999999D-001,
.00000000000000000D+000,          .00000000000000000D+000,
.00000000000000000D+000,          .00000000000000000D+000,
.00000000000000000D+000,          .00000000000000000D+000,
.00000000000000000D+000,          .00000000000000000D+000,
.00000000000000000D+000,          .00000000000000000D+000,
.00000000000000000D+000,          .00000000000000000D+000,
.00000000000000000D+000,          .00000000000000000D+000,
.00000000000000000D+000,          .00000000000000000D+000,
.00000000000000000D+000,          .00000000000000000D+000,
.00000000000000000D+000,          .00000000000000000D+000,
.00000000000000000D+000,          .00000000000000000D+000,
.00000000000000000D+000,          .00000000000000000D+000,
.00000000000000000D+000,          .00000000000000000D+000,
.00000000000000000D+000,          .00000000000000000D+000,
.00000000000000000D+000,          .00000000000000000D+000
NN      =           +68
TL      =    .00000000E+00
TR      =    .15000000E+03
YB      =    .00000000E+00
YT      =    .20000000E+01
YB1     =   -.10000000E+01
YT1     =    .10000000E+01
VARY    =    .10000000E+01
TOLER   =    .10000000E+05
ITERM4  =            +0
```

```
ITERM3  =                    +1
ITERM2  =                    +0

$END
```

INITIAL ESTIMATES FOR TWO-TERM EXPONENTIAL MODEL

I	F	A1	B1	A2	B2
1	.590963+01	.215863+01	.801143-01	.750000-01	-.148030-15
2	.196719+01	.152464+01	.718354-01	.750000-01	.000000
3	.500553+00	.104223+01	.626321-01	.750000-01	.000000
4	.339488+00	.687734+00	.524097-01	.750000-01	.000000
5	.751035+00	.436815+00	.410624-01	.750000-01	-.362522-16
6	.177415+01	.143947+00	.262821-01	.106386+00	.268911-02
7	.191917+01	.100484+00	.165219-01	.122265+00	.376475-02
8	.190504+01	.978026-01	.147689-01	.127769+00	.410700-02
9	.186429+01	.101178+00	.156778-01	.127769+00	.410700-02
10	.192071+01	.971186-01	.173734-01	.125225+00	.394904-02
11	.181897+01	.118473+00	.211404-01	.121696+00	.372338-02
12	.174347+01	.136673+00	.250453-01	.117949+00	.347515-02
13	.166707+01	.156334+00	.294937-01	.114336+00	.322693-02
14	.152889+01	.189219+00	.342206-01	.111003+00	.298965-02
15	.137230+01	.228518+00	.390007-01	.107997+00	.276820-02
16	.122094+01	.272315+00	.459700-01	.105314+00	.256422-02
17	.126961+01	.248492+00	.424049-01	.109156+00	.285635-02
18	.129204+01	.244959+00	.420009-01	.111471+00	.302838-02
19	.126053+01	.251692+00	.429768-01	.112668+00	.311640-02
20	.120205+01	.266993+00	.451249-01	.113065+00	.314554-02
21	.111543+01	.291385+00	.480171-01	.112902+00	.313353-02
22	.100508+01	.324914+00	.515887-01	.112356+00	.309294-02
23	.105643+01	.310743+00	.541731-01	.115105+00	.329661-02
24	.100863+01	.319902+00	.527675-01	.116911+00	.342858-02
25	.958134+00	.335119+00	.548990-01	.117980+00	.350610-02
26	.989749+00	.322318+00	.548993-01	.121272+00	.374201-02
27	.980938+00	.322770+00	.555774-01	.123609+00	.390656-02
28	.969877+00	.328668+00	.597746-01	.125166+00	.401511-02
29	.930089+00	.333333+00	.569731-01	.128359+00	.423481-02
30	.908869+00	.338491+00	.584712-01	.130677+00	.439175-02
31	.849165+00	.356912+00	.602814-01	.132265+00	.449834-02
32	.779104+00	.382289+00	.637355-01	.133255+00	.456452-02
33	.750235+00	.392667+00	.666284-01	.135511+00	.471442-02
34	.683900+00	.417103+00	.689071-01	.137105+00	.481945-02
35	.605232+00	.451429+00	.732927-01	.138145+00	.488777-02
36	.555520+00	.477088+00	.792927-01	.140173+00	.502024-02
37	.478854+00	.512865+00	.814716-01	.141618+00	.511405-02
38	.428466+00	.540754+00	.857185-01	.143836+00	.525707-02
39	.349652+00	.592374+00	.927223-01	.145473+00	.536184-02
40	.304460+00	.621841+00	.952586-01	.147729+00	.550522-02
41	.228019+00	.690254+00	.103980+00	.149422+00	.561205-02
42	.184256+00	.734845+00	.107960+00	.151620+00	.574981-02
43	.133032+00	.809925+00	.115381+00	.153282+00	.585332-02
44	.953342-01	.921104+00	.127343+00	.155362+00	.598209-02
45	.928580-01	.929952+00	.126726+00	.157760+00	.612939-02
46	.905191-01	.975847+00	.131825+00	.160397+00	.628977-02
47	.106840+00	.107050+01	.142246+00	.163206+00	.645887-02
48	.159916+00	.118219+01	.153529+00	.166133+00	.663315-02
49	.140773+00	.115206+01	.152772+00	.169134+00	.680981-02
50	.156104+00	.118221+01	.158558+00	.172752+00	.702008-02

51	.897793-01	.102191+01	.145858+00	.176847+00	.725439-02
52	.827253-01	.951721+00	.140500+00	.181307+00	.750509-02
53	.881064-01	.886495+00	.134412+00	.186036+00	.776607-02
54	.980894-01	.840049+00	.130080+00	.191381+00	.805513-02
55	.106007+00	.810961+00	.128233+00	.197582+00	.838273-02
56	.108451+00	.797879+00	.129563+00	.204789+00	.875336-02
57	.108707+00	.790074+00	.132336+00	.212778+00	.915198-02
58	.105861+00	.789423+00	.137549+00	.221650+00	.958047-02
59	.995807-01	.798078+00	.146680+00	.231457+00	.100378-01
60	.906543-01	.819589+00	.162410+00	.242216+00	.105212-01
61	.838456-01	.850950+00	.185905+00	.253707+00	.110178-01
62	.840578-01	.890918+00	.220787+00	.265941+00	.115260-01
63	.100611+00	.944801+00	.281730+00	.279245+00	.120566-01

THE INITIAL ESTIMATES USED ARE

I= 52

NON-LINEAR EXPONENTIAL REGRESSION ANALYSIS USING AN ITERATIVE CORRECTION PROCEDURE

ASSUMED MODEL IS TWO EXPONENTIALS
INITIAL ESTIMATES FOR PARAMETERS
A1= .951721+00 B1= .140500+00 A2= .181307+00 B2= .750509-02
OBSERVED RESPONSE, COMPUTED RESPONSE, AND RESIDUALS USING INITIAL ESTIMATES

OBSERVED	COMPUTED	RESIDUAL
.130000+01	.113303+01	.166972+00
.740000+00	.897182+00	-.157182+00
.610000+00	.718490+00	-.108490+00
.530000+00	.582961+00	-.529608-01
.470000+00	.480029+00	-.100290-01
.425000+00	.401719+00	.232815-01
.385000+00	.342007+00	.429933-01
.345000+00	.296346+00	.486540-01
.310000+00	.261303+00	.486970-01
.280000+00	.234285+00	.457150-01
.255000+00	.213335+00	.416654-01
.230000+00	.196973+00	.330265-01
.210000+00	.184086+00	.259145-01
.195000+00	.173828+00	.211721-01
.185000+00	.165564+00	.194359-01
.175000+00	.158813+00	.161866-01
.165000+00	.153213+00	.117873-01
.155000+00	.148488+00	.651209-02
.150000+00	.144432+00	.556824-02
.145000+00	.140888+00	.411223-02
.140000+00	.137738+00	.226237-02
.135000+00	.134892+00	.108072-03
.130000+00	.132283+00	-.228305-02
.125000+00	.129860+00	-.485984-02
.125000+00	.127584+00	-.258357-02
.120000+00	.125425+00	-.542489-02
.120000+00	.123362+00	-.336156-02
.115000+00	.121377+00	-.637667-02
.115000+00	.119457+00	-.445738-02
.110000+00	.117594+00	-.759387-02
.110000+00	.115779+00	-.577866-02
.105000+00	.114006+00	-.900600-02
.105000+00	.112271+00	-.727145-02
.105000+00	.110572+00	-.557158-02
.100000+00	.108904+00	-.890371-02
.100000+00	.107266+00	-.726572-02
.100000+00	.105656+00	-.565594-02
.100000+00	.104073+00	-.407302-02
.950000-01	.102516+00	-.751584-02
.950000-01	.100984+00	-.598351-02
.950000-01	.994752-01	-.447524-02
.900000-01	.979904-01	-.799037-02
.900000-01	.965283-01	-.652833-02

.900000-01	.950886-01	-.508860-02
.850000-01	.936707-01	-.867072-02
.850000-01	.922743-01	-.727427-02
.850000-01	.908989-01	-.589886-02
.850000-01	.895441-01	-.454411-02
.850000-01	.882097-01	-.320967-02
.850000-01	.868952-01	-.189521-02
.800000-01	.856004-01	-.560041-02
.800000-01	.843250-01	-.432495-02
.800000-01	.830685-01	-.306854-02
.800000-01	.818309-01	-.183088-02
.800000-01	.806117-01	-.611679-03
.800000-01	.794107-01	.589337-03
.800000-01	.782276-01	.177245-02
.800000-01	.770621-01	.293792-02
.800000-01	.759140-01	.408602-02
.800000-01	.747830-01	.521701-02
.800000-01	.736688-01	.633115-02
.750000-01	.725713-01	.242869-02
.750000-01	.714901-01	.350987-02
.750000-01	.704251-01	.457494-02
.750000-01	.693759-01	.562414-02
.750000-01	.683423-01	.665771-02
.750000-01	.673241-01	.767589-02
.750000-01	.663211-01	.867889-02

SIGYIN= .359525-01

CYCLE NUMBER = 1

B MATRIX

.100000+01	.000000	.100000+01	.000000
.755028+00	-.143715+01	.985102+00	-.357211+00
.570068+00	-.217018+01	.970426+00	-.703779+00
.430417+00	-.245782+01	.955968+00	-.103994+01
.324977+00	-.247430+01	.941726+00	-.136593+01
.245367+00	-.233521+01	.927696+00	-.168198+01
.185259+00	-.211578+01	.913875+00	-.198830+01
.139876+00	-.186372+01	.900260+00	-.228513+01
.105610+00	-.160818+01	.886848+00	-.257266+01
.797386-01	-.136600+01	.873636+00	-.285113+01
.602049-01	-.114597+01	.860620+00	-.312073+01
.454564-01	-.951761+00	.847799+00	-.338166+01
.343209-01	-.783934+00	.835168+00	-.363412+01
.259132-01	-.641217+00	.822726+00	-.387831+01
.195652-01	-.521378+00	.810469+00	-.411442+01
.147723-01	-.421773+00	.798394+00	-.434263+01
.111535-01	-.339681+00	.786500+00	-.456313+01
.842121-02	-.272498+00	.774782+00	-.477609+01
.635825-02	-.217846+00	.763240+00	-.498170+01
.480066-02	-.173618+00	.751869+00	-.518012+01
.362463-02	-.137986+00	.740667+00	-.537152+01
.273670-02	-.109392+00	.729633+00	-.555607+01
.206629-02	-.865273-01	.718763+00	-.573393+01
.156010-02	-.683201-01	.708055+00	-.590525+01
.117792-02	-.538106-01	.697506+00	-.607020+01

.889365-03	-.423214-01	.687114+00	-.622892+01
.671496-03	-.332320-01	.676878+00	-.638157+01
.506998-03	-.260561-01	.666794+00	-.652828+01
.382798-03	-.204018-01	.656860+00	-.666921+01
.289023-03	-.159540-01	.647074+00	-.680449+01
.218221-03	-.124611-01	.637433+00	-.693426+01
.164763-03	-.972212-02	.627937+00	-.705865+01
.124401-03	-.757726-02	.618582+00	-.717780+01
.939260-04	-.589983-02	.609366+00	-.729182+01
.709168-04	-.458952-02	.600288+00	-.740086+01
.535442-04	-.356714-02	.591345+00	-.750503+01
.404274-04	-.277024-02	.582535+00	-.760446+01
.305238-04	-.214971-02	.573856+00	-.769925+01
.230463-04	-.166696-02	.565307+00	-.778954+01
.174006-04	-.129172-02	.556885+00	-.787542+01
.131380-04	-.100029-02	.548588+00	-.795702+01
.991954-05	-.774132-03	.540415+00	-.803444+01
.748953-05	-.598748-03	.532364+00	-.810778+01
.565481-05	-.462835-03	.524433+00	-.817716+01
.426954-05	-.357580-03	.516620+00	-.824267+01
.322362-05	-.276119-03	.508923+00	-.830441+01
.243393-05	-.213111-03	.501341+00	-.836248+01
.183768-05	-.164402-03	.493872+00	-.841698+01
.138750-05	-.126770-03	.486514+00	-.846800+01
.104760-05	-.977086-04	.479266+00	-.851563+01
.790970-06	-.752783-04	.472126+00	-.855997+01
.597205-06	-.579740-04	.465092+00	-.860109+01
.450907-06	-.446303-04	.458163+00	-.863908+01
.340447-06	-.343452-04	.451338+00	-.867404+01
.257047-06	-.264208-04	.444614+00	-.870603+01
.194078-06	-.203179-04	.437990+00	-.873515+01
.146534-06	-.156195-04	.431464+00	-.876147+01
.110638-06	-.120038-04	.425036+00	-.878507+01
.835345-07	-.922218-05	.418704+00	-.880601+01
.630709-07	-.708306-05	.412466+00	-.882439+01
.476203-07	-.543855-05	.406321+00	-.884026+01
.359547-07	-.417470-05	.400268+00	-.885370+01
.271468-07	-.320369-05	.394305+00	-.886477+01
.204966-07	-.245789-05	.388430+00	-.887356+01
.154755-07	-.188523-05	.382643+00	-.888011+01
.116844-07	-.144564-05	.376943+00	-.888450+01
.882209-08	-.110829-05	.371327+00	-.888678+01
.666093-08	-.849473-06	.365795+00	-.888703+01

BT*W*B

.232595+01	-.587038+01	.390289+01	-.410829+01
-.587038+01	.408060+02	-.215654+02	.532206+02
.390289+01	-.215654+02	.294226+02	-.241823+03
-.410829+01	.532206+02	-.241823+03	.321212+04

INVERSE OF BT*W*B

.856977+00	.121971-01	-.255456+00	-.183380-01
.121971-01	.805688-01	.122212+00	.788133-02
-.255456+00	.122212+00	.362295+00	.249237-01
-.183380-01	.788133-02	.249237-01	.203365-02

INVERSE TIMES ORIGINAL MATRIX

.100000+01	-.607153-17	.693889-17	-.555112-16
.271051-18	.100000+01	-.173472-17	.277556-16

.000000	.693889-17	.100000+01	.000000
.135525-18	-.650521-18	.433681-18	.100000+01

DETERMINANT VALUE

.522963+06

OBSERVED RESPONSE, COMPUTED RESPONSE AND RESIDUALS

OBSERVED	COMPUTED	RESIDUAL
.130000+01	.115504+01	.144964+00
.740000+00	.895831+00	-.155831+00
.610000+00	.706661+00	-.966608-01
.530000+00	.568294+00	-.382939-01
.470000+00	.466786+00	.321428-02
.425000+00	.392026+00	.329741-01
.385000+00	.336684+00	.483163-01
.345000+00	.295444+00	.495561-01
.310000+00	.264452+00	.455478-01
.280000+00	.240914+00	.390862-01
.255000+00	.222802+00	.321982-01
.230000+00	.208646+00	.213536-01
.210000+00	.197381+00	.126187-01
.195000+00	.188233+00	.676698-02
.185000+00	.180640+00	.435966-02
.175000+00	.174196+00	.804076-03
.165000+00	.168604+00	-.360399-02
.155000+00	.163650+00	-.864970-02
.150000+00	.159177+00	-.917684-02
.145000+00	.155072+00	-.100718-01
.140000+00	.151252+00	-.112517-01
.135000+00	.147656+00	-.126562-01
.130000+00	.144241+00	-.142411-01
.125000+00	.140974+00	-.159740-01
.125000+00	.137831+00	-.128309-01
.120000+00	.134794+00	-.147944-01
.120000+00	.131851+00	-.118512-01
.115000+00	.128991+00	-.139913-01
.115000+00	.126207+00	-.112075-01
.110000+00	.123494+00	-.134937-01
.110000+00	.120846+00	-.108457-01
.105000+00	.118260+00	-.132597-01
.105000+00	.115733+00	-.107329-01
.105000+00	.113263+00	-.826289-02
.100000+00	.110848+00	-.108476-01
.100000+00	.108485+00	-.848530-02
.100000+00	.106174+00	-.617440-02
.100000+00	.103913+00	-.391349-02
.950000-01	.101701+00	-.670129-02
.950000-01	.995366-01	-.453659-02
.950000-01	.974183-01	-.241825-02
.900000-01	.953452-01	-.534522-02
.900000-01	.933164-01	-.331645-02
.900000-01	.913310-01	-.133096-02
.850000-01	.893878-01	-.438779-02
.850000-01	.874860-01	-.248603-02
.850000-01	.856248-01	-.624774-03
.850000-01	.838031-01	.119686-02
.850000-01	.820203-01	.297971-02
.850000-01	.802754-01	.472461-02

.800000-01	.785676-01	.143239-02
.800000-01	.768962-01	.310382-02
.800000-01	.752603-01	.473969-02
.800000-01	.736592-01	.634076-02
.800000-01	.720922-01	.790776-02
.800000-01	.705586-01	.944142-02
.800000-01	.690575-01	.109425-01
.800000-01	.675884-01	.124116-01
.800000-01	.661506-01	.138494-01
.800000-01	.647433-01	.152567-01
.800000-01	.633660-01	.166340-01
.750000-01	.620180-01	.129820-01
.750000-01	.606987-01	.143013-01
.750000-01	.594074-01	.155926-01
.750000-01	.581436-01	.168564-01
.750000-01	.569067-01	.180933-01
.750000-01	.556961-01	.193039-01
.750000-01	.545112-01	.204888-01

PARAMETER CORRECTIONS

DELA1= -.269235-01 DELB1= .202625-01 DELA2= .489308-01 DELB2= .324644-02

IMPROVED PARAMETER ESTIMATES

A1= .924798+00 B1= .160783+00 A2= .230238+00 B2= .107515-01

RESPONSE VARIABLE STANDARD DEVIATION

SIGY1= .359525-01 SIGY2= .341503-01

COEFFICIENT STANDARD DEVIATION

SGA1= .223921-01 SGB1= .534604-02 SGA2= .629584-02 SGB2= .602557-03

TOLER= -.126342-03

END OF CYCLE NUMBER 1

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APPROVAL

A METHOD FOR NONLINEAR EXPONENTIAL REGRESSION ANALYSIS

By Bobby G. Junkin

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

This document has also been reviewed and approved for technical accuracy.

H. HOELZER
Director, Computation Laboratory

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