Hindawi
Mathematical Problems in Engineering
Volume 2020, Article ID 8407830, 16 pages
https://doi.org/10.1155/2020/8407830

Research Article

A Method of Uncertainty Measurements for Multidimensional Z-number and Their Applications

Yanfei Li,[1] **Tao Wu**,[1,2] **Junjun Mao**,[1,2] **Haiyan Guo**,[1] **and Aiting Yao**[1]

[1]*School of Mathematical Sciences, Anhui University, Hefei 230601, China*
[2]*Key Laboratory of Intelligent Computing & Signal Processing, Ministry of Education, Anhui University, Hefei 230039, China*

Correspondence should be addressed to Tao Wu; wutao@ahu.edu.cn

Received 22 November 2019; Accepted 28 March 2020; Published 21 April 2020

Academic Editor: Vincenzo Vespri

Z-number provides the reliability of evaluation information, and it is widely used in many fields. However, people usually describe things from various aspects, so multidimensional Z-number has more advantages over traditional Z-number in describing evaluation information. In view of the uncertainty of the multidimensional Z-number, the entropy of multidimensional Z-number is defined and an entropy formula of multidimensional Z-number is established. Furthermore, the entropy is used to construct an average operator of multidimensional Z-numbers. In addition, a novel distance measure is introduced to measure the distance between two multidimensional Z-numbers. Moreover, the group decision model in the multidimensional Z-number environment is constructed by combining the average operator with the TOPSIS decision-making method. Finally, an illustrative example is given to verify the feasibility and effectiveness of the proposed method.

1. Introduction

In order to solve the problem of uncertain information, Zadeh [1] added membership function and proposed fuzzy sets (FSs) theory to solve the problem of quantitative calculation for uncertain information. However, merely adding membership degree cannot fully express the complexity of the practical problems. Thus, Atanassov [2] added nonmembership degree and hesitation and introduced intuitionistic fuzzy sets (IFSs). Torra [3] puts forward hesitant fuzzy sets (HFSs), and it changed membership from single to multiple and gave us the ability to express more possible situation. Mizumoto and Tanaka [4] proposeed type-2 fuzzy sets by replacing the given elements with intervals for the membership degree. The extension of fuzzy sets above has been successfully applied to multiattribute decision-making (MADM) problems [5–12]. However, the classical fuzzy set and its extension do not give the reliability measurement of evaluation information. In order to satisfy people's description of the fuzziness of complex uncertain problems, Zadeh [13] proposed the Z-number theory, and a Z-number can be denoted as $Z = (A, B)$. It combined the objective information of natural language with the subjective understanding of human beings by constraint A and reliability B.

After the concept of Z-number was proposed, many scholars have conducted in-depth research on Z-number, it can be roughly divided into two categories. The first category is theoretical research and expansion. In [14, 15], Aliev et al. developed basic arithmetic operations such as addition, subtraction, multiplication, division, and some algebraic operations such as maximum, minimum, square, and square root of discrete and continuous Z-numbers. Kang et al. [16] proposed a method of transforming Z-numbers to classical fuzzy number according to the fuzzy expectation. It contributed to the theory and methods on the classical fuzzy set which were applied to the Z-number environment. Yager and Ronald [17] discussed several special hidden probability distributions of Z^{+}-number. Banerjee and Pal [18, 19] proposed Z^{*}-number which extended the purpose of Zadeh's Z-number. It reinforced the capability of Z-numbers by virtue of parameters incorporating the context and time and affects embedded in natural language sentences. Sometimes the information was useful if some conditions were true; Allahviranloo and Ezadi [20] investigated Z-advance numbers, aiming to solve the uncertain information problem which was reliable depends on some conditions.

The second category is the practical application of Z-numbers; many scholars applied Z-number to linguistic calculation [21, 22], and some used it as a tool for fuzzy inference [23] and pattern recognition [24]. Z-number is more likely to be used for MADM. Wang et al. [25] discussed the application of linguistic Z-number and proposed a modified TODIM method by Choquet integral for MADM problems based on linguistic Z-numbers. To apply the classic VIKOR method to the Z-information, Shen and Wang [26] defined the comprehensive weighted distance measure of Z-numbers, and it not only considered the randomness but also considered fuzziness of Z-number simultaneously. Peng and Wang [27] raised some outranking relations of Z-numbers and defined the dominance degree of discrete Z-numbers according to the relations. Moreover, incorporating the advantages of ELECTRE III and QUALIFLEX, they developed a novel outranking method to address MADM problems. Peng and Wang [28, 29] proposed several MAGDM method based on cloud model and Z-numbers.

Technique for order preference by similarity to an ideal solution (TOPSIS) method is a kind of common multi-attribute group decision-making method of scheme ranking [30, 31]; it allows the scheme to get close to the positive ideal solution while moving away from the negative ideal solution. Zeng [32] proposed a new MADM method based on the nonlinear programming methodology, the TOPSIS method and IVIFVs. Yaakob and Gegov [33] developed an extended TOPSIS method to solve MADM problems which were based on Z-numbers called Z-TOPSIS. Qiao et al. [34] developed a new linear programming model for obtaining underlying probability distribution and used it to construct a comprehensive weighted crossentropy. Based on it, an extended TOPSIS approach was developed to solve a multi-criteria decision-making problem under discrete Z-context. It is on the basis of the TOPSIS method that this paper develops the decision-making method under multidimensional Z-number environment.

In uncertain sets, how to measure the uncertainty of the set is very important. Shannon [35] proposed the concept of shannon entropy based on probability. It was an uncertainty measure of information. Taking into account intuitionism and fuzziness of intuitionistic fuzzy sets, novel crossentropy and entropy models were investigated in [36], and it can be used to measure the discrimination among uncertain information. Considering the influence of fuzziness and the range of the fuzzy set, Kang et al. [37] developed a new measure of fuzziness and investigated a method of measuring the uncertainty of Z-number.

As an extension of Z-number, Shen et al. [38] proposed the concept of multidimensional Z-number ((about 3 min, about 3 km), usually). The multidimensional Z-number can be used to evaluate a certain phenomenon from multiple dimensions and provide reliability measure of comprehensive evaluation. It inherits the advantages of Z-number to describe qualitative information and characterizes the reliability of information. Compared to an original Z-number, multidimensional Z-number has several advantages as it has a much simpler expression; it contains more information, and it is more intuitive to evaluate things from multiple dimensions. However, there exists few studies about multidimensional Z-number now, and there is no method to measure the uncertainty of the multidimensional Z-number or apply it to decision-making problems.

This paper develops several theories and their applications in the context of multidimensional Z-number. The initial motivations and main contributions of this paper are as follows:

(1) In order to solve the problem that the uncertainty of multidimensional Z-number is difficult to measure, this paper introduces a novel entropy measure which incorporates the inherent fuzziness of fuzzy restriction and reliability measure and the fuzziness of reliability level of the reliability measure.

(2) This paper proposes a new distance measure which comprehensively considered the difference between fuzzy restriction and reliability measurement between two dimensional Z-numbers. Moreover, the differences of the membership value and element value are both taken into account, so the distance is about the order weight vector of each dimension of the two dimensional Z-numbers. It can be used to deal with the problem that the difference between two multidimensional Z-numbers is hard to measure.

(3) For the sake of the problem of group decision-making under multidimensional Z-number environment to be addressed, this paper develops a MAGDM method which combines TOPSIS method and multidimensional Z-number.

This paper is organized as follows. In Section 2, some of the definitions are briefly introduced covered in this article. In Section 3, two methods are developed to measure the fuzziness of multidimensional fuzzy sets; based on this, the entropy formula for multidimensional Z-number is proposed. In Section 4, a power weighted average operator is extended to situations in which the evaluation information consists of multidimensional Z-number. In Section 5, a distance measure of multidimensional Z-numbers about their order weight vector of each dimension are discussed. In Section 6, a novel GDM method is developed by combining multidimensional Z-numbers and TOPSIS method, and the feasibility and effectiveness of the method is discussed in Section 7. Finally, Section 8 gives some conclusions of this paper.

2. Preliminaries

This section reviews four basic definitions that are related to the research, including fuzziness measurement of fuzzy set, discrete Z-number, continuous Z-number, and multidimensional Z-number, and the research is based on these concepts.

A measure of fuzziness for fuzzy set In 1972, Luca and Termini introduced a measure of fuzziness for fuzzy set that is as follows.

Definition 1 (see [39]). There is a map

$$H: F(x) \longrightarrow [0,1], \tag{1}$$

which satisfies the following properties:

(1) $H(A) = 0$, if and only if A is a clear set

(2) $H(A) = 1$, if and only if $\mu_A(x) \equiv (1/2)$

(3) $\forall x \in X$, if $\mu_B(x) \leq \mu_A(x) \leq (1/2)$, $H(B) \leq H(A)$

(4) $\forall x \in X$, $H(A) = H(A^c)$

We denote mapping H as a measure of fuzziness on $F(x)$ and $H(A)$ as the measure of fuzziness on A.

Definition 2 (see [13], discrete Z-number). Let X be a random variable and A and B be two discrete fuzzy numbers, where

$$\begin{aligned} \mu_A &: x_1, x_2, \ldots, x_n \longrightarrow [0,1], \\ \mu_B &: b_1, b_2, \ldots, b_n \longrightarrow [0,1]. \end{aligned} \tag{2}$$

For the membership function of A and B, respectively, where $x_1, x_2, \ldots, x_n \in R$ and $b_1, b_2, \ldots, b_n \in [0,1]$, a discrete Z-number is defined as an ordered pair of discrete fuzzy numbers $Z = (A, B)$ on X, where A is the fuzzy restriction of X and B is the fuzzy restriction of the probability measure of A.

Definition 3 (see [13], continuous Z-number). A continuous Z-number is an ordered pair $Z = (A, B)$ where A is a continuous fuzzy number playing a role of a fuzzy constraint on the probability measure of A:

$$P(A) \text{ is } B. \tag{3}$$

In some cases, A and B are depicted in a natural language, such as (fair, unlikely) and(good, likely).

Definition 4 (see [38], multidimensional Z-number). Some random variables $X_i = (x_{i1}, x_{i2}, \ldots, x_{im_{k_i}})$ defined on sample space X and $A_i \subseteq X_i$; then, $(A_1, A_2, \ldots, A_n)$ is called the n-dimension restriction vector, where $X = \{(x_{1k_1}, x_{2k_2}, \ldots, x_{nk_n}) \mid x_{ik_i} \in X_i, k_i = 1, 2, \ldots, m_{k_i}, i = 1, 2, \ldots, n\}$. A multidimensional Z-number comprises multidimensional restriction vector, $(A_1, A_2, \ldots, A_n)$, and a fuzzy number, B, denoted as

$$MZ = ((A_1, A_2, \ldots, A_n), B). \tag{4}$$

Let $G = (A_1, A_2, \ldots, A_n)$, and a multidimensional Z-number can be expressed as $MZ = (G, B)$.

For example, in the case of nice weather with sun and a certain temperature, the undimensional Z-number cannot express fully the condition of weather. Furthermore, the weather cannot be judged from just one aspect of sun or breezy temperature. It could be good weather or bad weather. Hence, the information should be expressed in the form of multidimensional Z-number (comfort weather (sun and temperature around 26°C) and hot weather (sun and temperature above 30°C), likely).

3. Entropy for Multidimensional Z-number

3.1. Measure of Fuzziness for a Multidimensional Fuzzy Set.

In the fuzzy decision-making process under multidimensional Z-number environment, measuring the uncertainty of the multidimensional Z-number is an important problem. However, there is a lack of research on this aspect. In this paper, two kinds of uncertainty measure of multidimensional fuzzy set are developed by considering the possible factors which affect the fuzziness. Some properties of them are shown, and then this paper proposes a novel method of measuring the uncertainty of multidimensional Z-number.

The first uncertainty measure is constructed from the perspective of algebra and takes full advantage of the features of sine function. As membership value changes from the middle to both ends, the value of fuzziness measure decreases and the rate of the change decreases.

Definition 5 (the algebra measurements of fuzziness for multidimensional fuzzy set). Assume G is a continuous multidimensional fuzzy set, where the collection elements are normalized; the membership function of G is denoted by $\mu_G : X \longrightarrow [0,1]$, where value g belongs to domain X. The fuzziness measure for multidimensional fuzzy set is denoted by $H_1(G)$ as follows:

$$H_1(G) = \iint \cdots \int \sin(\mu_G(g) \cdot \pi) \mathrm{d}x_1 \cdots \mathrm{d}x_n, \tag{5}$$

where the upper and lower boundaries of every integration are 0 and 1.

Assume a discrete multidimensional fuzzy set as G; the membership function of G is denoted by $\mu_G : X \longrightarrow [0,1]$, where the value g belongs to the domain X. The measure of fuzziness for the multidimensional fuzzy set is denoted by $H_1(G)$ as follows:

$$H_1(G) = \sum_{X_1} \sum_{X_2} \cdots \sum_{X_n} \sin(\mu_G(g) \cdot \pi) \times \frac{1}{p}, \tag{6}$$

where p represents the cardinality of the discrete multidimensional fuzzy set, when the cardinality of the fuzzy set is infinite; this is a special case, the right side of the equation is an infinite series, p is a number that goes to infinity and can be expressed by the theory of limit, and n represents the dimension of the set.

Inspired by Definition 1, four properties of the fuzziness measure for multidimensional fuzzy set are given. Next, the fuzziness measure satisfies the given four properties which are proved. For convenience, this paper just proves the continuous case, and the discrete case has the same process of proof.

Property 1. $H(G) = 0$, if and only if G is a clear set.

Proof. Let G be a crisp set with membership values being either 0 or 1, it can be obtained that $H_1(G) = 0$ because

$\mu_G(g) \in [0,1]$, then $\sin(\mu_G(g)\cdot\pi) \ge 0$ always holds. G is a multidimensional fuzzy set and $\mu_G(g)$ is continuous in $[0,1]$; therefore, $\sin(\mu_G(g)\cdot\pi) = 0$, and $\mu_G(g) = 0$ or $\mu_G(g) = 1$ always holds, and it means that the multidimensional fuzzy set is a clear set. So, Property 1 has been proved.

□

Property 2. $H(G) = 1$, if and only if $\mu_G(g) \equiv (1/2)$.

Proof. Substitute $\mu_G(g) = (1/2)$ into the fuzziness formula, then $H_1(G) = \int\int\cdots\int \sin(\pi/2)\mathrm{d}x\cdots\mathrm{d}x_n = 1$; when $H_1(G) = 1$, $0 \le \sin(\mu_G(g)\cdot\pi) \le 1$, if there is a point g_0 where the membership value is not equal to 1 because this multidimensional fuzzy set is continuous, for a very small positive number ε, there exists an area D which is the neighborhood of g_0, in this area $1-\varepsilon < \sin(\mu_G(g)\cdot\pi) < 1$; it can be deduced that $\int_D \sin(\mu_G(g)\cdot\pi)\mathrm{d}X < \int_D \mathrm{d}X$; next inequality $\int_{X-D+D} \sin(\mu_G(g)\cdot\pi)\mathrm{d}X < 1$ can be obtained; according to this inequality, we know $\sin(\mu_G(g)\cdot\pi) = 1$ and $\mu_G(g) = (1/2)$. So, Property 2 has been proved.

□

Property 3. Assume G and G' are two multidimensional fuzzy sets, for all $g \in X$, if $\mu_G(g) \le \mu_{G'}(x) \le (1/2)$ and $H(G) \le H(G')$.

Proof. When $\mu_G(x) \le \mu_G'(x) \le (1/2)$, it can be obtained that $\sin(\mu_B(x)\cdot\pi) \le \sin(\mu_A(x)\cdot\pi)$. Thus, the following inequality is true:

$$\int_X \sin(\mu_G(g)\cdot\pi)\mathrm{d}X \le \int_X \sin(\mu_{G'}(g)\cdot\pi)\mathrm{d}X. \tag{7}$$

Then, inequation $H_1(G) \le H_1(G')$ is held.

□

Property 4. $\forall g \in X$, as $H(G) = H(G^c)$.

Proof. Because of $\mu_{G^c}(g) = 1-\mu_G(g)$, it can be acquired $\sin(\mu_{A^c}(x)\cdot\pi) = \sin(\pi-\mu_{G^c}(g)\cdot\pi) = \sin(\mu_G(g)\cdot\pi)$; then, the following inequality can be deduced:

$$\int_X \sin(\mu_G(g)\cdot\pi)\mathrm{d}X = \int_X \sin(\mu_{G^c}(g)\cdot\pi)\mathrm{d}X. \tag{8}$$

Therefore, inequality $H_1(G) = H_1(G^c)$ is true, and Property 4 can be proved.

□

Example 1. Let us consider two three-dimensional Z-numbers $MZ_1 = (G_1, B_1)$ and $MZ_2 = (G_2, B_2)$ as follows:

$$\begin{aligned}
G_1 &= \frac{0.1}{(1,1,1)} + \frac{0.25}{(1,1,2)} + \frac{0.5}{(1,1,3)} + \frac{0.75}{(1,2,1)} + \frac{1}{(1,2,2)} + \frac{0.8}{(1,2,3)} + \frac{0.6}{(1,3,1)} + \frac{0.4}{(1,3,2)} + \frac{0.2}{(1,3,3)},\\
G_2 &= \frac{0.25}{(1,1,1)} + \frac{0.3}{(1,1,2)} + \frac{0.5}{(1,1,3)} + \frac{0.7}{(1,2,1)} + \frac{0.8}{(1,2,2)} + \frac{0.9}{(1,2,3)} + \frac{1}{(1,3,1)} + \frac{0.5}{(1,3,2)} + \frac{0.3}{(1,3,3)},\\
B_1 &= \frac{0.1}{0} + \frac{0.2}{0.1} + \frac{0.5}{0.2} + \frac{0.8}{0.3} + \frac{1}{0.4} + \frac{0.8}{0.5} + \frac{0.7}{0.6} + \frac{0.6}{0.7} + \frac{0.4}{0.8} + \frac{0.2}{0.9} + \frac{0.1}{1},\\
B_2 &= \frac{0.05}{0} + \frac{0.2}{0.1} + \frac{0.6}{0.2} + \frac{0.9}{0.3} + \frac{1}{0.4} + \frac{0.7}{0.5} + \frac{0.5}{0.6} + \frac{0.4}{0.7} + \frac{0.3}{0.8} + \frac{0.1}{0.9} + \frac{0.05}{1}.
\end{aligned} \tag{9}$$

According to Definition 5, it can be calculated that the measure of fuzziness of G_1 for $H_1(G)$ is $H_1(G_1) = (\sin(0.05\pi) + \sin(0.25\pi) + \sin(0.5\pi) + \sin(0.75\pi) + \sin(\pi) + \sin(0.8\pi) + \sin(0.6\pi) + \sin(0.4\pi) + \sin(0.2\pi))/9 = 0.6445$. In the same way, $H_1(G_2) = 0.6701$. G_2 is more uncertain than G_1 and can be obtained.

For a one-dimensional fuzzy set, its uncertainty can still be measured according to Definition 5, and then let n be 1, and it can be calculated that $H_1(B_1) = 0.6073$ and $H_1(B_2) = 0.5489$. $H_1(B_1)$ and $H_1(B_2)$ only represent the inherent uncertainty of the fuzzy sets, independent of the uncertainty of the probability measure of A.

From the perspective of geometric, the second kind measure of fuzziness for multidimensional fuzzy set is constructed. There is a corresponding relation on the circle between the measure of fuzziness and membership degree, as shown in Figure 1.

As membership value changes from the middle to both ends, the value of fuzziness measure decreases but the rate of change increases.

Definition 6 (the geometric measurements of fuzziness for multidimensional fuzzy set). Assume that a continuous multidimensional fuzzy set is G, where the collection elements are normalized. The membership function of G is denoted by $\mu_G: X \longrightarrow [0,1]$, where the value g belongs to the domain X. The fuzziness measure for the multidimensional fuzzy set is denoted by $H_2(G)$ as follows:

$$H_2(G) = \int_X 2\times\sqrt{\mu_G(g) - \mu_G(g)^2}\mathrm{d}X, \tag{10}$$

where the upper and lower boundaries of every integration are 0 and 1.

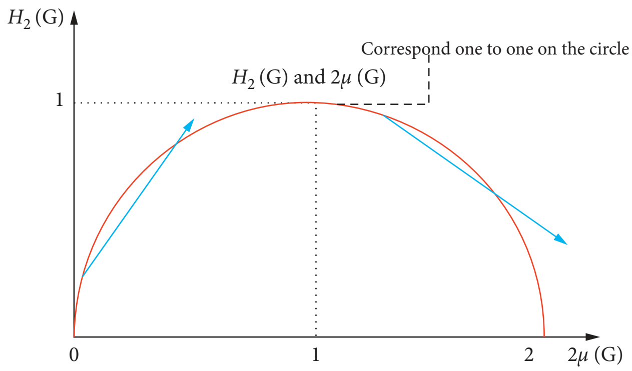

FIGURE 1: The geometric relationship between the fuzziness measure and the membership function.

Assume a discrete multidimensional fuzzy set is G, and the membership function of G is denoted by $\mu_G: X \longrightarrow [0,1]$, where the value g belongs to the domain X. The measure of fuzziness for the fuzzy set is denoted by $H_2(G)$ as follows:

$$H_2(G) = \sum_{X_1}\sum_{X_2}\cdots\sum_{X_n} 2\sqrt{\mu_G(g) - \mu_G(g)^2} \times \frac{1}{p}, \tag{11}$$

where p represents the cardinality of the discrete multidimensional fuzzy set, when the cardinality of the fuzzy set is infinite; this is a special case, the right side of the equation is an infinite series, p is a number that goes to infinity, which can be expressed by the theory of limit, and n represents the dimension of the set.

The first three properties of $H_1(G)$ and $H_2(G)$ have similar proof process, and now its Property 4 is proved as follows.

Proof

$$\begin{aligned} &\mu_{G^c}(g) - \mu_{G^c}(g)^2 \\ &= 1 - \mu_G(g) - (1 - \mu_G(g))^2 \\ &= 1 - \mu_G(g) - \left(1 - 2\mu_G(g) + \mu_G(g)^2\right) \\ &= \mu_G(g) - \mu_G(g)^2. \end{aligned} \tag{12}$$

It can be acquired that

$$\begin{aligned} H_2(G^c) &= \int_X 2\sqrt{\mu_{G^c}(g) - \mu_{G^c}(g)^2}\mathrm{d}X \\ &= \int_X 2\sqrt{\mu_G(g) - \mu_G(g)^2}\mathrm{d}X \\ &= H_2(G). \end{aligned} \tag{13}$$

The properties have been proved.

□

Example 2. Let us consider the two three-dimensional Z-numbers $MZ_1 = (G_1, B_1)$ and $MZ_2 = (G_2, B_2)$ in Example 1 and the geometric measurements of fuzziness for G_1 is $H_2(G_1) = 0.7657$; in the same way, $H_2(G_1) = 0.7795$ and G_2 is more uncertain than G_1. $H_2(B_1) = 0.7542$ and $H_2(B_2) = 0.6968$. B_1 is more uncertain than B_2. Results of comparison of the geometric measurements of fuzziness are the same with the results of the algebra measurements of fuzziness, and it proves that the comparison results of the two uncertainty measures are consistent for these sets.

3.2. Entropy Measure for Multidimensional Z-Number. Assume a multidimensional Z-number $MZ = (G, B)$; its first component G, is a fuzzy restriction on the multidimensional uncertain variable X. The second component B is a measure of reliability for the first component G. The membership function of G is denoted by $\mu_G(g): X \longrightarrow [0,1]$, where the value g belongs to the domain X; the membership function of B is denoted by $\mu_B(y): Y \longrightarrow [0,1]$, where the value y belongs to the domain Y.

In view of G and B, which are fuzzy sets, they have inherent uncertainty that will influence the uncertainty of multidimensional Z-number. It can be measured by the fuzziness measure formula H_1 or H_2. H_1 and H_2 are represented here by function H. $H(G)$ or $H(B)$ are used to represent the inherent uncertainty of G and B. B is a measure of reliability for first component G; the certainty of the reliability degree of G measured by B also influences the uncertainty of the multidimensional Z-number. It can be denoted by $V(B)$. It can be measured by the following equality:

$$V(B) = \frac{\int_Y y\mu(y)\mathrm{d}y}{\int_Y \mu(y)\mathrm{d}y}, \tag{14}$$

where $\int$ denotes an algebraic integration.

Definition 7. Now the entropy measure for multidimensional Z-number denoted by $E(MZ)$ is as follows:

$$E(MZ) = 1 - (1 - H(G))(1 - H(B))(2|V(B) - 0.5|). \tag{15}$$

Then, inspired by Definition 1, there are some properties of the proposed entropy measure for multidimensional Z-number.

Property 5. The range value of $E(MZ)$ is in $[0,1]$.

Proof. Because of $0 \le H(G) \le 1$, $0 \le H(B) \le 1$, and $0 \le V(B) \le 1$, then $0 \le (1 - H(G)) \le 1$, $0 \le 1 - H(B) \le 1$, $0 \le 2|V(B) - 0.5| \le 1$, so $0 \le E(MZ) \le 1$.

□

Property 6. $E(MZ) = 0$, if and only if G and B are clear sets, $V(B) = 0$ or 1.

Proof. Its sufficiency can be proved easily according to substitute $H(G) = 0$, $H(B) = 0$, $V(B) = 0$ or 1 entropy formula for multidimensional Z-number; if $E(MZ) = 0$, due to that $0 \le 1 - H(G) \le 1$, $0 \le 1 - H(B) \le 1$, $0 \le 2|V(B) - 0.5| \le 1$; it can be deduced that $1 - H(G) = 1$, $1 - H(B) = 1$, $2|V(B) - 0.5| = 1$. So, it can be obtained that $H(G) = 0$, $H(B) = 0$, $V(B) = 0$ or 1.

□

Property 7. $E(MZ) = 1$, if any one of the three equality $H(G) = 1$, $H(B) = 1$, and $V(B) = 0.5$ is true.

Proof. Substitute $H(G) = 1$ or $H(B) = 1$ or $V(B) = 0.5$ into entropy measure formula for multidimensional Z-number; $E(MZ) = 1$ can be obtained. □

Property 8. $E(MZ)$ increases as $H(G)$ and $H(B)$ increase, when $0 \le V(B) \le 0.5$, $E(Z)$ increases as $V(B)$ increases, $0.5 \le V(B) \le 1$, and $E(MZ)$ increases as $V(B)$ decreases.

Proof. Taking the derivative of the corresponding function, it can be acquired that inequalities $(\partial(E(MZ))/\partial H(G)) = -(1 - H(B))(2|V(B) - 0.5|) \ge 0$ and $(\partial(E(MZ))/\partial H(B)) = (1 - H(G))(2|V(B) - 0.5|) \ge 0$ are true, so $E(MZ)$ increases as $H(A)$ and $H(B)$ increase; when $0 \le V(B) \le 0.5$, it is easy to know that $(\partial(E(MZ))/\partial V(B)) = 2(1 - H(G))(1 - H(B)) \ge 0$; it means that $E(Z)$ increases as $V(B)$ increases; when $0.5 \le V(B) \le 1$, $(\partial(E(MZ))/\partial V(B)) = -2(1 - H(G))(1 - H(B)) \le 0$, so $E(MZ)$ increases as $V(B)$ decreases.

The variation tendency of the entropy of a multidimensional Z-number with the changing of $H(G)$, $H(B)$, and $V(B)$ is shown in Figures 2 and 3. □

Example 3. Kang et al. [37] developed a new entropy of Z-number, for Z-numbers $Z_1 = (A, B)$ and $Z_2 = (A^*, B^*)$, it can be obtained that $E(Z_1) = 0.6227$, $E(Z_2) = 0.5512$, and Z_1 is more ambiguous than Z_2, where fuzzy numbers A, B, A^*, and B^* are as follows:

$$\begin{aligned} A &= \frac{0}{1} + \frac{0.5}{2} + \frac{0.7}{3} + \frac{1}{4} + \frac{0.7}{5} + \frac{0.5}{6} + \frac{0}{7}, \\ B &= \frac{0}{0.1} + \frac{0.3}{0.2} + \frac{0.9}{0.3} + \frac{1}{0.4} + \frac{0.9}{0.5} + \frac{0.3}{0.6} + \frac{0}{0.7}, \\ A^* &= \frac{0}{1} + \frac{0.4}{2} + \frac{0.8}{3} + \frac{1}{4} + \frac{0.8}{5} + \frac{0.4}{6} + \frac{0}{7}, \\ B^* &= \frac{0}{0.1} + \frac{0.1}{0.2} + \frac{1}{0.3} + \frac{1}{0.4} + \frac{1}{0.5} + \frac{0.1}{0.6} + \frac{0}{0.7}. \end{aligned} \tag{16}$$

Applying the entropy measure, it can be calculated that $H_1(A)=0.2857$, $H_1(B)=0.3194$, $V(B)=0.4$, $H_1(A^*)=0.4397$, $H_1(B^*)=0.0883$, and $V(B^*)=0.4$, so $E(Z_1)=0.9028$, $E(Z_2)=0.8978$, and Z_1 is more ambiguous than Z_2. For the abovementioned example, the result of the proposed method is consistent with Kang's method, and it is concluded that the proposed entropy for Z-number can effectively represent the uncertainty of the Z-number.

Example 4. Let us consider the two three-dimensional Z-numbers $MZ_1 = (G_1, B_1)$ and $MZ_2 = (G_2, B_2)$ in Example 1; let function H equal to H_1, and it can obtained that $E(MZ_1) = 0.9948$ and $E(MZ_2) = 0.9836$. According to the entropy measure for multidimensional Z-number this paper proposed, it can be known that MZ_2 is more certain than MZ_1. From Examples 2 and 3, it can be seen that the proposed entropy can measure the fuzziness of classic and multidimensional Z-number simultaneously, which reflects the superiority of entropy proposed in this paper.

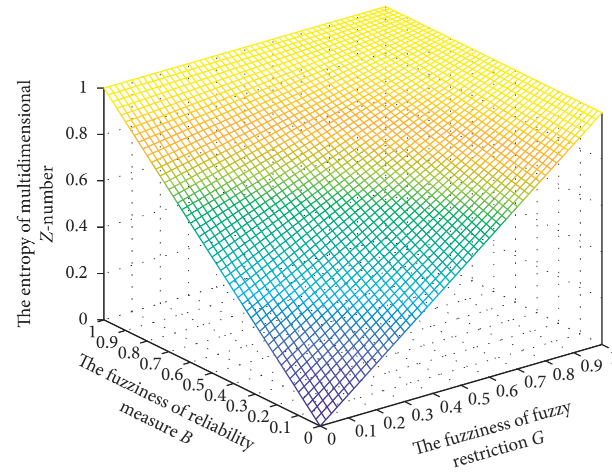

FIGURE 2: Variation tendency of entropy with the change with $H(G)$ and $H(B)$, when $V(B) = 1$.

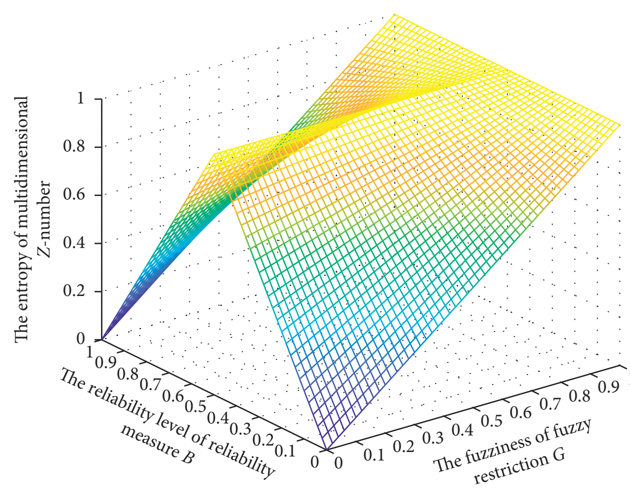

FIGURE 3: Variation tendency of entropy with the change with $H(G)$ and $V(B)$, when $H(B) = 0$.

4. Power Weighted Average Operator with Multidimensional Z-Number

In the process of group decision-making, evaluation information usually have to be fused. This section extends the power weighted average operator to situations in which the evaluation information consists of multidimensional Z-number.

Definition 8. Let $MZ_j = (G_j, B_j)(j = 1, 2, \ldots, m)$ be a collection of multidimensional Z-numbers. Suppose the weight vector is $\nu = \{\nu_1, \nu_2, \ldots, \nu_m\}$. It can be obtained as follows:

$$v_j = \frac{W\left(MZ_j\right)MZ_j}{\sum_{j=1}^{m} W\left(MZ_j\right)}, \tag{17}$$

where $W(MZ_j)$ is used to calculate the importance of MZ_j $(j = 1, 2, \ldots, m)$. This paper uses the entropy to estimate it, and the multidimensional Z-number with higher entropy is given a smaller weight because of the greater uncertainty. Therefore, $W(MZ_j)$ can be defined as follows:

$$W\left(MZ_j\right) = \frac{2\max_k E(MZ_k) - E\left(MZ_j\right) - \min_k E(MZ_k)}{2\max_k E(MZ_k) - 2\min_k E(MZ_k)}. \tag{18}$$

In particular, when $\min_k E(MZ_k) = \min_k E(MZ_k)$, it means that all the multidimensional Z-numbers have the same fuzziness, so they can be assigned the same weight.

Definition 9. The multidimensional Z-numbers power weighted average operator is the mapping $MZPWA\colon X^m \longrightarrow X$, which can be defined as follows:

$$\begin{aligned} &MZPWA(MZ_1, MZ_2, \ldots, MZ_m) \\ &= \frac{W(MZ_1)MZ_1}{\sum_{j=1}^{m} W\left(MZ_j\right)} \\ &\oplus \frac{W(MZ_2)MZ_2}{\sum_{j=1}^{m} W\left(MZ_j\right)} \oplus \cdots \oplus \frac{W(MZ_m)MZ_m}{\sum_{j=1}^{m} W\left(MZ_j\right)} \\ &= \sum_{j=1}^{m} \frac{W\left(MZ_j\right)MZ_j}{\sum_{j=1}^{m} W\left(MZ_j\right)}, \end{aligned} \tag{19}$$

where "⊕" has a special definition, and it means that the following equality is true:

$$\begin{aligned} aMZ_k \oplus bMZ_m = &\left(\int_X \frac{a\mu_{G_k}(g_k) + b\mu_{G_m}(g_m)}{(x_{j1}, x_{j2}, \ldots, x_{jn})}, \right. \\ &\left. \int_Y \frac{a\mu_{B_k}(y) + b\mu_{B_m}(y)}{y} \right), \end{aligned} \tag{20}$$

where $\int$ is a representation of a fuzzy set. According to the operations of multidimensional Z-number provided in equation (19), the following results can be obtained.

Theorem 1. *Let $MZ_j = (A_{j1}, A_{j2}, \ldots, A_{jn}, B_j)$ be a collection of multidimensional Z-numbers, the aggregated value calculated by the ZMPWA operator is also a multidimensional Z-number.*

Proof

$$\begin{aligned} &MZPWA(MZ_1, \ldots, MZ_m) \\ &= \left(\int_X \frac{\sum_{j=1}^{m}\left(W\left(MZ_j\right)\mu_{G_j}\left(g_j\right)/\sum_{j=1}^{m} W\left(MZ_j\right)\right)}{\left(x_{j1}, x_{j2}, \ldots, x_{jn}\right)}, \right. \\ &\left. \int_Y \frac{\sum_{j=1}^{m}\left(W\left(MZ_j\right)\mu_{B_j}(y)/\sum_{j=1}^{m} W\left(MZ_j\right)\right)}{y} \right). \end{aligned} \tag{21}$$

Theorem 1 can easily be proved according to the definition of multidimensional Z-number and the definition of the operation of ⊕.

□

Theorem 2. *This theorem reflects the boundary of the operator. Let $MZ_j = (A_{j1}, A_{j2}, \ldots, A_{jn}, B_j) = (G_j, B_j)$ be a collection of multidimensional Z-numbers, and let $p = (G_p, B_p) = (\cap_{j=1}^{m} G_j, \cap_{j=1}^{m} B_j)$, $q = (G_q, B_q) = (\cup_{j=1}^{m} G_j, \cup_{j=1}^{m} B_j)$; then, we can obtain $p \prec MZPWA(MZ_1, \ldots, MZ_m) \prec q$.*

Proof. For convenience, let $MZPWA(MZ_1, \ldots, MZ_m) = (A_1, A_2, \ldots, A_n, B) = (G, B)$, since $G_p \subseteq G_j \subseteq G_q$, $B_p \subseteq B_j \subseteq B_q$. Then, there are

$$\begin{aligned} G_p &\subseteq \int_X \frac{\sum_{j=1}^{m}\left(W\left(MZ_j\right)\mu_{G_j}\left(g_j\right)/\sum_{j=1}^{m} W\left(MZ_j\right)\right)}{\left(x_{j1}, x_{j2}, \ldots, x_{jn}\right)} = G, \\ B_p &\subseteq \int_Y \frac{\sum_{j=1}^{m}\left(W\left(MZ_j\right)\mu_{B_j}(y)/\sum_{j=1}^{m} W\left(MZ_j\right)\right)}{y} = B. \end{aligned} \tag{22}$$

Therefore, according to the inclusion relationship above, it can be proved that $p \prec MZPWA(MZ_1, \ldots, MZ_m)$). In the same way, $MZPWA(MZ_1, \ldots, MZ_m) \prec q$ can also be acquired. Thus, $p \prec MZPWA(MZ_1, \ldots, MZ_m) \prec q$.

□

Theorem 3. *Let $MZ_j = (A_{i1}, A_{i2}, \ldots, A_{in}, B_i) = (G_j, B_j)$ be a collection of multidimensional Z-numbers, and $(MZ_1', MZ_2', \ldots, MZ_m')$ be any permutation of $(MZ_1, MZ_2, \ldots, MZ_m)$; then, $MZPWA(MZ_1', MZ_2', \ldots, MZ_m') = MZPWA(MZ_1, M\ Z_2, \ldots, MZ_m)$.*

Proof

$$MZPWA(MZ_1,\ldots,MZ_m)$$
$$=\left(\int_X \frac{\sum_{j=1}^m\left(W\left(MZ_j\right)\mu_{G_j}\left(g_j\right)/\sum_{j=1}^m W\left(MZ_j\right)\right)}{\left(x_{j1},x_{j2},\ldots,x_{jn}\right)},\right.$$
$$\left.\int_Y \frac{\sum_{j=1}^m\left(W\left(MZ_j\right)\mu_{B_j}(y)/\sum_{j=1}^m W\left(MZ_j\right)\right)}{y}\right)$$
$$=\left(\int_X \frac{\sum_{j=1}^m\left(W\left(MZ_j'\right)\mu_{G_j}\left(g_j\right)/\sum_{j=1}^m W\left(MZ_j'\right)\right)}{\left(x_{j1},x_{j2},\ldots,x_{jn}\right)},\right.$$
$$\left.\int_Y \frac{\sum_{j=1}^m\left(W\left(MZ_j'\right)\mu_{B_j}(y)/\sum_{j=1}^m W\left(MZ_j'\right)\right)}{y}\right)$$
$$= MZPWA(MZ_1', MZ_2',\ldots,MZ_m'). \tag{23}$$

□

Theorem 4. *This theorem reflects the idempotency of the operator. Let MZ_1 and MZ_2 be two multidimensional Z-numbers, if $MZ_1 = MZ_1 = MZ$. Then, $MZPWA(MZ_1, M_2) = MZ$. The proof is similar to Theorem 3.*

Example 5. Let us consider the two three-dimensional Z-numbers $MZ_1 = (G_1, B_1)$ and $MZ_2 = (G_2, B_2)$ in Example 1; let function H equal to H_1, and it can be denoted that $MZ_3 = (G_3, B_3) = MZPWA(MZ_1, MZ_2)$. Because of $E(MZ_1) = 0.9948$, $E(MZ_2) = 0.9836$, then $W(MZ_1) = 0.5$, $W(MZ_2) = 1$; for the element (1, 1, 1), its membership value after fusion is $0.5/(1 + 0.5) \times 0.1 + 1/(1 + 0.5) \times 0.25 = 0.2$. In this way, the following results can be obtained:

$$G_3=\frac{0.2000}{(1,1,1)}+\frac{0.2833}{(1,1,2)}+\frac{0.5000}{(1,1,3)}+\frac{0.7167}{(1,2,1)}+\frac{0.8667}{(1,2,2)}+\frac{0.8667}{(1,2,3)}+\frac{0.8667}{(1,3,1)}+\frac{0.4667}{(1,3,2)}+\frac{0.2667}{(1,3,3)},$$
$$B_3=\frac{0.0667}{0}+\frac{0.2000}{0.1}+\frac{0.5667}{0.2}+\frac{0.8667}{0.3}+\frac{1.0000}{0.4}+\frac{0.7333}{0.5}+\frac{0.5667}{0.6}+\frac{0.4667}{0.7}+\frac{0.3333}{0.8}+\frac{0.1333}{0.9}+\frac{0.0667}{1}. \tag{24}$$

5. Distance Measure of Multidimensional Z-numbers about Their Order Weight Vector of Each Dimension

Distance measure is an important concept in fuzzy set and plays an important role in fuzzy decision-making and classification. Inspired by hamming distance, this paper proposes the distance measure of multidimensional Z-numbers about their order weight vector of each dimension as follows.

Definition 10. Let MZ_i and MZ_j be two continuous multidimensional Z-number and $\omega = \{\omega_1, \omega_2, \ldots, \omega_n\}$ be the order weight vector of each dimension of the multidimensional Z-number. The distance between MZ_i and MZ_j about the order weight vector ω is defined as follows:

$$d\left(MZ_i, MZ_j\right)=\sum_{k=1}^n \frac{\omega_k}{n}\int_{X_k}\left|\mu_{G_{ik}}\left(x_k\right)-\mu_{G_{jk}}\left(x_k\right)\right|\mathrm{d}x_k+\int_Y\left|\mu_{B_i}(y)-\mu_{B_j}(y)\right|\mathrm{d}y, \tag{25}$$

where X_k refers to the upper and lower limits of the multidimensional Z-number in the kth dimension; $\mu_{G_{ik}}(x_k) = \max\mu_{G_i}(g_{ik})$, x_k is value of the kth dimension of g_{ik}; in the same way $\mu_{G_{jk}}(x_k) = \max\mu_{G_j}(g_{jk})$, x_k is value of the kth dimension of g_{jk}.

Property 9. Let MZ_i, MZ_j, and MZ_t be three multidimensional Z-numbers, the distance measure defined in Definition 10 satisfies the following properties:

(1) $d(MZ_i, MZ_j) \geq 0$
(2) $d(MZ_i, MZ_j) = d(MZ_j, MZ_i)$
(3) if $MZ_i \leq MZ_j \leq MZ_t$, then $d(MZ_j, MZ_t) \leq d(MZ_i, MZ_t)$

Proof. It can be easily obtained that the distance $d(MZ_i, MZ_j)$ about the order weight vector ω satisfies (1) and (2) in Property 3, and the proof of (3) in Property 9 is as follows.

If $MZ_i \leq MZ_j \leq MZ_t$, it means that $G_i \subseteq G_j \subseteq G_t$ and $B_i \subseteq B_j \subseteq B_t$. With regard to the kth dimension of the multidimensional Z-numbers, it can be easily deduced that $\mu_{G_{ik}}(x_k) \leq \mu_{G_{jk}}(x_k) \leq \mu_{G_{tk}}(x_k)$ and $\mu_{B_i}(y) \leq \mu_{B_j}(y) \leq \mu_{B_t}(y)$. Thus, the following inequality is true:

$$\left|\mu_{G_{jk}}\left(x_k\right)-\mu_{G_{tk}}\left(x_k\right)\right|\leq\left|\mu_{G_{ik}}\left(x_k\right)-\mu_{G_{tk}}\left(x_k\right)\right|,\quad \left|\mu_{B_j}(y)-\mu_{B_t}(y)\right|\leq\left|\mu_{B_i}(y)-\mu_{B_t}(y)\right|. \tag{26}$$

Then, the following inequalities is true:

$$\begin{aligned}&\int_{X_k}\left|\mu_{G_{jk}}(x_k)-\mu_{G_{tk}}(x_k)\right|\mathrm{d}x_k\\&\le\int_{X_k}\left|\mu_{G_{jk}}(x_k)-\mu_{G_{tk}}(x_k)\right|\mathrm{d}x_k,\\&\int_{Y}\left|\mu_{B_i}(y)-\mu_{B_t}(y)\right|\mathrm{d}y\\&\le\int_{Y}\left|\mu_{B_j}(y)-\mu_{B_t}(y)\right|\mathrm{d}y.\end{aligned}\tag{27}$$

Therefore, according to abovementioned inequalities, when $MZ_i \le MZ_j \le MZ_t$, $d(MZ_j, MZ_t) \le d(MZ_i, MZ_t)$ can be proved.

□

6. A GDM Method Using Multidimensional Z-numbers and TOPSIS Method

In this section, a novel group decision-making method is developed by combining multidimensional Z-numbers and TOPSIS method.

Group decision-making problems in the multidimensional Z-number environment consist of a group of alternatives, denoted by $A_j\,(1 \le i \le m)$; a multidimensional Z-number has n dimensions; $\omega = \{\omega_1, \omega_2, \ldots, \omega_n\}$ is the order weight vector of each dimension of the multidimensional Z-number. For any ω_i that $\omega_i \in [0,1]$, $\sum_{i=1}^{n}\omega_i = 1$. Let $D = \{d_1, d_2, \ldots, d_q\}$ be a set of decision makers. All evaluation information is presented by multidimensional Z-numbers, and the evaluation information of decision maker d_k on alternative A_j is denoted as MZ_j^k.

Combining power weighted average operator, distance measure of multidimensional Z-numbers about the order weight vector ω, and TOPSIS method, this paper develops a new GDM method. The procedures can be described as follows.

Step 1. Normalize the fuzzy evaluation information.

The scale of evaluation information varies in different dimensions, so it is difficult to compare. If the value of ith dimension increases, a negative effect on the evaluation of the alternative occurs; we call it positive dimension, denoted as $i \in P$. If the value of ith dimension increases, a negative effect on the evaluation of the alternative occurs; we call it negative dimension, denoted as $i \in N$. x_{ji} represents the value of the ith dimension of the multidimensional Z-number MZ_j. In this paper, the evaluation information can be normalized as follows:

$$\widehat{x}_{ji} = \frac{x_{ji} - \min_j x_{ji}}{\max_j x_{ji} - \min_j x_{ji}}, \quad i \in P, \tag{28}$$

$$\widehat{x}_{ji} = \frac{\max_j x_{ji} - x_{ji}}{\max_j x_{ji} - \min_j x_{ji}}, \quad i \in N. \tag{29}$$

Step 2. Obtain the comprehensive evaluation information.

For alternative $A_j\,(1 \le i \le m)$, the evaluation information of q experts are written as $MZ_j^1, MZ_j^2, \ldots, MZ_j^q$. Comprehensive evaluation information for this alternative can be obtained by Definition 9 and denoted as MZ_j. The process is as follows:

$$MZ_j = MZPWA\left(MZ_j^1, MZ_j^2, \ldots, MZ_j^q\right). \tag{30}$$

Step 3. Computer the order weight vector $\omega = \{\omega_1, \omega_2, \ldots, \omega_n\}$ of each dimension of multidimensional Z-numbers.

Because the importance of each dimension is unknown, the maximizing deviation method can be used to solve this problem. In the decision-making process, if there is a big difference in one dimension, it is obviously easier to distinguish the alternatives, so it can be given more weight. Based on this idea, the following linear programming model is given in this paper to determine the order weight of each dimension.

Taking into account the comprehensive evaluation information $MZ_1, MZ_2, \ldots, MZ_m$, a maximizing deviation model can be established to derive the order weight of each dimension using distance measure as follows:

$$\begin{aligned}&\min F(\omega_k) = \sum_{i=1}^{m}\sum_{j=1, j\ne i}\frac{1 - d\left(MZ_i, MZ_j\right)}{2}\\&\text{s.t.}\begin{cases}\sum_{k=1}^{n}\omega_k = 1,\\ \omega_k \ge 0,\end{cases}\end{aligned}\tag{31}$$

where $d(MZ_i, MZ_j)$ is the distance measure between multidimensional Z-number MZ_i and MZ_j about the order weight vector ω and ω_k represents the order weight of the kth dimension of the multidimensional Z-numbers.

Step 4. Determine the positive ideal MZ^+ solution and negative ideal solution MZ^-.

In this step, the positive and negative ideal solution of the decision index is determined. The comprehensive evaluation information $\{MZ_1, MZ_2, \ldots, MZ_m\}$ is a set of multidimensional Z-numbers. $x_{ji}(\mu_G(g))$ means that the $\mu_G(g)$-cut set of the ith dimension of the multidimensional Z-number MZ_j, where the value of $\mu_G(g)$ is from 0 to 1. The positive and negative ideal solution of the set can be obtained according to the following equations:

$$\begin{aligned}MZ^+ = &\left(\left(\int\frac{\mu_G(g)}{\max_j \sup x_{j1}(\mu_G(g)), \ldots, \max_j \sup x_{jn}(\mu_G(g))},\right.\right.\\&\left.\int\frac{\mu_B(y)}{\max_j \sup y_j(\mu_B(y))}\right),\end{aligned}\tag{32}$$

where $\int$ is a representation of a fuzzy set, the value of $\mu_B(y)$ is from 0 to 1, $\sup y_j(\mu_B(y))$ refers to the supremum of the $\mu_G(g)$-cut set of the reliability measure B of the multidimensional Z-number MZ_j, and $\sup x_{ji}(\mu_G(g))$ refers to the supremum of the $\mu_G(g)$-cut set of the ith dimension of the multidimensional Z-number MZ_j.

$$MZ^- = \left(\int \frac{\mu_G(g)}{\min_j \inf x_{j1}(\mu_G(g)), \ldots, \min_j \inf x_{jn}(\mu_G(g))}, \int \frac{\mu_B(y)}{\min_j \inf y_j(\mu_B(y))}\right), \tag{33}$$

where $\int$ is a representation of a fuzzy set, the value of $\mu_B(y)$ is from 0 to 1, $\inf y_j(\mu_B(y))$ refers to the infimum of the $\mu_G(g)$-cut set of the reliability measure B of the multidimensional Z-number MZ_j, and $\inf x_{ji}(\mu_G(g))$ refers to the infimum of the $\mu_G(g)$-cut set of the ith dimension of the multidimensional Z-number MZ_j.

Step 5. Rank all the alternatives.

In this step, positive and negative ideal solutions already have been obtained. Following the steps of the TOPSIS method, the distance between each alternative can be acquired using the distance measure defined in Definition 10. The distance between alternative A_j and the positive ideal solution about their order weight vector of each dimension is calculated as follows:

$$D_j^+ = d\left(MZ_j, MZ^+\right). \tag{34}$$

The distance between alternative A_j and the negative ideal solution about their order weight vector of each dimension is calculated as follows:

$$D_j^- = d\left(MZ_j, MZ^-\right). \tag{35}$$

Then, the closeness coefficient of alternative A_j can be obtained:

$$D_j = \frac{D_j^-}{D_j^+ + D_j^-}. \tag{36}$$

The ranking of all alternatives can be obtained by the closeness coefficient D_j; the larger the value of D_j, the better the alternative A_j will be. The main process of the decision-making method is shown in Figure 4.

7. Illustrative Example

In this section, an example is given, combining comparative analysis and sensitivity analysis to verify the effectiveness and superiority of the proposed method jointly.

7.1. A Practical Example. Cloud services are patterns of growth, use, and interaction of internet-based related services; it often involves providing dynamically scalable and often virtualized resources over the Internet. Cloud is a metaphor for the Internet. The cloud used to be used in diagrams to represent the telecommunications network, and later it is used to represent abstractions of the Internet and underlying infrastructure. Cloud services are services that are available in an on-demand, scalable way over the network. This service can be related to IT, software, or other services. It means that computing power can also flow as a commodity over the Internet.

The emergence of cloud services cater to the development of network technology and the service needs of the customer. Therefore, how to choose the right cloud service provider is very important. After market research and preliminary screening, there are four potential cloud services A_1, A_2, A_3, and A_4 that need to be evaluated. The assessment team consists of three experts denoted by d_1, d_2, and d_3. They evaluate the cloud service provider in two dimensions: security and accuracy. The order weight vector of each dimension is $\omega = \{\omega_1, \omega_2\}$. x_{ji} represents the value of the ith dimension of the multidimensional Z-number MZ_j.

In this case, the fuzziness measure function of the multidimensional fuzzy set is $H = H_1$. The evaluation information of experts is shown in Table 1.

Step 6. Normalize the fuzzy evaluation information.

Because all the dimensions have a positive effect on the evaluation of alternatives, when the value of i_{th} dimension increases, all the evaluation information can be normalized by equation (28). The normalized evaluation information is denoted as $R^k = (MZ_j^{'k})_{1\times m}$.

Step 7. Obtain the comprehensive evaluation information.

The evaluation information given by the three experts is integrated by the $MZPWA$ operator, and the comprehensive evaluation information is shown in Table 2.

Step 8. Compute the order weight vector $\omega = \{\omega_1, \omega_2, \ldots, \omega_n\}$ of each dimension of multidimensional Z-numbers.

According to linear programming model (31), it can be obtained that $\omega = (0.3767, 0.6233)$.

Step 9. Determine the positive ideal MZ^+ solution and negative ideal solution MZ^-.

According to equations (32) and (33). The positive and negative ideal solutions of comprehensive evaluation information can be obtained as follows: $MZ^+ = (G^+, B^+)$, $MZ^- = (G^-, B^-)$, where

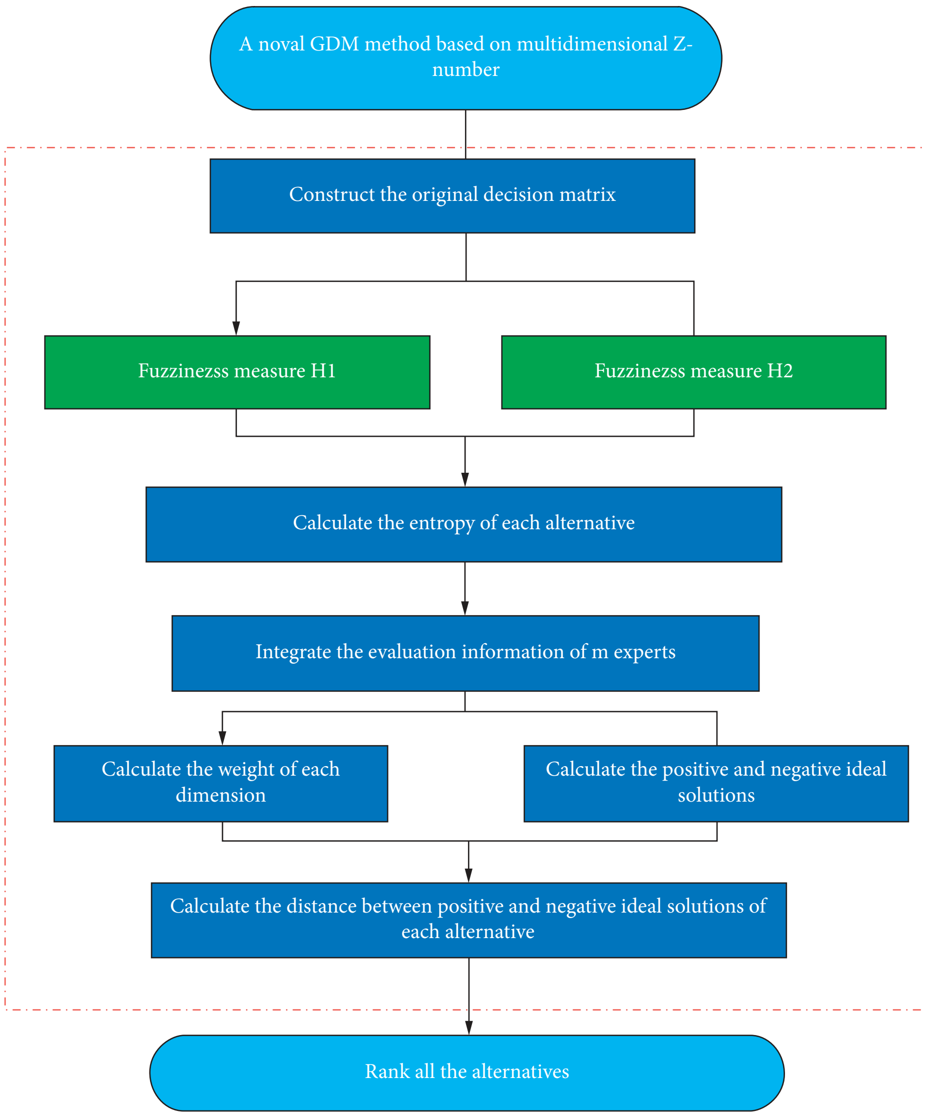

Figure 4: Procedure of the TOPSIS method based on multidimensional Z-number.

$$\begin{aligned}
G^{+} &= \frac{0}{(1/3,1/4)} + \frac{0}{(1/3,2/4)} + \frac{0.0596}{(1/3,3/4)} + \frac{0.2781}{(1/3,4/4)} + \frac{0}{(2/3,1/4)} + \frac{0.1720}{(2/3,2/4)} \\
&\quad + \frac{0.3797}{(2/3,3/4)} + \frac{0.6660}{(2/3,4/4)} + \frac{0.2440}{(3/3,1/4)} + \frac{0.5180}{(3/3,2/4)} + \frac{0.8303}{(3/3,3/4)} + \frac{1}{(3/3,4/4)}, \\
B^{+} &= \frac{0.1681}{0.6} + \frac{0.3267}{0.65} + \frac{0.5069}{0.7} + \frac{0.6290}{0.75} + \frac{0.7152}{0.8} + \frac{0.8653}{0.85} + \frac{0.7471}{0.9} + \frac{0.8569}{0.95} + \frac{0.4654}{0.1}, \\
G^{-} &= \frac{0}{(1,1)} + \frac{0.5655}{(1/3,2/4)} + \frac{0.1974}{(1/3,3/4)} + \frac{0}{(1/3,4/4)} + \frac{0.6543}{(2/3,1/4)} + \frac{1}{(2/3,2/4)} + \frac{0.6800}{(2/3,3/4)} \\
&\quad + \frac{0.3621}{(2/3,4/4)} + \frac{0}{(3/3,1/4)} + \frac{0.3655}{(3/3,2/4)} + \frac{0.1100}{(3/3,3/4)} + \frac{0}{(3/3,4/4)}, \\
B^{-} &= \frac{0.1602}{0.6} + \frac{0.2742}{0.65} + \frac{0.4316}{0.7} + \frac{0.5351}{0.75} + \frac{0.5894}{0.8} + \frac{0.7337}{0.85} + \frac{0.6772}{0.9} + \frac{0.5351}{0.95} + \frac{0.3266}{1}.
\end{aligned} \tag{37}$$

Table 1: Evaluation information of four alternatives under multidimensional Z-number environment.

d_1	A_1	$MZ_1^1 = (G_1^1, B_1^1)$	$G_1^1 = (0/(1,1)) + (0.35/(1,2)) + (0.1/(1,3)) + (0/(1,4)) + (0.75/(2,1)) + (1/(2,2)) + (0.6/(2,3)) + (0.4/(2,4)) + (0/(3,1)) + (0.3/(3,2)) + (0/(3,3)) + (0/(3,4))$	$B_1^1 = (0.12/0.6) + (0.25/0.65) + (0.43/0.7) + (0.66/0.75) + (0.85/0.8) + (1/0.85) + (0.73/0.9) + (0.46/0.95) + (0.22/1)$
	A_2	$MZ_2^1 = (G_2^1, B_2^1)$	$G_2^1 = (0/(1,1)) + (0.25/(1,2)) + (0/(1,3)) + (0/(1,4)) + (0.33/(2,1)) + (0.56/(2,2)) + (0.45/(2,3)) + (0.2/(2,4)) + (0.83/(3,1)) + (1/(3,2)) + (0.64/(3,3)) + (0.21/(3,4))$	$B_2^1 = (0.21/0.6) + (0.47/0.65) + (0.69/0.7) + (1/0.75) + (0.84/0.8) + (0.77/0.85) + (0.52/0.9) + (0.34/0.95) + (0.16/1)$
	A_3	$MZ_3^1 = (G_3^1, B_3^1)$	$G_3^1 = (0.14/(1,1)) + (0.25/(1,2)) + (0.75/(1,3)) + (1/(1,4)) + (0/(2,1)) + (0/(2,2)) + (0.35/(2,3)) + (0.68/(2,4)) + (0/(3,1)) + (0/(3,2)) + (0.27/(3,3)) + (0.54/(3,4))$	$B_3^1 = (0.15/0.6) + (0.22/0.65) + (0.36/0.7) + (0.47/0.75) + (0.52/0.8) + (0.66/0.85) + (0.81/0.9) + (1/0.95) + (0.55/1)$
	A_4	$MZ_4^1 = (G_4^1, B_4^1)$	$G_4^1 = (0/(1,1)) + (0/(1,2)) + (0/(1,3)) + (0.28/(1,4)) + (0/(2,1)) + (0.15/(2,2)) + (0.33/(2,3)) + (0.69/(2,4)) + (0.23/(3,1)) + (0.46/(3,2)) + (0.88/(3,3)) + (1/(3,4))$	$B_4^1 = (0.21/0.6) + (0.47/0.65) + (0.69/0.7) + (1/0.75) + (0.84/0.8) + (0.77/0.85) + (0.52/0.9) + (0.34/0.95) + (0.16/1)$
d_2	A_1	$MZ_1^2 = (G_1^2, B_1^2)$	$G_1^2 = (0/(1,1)) + (0.64/(1,2)) + (0.22/(1,3)) + (0/(1,4)) + (0.67/(2,1)) + (1/(2,2)) + (0.72/(2,3)) + (0.32/(2,4)) + (0/(3,1)) + (0.42/(3,2)) + (0.11/(3,3)) + (0/(3,4))$	$B_1^2 = (0.15/0.6) + (0.22/0.65) + (0.36/0.7) + (0.47/0.75) + (0.52/0.8) + (0.66/0.85) + (0.81/0.9) + (1/0.95) + (0.55/1)$
	A_2	$MZ_2^2 = (G_2^2, B_2^2)$	$G_2^2 = (0/(1,1)) + (0.22/(1,2)) + (0/(1,3)) + (0/(1,4)) + (0.43/(2,1)) + (0.76/(2,2)) + (0.36/(2,3)) + (0.15/(2,4)) + (0.74/(3,1)) + (1/(3,2)) + (0.66/(3,3)) + (0.25/(3,4))$	$B_2^2 = (0.12/0.6) + (0.25/0.65) + (0.43/0.7) + (0.66/0.75) + (0.85/0.8) + (1/0.85) + (0.73/0.9) + (0.46/0.95) + (0.22/1)$
	A_3	$MZ_3^2 = (G_3^2, B_3^2)$	$G_3^2 = (0.16/(1,1)) + (0.38/(1,2)) + (0.74/(1,3)) + (1/(1,4)) + (0/(2,1)) + (0/(2,2)) + (0.40/(2,3)) + (0.70/(2,4)) + (0/(3,1)) + (0/(3,2)) + (0.29/(3,3)) + (0.56/(3,4))$	$B_3^2 = (0.21/0.6) + (0.47/0.65) + (0.69/0.7) + (1/0.75) + (0.84/0.8) + (0.77/0.85) + (0.52/0.9) + (0.34/0.95) + (0.16/1)$
	A_4	$MZ_4^2 = (G_4^2, B_4^2)$	$G_4^2 = (0/(1,1)) + (0/(1,2)) + (0.12/(1,3)) + (0.26/(1,4)) + (0/(2,1)) + (0.17/(2,2)) + (0.42/(2,3)) + (0.66/(2,4)) + (0.24/(3,1)) + (0.52/(3,2)) + (0.79/(3,3)) + (1/(3,4))$	$B_4^2 = (0.15/0.6) + (0.22/0.65) + (0.36/0.7) + (0.47/0.75) + (0.52/0.8) + (0.66/0.85) + (0.81/0.9) + (1/0.95) + (0.55/1)$
d_3	A_1	$MZ_1^3 = (G_1^3, B_1^3)$	$G_1^3 = (0/(1,1)) + (0.62/(1,2)) + (0.24/(1,3)) + (0/(1,4)) + (0.55/(2,1)) + (1/(2,2)) + (0.68/(2,3)) + (0.40/(2,4)) + (0/(3,1)) + (0.33/(3,2)) + (0.20/(3,3)) + (0/(3,4))$	$B_1^3 = (0.21/0.6) + (0.47/0.65) + (0.69/0.7) + (1/0.75) + (0.84/0.8) + (0.77/0.85) + (0.52/0.9) + (0.34/0.95) + (0.16/1)$
	A_2	$MZ_2^2 = (G_2^2, B_2^2)$	$G_2^3 = (0/(1,1)) + (0.18/(1,2)) + (0/(1,3)) + (0.11/(1,4)) + (0.45/(2,1)) + (0.82/(2,2)) + (0.40/(2,3)) + (0.12/(2,4)) + (0.69/(3,1)) + (1/(3,2)) + (0.54/(3,3)) + (0.23/(3,4))$	$B_2^3 = (0.15/0.6) + (0.22/0.65) + (0.36/0.7) + (0.47/0.75) + (0.52/0.8) + (0.66/0.85) + (0.81/0.9) + (1/0.95) + (0.55/1)$
	A_3	$MZ_3^3 = (G_3^3, B_3^3)$	$G_3^3 = (0.13/(1,1)) + (0.44/(1,2)) + (0.67/(1,3)) + (1/(1,4)) + (0/(2,1)) + (0/(2,2)) + (0.52/(2,3)) + (0.76/(2,4)) + (0/(3,1)) + (0/(3,2)) + (0.34/(3,3)) + (0.63/(3,4))$	$B_3^3 = (0.15/0.6) + (0.22/0.65) + (0.36/0.7) + (0.47/0.75) + (0.52/0.8) + (0.66/0.85) + (0.81/0.9) + (1/0.95) + (0.55/1)$
	A_4	$MZ_4^2 = (G_4^2, B_4^2)$	$G_4^2 = (0/(1,1)) + (0/(1,2)) + (0.06/(1,3)) + (0.31/(1,4)) + (0/(2,1)) + (0.22/(2,2)) + (0.40/(2,3)) + (0.63/(2,4)) + (0.28/(3,1)) + (0.63/(3,2)) + (0.81/(3,3)) + (1/(3,4))$	$B_4^3 = (0.12/0.6) + (0.25/0.65) + (0.43/0.7) + (0.66/0.75) + (0.85/0.8) + (1/0.85) + (0.73/0.9) + (0.46/0.95) + (0.22/1)$

Table 2: Comprehensive evaluation information of the four alternatives.

A_1	$MZ_1 = (G_1, B_1)$	$G_1 = (0/(1/3,1/4)) + (0.5655/(1/3,2/4)) + (0.1974/(1/3,3/4)) + (0/(1/3,4/4)) + (0.6543/(2/3,1/4)) + (1/(2/3,2/4)) + (0.6800/(2/3,3/4)) + (0.3621/(2/3,4/4)) + (0/(3/3,1/4)) + (0.3655/(3/3,2/4)) + (0.1100/(3/3,3/4)) + (0/(3/3,4/4))$	$B_1 = (0.1602/0.6) + (0.2994/0.65) + (0.4720/0.7) + (0.6030/0.75) + (0.6907/0.8) + (0.8401/0.85) + (0.7072/0.9) + (0.6812/0.95) + (0.3590/1)$
A_2	$MZ_2 = (G_2, B_2)$	$G_2 = (0/(1/3,1/4)) + (0.2134/(1/3,2/4)) + (0/(1/3,3/4)) + (0.0547/(1/3,4/4)) + (0.4032/(2/3,1/4)) + (0.7149/(2/3,2/4)) + (0.4091/(2/3,3/4)) + (0.1548/(2/3,4/4)) + (0.7504/(3/3,1/4)) + (1/(3/3,2/4)) + (0.6013/(3/3,3/4)) + (0.2272/(1/3,4/4))$	$B_2 = (0.1648/0.6) + (0.3151/0.65) + (0.4921/0.7) + (0.6172/0.75) + (0.7044/0.8) + (0.8537/0.85) + (0.6900/0.9) + (0.6498/0.95) + (0.3407/1)$
A_3	$MZ_3 = (G_3, B_3)$	$G_3 = (0.1408/(1/3,1/4)) + (0.3446/(1/3,2/4)) + (0.7199/(1/3,3/4)) + (1/(1/3,4/4)) + (0/(2/3,1/4)) + (0/(2/3,2/4)) + (0.4202/(2/3,3/4)) + (0.7123/(2/3,4/4)) + (0/(3/3,1/4)) + (0/(3/3,2/4)) + (0.2988/(3/3,3/4)) + (0.5758/(3/3,4/4))$	$B_3 = (0.1630/0.6) + (0.2742/0.65) + (0.4316/0.7) + (0.5351/0.75) + (0.5894/0.8) + (0.7337/0.85) + (0.7471/0.9) + (0.8569/0.95) + (0.4654/1)$
A_4	$MZ_4 = (G_4, B_4)$	$G_4 = (0/(1/3,1/4)) + (0/(1/3,2/4)) + (0.0596/(1/3,3/4)) + (0.2781/(1/3,4/4)) + (0/(2/3,1/4)) + (0.1720/(2/3,2/4)) + (0.3797/(2/3,3/4)) + (0.6660/(2/3,4/4)) + (0.2440/(3/3,1/4)) + (0.5180/(3/3,2/4)) + (0.8303/(3/3,3/4)) + (1/(3/3,4/4))$	$B_4 = (0.1681/0.6) + (0.3267/0.65) + (0.5069/0.7) + (0.6290/0.75) + (0.7152/0.8) + (0.8653/0.85) + (0.6772/0.9) + (0.6257/0.95) + (0.3266/1)$

Step 10. Rank all the alternatives.

By equations (34)–(36), the distance of each alternative and the positive/negative ideal solution can be obtained as Table 3.

By comparing the values of closeness coefficient, the priority of the alternatives is as follows:

$$A_1 \prec A_3 \prec A_2 \prec A_4. \quad (38)$$

Therefore, the best result is alternative A_4 and the worst result is alternative A_1.

7.2. Comparison with Other Existing Methods

7.2. Comparison with Other Existing Methods. In order to testify the feasibility and effectiveness of the approach which are proposed in this paper, some previous methods are used to compare the ranking results. However, few scholars have carried out the group decision-making problem under the multidimensional Z-number environment before, so we have to change the data of the multidimensional Z-number to the ordinary Z-number. Making each dimension of the multidimensional Z-number an attribute of the evaluation alternative; it means that $((A_1, A_2, \ldots, A_n), B)$ will become $(A_1', B), (A_2', B), \ldots, (A_n', B)$, where A_i and A_i' have the same elements, and the maximum membership value corresponding to an element in A_i is the membership value of A_i'. For example, we are going to change $(0.2/(1,1) + 0.5/(1,2) + 0.4/(2,1) + 1/(2,2), 0.1/0.7 + 0.1/0.8 + 0.3/0.9)$ into $(0.5/1 + 1/2, 0.1/0.7 + 0.1/0.8 + 0.3/0.9)$ and $(0.4/1 + 1/2, 0.1/0.7 + 0.1/0.8 + 0.3/0.9)$.

Considering the randomness and fuzziness of Z-number, Shen and Wang [26] defined the comprehensive weighted distance measure of Z-number and presented a novel fuzzy VIKOR decision-making method based on Z-number context. As the goal is maximum entropy, Qiao et al. [34] developed a new linear programming model for obtaining underlying probability distribution and used it to construct a comprehensive weighted crossentropy. Based on it, one extended TOPSIS approach was developed to solve a multicriteria decision-making problem under discrete Z-context. Yao et al. [40] defined three different measurements dominance degree from three levels of geometry, algebra, and crossentropy based on the outranking relationship and established a multiattribute decision model on the basis of new grey association analysis and QUALIFLEX method. For the changed data in the example, applying the abovementioned three methods, the results are shown in Table 4.

It can be seen that the ranking of the four methods are essentially in agreement, and the results in [34] are completely consistent with the results of the proposed method. The results obtained by the other two methods are slightly different from the proposed method, which may be because the preference of the decision maker has different influence on the results in different methods; transforming multidimensional Z-number to Z-number has the loss of information that will also affect the ranking. To synthesize the ranking results of the four methods, we add the ranking values of the four methods, and the results are shown in Figure 5; it can be seen that the value of the sum from small to large, and their corresponding alternatives are A_1, A_3, A_2, and A_4. From the comparison above, the result illustrates the approach proposed in this paper is reasonable and scientific.

Table 3: The closeness coefficient of four alternatives.

Alternatives	D^+	D^-	D
A_1	0.5788	0.0732	0.1123
A_2	0.3659	0.4643	0.5593
A_3	0.3175	0.2793	0.3505
A_4	0.0802	0.5374	0.8702

Table 4: Ranking results obtained by different methods.

Method	Ranking results
Method in [26]	$A_3 \prec A_1 \prec A_4 \prec A_2$
Method in [34]	$A_1 \prec A_3 \prec A_2 \prec A_4$
Method in [40]	$A_2 \prec A_1 \prec A_3 \prec A_4$
Proposed method	$A_1 \prec A_3 \prec A_2 \prec A_4$

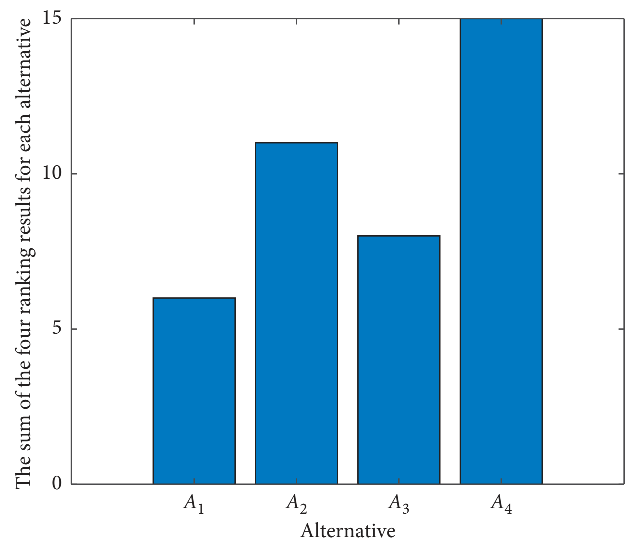

Figure 5: The sum of the ranking values of the four methods.

7.3. Sensitivity Analysis. In order to explore the effects of the different fuzziness of multidimensional fuzzy set, this paper substitutes H_1 with H_2 and applies the abovementioned example to make group decision again. The comparison of the closeness coefficient between each alternative and the ideal solution is shown in Figure 6.

As shown in Figure 6, the best alternative and the worst alternative, A_1 and A_4, do not change. However, the rank of A_2 and A_3 changes. Furthermore, the values of closeness coefficient under fuzziness measure H_2 is roughly bigger than that which is under fuzziness measure H_1. Therefore, it can be found that the ranking results will be affected by the selection of fuzziness measure.

The reason why Figure 6 appears is probably because H_1 and H_2 are considered in two different aspects, and with the

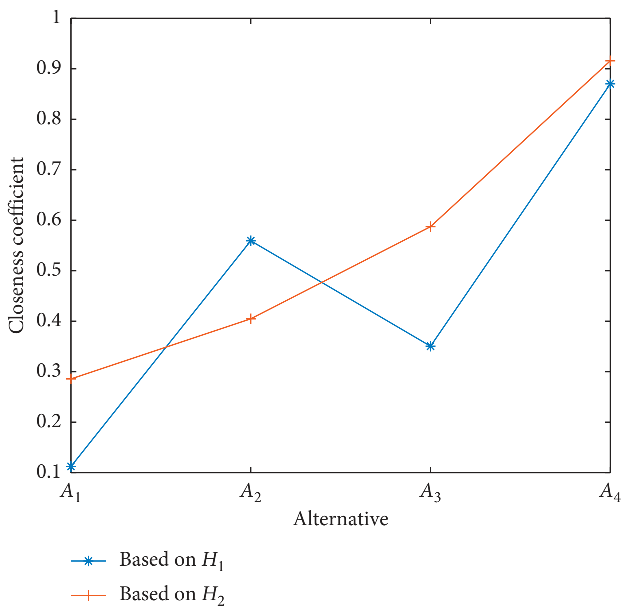

Figure 6: Closeness coefficient under different measures of fuzziness.

change of membership degree of multidimensional fuzzy set, the fuzziness value changes in different ways. For H_1, as the membership value changes from the middle to both ends, the value of fuzziness measure decreases and the rate of change also decreases. For H_2, as the membership value changes from the middle to both ends, the value of fuzziness measure decreases but the rate of change increases.

The preceding analysis reveals that different uncertainty measures will affect the ranking results; in the specific decision-making problem, decision makers can select appropriate fuzziness measures to ensure that the results are reasonable and accurate.

8. Conclusion

In order to solve the problem of group decision-making in the information environment of multidimensional Z-number, firstly, this study proposes two kinds of measure of fuzziness for multidimensional fuzzy set from the perspective of algebra and geometry, respectively. Then, the entropy for multidimensional Z-number is established by incorporating the inherent fuzziness of fuzzy restriction and reliability measure and the fuzziness of reliability level of the reliability measure. Next, the entropy is used to construct an average operator of multidimensional Z-numbers. In addition, a distance measure is introduced to measure the distance between two multidimensional Z-numbers about their order weight vector of each dimension. Furthermore, the group decision model in the multidimensional Z-number environment is constructed by combining the average operator with the TOPSIS decision-making method. Finally, the proposed method is applied to the issue of cloud service provider selection as an illustrative example to verify the feasibility and effectiveness of the proposed method.

This paper develops two kinds of fuzziness measure, which can be used to measure the fuzziness of the multidimensional fuzzy set from different aspects, and it is a complement to the measure of fuzziness of the multidimensional fuzzy set. The entropy for multidimensional Z-number fills the gap of uncertainty measure of multidimensional Z-number. The developed group decision-making method provides a solution to the decision-making problems under the multidimensional Z-number environment.

Further research will establish the dominance relationship and the crossentropy of multidimensional Z-number and develop other decision-making methods under multidimensional Z-number environment in the future.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

Acknowledgments

The work was supported by the National Nature Science Foundation of China (no. 71871001), National Nature Science Foundation for Young of China (no. 61806001), Anhui Provincial Natural Science Foundation of China, and Project of Graduate Academic Innovation of Anhui University.

References

[1] L. A. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, no. 3, pp. 338–353, 1965.
[2] K. T. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 20, no. 1, pp. 87–96, 1986.
[3] V. Torra, "Hesitant fuzzy sets," *International Journal of Intelligent Systems*, vol. 25, no. 6, pp. 529–539, 2010.
[4] M. Mizumoto and K. Tanaka, "Some properties of fuzzy sets of type 2," *Information and Control*, vol. 31, no. 4, pp. 312–340, 1976.
[5] F. E. Boran, S. Genç, M. Kurt, and D. Akay, "A multi-criteria intuitionistic fuzzy group decision making for supplier selection with TOPSIS method," *Expert Systems with Applications*, vol. 36, no. 8, pp. 11363–11368, 2009.
[6] H.-W. Liu and G.-J. Wang, "Multi-criteria decision-making methods based on intuitionistic fuzzy sets," *European Journal of Operational Research*, vol. 179, no. 1, pp. 220–233, 2007.
[7] M. Xia and Z. Xu, "Hesitant fuzzy information aggregation in decision making," *International Journal of Approximate Reasoning*, vol. 52, no. 3, pp. 395–407, 2011.
[8] S. P. Wan, F. Wang, and J. Y. Dong, "A group decision-making method considering both the group consensus and multiplicative consistency of interval-valued intuitionistic fuzzy preference relations," *Information Sciences*, vol. 466, pp. 109–128, 2018.
[9] S.-M. Chen and L.-W. Lee, "Fuzzy multiple attributes group decision-making based on the interval type-2 TOPSIS

method," *Expert Systems with Applications*, vol. 37, no. 4, pp. 2790–2798, 2010.

[10] D. Wu and J. M. Mendel, "A comparative study of ranking methods, similarity measures and uncertainty measures for interval type-2 fuzzy sets," *Information Sciences*, vol. 179, no. 8, pp. 1169–1192, 2009.

[11] K. Mei, Y. Peng, and J. Mao, "Comprehensive cross-entropy and entropy measures and their applications under interval type-2 fuzzy environment," *Journal of Intelligent & Fuzzy Systems*, vol. 37, no. 4, pp. 5143–5156, 2019.

[12] S. Wan, F. Wang, and J. Dong, "A group decision making method with interval valued fuzzy preference relations based on the geometric consistency," *Information Fusion*, vol. 40, pp. 87–100, 2018.

[13] L. A. Zadeh, "A note on z-numbers," *Information Sciences*, vol. 181, no. 14, pp. 2923–2932, 2011.

[14] R. A. Aliev, A. V. Alizadeh, and O. H. Huseynov, "The arithmetic of discrete Z-numbers," *Information Sciences*, vol. 290, pp. 134–155, 2015.

[15] R. A. Aliev, O. H. Huseynov, and L. M. Zeinalova, "The arithmetic of continuous Z-numbers," *Information Sciences*, vol. 373, pp. 441–460, 2016.

[16] B. Y. Kang, D. J. Wei, Y. Deng, and Y. Li, "A method of converting Z-number to classic fuzzy number," *Journal of Information and Computational Science*, vol. 9, no. 3, pp. 703–709, 2012.

[17] R. R. Yager and R. Ronald, "On Z-valuations using Zadeh's Z-numbers," *International Journal of Intelligent Systems*, vol. 27, no. 3, pp. 259–278, 2012.

[18] R. Banerjee and S. K. Pal, "A computational model for the endogenous arousal of thoughts through $Z*$-numbers," *Information Sciences*, vol. 405, pp. 227–258, 2017.

[19] R. Banerjee and S. K. Pal, "$Z*$-numbers: augmented Z-numbers for machine-subjectivity representation," *Information Sciences*, vol. 323, pp. 143–178, 2015.

[20] T. Allahviranloo and S. Ezadi, "Z-Advanced numbers processes," *Information Sciences*, vol. 480, pp. 130–143, 2019.

[21] S. K. Pal, R. Banerjee, S. Dutta, and S. S. Sarma, "An insight into the Z-number approach to CWW," *Fundamenta Informaticae*, vol. 124, no. 1-2, pp. 197–229, 2013.

[22] P. Patel, E. S. Khorasani, and S. Rahimi, "Modeling and implementation of Z-number," *Soft Comput*, vol. 20, no. 4, pp. 1–24, 2015.

[23] S. Cateni, M. Vannucci, and V. Colla, "Industrial multiple criteria decision making problems handled by means of fuzzy inference-based decision support systems," in *Proceedings of the International Conference on Intelligent Systems, Modelling and Simulation*, pp. 12–17, Bangkok, Thailand, January 2013.

[24] V. Khatibi and G. A. Montazer, "Intuitionistic fuzzy set vs. fuzzy set application in medical pattern recognition," *Artificial Intelligence in Medicine*, vol. 47, no. 1, pp. 43–52, 2009.

[25] J.-Q. Wang, Y.-X. Cao, and H.-Y. Zhang, "Multi-criteria decision-making method based on distance measure and choquet integral for linguistic Z-numbers," *Cognitive Computation*, vol. 9, no. 6, pp. 827–842, 2017.

[26] K.-W. Shen and J.-Q. Wang, "Z-VIKOR method based on a new comprehensive weighted distance measure of Z-number and its application," *IEEE Transactions on Fuzzy Systems*, vol. 26, no. 6, pp. 3232–3245, 2018.

[27] H. G. Peng and J. Q. Wang, "Outranking decision-making method with Z-number cognitive information," *Cognitive Computation*, vol. 10, no. 5, pp. 752–768, 2018.

[28] H. G. Peng and J. Q. Wang, "A multicriteria decision-making method for cloud service selection and ranking," *IEEE Transctions on Fuzzy Systems*, vol. 26, no. 6, pp. 139–147, 2018.

[29] H.-G. Peng, H.-Y. Zhang, J.-Q. Wang, and L. Li, "An uncertain Z-number multicriteria group decision-making method with cloud models," *Information Sciences*, vol. 501, pp. 136–154, 2019.

[30] S. Zeng and Y. Xiao, "A method based on TOPSIS and distance measures for hesitant fuzzy multiple attribute decision making," *Technological and Economic Development of Economy*, vol. 24, no. 3, pp. 969–983, 2018.

[31] C.-T. Chen, "Extensions of the TOPSIS for group decision-making under fuzzy environment," *Fuzzy Sets and Systems*, vol. 114, no. 1, pp. 1–9, 2000.

[32] S. Zeng, S.-M. Chen, and K.-Y. Fan, "Interval-valued intuitionistic fuzzy multiple attribute decision making based on nonlinear programming methodology and TOPSIS method," *Information Sciences*, vol. 506, pp. 424–442, 2020.

[33] A. M. Yaakob and A. Gegov, "Interactive TOPSIS based group decision making methodology using Z-numbers," *International Journal of Computational Intelligence Systems*, vol. 9, no. 2, pp. 311–324, 2016.

[34] D. Qiao, X.-K. Wang, J.-Q. Wang, and K. Chen, "Cross entropy for discrete z-numbers and its application in multicriteria decision-making," *International Journal of Fuzzy Systems*, vol. 21, no. 6, pp. 1786–1800, 2019.

[35] C. E. Shannon, "A mathematical theory of communication," *Bell System Technical Journal*, vol. 27, no. 3, pp. 379–423, 1948.

[36] J. Mao, D. Yao, and C. Wang, "A novel cross-entropy and entropy measures of ifss and their applications," *Knowledge-Based Systems*, vol. 48, no. 2, pp. 37–45, 2013.

[37] B. Kang, Y. Deng, K. Hewage, and R. Sadiq, "A method of measuring uncertainty for Z-number," *IEEE Transactions on Fuzzy Systems*, vol. 27, no. 4, pp. 731–738, 2019.

[38] K.-W. Shen, J.-Q. Wang, and T.-L. Wang, "The arithmetic of multidimensional z-number," *Journal of Intelligent & Fuzzy Systems*, vol. 36, no. 2, pp. 1647–1661, 2019.

[39] A. D. Luca and S. Termini, "A definition of a non-probabilistic entropy in the setting of fussy set theory," *Information and Control*, vol. 20, pp. 301–312, 1972.

[40] A. T. Yao, J. J. Mao, M. C. Wu, and T. Wu, "Dominance degree multiple attribute decision making based on z-numbers cognitive information," *Mathematical Problems in Engineering*, vol. 2019, Article ID 1545263, 19 pages, 2019.