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Controllable parameters in a power system include generator terminal voltage, generator input power and network admittances. These parameters can be controlled to damp mechanical rotor oscillation in the generators and thereby improve system transient stability. Explicit equations are derived in this thesis for the control of these parameters to introduce damping uniformly throughout a large power system. These equations are derived on the basis of minimizing a positive definite error function. Decision functions are included which inhibit damping action when it leads to system instability. This method can be used to coordinate the application of locally based damping techniques. A three-generator example is presented illustrating the damping method.

A Method of Improving Power System Transient Stability Using Controllable Parameters

bу

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## A METHOD OF IMPROVING POWER SYSTEM TRANSIENT STABILITY USING CONTROLLABLE PARAMETERS

#### I. INTRODUCTION

#### Introduction

To meet the growing demand for economical power, distant regions are interconnected electrically. A good example is the Pacific Northwest-Southwest Intertie (15, unnumbered preface).

These lines will tie together electric systems -public and private -- all the way from Vancouver,
B. C., and Seattle to Los Angeles and Phoenix, including the biggest hydro system in America (the
Bonneville Power Administration system), the biggest
municipal system (Los Angeles Department of Water and
Power), and one of the biggest private systems (the
private utilities of California).

As a power system grows, the difficulty of withstanding unexpected disturbances without disintegration increases.

Immediately following a disturbance, mechanical oscillation occurs in each generator relative to a reference axis rotating at nominal shaft speed. The rapid extinction of this oscillation, which is called damping, improves the system's ability to remain intact.

Transient stability is a condition which exists if a disturbance does not cause power system disintegration. The addition of damping will improve transient stability to include an enlarged class of system disturbances.

It is desirable that the mechanical oscillation of every generator be given a nearly equal rate of damping since system separation can occur if a single machine (generator) remains undamped. Uniform power system damping is that condition which exists if equal damping is given to every machine oscillation. Several techniques which introduce damping at local points in the power system are cited in the Literature Review. A method of coordinating these techniques is necessary to provide uniform system damping.

A method is presented in this thesis governing the control of basic power system parameters to improve transient stability with nearly uniform damping. The method can be employed to coordinate local damping techniques.

#### Statement of the Problem

#### Power System Model

Power systems are basically composed of a set of generating plants, a distribution network and a combination of industrial, commercial and residential loads. An example of a partial power system is given in Figure 1. In this diagram, a single line represents a full three phase transmission tie.  $E_i \angle A_i$  is the effective phase-to-neutral voltage and phasor angle of the ith machine.

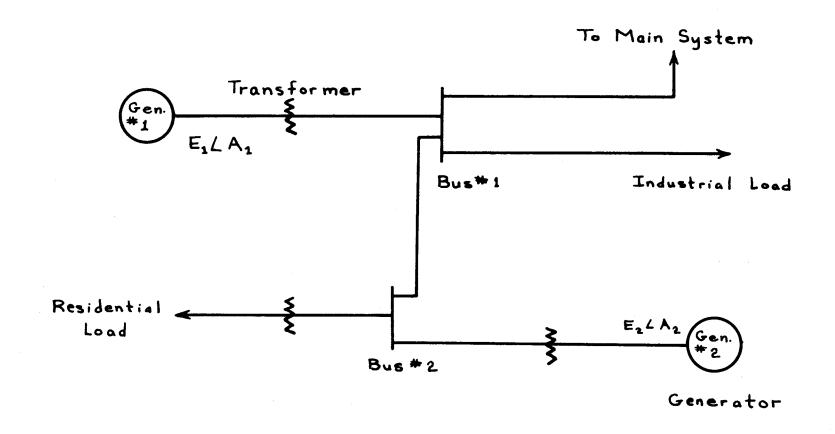


Figure 1. One-Line Diagram.

A positive sequence network model is used in this thesis as given in Figure 2. The positive sequence network representation is satisfactory when balanced loading appears on all three electrical phases. Under unbalanced conditions, the zero and negative sequence networks must also be employed. Power system disturbances can occur in balanced or unbalanced form. The positive sequence form can be used for damping purposes when the disturbed transmission line is removed quickly by circuit breaker action.

The "dynamic swing equation" which describes oscillation of the ith generator for a system composed of N generators is

$$M_{i} \ddot{\boldsymbol{\delta}}_{i} = W_{i} - P_{i} - C_{i} \dot{\boldsymbol{\delta}}_{i} \tag{1}$$

where

 $\mathbf{\delta_i}$  = the angular rotor displacement from a synchronously rotating reference axis,

M; = the generator inertia constant,

W = the mechanical input power to the ith machine minus
 all generator and prime mover losses,

 $C_{i}^{}$  = the internal generator damping coefficient and the electrical power output  $P_{i}^{}$  is given by

$$P_{i} = E_{i}^{2}G_{i} + \sum_{j=1}^{N} E_{j}E_{j}Y_{ij}\tilde{B}_{ij}$$
 (2)

where

$$G_{i} = \sum_{j=1}^{N} g_{ij} \cos \Theta_{ij}, \qquad (3)$$

$$Y_{ij} \angle T_{ij} = -Y_{ij} \angle \Theta_{ij} \quad \text{for } i \neq j,$$
 (4)

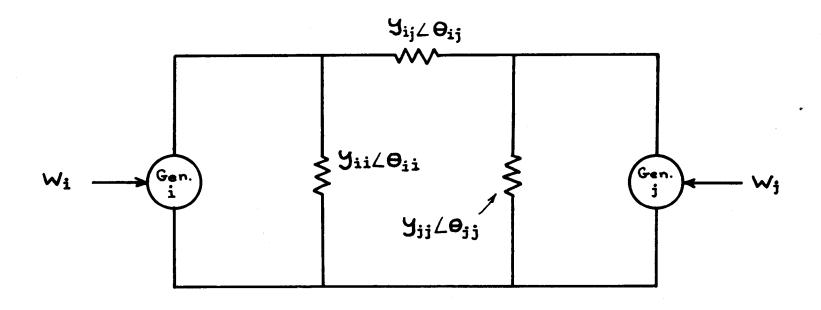


Figure 2. General Positive Sequence Model.

$$Y_{ii} \not \perp T_{ii} \stackrel{\triangle}{=} 0 , \qquad (5)$$

$$\widetilde{B}_{ij} \stackrel{\Delta}{=} \cos \left( T_{ij} - A_i + A_j \right) \tag{6}$$

and

$$y_{ij} \angle \Theta_{ij} = \text{a network admittance resulting from delta-star simplification.}$$
 (7)

The terms  $\mathbf{W_i}$ ,  $\mathbf{E_i}$  and  $\mathbf{G_i}$  can be independently controlled to introduce power system damping. This thesis provides the development of explicit equations for the control of  $\mathbf{W_i}$ ,  $\mathbf{E_i}$  and  $\mathbf{G_i}$  to give nearly uniform power system damping. This work provides the basis for the coordinated application of local damping techniques.

# Controllable Parameters

Local damping techniques, which are cited at the end of this chapter, can be shown to control  $W_i$ ,  $G_i$ ,  $E_i$  and  $Y_{ij} \angle T_{ij}$ .

Each power system generator has a system for controlling the mechanical input power which is  $W_i$  plus all mechanical and electrical machine losses. Since these losses are on the order of one per cent, damping action which controls the mechanical input power also controls  $W_i$ . Each generator also has an excitation system which is normally used to regulate the terminal voltage  $E_i$ . A supplementary signal may be introduced to control  $E_i$  to give power system damping.

Dynamic braking is a damping technique which involves the temporary application of special dissipative loads called braking

resistors and the limited removal of consumer loads for short periods which is called load shedding. Dynamic braking is principally reflected in the model presented as step changes in  ${\sf G}_{\tt i}$ .

Damping may be obtained by the switching of series capacitors in transmission lines which is reflected in the model as step changes in  $Y_{ij} \neq T_{ij}$ . No equation is developed for the control of  $Y_{ij} \neq T_{ij}$ , however, since step changes in this parameter do not uniquely define switching operations in the real system.

#### The Fundamental Error Quantity

Thus

To obtain uniform damping it is desirable to employ the angular acceleration of each generator  $(\mathbf{\ddot{s}_i})$  as a controlling error quantity. By this choice, equal emphasis is given to machines of large or small generating capacity. The "dynamic swing equation" may be written as

$$\dot{\delta}_{i} + \frac{C_{i}}{M_{i}} \dot{\delta}_{i} = \frac{W_{i} - P_{i}}{M_{i}}$$
(8)

Since internal generator damping is light, the right side of equation (8) represents a good approximation of  $\ddot{\delta}_i$  and is defined as the fundamental error quantity  $\epsilon_i$ .

$$\epsilon_{i} = \frac{W_{i} - P_{i}}{M_{i}} . \tag{9}$$

When the magnitude of  $\boldsymbol{\epsilon}_i$  is large, it closely approximates  $\boldsymbol{\delta}_i$ , and forcing  $\boldsymbol{\epsilon}_i$  toward zero also results in forcing  $\boldsymbol{\delta}_i$  toward zero. As the magnitude of  $\boldsymbol{\epsilon}_i$  becomes small, equation (8) may be approximated by

$$\ddot{\boldsymbol{\delta}}_{i} + \frac{C_{i}}{M_{i}} \dot{\boldsymbol{\delta}}_{i} = 0 \tag{10}$$

which has a stable solution for  $\dot{\delta}_1$  of

$$\dot{\delta}_{i} = k_{i} \exp \left(-C_{i} t/M_{i}\right) \tag{11}$$

and for  $\overset{\cdot \cdot \cdot}{\phi_{\mathbf{i}}}$  of

$$\dot{\delta}_{i} = -\frac{C_{i}}{M_{i}} \exp\left(-C_{i}t/M_{i}\right) \tag{12}$$

where  $C_{i}/M_{i}$  must be positive.

Thus for large and small magnitudes of  $\epsilon_i$ , control action which forces  $\epsilon_i$  to zero also forces the system to a stable equilibrium condition.

#### The Fundamental Error Function

The fundamental error function is defined as

$$\emptyset \stackrel{\triangle}{=} \frac{1}{2} \sum_{i=1}^{N} \epsilon_{i}^{2}. \tag{13}$$

This positive definite function is employed to determine regulation of controllable parameters which gives uniform power system damping. The time derivative of equation (13) is

$$\dot{\emptyset} = \sum_{j=1}^{N} \epsilon_{i} \left( \dot{w}_{i} - \dot{P}_{i} \right) \frac{1}{M_{i}} . \tag{14}$$

If all components of  $\mathring{\phi}$  are continuously negative,  $\emptyset$  asymptotically approaches zero which implies that  $\pmb{\epsilon}_i$  approaches zero also. Explicit equations are derived in Chapter II which cause as many terms of  $\mathring{\phi}$  to be negative as possible.

# Measurement of the Fundamental Error Quantity

To apply the damping control equations which have been developed,  $\epsilon_i$  must be measured at each generator in the power system. Currently rotor angular acceleration,  $\epsilon_i$ , is estimated by using the approximate time derivative of generator frequency as given in Figure 3 (5, 11). These devices are used on a limited scale for damping low frequency tie-line oscillations. The response of this instrumentation system is limited by the slow frequency transducers.

Blythe has presented a method for estimating  $\boldsymbol{\epsilon}_{\mathbf{i}}$  as given in Figure 4 (2).

Under transient conditions,  $P_i$  oscillates about  $W_i$ . By employing the filter, G(s), the highest oscillation frequencies are attenuated, thus giving an approximation of  $W_i$ . The estimated  $\boldsymbol{\epsilon}_i$  is the difference between the approximate  $W_i$  and  $P_i$ . Although Blythe used a first order filter of time constant  $\boldsymbol{\rho}$ , high order forms may also be used. To measure  $P_i$ , a high speed power transducer must be used. Although a high speed power

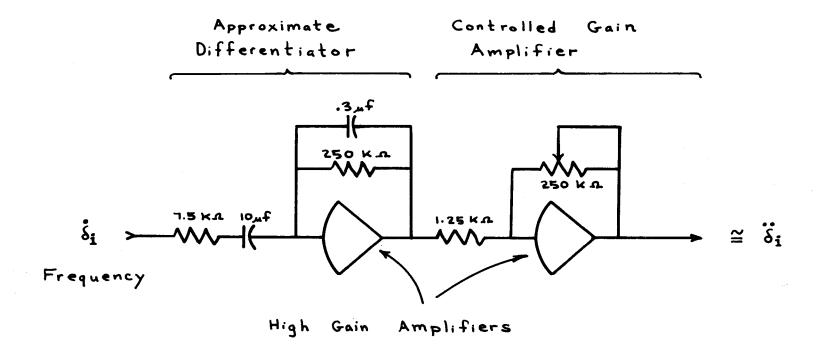


Figure 3. Approximate Differentiator Circuit for Estimation of  $\ddot{\xi}_{i}$ .

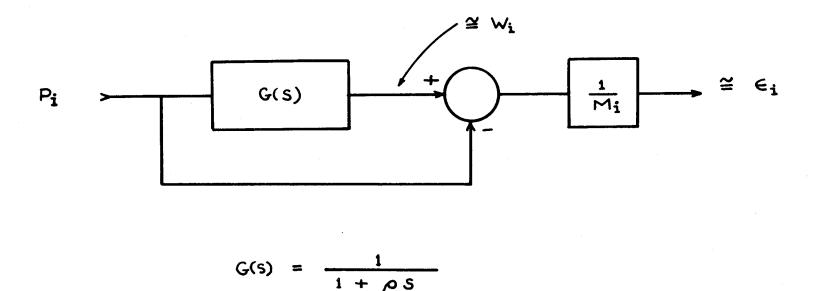


Figure 4. Block Diagram of a Method for Approximating  $\boldsymbol{\epsilon}_{i}$ .

transducer is considered to have lower accuracy than the slow responding thermocouple model, no difficulty is expected since only the relative magnitude of  $\boldsymbol{\epsilon}_{\mathbf{i}}$  is necessary.

# Statement of Results

Explicit equations governing the control of  $\mathring{w}_i$ ,  $\mathring{G}_i$  and  $\mathring{E}_i$  are now given. The expression for  $\mathring{w}_i$  control is

$$\hat{\mathbf{w}}_{\mathbf{i}} = -\mathbf{Y}_{\mathbf{M}_{\mathbf{i}}} \boldsymbol{\psi}_{\mathbf{i}} \boldsymbol{\epsilon}_{\mathbf{i}} \tag{31}$$

where

Y = a positive system coefficient

and

$$\psi_{i} = \begin{cases} 1 & \text{for } d|\epsilon_{i}|/dt < 0 \\ 0 & \text{for } d|\epsilon_{i}|/dt > 0 \end{cases}$$
(30)

The general expression for  $\overset{\bullet}{\mathsf{G}}_{\mathbf{i}}$  control is

$$\dot{G}_{i} = \lambda \underbrace{\frac{M_{i} \in i}{E_{i}^{2}}}_{E_{i}}$$
(34)

Where

 $\lambda$  = a positive system coefficient.

For discretely operated controllers this becomes

$$\dot{G}_{i} = \frac{\sum_{i=1}^{M} U_{o}(t-T_{i})}{E_{i}^{2}}$$
(43)

where

$$\Lambda_{i} = \begin{cases}
1 & \epsilon_{i} > 0 \\
& \text{and} \\
-1 & \epsilon_{i} < 0
\end{cases}$$

$$\Delta G_{i} E_{i}^{2} / M_{i} < |\epsilon_{i}|$$

$$0 \quad \Delta G_{i} E_{i}^{2} / M_{i} > |\epsilon_{i}|,$$
(44)

 $U_{o}(t-T_{i}) = a \text{ unit impulse occurring at } t = T_{i}$ 

and

 $\Delta G_{\underline{i}}$  = the magnitude of the change in  $G_{\underline{i}}$  .

The term  $T_i$  is defined as the time at which  $|\epsilon_i|$  reaches a maxima.

The general expression for  $\dot{E}_{i}$  control is

$$\dot{\mathbf{E}}_{i} = \frac{\mu \, \boldsymbol{\psi}_{i} \, \boldsymbol{\epsilon}_{i}}{\boldsymbol{\xi}_{Ri} + \boldsymbol{\xi}_{Li}} \tag{52}$$

where

 $\mu$  = a positive system coefficient,

$$\mathcal{E}_{Ri} \stackrel{\triangle}{=} \sum_{j=1}^{N} \frac{E_{j}Y_{ij}}{M_{i}} \stackrel{\widetilde{B}_{ij}}{\leftarrow} \frac{\widetilde{B}_{ji}}{\leftarrow} \frac{\varepsilon_{j}}{\leftarrow} , \qquad (46)$$

$$\widetilde{B}_{ij} \stackrel{\triangle}{=} Cos (T_{ij} - A_i + A_j)$$
(6)

and

$$\mathcal{E}_{Li} \stackrel{\triangle}{=} \frac{2 E_i G_i}{M_i} . \tag{45}$$

The form may be reduced to

$$\dot{\mathbf{E}}_{\mathbf{i}} = \frac{\mu \, \mathbf{\psi}_{\mathbf{i}} \, \mathbf{\epsilon}_{\mathbf{i}}}{\mathcal{E}_{\mathbf{i}\mathbf{i}}} \tag{53}$$

at generators having a relatively large driving point conductance,  $G_i$ . An expression controlling  $Y_{ij} \neq T_{ij}$  is not presented since its interpretation in the actual network is not unique.

## Discussion of Results

The equations presented may be used explicitly to govern the application of local damping devices. It is not necessary that the parameter responses exactly follow the given equations although sign agreement should be maintained.

All control equations are based around the fundamental error quantity  $\epsilon_i$ . A practical method of estimating this quantity is given on page 9.

The equation for control of  $W_i$  is the simplest of those given. Furthermore, this parameter is the most desirable to employ for system damping since it does not directly introduce voltage fluctuation (as does  $E_i$  control) and it does not require temporary dropping of any customer loads (as can  $G_i$  control). Unfortunately, this is the most difficult parameter to control in practice because of the slow response of the generator and turbine systems.

Some success has been achieved by Schleif, Martin and Angell for low frequency oscillations (11). Fruitful ideas

leading to rapid control of  $W_i$  are highly desirable.

The equation for control of  $\mathbf{G}_{\mathbf{i}}$  may be directly employed in the application of dynamic braking. This provides the solution to the three primary difficulties:

- 1. Relative brake resistor sizes are given for each generating plant by equation (35).
- 2. The time of application is given for each switching operation in terms of a locally measurable parameter.
- 3. A decision function is provided by equation (42) to determine when braking operations must be terminated. It is most desirable if braking resistors are applied at the generator sites. If load shedding is employed, it must be determined which loads within the system selectively influence the driving point conductances.

With the introduction of improved exciter systems, the control of  $\mathbf{E}_i$  has become a reasonable method of introducing damping. Two equations are given for the control of  $\mathbf{E}_i$  of which one requires the knowledge of many system parameters, and the other, only local parameters. From the three-generator example of Chapter III, it was found that the local equation gave results almost as satisfactory as the total system equation. By examination of equations (45) and (46) it appears that generators with large driving point conductance values are most amenable to the local voltage control equation.

The derived equations give instruction for the control of  $W_i$ ,  $G_i$  and  $E_i$  to obtain nearly uniform power system damping. Further extension of this method is intended by the author.

# Recommendation for Further Study

The following topics are recommended for further study:

- The extent of applicability of the local voltage control equation should be determined.
- 2. Further work should be conducted on circuits for rapid measurement of the fundamental error quantity,  $\boldsymbol{\epsilon}_{i}$ .
- 3. The most desirable range of system coefficients  $\mathbf{Y}$ ,  $\mathbf{\lambda}$  and  $\mu$  should be determined.
- 4. Procedures should be prepared for determining how shed loads influence G.
- 5. The damping method should be simulated on a large power system model.

## Literature Review

The subject of power system transient stability is very active in the literature. Methods have been proposed for determining the boundary region of system stability using the direct method of Liapunov. Other emphasis has been on locally controlled damping techniques.

By the direct method of Liapunov, a positive definite (always positive) function V is defined in terms of system state variables. The time derivative  $\mathring{V}$  may be explicitly determined by using the system differential equations. Asymptotic stability of the system response is guaranteed over the region for which  $\mathring{V}$  is always negative.

If a good Liapunov function is chosen, the negative region of  $\mathring{V}$  closely corresponds to the true region of system stability. Liapunov functions have been derived for several degrees of refinement (3, 6, 17). A useful resulting concept is the determination of the maximum time within which a disturbance must be cleared to maintain stability (3).

The local damping techniques presented are intended for "on-line" operation. Much work has been done by Schleif of the U. S. Bureau of Reclamation on damping by prime mover control. This work was prompted by serious oscillations in Northwest-Southwest tie lines through Utah and Colorado. Frequent line tripping occurred as a result of drifting tie-line load and periodic swings at six cycles per minute (5). The work by Schleif has resulted in prime mover control at Grand Coulee and McNary dams based on the time derivative of local frequency (11, 12). Satisfactory damping of low frequency tie-line oscillations was obtained.

The introduction of damping by generator terminal voltage control has received interest because of the low modification

cost. The generator voltage may be controlled by changing the current of the field winding located on the rotating generator shaft. The exciter system which provides the field winding current must be driven to high voltage magnitudes to give a rapid change in generator terminal voltage. Techniques of bang-bang (discrete output) exciter control have been developed by O. J. M. Smith and G. A. Jones (7, 13). Smith uses local shaft angle, shaft velocity, field current and power flow as inputs to decision making controllers which command exciter voltage to be maximum positive, maximum negative, or normal.

Much work has also been done by Blythe on generator voltage control of the Peace River Transmission System in Canada. Preliminary digital simulation studies were conducted using frequency deviation from nominal 60 cycles per second as a control signal (4). Blythe and Shier have also given a comparison of damping possible with rotating and static (thyrister) excitation systems. The static exciter gives a significant improvement in the ability to control terminal voltage for this purpose (2, 16).

A damping technique which has received much discussion pro and con is dynamic braking (10). Although the application of discrete braking resistors is considered by some to be a drastic measure, others maintain that this is necessary for the severe oscillations which may occur in interconnected systems. It has been suggested that damping resistors may be applied for one to

one and one-half seconds following a disturbance (4). The feasibility of employing braking resistors is being considered by Bonneville Power Administration.

The switching of series capacitors is being employed in the Pacific Northwest-Southwest Intertie (15). The insertion of series capacitors in transmission lines increases line admittance which may be used to improve transient stability (8, 10). One plan of application is to insert series capacitors immediately following a disturbance and leaving them in until system conditions return to normal operation. Another method is to insert series capacitors when the electrical phase angle between transmission line terminals is increasing and removal of capacitors when the angle is decreasing (10).

#### Nomenclature

#### English Symbols

- A. = the phase angle of the phase-to-neutral voltage for the ith generator, in radians.
- $\tilde{\mathbf{B}}_{i,i}$  = a variable defined by equation (6).
- c = the internal damping coefficient for the ith generator, in per-unit power second/radian.
- $\mathbf{E}_{i}$  = the magnitude of the phase-to-neutral voltage of the ith generator, in per-unit voltage.
- $\dot{\tilde{E}}_{i}$  = the value used for  $\dot{\tilde{E}}_{i}$  in the Runge Kutta numerical integration subroutine, in units of per-unit voltage/second.

- $\xi_{Li}$  = the local voltage control variable defined by equation (45).
- $\boldsymbol{\xi}_{Ri}$  = the remote voltage control variable defined by equation (46).
- G(s) = a transfer function.
- G = the driving point conductance of the ith generator defined by equation (3), in per-unit admittance.
- $\Delta G_{i}$  = the magnitude of the change in  $G_{i}$  resulting from dynamic braking.
- $G_{Ni}$  = the nominal value of  $G_{i}$  when dynamic braking is not applied, in per-unit admittance.
- Ifi = the d.c. field current of the ith generator, in per-unit
  current.
- $K_i = a \text{ generator model constant.}$
- k; = a constant of integration.
- $L_{fi}$  = the inductance of the field winding of the ith generator.
- $M_i$  = the inertia constant for the ith generator, in per-unit power second  $^2$ /radian.
- N = the number of generators in the power system.
- P = the electrical power output of the ith generator given by equation (2), in per-unit power.
- $\bar{Q}_{i}$  = a Runge Kutta vector defined by equation (61).
- $\bar{R}_{i}$  = a Runge Kutta vector defined by equation (62).

- R<sub>fi</sub> = the field winding resistance of the ith generator, in per-unit resistance.
- $\bar{s}_{i}$  = a Runge Kutta vector defined by equation (63).
- T = the time at which a dynamic braking switching operation occurs, in seconds.
- $T_{ij}$  = the phasor angle of the transfer admittance defined by equation (4), in radians.
- $\overline{V}_{i}$  = a Runge Kutta vector defined by equation (64).
- w = the mechanical input power to the ith generator minus all generator and prime mover losses, in per-unit power.
- $\bar{X}_{i}$  = a Runge Kutta vector defined by equations (57) and (60).
- Y = a transfer admittance magnitude defined by equation (4), in per-unit admittance.
- $y_{i,j}$  = an admittance magnitude resulting from delta-star network reduction, in per-unit admittance.
- 3 = a variable defined in equation (28).

## Greek Symbols

- $\alpha_i$  = a coefficient used in the development of the  $W_i$  control equation.
- $\beta_i$  = a coefficient used in the development of the  $E_i$  control equation.
- Y = the system coefficient for control of  $W_i$ , in seconds<sup>-1</sup>.

- the angular displacement of the rotor of the ith generator from a synchronously rotating reference axis, in radians.
- the fundamental error quantity defined by equation (9),
   in units of radians/second<sup>2</sup>.
- $\epsilon_{i}'$  = the value of  $\epsilon_{i}$  immediately following a dynamic braking switching operation.
- $\Delta \in \mathcal{E}_{i}$  = the change in  $\mathcal{E}_{i}$  resulting from a dynamic braking switching operation.
- $\theta_{ij}$  = a phasor admittance angle resulting from delta-star network reduction.
- $\lambda$  = the system coefficient for control of  $G_i$ , in seconds<sup>-1</sup>.
- $\mu$  = the system coefficient for  $E_i$  control, in seconds<sup>-1</sup>.
- = the time constant of the first order filter, G(s), in seconds.
- = the delay time between a disturbance and the application of dynamic braking, in seconds.
- $\emptyset$  = the fundamental error function defined by equation (13), in radians<sup>2</sup>/second<sup>4</sup>.
- $\dot{p}_{E}$  = a component of  $\dot{p}$  defined by equation (22).
- $\phi_{G}$  = a component of  $\phi$  defined by equation (21).
- $\overset{\bullet}{p}_{w}$  = a component of  $\overset{\bullet}{p}$  defined by equation (20).
- $\dot{\phi}_{VB}$  = a component of  $\dot{\phi}$  defined by equation (23).
- $\psi_i$  = the decision function defined by equation (30).
- $\Omega_i$  = a decision function defined by equation (44).

# II. DERIVATION OF DAMPING EQUATIONS

#### Expansion of Ø

The basic power system model and damping criteria are presented in Chapter I. Details of the derivation of parameter control equations are given in this chapter.

The fundamental error quantity is defined as

$$\boldsymbol{\epsilon}_{i} \stackrel{\triangle}{=} (\boldsymbol{W}_{i} - \boldsymbol{P}_{i})/\boldsymbol{M}_{i} \tag{15}$$

and may be expanded by equation (2) to

$$\boldsymbol{\epsilon}_{i} = (\mathbf{W}_{i} - \mathbf{E}_{i}^{2} \mathbf{G}_{i} - \sum_{j=1}^{N} \mathbf{E}_{i} \mathbf{E}_{j} \mathbf{Y}_{ij} \widetilde{\mathbf{B}}_{ij}) / \mathbf{M}_{i}. \tag{16}$$

The time derivation of  $\boldsymbol{\epsilon}_{i}$  is

$$\dot{\boldsymbol{\epsilon}}_{\mathbf{i}} = \frac{\dot{\mathbf{W}}_{\mathbf{i}}}{\dot{\mathbf{M}}_{\mathbf{i}}} - \frac{2\mathbf{E}_{\mathbf{i}}\mathbf{G}_{\mathbf{i}}\dot{\mathbf{E}}_{\mathbf{i}}}{\dot{\mathbf{M}}_{\mathbf{i}}} - \frac{\mathbf{E}_{\mathbf{i}}^{2}\dot{\mathbf{G}}_{\mathbf{i}}}{\dot{\mathbf{M}}_{\mathbf{i}}}$$

$$-\sum_{\mathbf{j}=1}^{N} \frac{(\mathbf{E}_{\mathbf{i}}\mathbf{Y}_{\mathbf{i}}\mathbf{\tilde{B}}_{\mathbf{i}}\mathbf{\tilde{E}}_{\mathbf{j}}}{\mathbf{M}_{\mathbf{i}}} + \frac{\mathbf{E}_{\mathbf{j}}\mathbf{Y}_{\mathbf{i}}\mathbf{\tilde{B}}_{\mathbf{i}}\mathbf{\tilde{E}}_{\mathbf{i}}}{\mathbf{M}_{\mathbf{i}}})$$
(17)

$$-\sum_{j=1}^{N} \frac{E_{i}E_{j}}{M_{i}} \frac{d^{(Y_{ij}\widetilde{B}_{ij})}/dt}.$$

The time derivative of the fundamental error function is

$$\dot{\emptyset} = \sum_{i=1}^{N} \in_{i} \dot{\epsilon}_{i}$$
 (18)

which may be expanded by equation (17) to give

$$\dot{\phi} = \dot{\phi}_{W} + \dot{\phi}_{G} + \dot{\phi}_{E} + \dot{\phi}_{YB} \tag{19}$$

where

$$\dot{\phi}_{\mathbf{W}} \stackrel{\triangle}{=} \sum_{i=1}^{N} \frac{\boldsymbol{\epsilon}_{i} \dot{\mathbf{W}}_{i}}{\mathbf{M}_{i}} , \qquad (20)$$

$$\dot{\phi}_{G} \stackrel{\triangle}{=} - \sum_{i=1}^{N} \frac{E_{i}^{2} \boldsymbol{\epsilon}_{i} \dot{\mathbf{G}}_{i}}{M_{i}}, \qquad (21)$$

$$\overset{\bullet}{\phi}_{\mathbf{E}} \stackrel{\triangle}{=} - \sum_{\mathbf{i}=1}^{\mathbf{N}} \overset{\bullet}{\epsilon}_{\mathbf{i}} \left( \frac{2\mathbf{E}_{\mathbf{i}}\mathbf{G}_{\mathbf{i}}}{\mathbf{M}_{\mathbf{i}}} + \sum_{\mathbf{j}=1}^{\mathbf{N}} \frac{\mathbf{E}_{\mathbf{j}}\mathbf{Y}_{\mathbf{i}\mathbf{j}}}{\mathbf{M}_{\mathbf{i}}} \left( \widetilde{\mathbf{B}}_{\mathbf{i}\mathbf{j}} + \widetilde{\mathbf{B}}_{\mathbf{j}\mathbf{i}} \underbrace{\boldsymbol{\epsilon}_{\mathbf{j}}}_{\boldsymbol{\epsilon}} \right) \overset{\bullet}{\mathbf{E}}_{\mathbf{i}} \right) \tag{22}$$

and

$$\overset{\bullet}{p}_{YB} \stackrel{\triangle}{=} - \sum_{i=1}^{N} \in_{i} \sum_{j=1}^{N} \frac{E_{i}E_{j}}{M_{i}} d(Y_{ij}\widetilde{B}_{ij})/dt .$$
(23)

The basic plan is to control the parameters  $\hat{\mathbf{w}}_i$ ,  $\hat{\mathbf{g}}_i$  and  $\hat{\mathbf{E}}_i$  to force  $\hat{\mathbf{g}}$  to be as negative as possible.

# wi Control Equation

It can be shown by equation (20) that  $\phi_{W}$  is always negative if  $\hat{W}_{i}$  is defined as

$$\dot{\mathbf{W}}_{\mathbf{i}} \stackrel{\triangle}{=} -\alpha_{\mathbf{i}} \boldsymbol{\epsilon}_{\mathbf{i}} \tag{24}$$

where  $\alpha_i$  is a positive coefficient. Control action is applied equally to all generators having equal  $\epsilon_i$  if  $\alpha_i$  is defined as

$$\boldsymbol{\mathsf{A}}_{\mathbf{i}} \stackrel{\Delta}{=} \boldsymbol{\mathsf{Y}} \, \mathbf{\mathsf{M}}_{\mathbf{i}}$$
(25)

where  $\mathbf{Y}$  is a positive coefficient. The resulting equations for  $\overset{\bullet}{p}_{\mathbf{W}}$  and  $\overset{\bullet}{\mathbf{W}}$  are

$$\dot{\phi}_{\mathbf{W}} = -\sum_{i=1}^{N} \mathbf{Y} \in \mathbf{i}^{2} \tag{26}$$

and

$$\dot{\mathbf{W}}_{\mathbf{i}} = -\mathbf{Y} \,\mathbf{M}_{\mathbf{i}} \,\boldsymbol{\epsilon}_{\mathbf{i}} \,. \tag{27}$$

It may be shown by substitution of equation (27) into equation (1) that this action acts continuously to reduce angular acceleration of  $\delta$  at each generator. Strictly from the standpoint of damping oscillation this action is desirable, but in terms of transient stability, a further modification is necessary.

This modification is a result of consideration of power system equilibrium points. El-Abiad and Nagappan have derived

equations for estimating stable and unstable equilibrium points

(3). The relative rotor angle of a stable equilibrium condition

between the ith and jth machine is approximately

$$(\delta_i - \delta_j) = \sin^{-1} (3)$$
(28)

where

$$7 = \frac{M_{j}W_{i} - M_{i}W_{j} - M_{j}E_{i}^{2}G_{i} + M_{i}E_{j}^{2}G_{j}}{(M_{i} + M_{j}) E_{i}E_{j}Y_{ij}Sin(T_{ij})}$$

and the unstable condition is given by

$$(\delta_i - \delta_j) = \pi - \sin^{-1}(3).$$
 (29)

By examining the principal angles of  $\sin^{-1}(\gamma)$ , it may be shown that for a stable system equilibrium point, the angles  $(d_i - d_j)$  should lie within  $\pm$  90 degrees. An unstable equilibrium condition exists if any  $(\delta_i - \delta_j)$  lies between 90 and 270 degrees. Figure 5 illustrates the set of principal equilibrium points for the three-generator system considered in Chapter III.

It is desirable that damping action does not drive the relative system angles toward the region of unstable equilibrium points since stability may be lost. For this reason, the retarding angular acceleration should not be diminished when  $\frac{d \mid \boldsymbol{\xi} \mid \mid}{dt}$  is positive.

The derivation of the  $\overset{ullet}{\mathbf{W}}_{\mathbf{i}}$  control equation is complete with the addition of

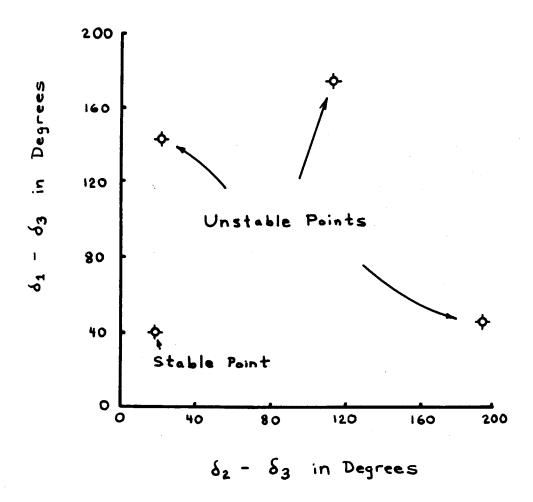


Figure 5. Principal Equilibrium Points for the Undamped Three-Generator Model of Chapter III.

$$\psi_{i} \stackrel{\triangle}{=} 
\begin{cases}
1 & \text{for } d \in |A| / dt < 0 \\
0 & \text{for } d \in |A| / dt > 0
\end{cases}$$
(30)

thereby giving

$$\dot{\mathbf{W}}_{\mathbf{i}} = -\mathbf{Y}\mathbf{M}_{\mathbf{i}}\boldsymbol{\psi}_{\mathbf{i}}\boldsymbol{\epsilon}_{\mathbf{i}} . \tag{31}$$

# Gi Control Equation

It can be shown by equation (21) that  $\overset{\bullet}{\phi}_{G}$  is always negative if  $\overset{\bullet}{G}_{i}$  is defined as

$$\dot{\mathbf{G}}_{\mathbf{i}} \stackrel{\triangle}{=} \boldsymbol{\beta}_{\mathbf{i}} \boldsymbol{\epsilon}_{\mathbf{i}} \tag{32}$$

where  $\beta_i$  is a positive coefficient. Control action is applied equally to all generators having equal  $\epsilon_i$  if  $\beta_i$  is defined as

$$\beta_{i} \stackrel{\Delta}{=} \frac{M_{i} \lambda}{E_{i}^{2}}$$
 (33)

where  $\lambda$  is a positive system coefficient.

In equation (33),  $E_i$  can be approximated by its nominal value since deviations from the nominal value are generally small.

The technique of dynamic braking appearing in the literature involves step changes in G rather than continuous control as given by

$$\overset{\bullet}{G}_{i} = \frac{\overset{M}{i} \lambda}{E_{i}^{2}} \in i$$
 (34)

To provide for discrete changes in  $G_{i^{\bullet}}$  a unit impulse term  $U_{o}(t-T_{i}) \text{ is employed which occurs when the argument } t-T_{i} \text{ is zero.}$  If only one size braking resistor is available at each generation site, the magnitude should be determined by

$$\Delta G = \frac{M_i \lambda}{E_i^2}$$
(35)

where the nominal terminal voltage is used for  $\mathbf{E_i}$ . With a fixed magnitude discrete braking operation, the resulting change in  $\emptyset_G$  is given by

$$\Delta \phi_{G} = - \lambda U_{-1} (t-T_{i})$$
(36)

where  $U_{-1}(t-T_i)$  is the unit step occurring at  $t=T_i$ .

It can be seen from equation (21) that a maximum decrease in  $\emptyset_G$  is sustained if the switching occurs when  $|\xi_i|$  is at a maximum value. Studies have indicated that it is desirable to apply dynamic braking immediately following a disturbance (4). This is compatible with the maximum  $|\xi_i|$  criterion since  $|\xi_i|$  always reaches a maxima immediately following a disturbance.

Since a switched or discretely controlled operation is employed, it is necessary to develop a criterion to distinguish when dynamic braking should be applied and when it should be

discontinued following an application. If a switching operation will result in a reversal of the sign of  $\boldsymbol{\epsilon}_i$  following a fault, no switching action is taken. Furthermore, dynamic braking is discontinued following a sustained application if further switching will result in a reversal of the sign of  $\boldsymbol{\epsilon}_i$ .

To determine the switching criterion, equation (2) is substituted into equation (9) giving

$$\epsilon_{i} = \frac{1}{M_{i}} (W_{i} - E_{i}^{2}G_{i} - \sum_{j=1}^{N} E_{i}^{E} E_{j}^{Y} i j^{\widetilde{B}} i j).$$
(37)

The term  $\Delta G_i$  is added to  $G_i$  if a braking resistor is applied, and is substracted from  $G_i$  if load shedding occurs. Immediately following a switching operation, equation (37) becomes

$$\boldsymbol{\epsilon}_{i}' = \frac{1}{M_{i}} \left( \mathbf{W}_{i} - \mathbf{E}_{i}^{2} \left( \mathbf{G}_{i} + \Delta \mathbf{G} \right) - \sum_{j=1}^{N} \mathbf{E}_{i} \mathbf{E}_{j}^{Y} \mathbf{i}_{j} \widetilde{\mathbf{B}}_{ij} \right)$$
(38)

where  $\epsilon_i'$  is the resulting fundamental error quantity. The change in  $\epsilon_i$ ,  $\Delta \epsilon_i$ , is

$$\Delta \boldsymbol{\epsilon}_{i} = \boldsymbol{\epsilon}_{i}' - \boldsymbol{\epsilon}_{i} = \boldsymbol{\tau} \frac{\mathbf{E}_{i}^{2} \Delta \mathbf{G}_{i}}{\mathbf{M}_{i}}. \tag{39}$$

The braking resistor is applied when  $\epsilon_i$  reaches a maximum positive value providing a sign reversal of  $\epsilon_i$  will not occur. Thus if the inequality

$$|\epsilon_{i}| - \frac{E_{i}^{2} \Delta G_{i}}{M_{i}} > 0 \tag{40}$$

is satisfied, a braking resistor may be applied. Load shedding may be applied when  $\boldsymbol{\epsilon}_i$  reaches a maximum negative value if a sign reversal of  $\boldsymbol{\epsilon}_i$  does not result. Thus, if the inequality

$$- |\epsilon_{i}| + \frac{E_{i}^{2}\Delta G_{i}}{M} < 0$$
 (41)

is satisfied, load shedding may be applied. By inspection it is seen that equation (40) and equation (41) are the same inequality and may be written as

$$\mathsf{I} \in_{\mathbf{i}} \mathsf{I} > \frac{\mathsf{E}_{\mathbf{i}}^{2} \Delta \mathsf{G}_{\mathbf{i}}}{\mathsf{M}_{\mathbf{i}}} \tag{42}$$

and is called the dynamic braking switching criterion.

When the conditions of this inequality are not satisfied at the switching time  $T_i$ , dynamic braking must not be continued or initiated. With the inclusion of the switching criterion,  $\hat{G}_i$  may be written as

$$\dot{G}_{i} = \frac{M_{i} \lambda \Omega_{i}^{U} o(t-T_{i})}{E_{i}^{2}}$$
(43)

where

$$\boldsymbol{\Lambda}_{i} = \begin{cases}
1 & \boldsymbol{\epsilon}_{i} > 0 \text{ and } \Delta G_{i} E_{i}^{2} / M_{i} < I \boldsymbol{\epsilon}_{i} \\
0 & \Delta G_{i} E_{i}^{2} / M_{i} > I \boldsymbol{\epsilon}_{i} \\
-1 & \boldsymbol{\epsilon}_{i} < 0 \text{ and } \Delta G_{i} E_{i}^{2} / M_{i} < I \boldsymbol{\epsilon}_{i} \end{cases} \tag{44}$$

and  $T_i$  is equal to the time at which  $d \in 1/dt$  reaches a maxima.

Figure 6 illustrates the application of dynamic braking at one generator for a hypothetical case. Braking resistors and load shedding are specified by equal conductance magnitudes.

If a disturbance occurs on a transmission line, the line is opened by circuit breaker action and generally followed by an attempted reclosure. If the disturbance is of a temporary nature, such as a lightning stroke, the reclosure is successful and dynamic braking unnecessary. To allow for this possibility a delay time  $\boldsymbol{\tau}$  may be introduced at the outset of a disturbance. If  $\boldsymbol{\epsilon}_i$  remains large enough to warrant switching following the delay, normal switching is begun. This case is illustrated in Figure 7. If fast circuit breaker action is employed this delay is not detrimental because the  $\boldsymbol{\epsilon}_i$  function changes slowly at the outset due to generator rotor inertia.

# $\frac{\dot{\mathbf{E}}}{\dot{\mathbf{E}}}$ Control Equation

Equation (22) for  $\overset{\bullet}{p}_{\rm E}$  contains two terms which may be defined as

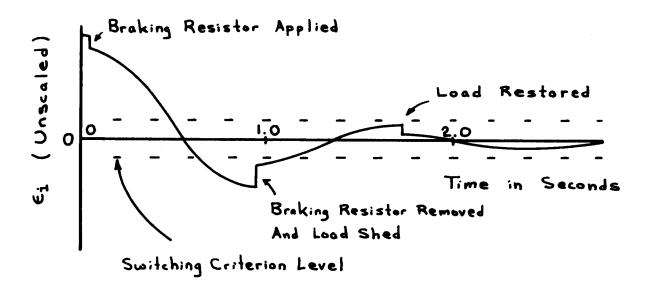


Figure 6. Dynamic Braking Following a Simple Power System Disturbance.

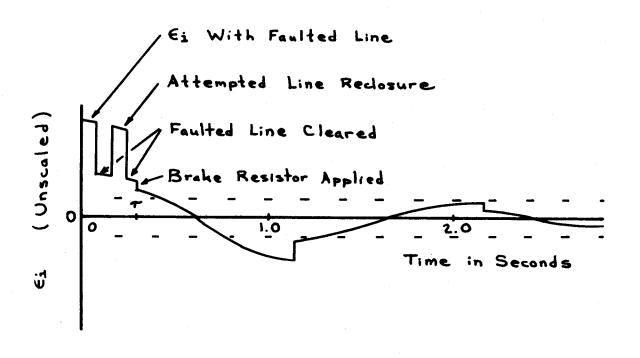


Figure 7. Dynamic Braking Applied Following a au Second Delay. Allowed for Attempted Line Reclosure.

$$\mathcal{E}_{Li} \stackrel{\Delta}{=} \frac{2E_{i}G_{i}}{M_{i}} \tag{45}$$

and

$$\mathcal{E}_{Ri} \stackrel{\Delta}{=} \sum_{j=1}^{N} \frac{\mathbf{E}_{j} \mathbf{Y}_{ij}}{\mathbf{M}_{i}} (\widetilde{\mathbf{B}}_{ij} + \widetilde{\mathbf{B}}_{ji} \boldsymbol{\epsilon}_{j} / \boldsymbol{\epsilon}_{i}). \tag{46}$$

Thus the equation for  $\boldsymbol{\mathring{\phi}}_{E}$  may be written as

$$\dot{\phi}_{E} = -\sum_{i=1}^{N} \epsilon_{i} (\mathcal{E}_{Li} + \mathcal{E}_{Ri})\dot{E}_{i}. \tag{47}$$

 $\overset{ullet}{ extstyle arphi}_{ ext{E}}$  is always negative if  $\overset{ullet}{ ext{E}}_{ ext{i}}$  is defined as

$$\stackrel{\bullet}{E}_{i} \stackrel{\triangle}{=} \frac{\mu \, \epsilon_{i}}{\epsilon_{Li} + \epsilon_{Ri}} \tag{48}$$

where  $\mu$  is a positive system coefficient. Control applied to E by equation (48) is called total voltage control.

It can be seen that as  $\boldsymbol{\epsilon}_i$  goes to zero,  $\boldsymbol{\mathcal{E}}_{Ri}$  becomes infinite. This, however, does not present a problem since

$$\lim_{\xi_{i} \to 0} \left\{ \mu \in \left[ \left( \xi_{L_{i}} + \xi_{R_{i}} \right) \right] = 0.$$

$$(49)$$

The use of nominal voltages for  $E_i$  and  $E_j$  can lead to a serious error in the computation of equation (48) since at any instant the denominator can involve the combination of positive and negative numbers.

To employ  $\mathcal{E}_{\mathrm{Ri}}$ , remote system parameters must be telemetered to each generation site. It is desirable, however, if this complication can be avoided.

If the inequality

$$|\varepsilon_{Li}| > |\varepsilon_{Ri}|$$
 (50)

is valid when  $oldsymbol{\xi}_{ ext{Li}}$  and  $oldsymbol{\xi}_{ ext{Ri}}$  are of opposite sign, then the simplified form

$$\dot{\mathbf{E}}_{\mathbf{i}} \stackrel{\triangle}{=} \frac{\mu \, \mathbf{E}_{\mathbf{i}}}{\mathbf{E}_{\mathbf{L},\mathbf{i}}} \tag{51}$$

also gives a continuously negative  $\oint_{\mathbf{E}}$ . The control of  $\mathbf{E_i}$  by equation (51) is called local voltage control. In the expansion of equation (51),  $\mathbf{E_i}$  can be approximated by its nominal value since deviations from the nominal are generally small and the denominator involves only one term. Further investigation of the extent of applicability of equation (51) is warranted. It may be shown by substitution of equation (48) into equation (1) that  $\mathbf{\hat{E}_i}$  control acts continuously to reduce the angular acceleration of  $\mathbf{\mathcal{J}_i}$ . On the basis of the same argument presented for  $\mathbf{\hat{W}_i}$  control, the function

$$\psi_{i} \stackrel{\triangle}{=} \begin{cases} 1 & \text{for } d | \epsilon_{i}| / dt < 0 \\ 0 & \text{for } d | \epsilon_{i}| / dt > 0 \end{cases}$$
(30)

is used in the total system voltage control equation to give

$$\dot{\mathbf{E}}_{i} = \frac{\mu \, \boldsymbol{\psi}_{i} \, \boldsymbol{\epsilon}_{i}}{\boldsymbol{\xi}_{Ri} + \boldsymbol{\xi}_{Li}} \tag{52}$$

and in the local voltage control equation, giving

$$\dot{\mathbf{E}}_{i} = \frac{\mu \, \boldsymbol{\psi}_{i} \, \boldsymbol{\epsilon}_{i}}{\boldsymbol{\xi}_{Li}} . \tag{53}$$

A comparison of the two forms is given by example in Chapter III.

## $\frac{d(Y_{ij}\tilde{B}_{ij})/dt}{Control}$ Equation

The expression  $d(Y_{ij}\tilde{B}_{ij})/dt$  may be further expanded as

$$d(Y_{ij}\tilde{B}_{ij})/dt = Y_{ij}cos(T_{ij}-A_{i}+A_{j})$$

$$-Y_{ij}(\tilde{T}_{ij}-\tilde{A}_{i}+\tilde{A}_{j})sin(T_{ij}-A_{i}+A_{j}).$$
(54)

The terms  $\dot{Y}_{ij}$  and  $\dot{T}_{ij}$  reflect changes in the network configuration such as switched series capacitors. A very serious difficulty arises from attempting to employ these parameters for damping control. Any specified values for  $Y_{ij}$  and  $T_{ij}$  cannot be related uniquely to the real physical network (9, p. 87). For this reason, a control equation for  $d(Y_{ij}\tilde{B}_{ij})/dt$  is not presented.

Conceptually, however, the  $d(Y_{ij}\tilde{B}_{ij})/dt$  term occupies an interesting role. If a network disturbance occurs such as an opened transmission line, the initial displacement in  $\emptyset$  is a result of an impulse in  $d(Y_{ij}\tilde{B}_{ij})/dt$ . The terms  $\hat{A}_i$  and  $\hat{A}_j$  are dependent variables which constitute the dynamic response of  $\emptyset$  which must be damped to a steady-state condition.

# III. THE DAMPING METHOD APPLIED TO A THREE-GENERATOR MODEL

#### Model System

The three generator system of Figure 8 is used to demonstrate the damping method. This system is chosen since it displays interactions similar to those of large systems for which the method is intended. System constants are expressed in the per-unit system for which

1 per-unit power = 10 MVA,

1 per-unit voltage = 12 KV

and

1 per-unit admittance = 0.0694 mhos.

The numerical values used for the parameters of Figure 8 are:

 $W_i = 2.127$  per-unit for i = 1 to 3,

 $E_i = 1.100$  per-unit for i = 1 to 3,

 $M_i = 0.100$  per-unit for i = 1 to 3,

 $y_{11} \angle \theta_{11} = 0.718 \angle -15.8^{\circ} \text{ per-unit},$ 

 $y_{22} \neq \theta_{22} = 1.930 \neq -13.2^{\circ} \text{ per-unit},$ 

 $y_{33} \neq \theta_{33} = 3.100 \neq -17.5^{\circ} \text{ per-unit},$ 

 $y_{12} \angle \theta_{12} = 1.219 \angle -101.2^{\circ} \text{ per-unit},$ 

 $y_{13} \neq \theta_{13} = 1.192 \neq -108.1^{\circ} \text{ per-unit}$ 

and

$$y_{23} \neq \theta_{23} = 1.183 \neq -115.3^{\circ}$$
 per-unit.

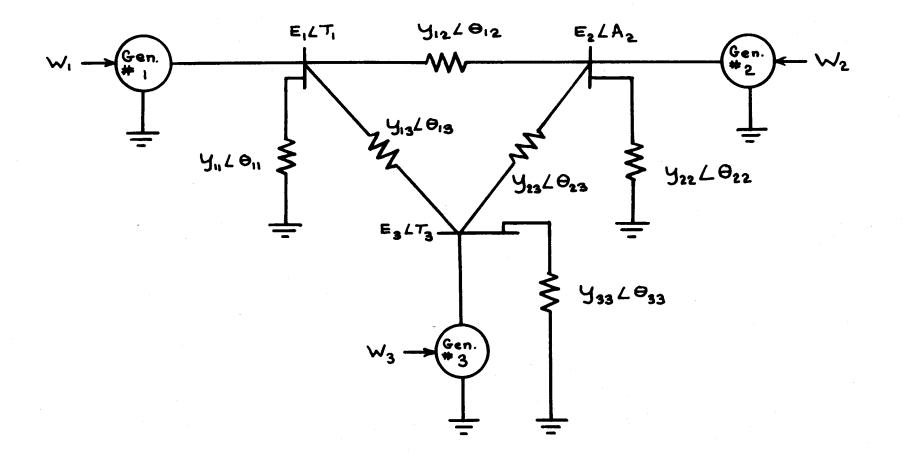


Figure 8. Positive Sequence Diagram of the Three-Generator Model System.

The resulting driving point conductance and transfer admittance values as determined by equations (3) and (4) are:

$$G_1$$
 = 0.0866 per-unit,  $G_2$  = 1.149 per-unit,  $G_3$  = 2.085 per-unit,  $Y_{12} \angle T_{12}$  = 1.219  $\angle 78.78^\circ$  per-unit,  $Y_{13} \angle T_{13}$  = 1.191  $\angle 71.91^\circ$  per-unit

and

$$Y_{23} \angle T_{23} = 1.130 \angle 64.74^{\circ} \text{ per-unit.}$$

An ideal generator model is used for this example having a voltage source of magnitude and angle  $E_i \neq A_i$ , where the rotor angle,  $\delta_i$ , corresponds identically with  $A_i$ . Although this represents a significant simplification of the real generator, the dynamic characteristics of the damping method are demonstrated. The magnitude of the source voltage is directly related to the rotor field current by

$$\mathbf{E}_{\mathbf{i}} = \mathbf{K}_{\mathbf{i}} \mathbf{I}_{\mathbf{f}_{\mathbf{i}}} \tag{55}$$

where

 $\mathbf{K}_{\mathbf{i}}$  = a positive generator constant and

I = the d.c. field current.

The generator field is driven by a static exciter having the differential equation

$$\mathbf{E}_{\mathbf{x}i} = \mathbf{L}_{\mathbf{f}i} \mathbf{I}_{\mathbf{f}i} + \mathbf{R}_{\mathbf{f}i} \mathbf{I}_{\mathbf{f}i} \tag{56}$$

where

 $E_{xi}$  = the exciter output voltage,

 $L_{f\,i}$  = the field winding inductance

and

 $R_{fi}$  = the field winding resistance.

The field winding time constant,  $L_{\rm fi}/R_{\rm fi}$ , is chosen to be six seconds which is representative of that found in large generators.

#### Simulation Program

A FØRTRAN program was developed to simulate a power system of N generators including the generator model described on page 41. The flow chart is found as follows:

Main Program	Appendix A
Voltage Damping Subroutine	Appendix B
Dynamic Braking Subroutine	Appendix C
Numerical Integration Subroutine	Appendix D

The fourth order Runge Kutta method is used to obtain a numerical solution to the differential equation

$$\begin{bmatrix} \dot{x}_{1i} \\ \dot{x}_{2i} \\ \dot{x}_{3i} \end{bmatrix} = \begin{bmatrix} \dot{e}_{i}^{*} \\ x_{3i} \\ f_{i}(x_{1k}, x_{2k}) \end{bmatrix}$$

$$(57)$$

for the ith generator where

$$X_{1i} = E_{i}$$
,  
 $X_{2i} = \delta_{i} = A_{i}$ ,  
 $X_{3i} = \dot{\delta}_{i} = \dot{A}_{i}$ ,  
 $\dot{E}_{i}^{*}$  = the required  $\dot{E}_{i}$  to be held constant over the integration period  $\Delta t$ 

and

$$f_{i}(x_{1k}, x_{2k}) = \frac{1}{M_{i}} \left\{ w_{i} - x_{1i}^{2} G_{i} - \sum_{j=1}^{3} x_{1i} x_{1j} Y_{ij} cos(T_{ij} - X_{2i} + X_{2j}) \right\}$$
(58)

The Runge Kutta recursion formula is

$$\bar{X}_{i}(m+1) = \bar{X}_{i}(m) + \frac{1}{6} \bar{Q}_{i} + \frac{1}{3} \bar{R}_{i} + \frac{1}{3} \bar{S}_{i} + \frac{1}{6} \bar{V}_{i}$$
 (59)

where

$$\bar{\mathbf{x}}_{i} = \begin{bmatrix} \mathbf{x}_{1i} \\ \mathbf{x}_{2i} \\ \mathbf{x}_{3i} \end{bmatrix} \tag{60}$$

$$\overline{Q}_{i} = \begin{bmatrix} Q_{1i} \\ Q_{2i} \\ Q_{3i} \end{bmatrix} = \begin{bmatrix} \dot{E}_{i}^{*} \\ X_{3i} \\ f_{i}(X_{1k}, X_{2k}) \end{bmatrix} \Delta t , \qquad (61)$$

$$\vec{R}_{i} = \begin{bmatrix} R_{1i} \\ R_{2i} \\ R_{3i} \end{bmatrix} = \begin{bmatrix} \dot{E}_{i}^{*} \\ X_{3i} + Q_{3i/2} \\ f_{i}(X_{1k} + Q_{1k}, X_{2k} + Q_{2k}) \end{bmatrix} \Delta t ,$$
(62)

$$\bar{S}_{i} = \begin{bmatrix} s_{1i} \\ s_{2i} \\ s_{3i} \end{bmatrix} = \begin{bmatrix} \dot{E}_{i}^{*} \\ x_{3i} + R_{3i/2} \\ f_{i} (x_{1k} + R_{1k}, x_{2k} + R_{2k}) \\ \frac{1}{2} & (63) \end{bmatrix}$$

$$\overline{V}_{i} = \begin{bmatrix} V_{1i} \\ V_{2i} \\ V_{3i} \end{bmatrix} = \begin{bmatrix} \dot{E}_{i}^{*} \\ X_{3i} + S_{3i} \\ f_{i} (X_{1k} + S_{1k}, X_{2k} + S_{2k}) \end{bmatrix} \Delta t$$
(64)

and

m = the number of the last computed period.

For all cases given, a step size,  $\Delta t$ , of 0.01 seconds is used. The accuracy of these solutions have been verified by using a step size of 0.005 seconds.

#### Reference Case

A disturbance may be introduced in the system by off-setting the initial relative angles  $\delta_1$ -  $\delta_3$  and  $\delta_2$ -  $\delta_3$  from the steady-state equilibrium point for this system illustrated in Figure 5. The undamped dynamic response with initial relative angles of zero degrees is given in Figure 9. Damped cases are to be given for this disturbance to demonstrate and compare the damping methods.

#### Voltage Control Damping Cases

The damping equation for total voltage control is

$$\dot{\mathcal{E}}_{i} = \frac{\mu \, \psi_{i} \, \boldsymbol{\epsilon}_{i}}{\mathcal{E}_{L_{i}} + \mathcal{E}_{R_{i}}} \tag{52}$$

and for local voltage control is

$$E_{i} = \frac{\mu \psi_{i} \epsilon_{i}}{\mathcal{E}_{Li}} . \tag{53}$$

The required exciter output voltage,  $E_{xi}$ , may be found in terms of  $E_i$  by substitution of equation (55) into equation (56) to give

$$E_{xi} = \left\{ \left( \frac{L_{fi}}{R_{fi}} \right) \stackrel{\dot{E}}{E}_{i} + E_{i} \right\} \frac{R_{fi}}{K_{i}} . \tag{65}$$

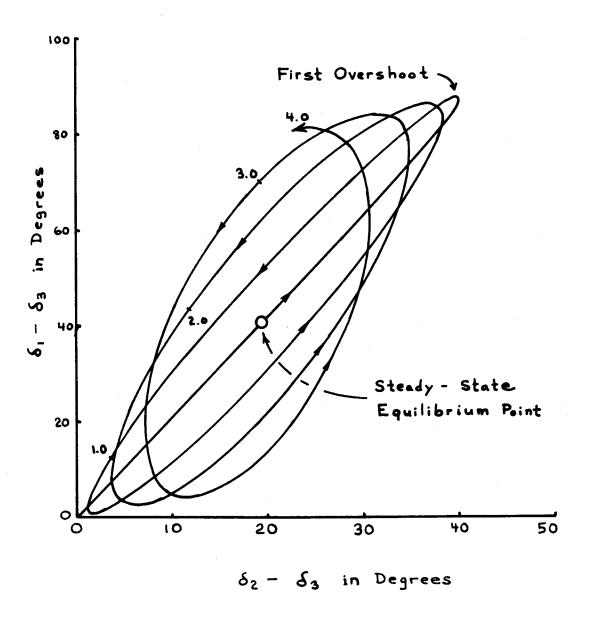


Figure 9. The Undamped Three-Generator Reference Case with Time Intervals Marked in Seconds on the Trajectory.

If the magnitude of the required exciter output voltage exceeds five times its nominal value, a limit is imposed. The flow chart for the voltage control subroutine is given in Appendix B.

A  $\mu$  of ten is used for this system on both local and total voltage control cases. It has been determined from other cases not included in this thesis that damping increases as  $\mu$  is made large although a limit is reached due to exciter saturation. The dynamic response with total voltage control is given in Figure 10 and the response with local voltage control is given in Figure 11. It is noticed that these cases are quite similar although somewhat better damping is obtained for total voltage control.

For the purpose of comparison, the norm is defined as the linear angular distance between the undamped steady-state equilibrium point of Figure 9, and the maximum excursion of the first overshoot. In both cases the norm is reduced by 59 per cent from the undamped case.

The voltage response of each generator for both forms of control is illustrated in Figure 12. The greatest difference between the response with total and local voltage control occurs at generator number two which has the least oscillation of all three generators.

System responses not included in this thesis were also made for initial relative angles of:

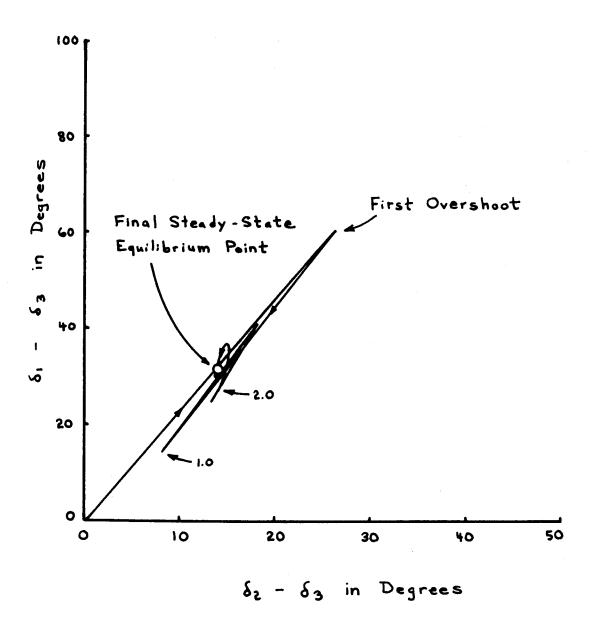


Figure 10. Total Voltage Control Applied to the Three-Generator Reference Case with Time Intervals Marked in Seconds on the Trajectory.

 $(\mu = 10 \text{ per-unit})$ 

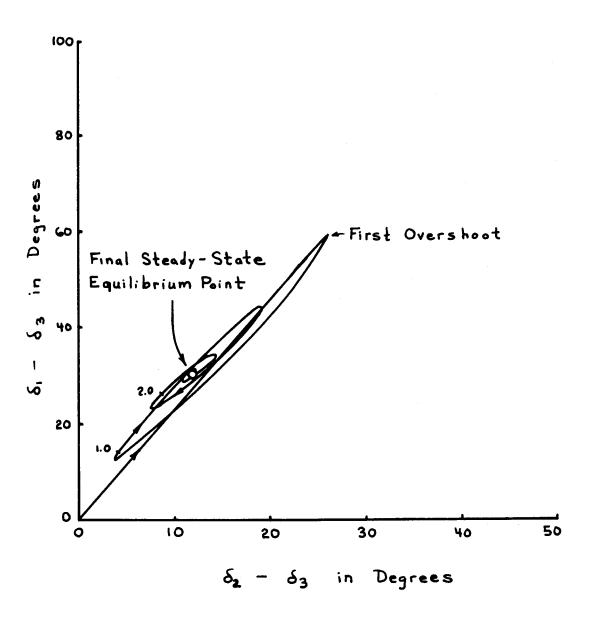
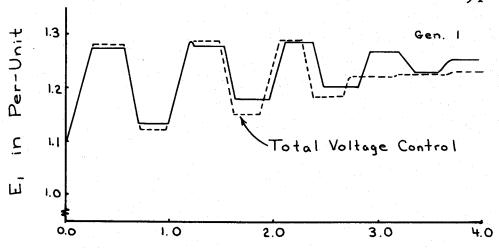


Figure 11. Local Voltage Control Applied to the Three-Generator Reference Case with Time Intervals Marked in Seconds on the Trajectory.

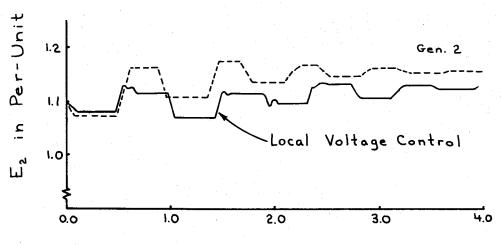
 $(\mu = 10 \text{ per-unit})$ 

Figure 12. The Voltage Response of Each Generator with Local and Total Voltage Control Applied to the Three-Generator Reference Case.

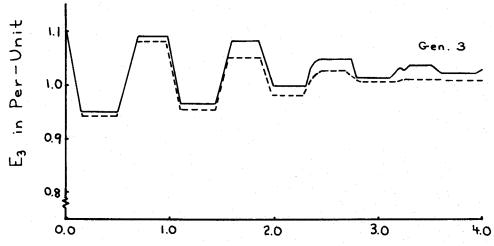
 $(\mu = 10 \text{ per-unit})$ 



Time in Seconds



Time in Seconds



Time in Seconds

$\delta_1$ - $\delta_3$	$\delta_2^- \ \delta_3$
40°	0°
40°	40°
80°	20°

Results of damping for these cases are comparative to the damping for the case given.

#### Dynamic Braking Case

Control is directly exercised on each driving point conductance in the three-generator model. For the application of load resistors,  $\Delta G_i$  is added to  $G_i$ , and for load shedding  $\Delta G_i$  is subtracted from  $G_i$ . The required size of  $\Delta G_i$  is given as

$$\Delta G_{i} = \lambda M_{i} / E_{i}^{2}. \tag{35}$$

Since the nominal value of  $\mathbf{E_i}$  and  $\mathbf{M_i}$  are identical for all generators,  $\Delta \mathbf{G_i}$  must also be identical. The coefficient is chosen such that the application of  $\Delta \mathbf{G_i}$  causes a step change in  $\mathbf{P_i}$  equal to ten per cent of the nominal  $\mathbf{W_i}$ .

Thus

$$\lambda = 0.1 W_i/M_i = 2.127$$

and

$$\Delta G_{i} = 0.179 \text{ for } i = 1,2,3.$$

Although it is noticed that  $G_1$ - $\Delta G_1$  results in a negative conductance, this is permitted since the main concern is to verify the damping method. Dynamic braking is applied in accordance with equation (43) for which a flow chart is given in Appendix C.

The dynamic response for initial relative angles of zero degrees is given in Figure 13. Curves illustrating the response of  $\boldsymbol{\epsilon}_i$  are given in Figure 14. The 35 per cent reduction in norm from the undamped case is not as good as voltage control although the oscillation settles to zero more rapidly. In Figure 14 it is seen that dynamic braking did not occur at generator number two since the switching criterion given by equation (42) is not satisfied for that machine.

System responses not included in this thesis were also made for initial relative angles of:

_ <b>ઠ</b> 1 - ઠ3	<b>ઠ</b> 2 - <b>ઠ</b> 3
40°	0°
40°	40°
80°	20°

Results of damping for these cases are comparative to that obtained for the case given.

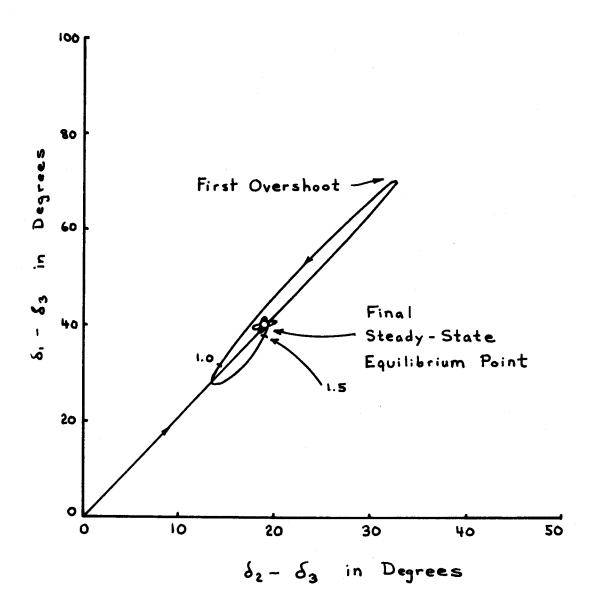


Figure 13. Dynamic Braking Applied to the Three-Generator Reference Case with Time Intervals in Seconds Marked on the Trajectory.

 $(\lambda = 2.127 \text{ per-unit})$ 

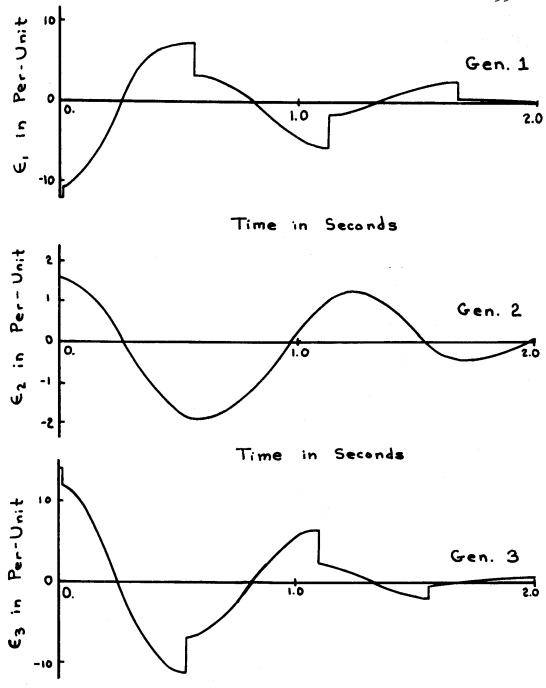


Figure 14. Response of **€** with Dynamic Braking Applied to the Three-Generator Reference Case. For Interpretation of Discontinuities see Figure 6.

 $(\lambda = 2.127 \text{ per-unit})$ 

### Prime Mover Control

A case is not given for the control of  $\mathbf{W}_{\mathbf{i}}$  to introduce damping by

$$\dot{\mathbf{W}}_{\mathbf{i}} = -\mathbf{Y}\mathbf{M}_{\mathbf{i}}\boldsymbol{\varphi}_{\mathbf{i}}\boldsymbol{\in}_{\mathbf{i}} \tag{31}$$

since present governor and turbine systems cannot follow the equation for most oscillation frequencies. This equation has been developed, however, in anticipation of improvements in the prime mover response.

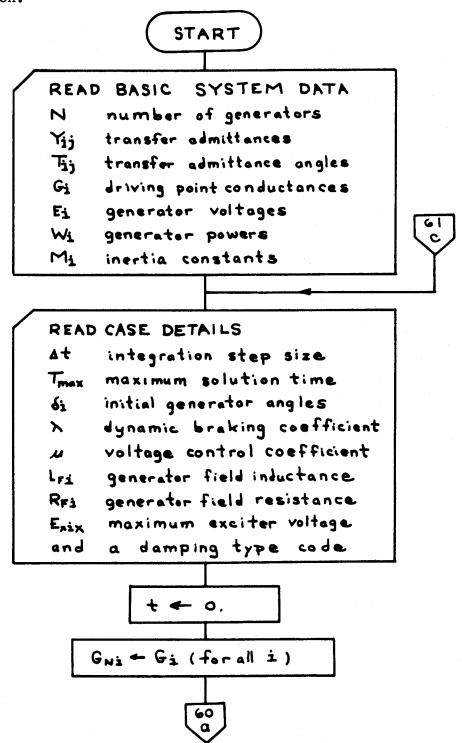
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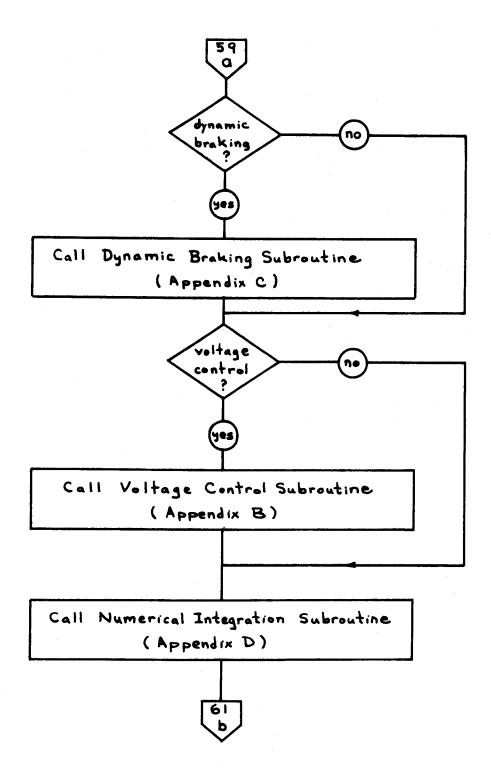
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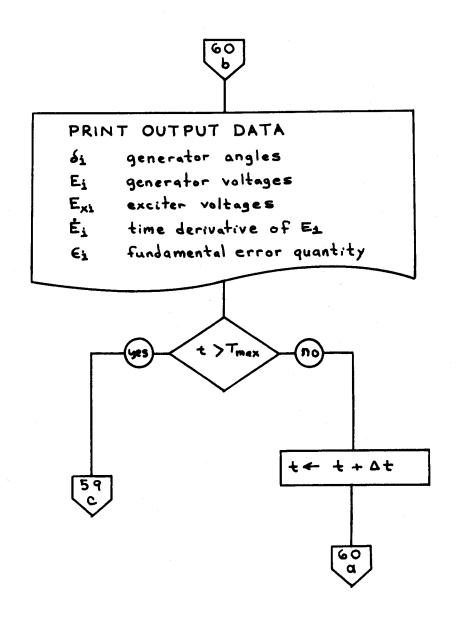
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### APPENDICES

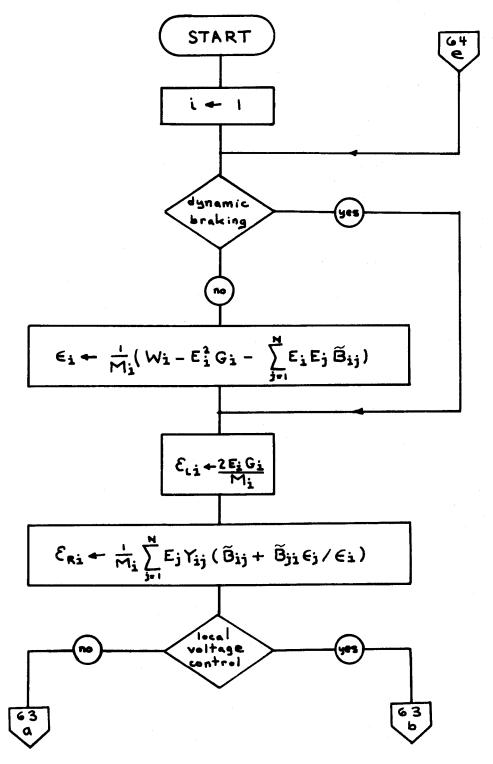
Appendix A. The Main Program Flow Chart for Power System Simulation.

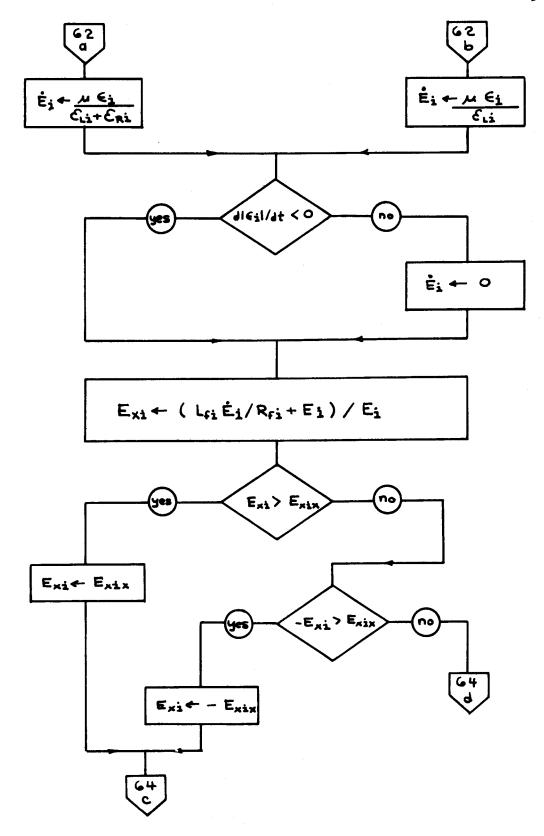


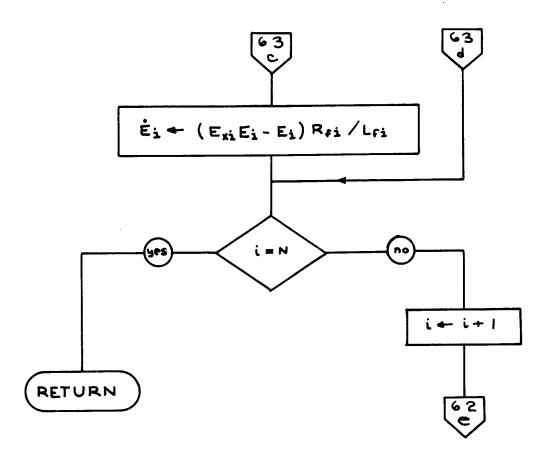




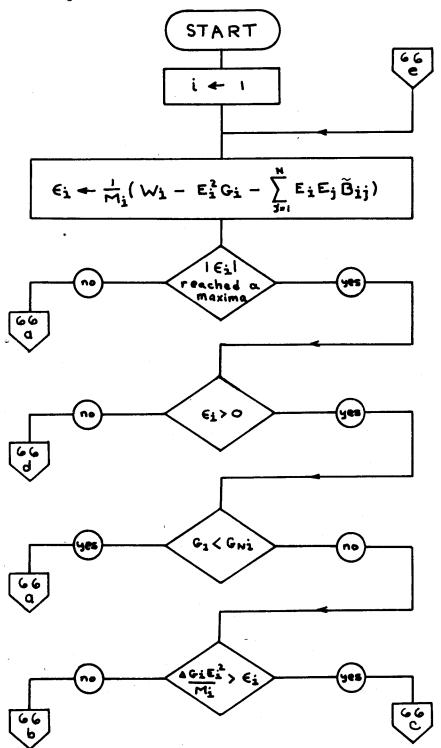
Appendix B. The Subroutine Flow Chart for Damping by Generator Voltage Control.

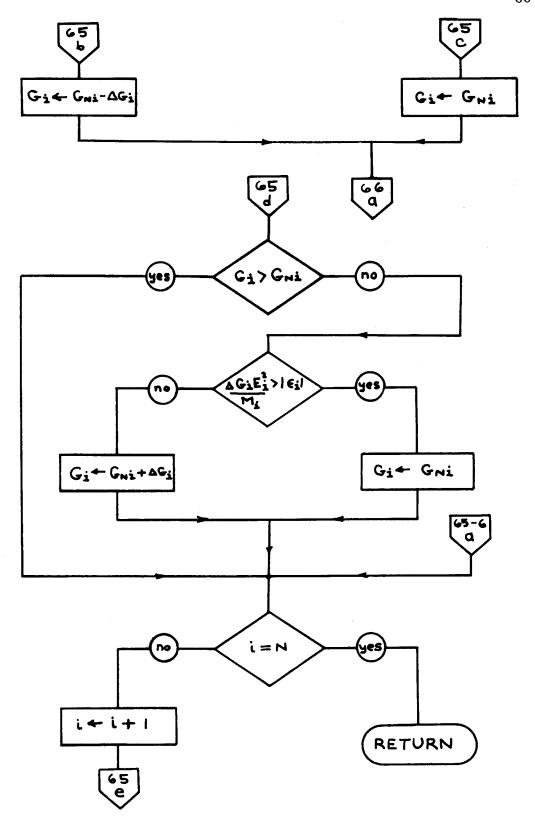






Appendix C. The Subroutine Flow Chart for Application of Dynamic Braking.





Appendix D. The Subroutine Flow Chart for Fourth Order Runge Kutta Numerical Integration.

