DELFT UNIVERSITY OF TECHNOLOGY

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A METHOD TO DERIVE ANGLE OF PITCH, FLIGHT-PATH ANGLE AND ANGLE OF ATTACK FROM MEASUREMENTS IN NONSTEADY FLIGHT

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SUMMARY

In this report a method is described to determine the angle of pitch Θ, the flight-path angle γ and the angle of attack α of an aircraft during steady or nonsteady flight. These angles are determined by integration of the rate of pitch q and the specific forces A_x and A_z and not by directly measuring these angles.

Errors in the estimates of the initial conditions of the integrations and of the zeroshifts in the measurements of q, A_x and A_z are corrected for by comparing the computed and the measured values of the change in altitude and airspeed.

An error analysis is carried out, to determine the accuracy with which the angles Θ, γ and α are determined by the described method for steady as well as for nonsteady flight.

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1. List of symbols

A	covariance matrix of θ, γ and α
a_{hor_x}	acceleration along the horizontal X_g-axis, positive in the positive direction of the X_g-axis
a_{hor_y}	acceleration along the horizontal Y_g-axis, positive in the positive direction of the Y_g-axis
a_{vert}	vertical acceleration, positive upwards
A_x	specific force in X-direction
ΔA_{x_o}	zeroshift in the measured A_x ; defined in (4.1)
A_y	specific force in Y-direction
A_z	specific force in Z-direction
ΔA_{z_o}	zeroshift in the measured A_z ; defined in (4.1)
B	covariance matrix of the systematic errors
$\bar{c}$	mean aerodynamic chord
C	rate of climb
ΔC	error in the rate of climb ; defined in (4.1)
C_Z	dimensionless coefficient of the aerodynamic force in Z-direction
g	acceleration due to gravity
h	pressure altitude
Δh	change in altitude with respect to h at $t = t_o$
$\Delta\Delta h$	difference between the computed and measured values of Δh ; defined in (4.2)
m	aircraft mass

q	rate of pitch
Δq_o	zeroshift in q ; defined in (4.1)
r	rate of yaw
S	sensitivity matrix
t	time
u	component of V along the X-axis
U_a	component of V along the X_g-axis
ΔU_a	difference between the computed and measured values of U_a ; defined in (4.3)
V	velocity of the centre of gravity of the aircraft with respect to the undisturbed air
W	aircraft weight
w	component of V along the Z-axis
w_w	vertical wind component
X	aerodynamic force component in X-direction
Z	aerodynamic force component in Z-direction
α	angle of attack ; defined in (3.10)
α_{tot}	total angle of attack ; defined in Fig. 2
α_w	$\alpha_{tot} - \alpha$
γ	flight path angle
θ	angle of pitch
$\Delta\theta$	error in θ ; defined in (4.1)
ρ	air density
ψ	angle of yaw
φ	angle of roll

Subscripts

j	any time during the manoeuvre at which all required variables are given
o	1. the time at which the computation is started ; 2. subscript for systematic errors
f	the time at which the computation is ended

Aircraft body axes

The origin of the frame of aircraft body axes is the aircraft's centre of gravity. The X-axis is in the plane of symmetry and parallel to the mean chord of the wing. The positive direction is forward. The Y-axis is perpendicular to the plane of symmetry and positive to starboard. The Z-axis is perpendicular to the XOY plane and positive downwards.

Geodetical axes

The origin of the frame of geodetical axes is the position of the aircraft's centre of gravity at $t = t_o$. The X_g-axis is in the horizontal plane through the origin. The positive direction coincides with the orthogonal projection of the vector V on the horizontal plane at $t = t_o$. The Y_g-axis is in the horizontal plane perpendicular to the X_g-axis. The positive direction is 90^o to the right in the direction of the positive Z_g-axis. The Z_g-axis is perpendicular to the horizontal plane, positive in the direction of the centre of the earth.

2. Introduotion.

For several years the Department of Aeronautics of the Delft University of Technology has been investigating the determination of performance, stability and control characteristics of aircraft from measurements in nonsteady flight. One part of the analysis of these measurements is discussed in this report. The different aspects of the method are briefly described below, to relate more clearly the contents of this report to the method itself.

To determine the characteristics of the aircraft a special nonsteady manoeuvre is flown. During this manoeuvre the measurements are carried out by an accurate instrumentation system. Recording is done on magnetic tape in a digital form at a rate of 80 measurements per second. For further details about the manoeuvre itself and the instrumentation system see Chapter 8 and Ref. 1. The analysis of the measurements can be divided into two parts.

1. Reconstruction of the aircraft's motion during the test manoeuvre as accurately as possible.
2. Determination of the characteristics of the aircraft from the reconstructed motion.

This report describes the method for reconstructing the aircraft's motion from the measured variables. For the determination of the aircraft characteristics, see Ref. 1.

By the method described in this report the angles θ, γ and α during the manoeuvre are computed without direct measurements of these angles. This avoids a drawback inherent in the use of an angle of attack meter, in that the installation has to be callibrated in flight. The advantage of the method described is that the angle of attack meter can be completely dispensed with.

To compute the aircraft's motion the following variables are measured :

a. the specific forces along the body axes A_x, A_y and A_z.
b. the rate of pitch q and the rate of yaw r.

c. the change in altitude Δh, determined from pressure measurements.

d. the airspeed V.

e. the angle of roll φ.

The change in altitude Δh, the airspeed V as well as the angles Θ, γ and α are computed by integrating the measured specific forces A_x, A_y and A_z and the angular rates q and r. Errors in the initial conditions of the integrations and certain zeroshifts in the transducers give rise to errors in the computed motion. In addition, the changes in altitude and airspeed are determined by pressure measurements. Comparison of the computed and measured values of Δh and V allows corrections to be made for the already mentioned ''systematic'' errors.

The angle of attack α is determined as the difference of the angle of pitch Θ and the flightpath angle γ. If, due to weather conditions, the air around the aircraft moves vertically or if the horizontal component of the wind is not constant, errors appear in the value of α. In this report it is shown that these errors can not be corrected for by the described method.

In the first part of this report it will be shown how the computations are carried out and how the systematic errors are determined. In the second part an error analysis, carried out on the results of measurements during steady and nonsteady flights, is discussed. The influence, of the systematic errors on the accuracy of Θ, γ and α is determined.

3. Principle of the computation of Θ, γ and α.

In this chapter a general description of the method to determine Θ, γ and α is given.

3.1. The angle of pitch, Θ.

The angle of pitch Θ is found by integrating $\dot{\Theta}$:

$$\Theta = \Theta_o + \int_{t_o}^{t} \dot{\Theta}\, dt \tag{3.1}$$

The exact expression for $\dot{\Theta}$ has to be used in (3.1), since small deviations from symmetric flight will occur during the measurements:

$$\dot{\Theta} = q \cos \varphi - r \sin \varphi \tag{3.2}$$

An estimated value of Θ_o as required in (3.1) is obtained as follows. The equations of motion of the rigid aircraft for the translations along the X- and Z-axes are :

$$-W \sin \Theta + X = m(\dot{u} + wq)$$

$$W \cos \Theta + Z = m(\dot{w} - uq)$$

Dividing by m and introducing the specific forces A_x and A_z :

$$A_x = \frac{X}{m} \quad \text{and} \quad A_z = \frac{Z}{m}$$

results in :

$$g \sin \Theta + A_x = \dot{u} + wq \tag{3.3}$$

$$g \cos \Theta + A_z = \dot{w} - uq \tag{3.4}$$

The assumption is made that the manoeuvre starts at t_o from a condition of reasonably steady symmetric flight. Then (3.3) and (3.4) can be simplified for t_o by letting :

$$\dot{u} = q = \dot{w} = 0$$

For the equations thus simplified, an estimate of Θ_o follows :

$$\theta_o = -\text{arctg}\left(\frac{A_x}{A_z}\right)_{t_o} \tag{3.5}$$

An estimate of θ at any t is now obtained by substituting (3.5) and (3.2) in (3.1) :

$$\theta = -\text{arctg}\left(\frac{A_x}{A_z}\right)_{t_o} + \int_{t_o}^{t} (q \cos \varphi - r \sin \varphi)dt$$

3.2. The flight path angle, γ.

The flight path angle γ, see Fig.1, is determined from :

$$\gamma = \arcsin \frac{C}{V} \tag{3.6}$$

were the rate of climb C is obtained by integrating the vertical acceleration :

$$C = C_o + \int_{t_o}^{t} a_{vert} dt \tag{3.7}$$

The vertical acceleration a_{vert}, relative to the earth, is determined from the specific forces A_x, A_y, A_z, the angle of pitch θ and the angle of roll φ, according to :

$$a_{vert} = A_x \sin \theta - A_y \sin \varphi \cos \theta - A_z \cos \varphi \cos \theta - g \tag{A1-7}$$

as derived in Appendix 1.

In (3.7) an estimate of the initial value of the rate of climb, C_o, is obtained by making use of the change of altitude Δh. This change of altitude is a function of C_o and a_{vert}.

$$\Delta h = \Delta h_o + \int_{t_o}^{t} C_o . dt + \iint_{t_o}^{t} a_{vert} dt^2 \tag{3.8}$$

The difference between the altitude at the initial moment t_o and the final moment t_f of the manoeuvre is :

$$\Delta h_f - \Delta h_o = \int_{t_o}^{t_f} C_o dt + \iint_{t_o}^{t_f} a_{vert} dt^2$$

As was mentioned in the Introduction, the change in altitude during the manoeuvre has also been determined directly from measurements of the static pressure.

Using $\Delta h_f - \Delta h_o$ as obtained from the pressure measurements, an estimate of C_o is obtained from the above expression :

$$C_o = \frac{\Delta h_f - \Delta h_o - \iint_{t_o}^{t_f} a_{vert} dt^2}{t_f - t_o} \qquad (3.9)$$

Now C at any time during the manoeuvre can be derived, using (3.7), (3.9) and (A1-7).

Given the velocity V of the aircraft's centre of gravity relative to the undisturbed air, the flight path angle γ, relative to the air, is formed from equation (3.6).

3.3. The angle of attack, α.

Finally the angle α is obtained as the difference between the angle of pitch Θ and the flight path angle γ :

$$\alpha = \Theta - \gamma \qquad (3.10)$$

This simple geometric relationship is true under all conditions of symmetric flight.

If, however, the angle of attack expressed by the above relation is to be related to the aerodynamic characteristics of the aircraft two assumptions have to be made :

a. The air around the aircraft has an average vertical velocity w_w equal to zero.

b. The air has a constant horizontal velocity relative to the earth.

Actually the air around the aircraft mostly has a small vertical velocity, $w_w + w_g$, while the horizontal velocity of the air shows small fluctuations. The average component of the vertical velocity, w_w, is of interest here; small fluctuations about the average due to turbulence, w_g, can be considered equivalent to random errors which are neutralised in the computation process. The variations in the horizontal velocity of the air about the average windspeed have to be small relative to the velocity V of the aircraft in order to be negligible. This latter condition appears to be met quite easily under flight test conditions.

In Fig.2 it is shown how the vertical component of the wind w_w gives rise to an angle of attack α_w :

$$\alpha_w = \arcsin \frac{w_w}{V}$$

Neglecting the angle of attack due to turbulence for the reason stated above, the total angle of attack of the aircraft is :

$$\alpha_{tot} = \theta - \gamma + \alpha_w$$

Or, using (3.10) :

$$\alpha_{tot} = \alpha + \alpha_w$$

It is of fundamental importance to note that the aerodynamic forces and moments on the aircraft and hence its performance, stability and control characteristics are functions of α_{tot} rather than of α. Since, unfortunately no feasible method of determining w_w is known, the contribution of α_w to α_{tot} remains unknown and it has to be assumed that :

$$w_w = 0$$

$$\alpha_{tot} = \alpha$$

$$= \theta - \gamma$$

To make this assumption more acceptable, the flight tests have to be carried out -as usual- in carefully selected calm weather conditions. The influence of the vertical component of the wind w_w is further discussed in the Chapters 7 and 9.

From the foregoing it may be seen that the measured variables A_x, A_z, q, Δh and V are the principal variables, describing the aircraft's essentially symmetric flight. The remaining measured variables, i.e. A_y, r and φ are required in order to correct for the inevitable small deviations from symmetric flight during the manoeuvre.

4. Corrections for systematic errors.

The measured variables mentioned in Chapter 3 generally contain errors due to the measurements themselves. These errors mostly are of two essentially different types.

In the first place there are systematic errors. They are assumed to be constant throughout the manoeuvre. The systematic errors to be considered here have the same effect as and indeed will often be due to zero-shifts of the instruments used. Hence this type of errors will further be indicated as zero-shifts. The systematic error in a variable is equated to the time average of the total measurement-error in that variable during a manoeuvre.

In the second place there are errors having a more or less random character. They are considered to be caused by measurement noise.

The errors of the first type, the systematic errors, directly influence the computed aircraft's motion.

If, in the course of the analysis, a measured variable has to be integrated, a zero-shift in the variable may lead to unacceptable large errors. Consequently these errors have to be eliminated by correcting for the zero-shift.

In the subsequent part of this report it will be shown how the more important zero-shifts ΔA_{x_o}, ΔA_{z_o} and Δq_o in the measured variables A_x, A_z and q can be determined and corrected for. Errors in the initial estimates Θ_o and C_o of the integrations (3.1) and (3.7) will be shown to be amenable to a similar treatment. An error, such as the systematic errors ΔA_{x_o}, ΔA_{z_o}, Δq_o, $\Delta\Theta_o$ and ΔC_o, in a variable y is defined as follows. Let y be the actual value of the variable and y' the measured or the computed value. Then the error in y' is defined by :

$$\Delta y = y' - y \tag{4.1}$$

In 3.3 of Chapter 3 the difference between the computed value α and α_{tot} due to the vertical component of the wind has already been mentioned Therefore w_w is taken into account as a systematic error. It should be noted that Θ and γ are measured independently of w_w, although for a given engine power setting and airspeed and, therefore, at a given value of α_{tot} the flight path angle actually achieved by the aircraft does depend on w_w. The vertical wind component w_w will be neglected in the following Chapters. In the error analysis (Chapter 7) the influence of w_w will be discussed.

The possibility to correct for the remaining systematic errors arises in principle from the fact that the ensemble of all measurements contains a certain redundancy. This redundancy may be found in the change in altitude Δh and the horizontal velocity U_a, both of which can be determined in two different ways :

1. In the first place Δh and U_a can be calculated directly as functions of time from the measured static and total pressure and the air temperature. In order to simplify the ensueing computations, the horizontal velocity relative to the surounding air, U_a, is used instead of the velocity V. The relation between U_a and V is :

$$U_a = \sqrt{V^2 - C^2}$$

The assumption has already been made, that the airmass surounding the aircraft has a constant horizontal speed relative to the earth.

2. In the second place Δh and U_a are obtained in a more indirect way be integrating the horizontal and vertical accelerations of the aircraft. Both accelerations are derived from measurements, see Appendix 1 :

$$a_{hor} = A_x \cos \Theta + A_y \sin \varphi \sin \Theta + A_z \sin \Theta \cos \varphi \qquad (A1\text{-}8)$$

$$a_{vert} = A_x \sin \Theta - A_y \cos \Theta \sin \varphi - A_z \cos \Theta \cos \varphi - g \qquad (A1\text{-}7)$$

where the angle of pitch Θ is obtained according to (3.1).

The discrepancies in Δh and U_a arising between the values deduced from pressuremeasurements and those obtained from integrations, are attributed to the two types of errors mentioned earlier :

a. The systematic errors due to zero-shifts in A_x, A_z, q, Θ and C.

Since the zero-shifts were assumed to be constant during the manoeuvre, the errors in Δh and U_a caused by the zero-shifts for a given manoeuvre are deterministic functions of time.

b. The random errors due to noise in the pressure measurements. The time averages of these random errors are assumed to be zero.

In addition to the above errors, another error has yet to be mentioned. The static pressure is found by applying the conventional position error correction (P.E.C.). This correction, however, appears to be very sensitive to the fact that the airflow around the aircraft is nonsteady during the manoeuvre. This effect has been discussed in Appendix 4, where a correction for this nonsteady effect is derived.

In the following Chapters the change of altitude determind by means of integration from the uncorrected values of C_o and a_{vert} is denoted by $\Delta h'$, whereas the change of altitude found from pressure measurements, is indicated by Δh^*. Then :

$$\Delta\Delta h = \Delta h' - \Delta h^* \qquad (4.2)$$

In the same way the horizontal components U_a' and U_a^* of the velocity V relative to the undisturbed air are defined. U_a' is based on the integration of uncorrected values of a_{hor}, whereas U_a^* is based on the pressure measurements. Then :

$$\Delta U_a = U_a' - U_a^* \qquad (4.3)$$

The essential distinction between $\Delta h'$ and U_a' on the one hand and Δh^* and U_a^* on the other is, that $\Delta h'$ and U_a' are assumed to be in error only due to the zero-shift effects, whereas Δh^* and U^* are assumed to be susceptible to the random errors only.

5. The relation between $\Delta\Delta h$, ΔU_a and the systematic errors.

In this Chapter expressions are derived for the relations between the differences $\Delta\Delta h$ and ΔU_a introduced in (4.2) and (4.3) of the previous Chapter and the systematic errors ΔA_{x_o}, ΔA_{z_o}, Δq_o, $\Delta\theta_o$ and ΔC_o, neglecting for a moment the random errors.

5.1. The expression for $\Delta\Delta h$.

The altitude change Δh determined by integrating C_o and a_{vert} is :

$$\Delta h = \Delta h_o + \int_{t_o}^{t} C_o . dt + \iint_{t_o}^{t} a_{vert} dt^2 \qquad (3.8)$$

When actually performing the integration, Δh_o in (3.8) has to be replaced by Δh_o^* as obtained from pressure measurements, since no other means of obtaining Δh_o is available. Instead of the exact values of C_o and a_{vert}, computed values of C'_o and a'_{vert} as derived from measured variables according to (3.9) and (A1-7) have to be used in (3.8). The result is $\Delta h'$:

$$\Delta h' = \Delta h_o^* + \int_{t_o}^{t} C'_o . dt + \iint_{t_o}^{t} a'_{vert} dt^2$$

Δh_o^*, C'_o and a'_{vert} are in error by $\Delta\Delta h_o$, ΔC_o and Δa_{vert} respectively. Therefore using (4.2), $\Delta\Delta h$ can be written as :

$$\Delta\Delta h = \Delta h_o + \Delta\Delta h_o + \int_{t_o}^{t} (C_o + \Delta C_o) dt + \iint_{t_o}^{t} (a_{vert} + \Delta a_{vert}) dt^2 - \Delta h^*$$

After rearranging :

$$\Delta\Delta h = \Delta h_o + \int_{t_o}^{t} C_o dt + \iint_{t_o}^{t} a_{vert} dt^2 - \Delta h^* + \Delta\Delta h_o + \Delta C_o (t - t_o) + \iint_{t_o}^{t} \Delta a_{vert} dt^2 \qquad (5.1)$$

The pressure measurements have been assumed to contain random errors only. Since these errors are neglected in this Chapter, it follows :

$$\Delta h^{*} = \Delta h_o + \int_{t_o}^{t} C_o dt + \iint_{t_o}^{t} a_{vert} dt^2 \qquad (5.2)$$

Substituting (5.2) in (5.1) results in :

$$\Delta\Delta h = \Delta\Delta h_o + \Delta C_o (t-t_o) + \iint_{t_o}^{t} \Delta a_{vert} dt^2 \qquad (5.3)$$

The error in the vertical acceleration Δa_{vert} is supposed to be the result of the systematic errors ΔA_{x_o} , ΔA_{z_o} , $\Delta\Theta_o$ and Δq_o. From Appendix 1 it follows :

$$a_{vert} = A_x \sin\Theta - A_y \cos\Theta \sin\varphi - A_z \cos\Theta \cos\varphi - g \qquad (A1\text{-}7)$$

Partial differentiation of a_{vert} with respect to the measured variables leads to :

$$\partial a_{vert} = \partial A_x \sin\Theta - \partial A_z \cos\varphi \cos\Theta + \partial\Theta (A_x \cos\Theta$$

$$+ A_y \sin\varphi \sin\Theta + A_z \cos\varphi \sin\Theta) \qquad (5.4)$$

In (5.4) use can be made of (A1-8) :

$$a_{hor} = A_x \cos\Theta + A_y \sin\varphi \sin\Theta + A_z \cos\varphi \sin\Theta$$

resulting in :

$$\partial a_{vert} = \partial A_x \sin\Theta - \partial A_z \cos\varphi \cos\Theta + \partial\Theta . a_{hor} \qquad (5.5)$$

The angle of pitch Θ has been obtained in (3.1) by integrating the rate of pitch q :

$$\Theta = \Theta_o + \int_{t_o}^{t} (q \cos \varphi - r \sin \varphi) dt \qquad (3.1)$$

Since the angle of roll remains small during the manoeuvre it is assumed in (5.5) and (3.1) that $\cos \varphi = 1$. In the foregoing Chapter systematic errors in Θ_o and q_o were assumed to exist. Therefore :

$$\partial \Theta = \partial \Theta_o + \int_{t_o}^{t} \partial q_o dt$$

or :

$$\partial \Theta = \partial \Theta_o + \partial q_o (t-t_o) \qquad (5.6)$$

If the systematic errors in A_x and A_z as well as those in Θ_o and q_o are assumed to be sufficiently small, the resulting expression for Δa_{vert} is :

$$\Delta a_{vert} = +\Delta A_{x_o} \sin \Theta - \Delta A_{z_o} \cos \Theta + \Delta\Theta . a_{hor} + \Delta q_o . a_{hor}(t-t_o) \qquad (5.7)$$

Substitution of Δa_{vert} in (5.1) yields the relation between $\Delta\Delta h$ and the systematic errors.

$$\Delta\Delta h = \Delta\Delta h_o + \Delta C_o (t-t_o) + \Delta A_{x_o} \iint_{t_o}^{t} \sin \Theta \, dt^2$$

$$- \Delta A_{z_o} \iint_{t_o}^{t} \cos \Theta \, dt^2 + \Delta\Theta_o \iint_{t_o}^{t} a_{hor} dt^2 + \Delta q_o \iint_{t_o}^{t} a_{hor} t . dt^2 \qquad (5.8)$$

In view of the use to be made of (5.8) later on, this expression will be written as :

$$Y = a_o + a_1 X_1 + a_2 X_2 + a_3 X_3 + a_4 X_4 + a_5 X_5 \qquad (5.9)$$

where :

$$a_o = \Delta\Delta h_o \qquad Y = \Delta\Delta h$$

$$a_1 = \Delta C_o \qquad X_1 = t - t_o$$

$$a_2 = \Delta A_{x_o} \qquad X_2 = \iint_{t_o}^{t} \sin\Theta \, dt^2$$

$$a_3 = \Delta A_{z_o} \qquad X_3 = -\iint_{t_o}^{t} \cos\Theta \, dt^2$$

$$a_4 = \Delta\Theta_o \qquad X_4 = \iint_{t_o}^{t} a_{hor} dt^2$$

$$a_5 = \Delta q_o \qquad X_5 = \iint_{t_o}^{t} a_{hor} t . dt^2$$

5.2. The expression for ΔU_a.

The horizontal component of the aircraft speed U_a is determined by integrating the horizontal accelerations a_{hor_x} and a_{hor_y} :

$$U_a = \sqrt{(U_{a_o} + \int_{t_o}^{t} a_{hor_x} dt)^2 + (\int_{t_o}^{t} a_{hor_y} dt)^2} \qquad (5.10)$$

When actually performing the integrations implied in (5.10), the latter equation has to be modified in the same way as (5.1) for $\Delta\Delta h$. This means that U_{a_o}, a_{hor_x} and a_{hor_y} have to be replaced by $U^{*}_{a_o}$, a'_{hor_x} and a'_{hor_y} as derived from measurements :

$$U'_a = \sqrt{(U^*_{a_o} + \int_{t_o}^{t} a'_{hor_x} dt)^2 + (\int_{t_o}^{t} a'_{hor_y} dt)^2}$$

$U^*_{a_o}$, a'_{hor_x} and a'_{hor} contain the errors ΔU_{a_o}, Δa_{hor_x} and Δa_{hor_y} respectively.

Therefore :

$$U'_a = \sqrt{\left\{U_{a_o} + \Delta U_{a_o} + \int_{t_o}^{t} (a_{hor_x} + \Delta a_{hor_x})dt\right\}^2 + \left\{\int_{t_o}^{t} (a_{hor_y} + \Delta a_{hor_y})dt\right\}^2}$$

Since the manoeuvre of the aircraft is nearly symmetric, it is assumed that the second integral in the above expression may be neglected in this error analysis.

Therefore :

$$U'_a = U_{a_o} + \Delta U_{a_o} + \int_{t_o}^{t} (a_{hor_x} + \Delta a_{hor_x})dt \qquad (5.11)$$

Again neglecting the random errors, the difference between the integrated horizontal speed U'_a and U^*_a as derived from pressure measurements is, according to (4.3) :

$$\Delta U_a = U_{a_o} + \Delta U_{a_o} + \int_{t_o}^{t} a_{hor_x} dt + \int_{t_o}^{t} \Delta a_{hor_x} dt - U^*_a$$

or :

$$\Delta U_a = \Delta U_{a_o} + \int_{t_o}^{t} \Delta a_{hor_x} dt \qquad (5.12)$$

The error in the horizontal acceleration Δa_{hor_x} in (5.12) is considered to be the result of the systematic errors, ΔA_{x_o}, ΔA_{z_o}, $\Delta\theta_o$ and Δq_o. According to (A1-8) of Appendix 1 :

$$a_{hor_x} = A_x \cos \Theta + A_y \sin \varphi \sin \Theta + A_z \cos \varphi \sin \Theta \qquad \text{(A1-8)}$$

The error Δa_{hor_x} as a function of the systematic errors, is derived in a similar way to that of Δa_{vert} in de preceding paragraph. The result is :

$$\Delta a_{hor_x} = \Delta A_{x_o} \cos \Theta + \Delta A_{z_o} \sin \Theta - \Delta\Theta_o (a_{vert}+g) - \Delta q_o (a_{vert}+g)(t-t_o)$$

Substitution of Δa_{hor_x} in (5.12) finally provides the expression of ΔU_a as a function of the systematic errors :

$$\Delta U_a = \Delta U_{a_o} + \Delta A_{x_o} \int_{t_o}^{t} \cos \Theta dt + \Delta A_{z_o} \int_{t_o}^{t} \sin \Theta dt - \Delta\Theta_o \int_{t_o}^{t} (a_{vert}+g) dt$$

$$- \Delta q_o \int_{t_o}^{t} (a_{vert}+g) t \, dt \qquad (5.13)$$

This expression can be written in a form similar to (5.9) :

$$Y = a_o + a_1 X_1 + a_2 X_2 + a_3 X_3 + a_4 X_4 \qquad (5.14)$$

where :

$$a_o = \Delta U_{a_o} \qquad Y = \Delta U_a$$

$$a_1 = \Delta A_{x_o} \qquad X_1 = \int_{t_o}^{t} \cos \Theta \, dt$$

$$a_2 = \Delta A_{z_o} \qquad X_2 = \int_{t_o}^{t} \sin \Theta \, dt$$

$$a_3 = \Delta\theta_o \qquad X_3 = -\int_{t_o}^{t} (a_{vert}+g)dt$$

$$a_4 = \Delta q_o \qquad X_4 = -\int_{t_o}^{t} (a_{vert}+g)t.dt$$

6. Estimation of the systematic errors.

6.1. Introduction.

The expressions (5.8) and (5.13) for $\Delta\Delta h$ and ΔU_a previously derived, will now be treated along identical lines. Therefore they are written as :

$$Y = a_o + a_1X_1 + a_2X_2 \; .... + a_mX_m \qquad (6.1)$$

in accordance to (5.9) and (5.14).

The variables Y and X_i (i = 1,2,...,m) are supposed to be known from measurements at a sufficiently large number (n) of instants in time t_j ($j = 1,2,...,n \gg m$).
In the actual tests to be described in Section 8,

$$t_{j+1} - t_j = 0{,}1 \text{ sec}$$

and $n \geqslant 600$.

The problem to be solved is the determination of the unknown coefficients a_i (i=0,1,2,...,m) in (6.1). Use is made of the method of regression analysis, which is based on the principle of least squares. The purpose of the method can be described very briefly by considering the n sets of data Y, X_i. In general it will not be possible to fit the expression (6.1) exactly to all combinations of the variables Y, X_i by means of a single set of regression coefficients a_i. Therefore (6.1) is made exact by adding a residual term ΔY, which accounts for the random errors in the measurements :

$$Y = a_o + a_1X_1 + a_2X_2 + \; ..... + a_mX_m + \Delta Y \qquad (6.2)$$

It is the purpose of the regression analysis to chose the coefficients a_i such as to minimize the sum :

$$S = \sum_1^n \Delta Y^2$$

An aspect of great practical importance should be mentioned here. Suppose a set of measured data Y, X_i of given accuracy is available. The accuracy of the regression coefficients a_i to be determined from these data, strongly depends on the degree to which the variables X_i are linearly related. If the variables X_i happen to be highly linearly related, some or all of the coefficients a_i can be found with poor accuracy only. For further details about regression analysis see Ref. 2 and Ref. 3.

6.2. Application of regression analysis.

As mentioned in the previous paragraph, the variables X_i (i = 1, ..., m) in (6.2) should be as linearly independent as possible. The linear dependency that actually occurs in practice proves to be one of the main difficulties in the application of regression analysis to the problem at hand.
Steady symmetric flight will be discussed as a first example. In this case the expressions (5.8) and (5.13) can be written more explicitely since Θ is constant :

$$\Delta\Delta h_j = \Delta\Delta h_o + \Delta C_o t_j + \Delta A_{x_o} \sin \Theta . \tfrac{1}{2}\, t_j^2 - \Delta A_{z_o} \cos \Theta . \tfrac{1}{2}\, t_j^2 \qquad (6.3)$$

$$\Delta U_{a_j} = \Delta U_{a_o} + \Delta A_{x_o} \cos \Theta . t_j + \Delta A_{z_o} \sin \Theta . t_j - \Delta\Theta_o . g . t_j - \Delta q_o g . \tfrac{1}{2}\, t_j^2 \qquad (6.4)$$

In (6.3) the variables $\sin \Theta . \frac{1}{2}\, t_j^2$ and $\cos \Theta . \frac{1}{2}\, t_j^2$ differ by a constant factor, because Θ is constant. This means that the two variables are exactly linearly related. As a consequence, the two coefficients ΔA_{x_o} and ΔA_{z_o} can not be determined separately from (6.3). In the same way, the terms in (6.4) containing ΔA_{x_o}, ΔA_{z_o} and $\Delta\Theta_o$ are exactly linearly related. Therefore, it is impossible to determine the three errors ΔA_{x_o}, ΔA_{z_o} and $\Delta\Theta_o$ separately from measurements in steady flight.

The only way to escape from this situation is to neglect one of the three errors.

A little reflection will show that it is necessary to equate ΔA_{x_o} to zero, since it has the smallest influence on $\Delta\Delta h$. If such is done, it is quite easy to determine the remaining systematic errors $\Delta\Delta h_o$, ΔU_{a_o}, ΔC_o, $\Delta\Theta_o$, ΔA_{z_o} and Δq_o from measurements made in steady flights.

As a next case nonsteady symmetric flight is considered. Angle of pitch Θ now varies with time. The variables in (5.8) and (5.13) then prove to be linearly independent to a sufficient degree only if Θ changes over a rather wide range, e.g. $\Theta_{max} - \Theta_{min} = 25^o$. Even then it is still necessary to employ a certain iterative scheme of regression analyses in order to determine the various systematic errors.

This scheme can be described as follows.

The basic idea is to find an estimate of the most important errors first by initially neglecting smaller contributions to $\Delta\Delta h$ and ΔU_a. Starting with $\Delta\Delta h$, the smaller terms in (5.8) are those containing ΔA_{x_o}, $\Delta\Theta_o$ and Δq_o. To a first approximation, therefore, $\Delta\Delta h_j$ can be written as :

$$\Delta\Delta h_j = \Delta\Delta h_o + \Delta C_o \cdot t_j - \Delta A_{z_o} \cdot \iint_{t_o}^{t_j} \cos\Theta \, dt^2 \qquad (6.5)$$

Applying regression analysis to this expression yields an initial estimate of $\Delta\Delta h_o$, ΔC_o and ΔA_{z_o}.

As a next step, ΔU_a is considered. Neglecting for a moment in (5.13) the term containing ΔA_{x_o}, an approximated ΔU_{a_j} is used :

$$\Delta U_{a_j}^{*} \doteq \Delta U_{a_o} - \Delta\Theta_o \int_{t_o}^{t} (a_{vert} + g)\,dt - \Delta q_o \int_{t_o}^{t_j} (a_{vert} + g)\,t\,dt \qquad (6.6)$$

where :

$$\Delta U_{a_j}^{*} = \Delta U_{a_j} - \Delta A_{z_o} . \int_{t_o}^{t_j} \sin\Theta \, dt$$

An initial estimate of ΔA_{z_o} has been derived from (6.5).
Regression analysis applied to (6.6) provides estimates of ΔU_{a_o}, $\Delta\Theta_o$ and Δq_o.

The latter two systematic errors are used in a version of the $\Delta\Delta h$ equation (5.8) which is more refined than (6.5) :

$$\Delta\Delta h_j^{**} = \Delta\Delta h_o + \Delta C_o . t_j + \Delta A_{x_o} . \iint_{t_o}^{t_j} \sin\Theta \, dt^2 - \Delta A_{z_o} . \iint_{t_o}^{t_j} \cos\Theta \, dt^2 \qquad (6.7)$$

where $\Delta\Delta h_h^{**}$ is calculated as follows :

$$\Delta\Delta h_j^{**} = \Delta\Delta h_j - \Delta\Theta_o . \iint_{t_o}^{t_j} a_{hor} dt^2 - \Delta q_o . \iint_{t_o}^{t_j} a_{hor} t . dt^2$$

From (6.7), values of $\Delta\Delta h_o$, ΔC_o, ΔA_{x_o} and ΔA_{z_o} are derived, again by regression analysis.

As a last step, the errors ΔA_{x_o} and ΔA_{z_o} thus found are used in a version of the ΔU_a equation (5.13) more refined than (6.6) :

$$\Delta U_{a_j}^{**} = \Delta U_{a_o} - \Delta\Theta_o . \int_{t_o}^{t_j} (a_{vert} + g) dt - \Delta q_o . \int_{t_o}^{t_j} (a_{vert} + g) t . dt \qquad (6.8)$$

where :

$$\Delta U_{a_j}^{**} = \Delta U_{a_j} - \Delta A_{x_o} . \int_{t_o}^{t_j} \cos\Theta \, dt - \Delta A_{z_o} . \int_{t_o}^{t_j} \sin\Theta \, dt$$

If so desired, $\Delta\Theta_o$ and Δq_o found by regression analysis from (6.8) can be used again to determine an improved $\Delta\Delta h_j^{**}$ in (6.7), in order to obtain more accurate values of ΔA_{x_o} and ΔA_{z_o}. The latter may again be used in ΔU_{a_j} of (6.8) to improve $\Delta\theta_o$ and Δq_o and so on. Experience with the flight test data described in Section 8 has shown that the iterative process converges after two or three iterations.

Each of the regression analyses just mentioned not only provides a set of regression coefficients, a_i (i = 0, 1, ..., m) in (6.1), but also an estimate of the accuracy of each a_i in the form of a set of expected values of the standard deviations σ_{a_i}.

In cases where the standard deviation σ_{a_i} is much larger than the value of the corresponding coefficient a_i, it may safely be assumed that the term containing a_i may well be neglected in the regression analysis. This particular feature of regression analysis is made use of, in order to employ the iterative scheme just described not only to nonsteady flight, but to steady flight as well. It has been argued previously that the error ΔA_{x_o}, which is one of the regression coefficients in (6.7), can not be determined from measurements in steady flight. This fact can now be expressed more precisely by saying that $\left|\Delta A_{x_o}\right| \ll \sigma_{\Delta A_{x_o}}$ when analyzing measurements taking in steady flight. Consequently, in the iterative schema, ΔA_{x_o} is set equal to zero, if the following condition is satisfied :

$$\left|\Delta A_{x_o}\right| < k . \sigma_{\Delta A_{x_o}}$$

where k has been set somewhat arbitrarily at 5.

The entire iterative schema as just described has been presented in the flow diagram of Fig. 3. It should be remarked, that each time a set of systematic errors is obtained from the regression analyses in the scheme, the variables in the righthand sides of the regression analyses are recomputed in order to correct for the newly found systematic errors. Additional values of the systematic errors then are determined by restarting the regression procedure.

The angles θ, γ and α finally can be determined according to (3.1), (3.6) and (3.10), as soon as the systematic errors ΔC_o, ΔA_{x_o}, ΔA_{z_o}, $\Delta\theta_o$ and Δq_o have reached their final estimates.
In Appendix 2 the operations actually performed in the θ, γ and α computation are summarized.

7. A method for evaluating the accuracy of the results of the Θ, γ and α calculation.

7.1. Introduction.

In this Chapter error analysis is applied to the previously described method of determining Θ, γ and α. In addition an independent check on the error analysis is given for measurements in steady symmetric flight.

As already indicated in Chapter 4 the calculation of Θ, γ and α is based on the assumption that two distinct types of errors occur in the measurements. Briefly, these types are :

1. Errors of a random character, having a mean value of zero during the manoeuvre. Their influence on the results is neglected, see Chapter 4.
2. Systematic errors, ΔC_o, ΔA_{x_o}, ΔA_{z_o}, $\Delta\Theta_o$, Δq_o and w_w, assumed constant during the manoeuvre.
   The vertical component of the wind w_w excepted, the systematic errors are estimated by means of regression analysis, see Chapter 6.

It might be expected that the systematic errors can not be obtained exactly from regression analysis. They are found with a remaining uncertainty. In the subsequent part of this report it is assumed that the remaining parts of the systematic errors, after application of the regression analyses, posess a multinormal distribution with zero mean. Such a distribution is described entirely by a covariance matrix, which will be denoted by B, see Ref. 3 and 4.

The joint probability density can be given as :

$$\phi(x) = \frac{1}{(2\pi)^{\frac{1}{2}p} \cdot |B|^{\frac{1}{2}}} \exp\left[-\frac{1}{2}(x-\mu)' B^{-1} (x-\mu)\right]$$

in which :

$$x = \begin{vmatrix} x_1 \\ x_2 \\ \vdots \\ x_p \end{vmatrix} \qquad \mu = \begin{vmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \vdots \\ \bar{x}_p \end{vmatrix}$$

and B is the covariance matrix of x.
The regression analyses, used to estimate the systematic errors, also provide an estimate of this covariance matrix B.

The remaining parts of ΔC_o, ΔA_{x_o}, ΔA_{z_o}, $\Delta\theta_o$, and Δq_o and in addition w_w, being small it is reasonable to expect a lineair relation between the remaining parts of the systematic errors and the resulting errors $\Delta\theta$, $\Delta\gamma$ and $\Delta\alpha$ in θ, γ and α respectively.
In that case :

$$\Delta\theta = s_{11}\Delta C_o + s_{12}\Delta A_{x_o} + s_{13}\Delta A_{z_o} + s_{14}\Delta\theta_o + s_{15}\Delta q_o + s_{16}w_w \qquad (7.1)$$

where ΔC_o, ΔA_{x_o} etc. are the remaining systematic errors after application of the θ, γ and α calculation.
Corresponding expressions for $\Delta\gamma$ and $\Delta\alpha$ hold where :

$$\Delta\theta = \Delta\gamma + \Delta\alpha$$

In vector notation the relations such as (7.1) are :

$$\begin{vmatrix} \Delta\theta \\ \Delta\gamma \\ \Delta\alpha \end{vmatrix} = S \cdot \begin{vmatrix} \Delta C_o \\ \Delta A_{x_o} \\ \Delta A_{z_o} \\ \Delta\theta_o \\ \Delta q_o \\ w_w \end{vmatrix} \qquad (7.2)$$

S is a sensitivety matrix, the elements s_{ij} - already given in (7.1) for i = 1 - are to be determined below.

From the preceeding assumptions it follows that the distribution of the errors $\Delta\theta$, $\Delta\gamma$ and $\Delta\alpha$ will be multinormal as well. According to Ref. 3 this distribution is characterized by a covariance matrix denoted here by A.

The relation between the covariance matrices A and B, thus between $\Delta\theta$, $\Delta\gamma$ and $\Delta\alpha$ on the one side and the systematic errors on the other is, according to Ref. 3 and 4 :

$$A = S.B.S^T \qquad (7.3)$$

The aim of the following is to obtain A from S en B.

In addition the covariance matrix A can be determined for steady symmetric flights directly from the results of θ, γ and α, so that for steady flights a check on the matrix A computed according to (7.3) can be made.

7.2. The sensitivity matrix S.

The elements of the matrix S are found as partial derivatives of θ, γ and α to the systematic errors. The expressions for $\Delta\Delta h$ and ΔU_a (5.8) and (5.13) are used when determining these derivatives. First the matrix S is determined for steady flights.

7.2.1. The sensitivity matrix S for steady symmetric flight.

In steady symmetric flight the angle of pitch θ is constant and $a_{hor} = a_{vert} = 0$. Eqs.(5.8) and (5.13) then change into :

$$\Delta\Delta h = \Delta\Delta h_o + \Delta C_o.t + \Delta A_{x_o} \sin\theta.\tfrac{1}{2}t^2 - \Delta A_{z_o} \cos\theta.\tfrac{1}{2}t^2 \qquad (7.4)$$

$$\Delta U_a = \Delta U_{a_o} + \Delta A_{x_o} \cos\theta.t + \Delta A_{z_o} \sin\theta.t - \Delta\theta_o g.t - \Delta q_o.g.\tfrac{1}{2}t^2 \qquad (7.5)$$

Differentiation of (7.4) with respect to the time t results in :

$$\Delta\dot{\Delta h} = \Delta C = \Delta C_o + \Delta A_{x_o} \sin\theta.t - \Delta A_{z_o} \cos\theta.t \qquad (7.6)$$

The flight path angle is determined by :

$$\gamma = \text{arctg}\,\frac{C}{U_a}$$

This angle remains continuously small, so for the error analysis :

$$\gamma = \frac{C}{U_a}$$

is used.

With the above mentioned equations and (3.10) and (5.6) :

$$\partial\theta = \partial\theta_o + \partial q_o . t \qquad (5.6)$$

and :

$$\alpha = \theta - \gamma \qquad (3.10)$$

the partial derivatives of θ, γ and α to a variable x of the systematic errors, can now be derived.

$$\frac{\partial\theta}{\partial x} = \frac{\partial\theta_o}{\partial x} + t . \frac{\partial q_o}{\partial x} \qquad (7.8)$$

$$\frac{\partial\gamma}{\partial x} = \frac{U_a . \frac{\partial C}{\partial x} - C . \frac{\partial U_a}{\partial x}}{U_a^2} \qquad (7.9)$$

$$\frac{\partial\alpha}{\partial x} = \frac{\partial\theta}{\partial x} - \frac{\partial\gamma}{\partial x} \qquad (7.10)$$

To simplify the elements of S, in the above equations, U_a is set equal to V.

In Chapter 3 it has already been mentioned that the average vertical speed w_w of the air around the aircraft affects the aircraft's flight path angle γ. In the case of steady or quasi steady flight this does not hold for θ, see Fig. 2. The flight tests were carried out in very smooth air, therefore fluctuations of the vertical velocity about the average value w_w will be neglected.

With Fig.2 it follows at a constant engine powersetting and airspeed and, therefore, at a constant $\alpha_{tot} = \theta - \gamma + \alpha_w$:

$$\frac{\partial \theta}{\partial w_w} = 0 \qquad , \qquad \frac{\partial \gamma}{\partial w_w} = \frac{1}{V}$$

and consequently :

$$\frac{\partial \alpha}{\partial w_w} = - \frac{1}{V} \tag{7.11}$$

In Appendix 3 the elements of S are determined, using (7.4) up to (7.10). The resulting matrix S for steady flights is :

$$S = \begin{array}{c} \\ \\ \\ \end{array}\begin{array}{cccccc} (\Delta C_o) & (\Delta A_{x_o}) & (\Delta A_{z_o}) & (\Delta\theta_o) & (\Delta q_o) & (w_w) \end{array}$$

$$S = \left| \begin{array}{cccccc} 0 & \dfrac{\cos\theta}{g} & \dfrac{\sin\theta}{g} & 1 & t & 0 \\ \dfrac{1}{V} & 0 & -\dfrac{t}{V}\cos\theta & \dfrac{\gamma g t}{V} & \dfrac{\gamma g t^2}{2V} & \dfrac{1}{V} \\ -\dfrac{1}{V} & \dfrac{\cos\theta}{g} & \dfrac{\sin\theta}{g} + \dfrac{t}{V}\cos\theta & 1 - \dfrac{\gamma g t}{V} & t - \dfrac{\gamma g t^2}{2V} & -\dfrac{1}{V} \end{array} \right| \begin{array}{c} (\theta) \\ \\ (\gamma) \\ \\ (\alpha) \end{array} \tag{7.12}$$

7.2.2. <u>The sensitivity matrix S for nonsteady symmetric flight.</u>

In the case of nonsteady symmetric flight the elements of S are again determined using eqs (5.8) and (5.13) :

$$\Delta\Delta h = \Delta\Delta h_o + \Delta C_o . t + \Delta A_{x_o} . \iint_{t_o}^{t} \sin\theta \, dt^2 - \Delta A_{z_o} . \iint_{t_o}^{t} \cos\theta \, dt^2$$

$$+ \Delta\theta_o . \iint_{t_o}^{t} a_{hor} dt^2 + \Delta q_o . \iint_{t_o}^{t} a_{hor} t . dt^2 \tag{5.8}$$

$$\Delta U_a = \Delta U_{a_o} + \Delta A_{x_o} \cdot \int_{t_o}^{t} \cos\Theta \, dt + \Delta A_{z_o} \cdot \int_{t_o}^{t} \sin\Theta \, dt$$

$$- \Delta\Theta_o \cdot \int_{t_o}^{t} (a_{vert} + g) \, dt - \Delta q_o \cdot \int_{t_o}^{t} (a_{vert} + g) \, t \, . dt \qquad (5.13)$$

An equation for ΔC is obtained by differentiating (5.8) with respect to the time t :

$$\Delta C = \Delta C_o + \Delta A_{x_o} \cdot \int_{t_o}^{t} \sin\Theta \, dt - \Delta A_{z_o} \cdot \int_{t_o}^{t} \cos\Theta \, dt + \Delta\Theta_o \cdot \int_{t_o}^{t} a_{hor} \, dt$$

$$+ \Delta q_o \cdot \int_{t_o}^{t} a_{hor} \, . t \, . dt \qquad (7.13)$$

During the special nonsteady manoeuvre, which is described in Chapter 8, a_{vert} remains relatively small with respect to g and a_{hor} differs little from the mean value of $a_{hor} = 0.5$ m/sec^2. This allows some simplifications to be made. The elements of the matrix S are written out in Appendix 3. The result for nonsteady flights is :

$$S = \begin{array}{c} \\ \\ \\ \end{array}\left|\begin{array}{cccccc} (\Delta C_o) & (\Delta A_{x_o}) & (\Delta A_{z_o}) & (\Delta\Theta_o) & (\Delta q_o) & (w_w) \\ 0 & \frac{1}{g} & 0 & 1 & t & 0 \\ \frac{1}{V} & \frac{1}{V}\int_{t_o}^{t} \sin\Theta dt & -\frac{t}{V}-\frac{\gamma}{V}\int_{t_o}^{t} \sin\Theta dt & \frac{1}{V}\left\{\frac{t}{2}+\gamma g t\right\} & \frac{t^2}{2V}\left\{\frac{1}{2}+\gamma g\right\} & \frac{1}{V} \\ -\frac{1}{V} & \frac{1}{g}-\frac{1}{V}\int_{t_o}^{t} \sin\Theta dt & +\frac{t}{V}+\frac{\gamma}{V}\int_{t_o}^{t} \sin\Theta dt & 1-\frac{1}{V}\left\{\frac{t}{2}+\gamma g t\right\} & t-\frac{t^2}{2V}\left\{\frac{1}{2}+\gamma g\right\} & -\frac{1}{V} \end{array}\right| \begin{array}{c} \\ (\Theta) \\ (\gamma) \\ (\alpha) \end{array}$$

(7.14)

If the covariance matrix of the systematic errors is known, the sought for covariance matrix A of θ, γ and α can be computed using (7.3) and the expressions (7.12) or (7.14) for S.

7.3. The covariance matrix A' from the results of the steady symmetric flights.

Applying the method presented in paragraph 7.1, the covariance matrix A of θ, γ and α can be computed, provided S and B are known. For a series of steady flights there is a second way to determine A which will now be discussed.

This method is based on the fact that for a series of steady flights (climbs and descents) performed at one engine power setting, there is a unique relation between θ and C_Z, γ and C_Z and of course between α and C_Z, see Fig.4. If a sufficient number of measurements is available, the angles θ, γ and α can be expressed analytically as functions of C_Z from these measurements with the aid of regression analysis. In the case of the flight tests to be described in Chapter 8, fifth degree polynomials in C_Z were used.

For all n steady flights θ_i (i = 1, 2, ..., n) is expressed as :

$$\theta_i = a_o + a_1 C_{Z_1} + a_2 C_{Z_i}^2 + \dots + a_5 C_{Z_i}^5 + \Delta\theta_i$$

Here each of the θ_i and C_{Z_i} are the average values of θ and C_Z during a nominally steady flight lasting about 60 sec. From the above relation follows :

$$\Delta\theta_i = \theta_i - (a_o + a_1 C_{Z_i} + a_2 C_{Z_i}^2 + \dots + a_5 C_{Z_i}^5) \qquad (7.15)$$

in which $\Delta\theta_i$ is the deviation of θ_i relative to the mean relation between θ and C_Z resulting from the regression analysis. For all observations $\Delta\theta_i$, $\Delta\gamma_i$ and $\Delta\alpha_i$ can be obtained in this way.

The presence of a_o in (7.15) garantees that the mean value of $\Delta\theta_i$ equals zero :

$$\overline{\Delta\theta_i} = \overline{\Delta\gamma_i} = \overline{\Delta\alpha_i} = 0$$

The covariance matrix A' of θ, γ and α for steady flights is, according to Ref.2 :

$$A' = \frac{1}{n-1} \begin{vmatrix} \Delta\theta_1, & \Delta\theta_2, & \ldots\ldots & \Delta\theta_n \\ \Delta\gamma_1, & \Delta\gamma_2, & \ldots\ldots & \Delta\gamma_n \\ \Delta\alpha_1, & \Delta\alpha_2, & \ldots\ldots & \Delta\alpha_n \end{vmatrix} \cdot \begin{vmatrix} \Delta\theta_1 & \Delta\gamma_1 & \Delta\alpha_1 \\ \Delta\theta_2 & \Delta\gamma_2 & \Delta\alpha_2 \\ \vdots & \vdots & \vdots \\ \Delta\theta_n & \Delta\gamma_n & \Delta\alpha_n \end{vmatrix} \qquad (7.16)$$

For steady symmetric flight it is evident that a comparison of the covariance matrix A' of (7.16) with the covariance matrix A from the error analysis according to (7.3) is possible. Thus it can be checked whether the assumptions of paragraph 7.1 on which the error analysis is based are correct.

8. The flight tests.

8.1. Introduction.

This Chapter gives a brief description of the flight tests to which the discussed method to reconstruct the measured aircraft's motions was applied.

8.2. The aircraft.

The flight tests were carried out with the laboratory aircraft of the Delft University of Technology, a De Havilland DHC-2''Beaver'', PH-VTH, see Fig.5. In Tabel 1 some general data of this aircraft are given.

8.3. The instrumentation system.

A simplified scheme of the instrumentation system used for the flight measurements is given in Fig.6.

The system contains 18 transducers, the electric output signals of which are all between 0 and 10 Volt DC. These output signals are fed to an analog-digital converter via active filters. The digital output signal of the converter is recorded on magnetic tape. A total of 80 measurements per second is taken, the most important variables as A_x,A_z and q having a scanning rate of 10 per sec. Other variables have lower scanning rates.

The analog-digital converter has a resolution of 1 part in 10.000 corresponding to 0.002 m/sec^2 for A_x and A_z and 0.004^o/sec for q. The calibration characteristics of the instruments relating the recorded voltages to the measured variables are expressed as polynomials, see Ref. 5. For a more detailed description of the instrumentation system, see Ref. 6.

8.4. The flight test programme.

In the first three months of 1967 six test flights were carried out, each of about three hours duration. During these flights 119 recordings, of one minute each, were made in steady as well as in nonsteady flight. During steady flights 75 recordings were made. They can be divided as follows :

1. 39 recordings in steady symmetric flight to determine the position error correction of the swiveling state tube. The position error correction is elaborated upon in Appendix 4.
2. 36 recordings in steady symmetric climbs. These measurements were used to determine the performance of the aircraft in the conventional way, see Ref. 1. Thus the performance characteristics obtained from measurements in nonsteady flight could be verified. The results of 28 of these recordings appeared suitable to be used for the determination of the covariance matrix A' of Θ, γ and α as described in paragraph 7.3.

The remaining 44 recordings consisted of measurements during nonsteady flight. The nonsteady manoeuvres which were recorded were of several different types. All these manoeuvres were carried out at one engine power setting at an altitude of about 6000 ft. Only one type of manoeuvre was chosen for further analysis. Of this particular type 7 manoeuvres were recorded, the results of which were used for the error analysis described in Chapter 7. The shape of these manoeuvre is described in the following paragraph.

8.5. The nonsteady manoeuvre.

The nonsteady manoeuvre consists of a symmetric flight which is started from a climb at low speed. From this state the aircraft is accelerated forward at constant engine power to maximum speed. For this purpose the aircraft's nose is lowered by gradually pushing the control wheel forward. During this accelerated motion the aircraft is forced four times to oscillate about the lateral axis, again by moving the elevator. In Fig.7 the elevator angle δ_e, the change of altitude Δh, the speed V, the angle of pitch Θ, the flight path angle γ and the angle of attack α are plotted as a function of time. A more detailed discussion of the choice of the shape of this manoeuvre is given in Ref. 1.

9. Results.

9.1. Introduction.

In Chapter 7 a method was described with which an estimate of the accuracy of the angles Θ, γ and α can be determined. The results of this error analysis will be discussed in this Chapter. In particular the covariance matrix A of Θ, γ and α and the covariance matrix B of the systematic error are of interest here.

To obtain a better expression of the values of the elements of these matrices they are not only presented in the conventional way but also in the following somewhat modified form.

Suppose a covariance matrix X of the variables x_1, x_2, x_3, ... x_n is given. The elements on the main diagonal of X are the variances of x_1, x_2, ... , x_n :

$$\sigma_{x_i}^2 = \frac{\sum_{k=1}^{m} (x_{i_k} - \bar{x}_i)^2}{m - 1} \tag{9.1}$$

The other elements of the matrix X are the covariances of x_1, x_2, ..., x_n :

$$\sigma_{x_i x_j} = \frac{\sum_{k=1}^{m} (x_{i_k} - \bar{x}_i)(x_{j_k} - \bar{x}_j)}{m - 1} \tag{9.2}$$

So the covariance matrix X is :

$$X = \begin{vmatrix} \sigma_{x_1}^2 & \sigma_{x_1 x_2} & \sigma_{x_1 x_n} \\ \sigma_{x_2 x_1} & \sigma_{x_2}^2 & \\ \sigma_{x_n x_1} & & \sigma_{x_n}^2 \end{vmatrix} \tag{9.3}$$

In the modified form of X the elements on the main diagonal are the standard deviations σ_{x_i} and the nondiagonal elements are the elementary correlation coefficients r_{ij}, defined as :

$$r_{ij} = \frac{\sigma_{x_i x_j}}{\sigma_{x_i} \cdot \sigma_{x_j}} \qquad (9.4)$$

The elementary correlation coefficient has the following qualities :

$|r_{ij}|$ = 1 in the case of exact lineair dependency of x_i and x_j

$|r_{ij}|$ = 0 in the case of exact lineair independency of x_i and x_j.

The modified form of the covariance matrix X will be denoted in the following by X^* :

$$X^* = \begin{vmatrix} \sigma_{x_1} & r_{12} & r_{13} & \cdots & r_{1n} \\ r_{21} & \sigma_{x_2} & & & \vdots \\ r_{31} & \vdots & \sigma_{x_3} & & \vdots \\ \vdots & \vdots & \vdots & & \vdots \\ r_n & \vdots & \vdots & \cdots & \sigma_{x_n} \end{vmatrix} \qquad (9.5)$$

9.2. <u>Steady symmetric flight.</u>

Chapter 7 described how the covariance matrix A of Θ, γ and α can be estimated with (7.3) if the sensitivity matrix S and the covariance matrix B of the systematic errors are known.

Moreover, it was found that the covariance matrix A' of Θ, γ and α could be determined experimentally from the results of the steady flights with (7.16). If the error analysis on which the computation of A is based is correct than the covariance matrix A in principle should agree with the covariance matrix A'. Comparison of these two matrices thus provides a check on the error analysis on which the estimate of A is based.

9.2.1. The sensitivity matrix S.

In paragraph 7.2.1. an expression for the matrix S was derived. From (7.12) it appears that S depends on the aircraft's motion and thus on V, θ and γ. In addition some elements of the sensitivity matrix S depend on time t. For one steady symmetric flight at constant V, θ and γ the matrix S changes as a function of t.

From (7.3) it follows that the covariance matrix A depends as well on the variables just mentioned. To compare A with A', A and consequently S have to be computed for the same set of conditions as A' has been determined for.

The values of θ_i, γ_i, α_i and C_{Z_i} (i = 1, 2, ..., n), derived from the steady flights, are the averages of θ, γ, α and C_Z during each of the n steady symmetric flights, see paragraph 7.3. The steady flights were carried out at different speeds averaging 42 m/sec approximately. When computing S for all steady flights together, the following average values of V, θ and γ were used.

$$V = 42 \text{ m/sec}$$
$$\theta = 0.2046 \text{ rad}$$
$$\gamma = 0.0585 \text{ rad}$$

The recordings of the steady flights all lasted about 60 sec. For a comparison of A with A' the time t, which is a variable in the sensitivity matrix S, was chosen half way the steady flight, i.e. t=30 sec. The values of θ, γ, V and t just mentioned did indeed occur together during one of the steady flights and are in good correspondence with the average conditions for which A' was determined.

From (7.12) the sensitivity matrix S for an average of the steady symmetric flights at t = 30 sec is now :

$$S = \begin{array}{c} \\ \\ \\ \\ \end{array} \begin{array}{cccccc} (\Delta C_o) & (\Delta A_{x_o}) & (\Delta A_{z_o}) & (\Delta\theta_o) & (\Delta q_o) & (w_w) \end{array}$$

	(ΔC_o)	(ΔA_{x_o})	(ΔA_{z_o})	$(\Delta\theta_o)$	(Δq_o)	(w_w)	
	0	0.1	0.015	1	30	0	(θ)
S =	0.024	0	-0.716	0.455	6.82	0.024	(γ)
	-0.024	0.1	0.731	0.545	23.18	-0.024	(α)

(9.6)

9.2.2. The covariance matrix B of the systematic errors for steady flight.

The covariance matrix B of the systematic errors for steady symmetric flight is built up from results of regression analyses executed in accordance with (6.7) and (6.8) :

$$\Delta\Delta h_j^{**} = \Delta\Delta h_o + \Delta C_o . t_j + \Delta A_{x_o} \iint_{t_o}^{t_j} \sin \theta dt^2 - \Delta A_{z_o} \iint_{t_o}^{t_j} \cos \theta dt^2$$

$$= \Delta\Delta h_j - \Delta\theta_o \iint_{t_o}^{t_j} a_{hor} dt^2 - \Delta q_o \iint_{t_o}^{t_j} a_{hor} .t.dt^2 \qquad (6.7)$$

resulting in values of ΔC_o, ΔA_{x_o} and ΔA_{z_o} and :

$$\Delta U_{a_j}^{xx} = \Delta U_{a_o} - \Delta\theta_o \int_{t_o}^{t_j} (a_{vert}+g)\,dt - \Delta q_o \int_{t_o}^{t_j} (a_{vert}+g).t.dt$$

$$= \Delta U_{a_j} - \Delta A_{x_o} \int_{t_o}^{t_j} \cos \theta dt - \Delta A_{z_o} \int_{t_o}^{t_j} \sin \theta dt \qquad (6.8)$$

resulting in values of $\Delta\theta_o$ and Δq_o.

Since the systematic errors are determined from two separate regression analyses, no covariance is found between ΔC_o, ΔA_{x_o} and ΔA_{z_o} on the one hand and $\Delta\theta_o$ and Δq_o on the other.

Errors in the initial estimates of $\Delta\theta_o$ and Δq_o will influence the computed values of $\Delta\Delta h_j^{**}$ and the remaining errors in ΔA_{x_o} and ΔA_{z_o} will influence the computed $\Delta U_{a_j}^{**}$. As a consequence the final estimates of ΔC_o, ΔA_{x_o}, ΔA_{z_o}, $\Delta\theta_o$ and Δq_o will contain errors which are mutually dependent. This means that the covariances in the matrix B between ΔC_o, ΔA_{x_o}, ΔA_{z_o} and $\Delta\theta_o$, Δq_o differ from zero. Since these covariances are unknown they must be assumed to be zero. The elements of the sensitivity matrix S, however, are corrected in such a way as to compensate for this effect. This correction is discussed in Appendix 3.

The matrix B for steady flight determined from the results of regression analyses according to (6.7) and (6.8) proves to be as follows :

$$B = \left| \begin{array}{cccccc} 0.401\cdot10^{-5} & 0 & 0.142\cdot10^{-3} & 0 & 0 & 0 \\ 0 & 0.643\cdot10^{-3} & 0 & 0 & 0 & 0 \\ 0.142\cdot10^{-6} & 0 & 0.537\cdot10^{-8} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.541\cdot10^{-7} & 0.186\cdot10^{-8} & 0 \\ 0 & 0 & 0 & 0.186\cdot10^{-8} & 0.681\cdot10^{-10} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0484 \end{array} \right| \begin{array}{l} (\Delta C_o) \\ (\Delta A_{x_o}) \\ (\Delta A_{z_o}) \\ (\Delta\theta_o) \\ (\Delta q_o) \\ (w_w) \end{array} \quad (9.7)$$

and in the modified form introduced in paragraph 9.1, B^* is :

$$B^* = \left| \begin{array}{cccccc} 0.200\cdot10^{-2} & 0 & 0.968 & 0 & 0 & 0 \\ 0 & 0.252\cdot10^{-1} & 0 & 0 & 0 & 0 \\ 0.968 & 0 & 0.732\cdot10^{-4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.232\cdot10^{-3} & 0.968 & 0 \\ 0 & 0 & 0 & 0.968 & 0.825\cdot10^{-5} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.22 \end{array} \right| \begin{array}{l} (\Delta C_o) \\ (\Delta A_{x_o}) \\ (\Delta A_{z_o}) \\ (\Delta\theta_o) \\ (\Delta q_o) \\ (w_w) \end{array} \quad (9.8)$$

In this matrix B of (9.7) a value of $\sigma^2_{\Delta A_{x_o}}$ has been given although, as discussed in Chapter 6, ΔA_{x_o} cannot be determined from steady flights. The variance of ΔA_{x_o} resulting from a neglection of the actual zeroshift has been estimated, however, using the values of ΔA_{x_o} computed for the nonsteady flights. See Table 2. From these values - to be discussed in 9.6.1. - the variance of ΔA_{x_o} appears to be :

$$\sigma^2_{\Delta A_{x_o}} = 0.634 \cdot 10^{-3} \ m^2/sec^4$$

This value was inserted in (9.7).

Although the results of the regression analyses, on wich Table 2 is based, suggest a correlation to excist between ΔA_{x_o} and ΔA_{z_o} for nonsteady flights, it has been assumed in (9.7) that the correlation between ΔA_{x_o} and ΔA_{z_o} for the steady flights is equal to zero.

Furthermore, in (9.7) a value of $\sigma_{w_w}^2$ has been given, although in the previous discussions the influence of w_w was neglected most of the time. The quantitative estimation of this $\sigma_{w_w}^2$ will be discussed in 9.2.3. It can be remarked here, however, that the systematic error w_w is assumed to be independent of the other systematic errors, which means in (9.7) that the covariances between these errors and w_w have to be put equal to zero.

9.2.3. The covariance matrix A of Θ, γ and α for steady symmetric flight.

Knowing the sensitivity matrix S and the covariance matrix B as discussed in 9.2.1. and 9.2.2., an estimate of the covariance matrix A can be calculated using (7.3). In this context the quantitative determination of $\sigma_{w_w}^2$ should also be further discussed.

Assuming first that w_w has been equal to zero, A can be calculated by letting $\sigma_{w_w}^2 = 0$. The matrix A then becomes :

$$A_{w_w = \bar{\bar{0}}} = \begin{vmatrix} 0.684\cdot10^{-5} & 0.503\cdot10^{-9} & 0.684\cdot10^{-5} \\ 0.503\cdot10^{-9} & 0.301\cdot10^{-8} & -0.251\cdot10^{-8} \\ 0.684\cdot10^{-5} & -0.251\cdot10^{-8} & 0.684\cdot10^{-5} \end{vmatrix} \begin{matrix} (\theta) \\ (\gamma) \\ (\alpha) \end{matrix} \qquad (9.9)$$

If the modified matrix A^{*} resulting from (9.9) is formed, the elements on the main diagonal are the standard deviations of θ, γ and α. In (9.10) this matrix A^{*} is given, where the three standard deviations are expressed in degrees.

$$A^{*}_{w_w = \bar{\bar{0}}} = \begin{vmatrix} 0.150^{o} & 0.0035 & 0.999 \\ 0.0035 & 0.0031^{o} & -0.0175 \\ 0.999 & -0.0175 & 0.150^{o} \end{vmatrix} \begin{matrix} (\theta) \\ (\gamma) \\ (\alpha) \end{matrix} \qquad (9.10)$$

In this matrix the standard deviation of the flightpath angle σ_{γ} is very small. Furthermore $\sigma_{\theta} = \sigma_{\alpha}$ and $r_{\theta\alpha} = 1$. This means that the error in α is practically caused only by the error in θ.

Attention is now given to the experimentally derived covariance matrix A. In Fig. 4 the angles θ, γ and α of the different steady flights have been plotted as functions of C_Z. The curves drawn through the datapoints are fifth degree polynomials of C_Z, the coefficients of which were determined as discussed in paragraph 7.3. With (7.16) the covariance matrix A', based on the measurements in steady flights of θ, γ and α is now determined using the deviations $\Delta\theta$, $\Delta\gamma$ and $\Delta\alpha$ of θ, γ and α, relative to the curves of Fig. 4. The resulting values of θ, γ, α, $\Delta\theta$, $\Delta\gamma$ and $\Delta\alpha$ employed in this analysis have been given in Table 3.

The resulting covariance matrix A' is :

$$A' = \begin{vmatrix} 0.881\cdot10^{-5} & 0.931\cdot10^{-6} & 0.730\cdot10^{-5} \\ 0.931\cdot10^{-6} & 0.272\cdot10^{-4} & -0.273\cdot10^{-4} \\ 0.730\cdot10^{-5} & -0.273\cdot10^{-4} & 0.355\cdot10^{-4} \end{vmatrix} \begin{matrix} (\theta) \\ (\gamma) \\ (\alpha) \end{matrix} \qquad (9.11)$$

where θ, γ and α are expressed in radians.

In the modified form, expressing the standard deviations of θ, γ and

α again in degrees, A' becomes :

$$A'^{*} = \begin{vmatrix} 0.170^{o} & 0.0589 & 0.413 \\ 0.0589 & 0.299^{o} & -0.878 \\ 0.413 & -0.878 & 0.342^{o} \end{vmatrix} \quad \begin{matrix} (\theta) \\ (\gamma) \\ (\alpha) \end{matrix} \qquad (9.12)$$

If, during the measurements in steady flight w_W had indeed been equal to zero, than the covariance matrix A in (9.9) should be in agreement with the matrix A' in (9.11). From the modified presentation of these matrices (9.10) and (9.12) it appears very clearly that the standard deviation of θ agrees reasonably well. The standard deviation of γ in (9.10) is, however, not at all equal to the corresponding value in (9.12). In addition the remaining elements of (9.10) are all different of their counterparts in (9.12).

Since the actual standard deviations of γ in (9.12) - being 0.299^{o} - is much larger than the calculated standard deviation 0.0031^{o} in $A^{*}_{w_W=0}$, it is obvious to assume that w_W has not been equal to zero during the flight tests.

The value of the variance of w_W which has to be inserted in the covariance matrix B is chosen such, that the variance of γ in the covariance matrix A for $w_W \neq 0$ becomes equal to the experimentally derived value of $0.272 \cdot 10^{-4}$ in stead of $0.301 \cdot 10^{-8}$ in (9.9). The required value in B is $\sigma_{w_W}^{2} = 0.0484\ m^2/sec^2$. Then the covariance matrix $A_{w_W \neq 0}$, replacing (9.9), becomes :

$$A_{w_W \neq 0} = \begin{vmatrix} 0.684 \cdot 10^{-5} & 0.503 \cdot 10^{-9} & 0.684 \cdot 10^{-5} \\ 0.503 \cdot 10^{-9} & 0.279 \cdot 10^{-4} & -0.279 \cdot 10^{-4} \\ 0.684 \cdot 10^{-5} & -0.279 \cdot 10^{-4} & 0.347 \cdot 10^{-4} \end{vmatrix} \quad \begin{matrix} (\theta) \\ (\gamma) \\ (\alpha) \end{matrix} \qquad (9.13)$$

and in the modified form :

$$A^{*}_{w_W \neq 0} = \begin{vmatrix} 0.150^{o} & 0.364 \cdot 10^{-4} & 0.444 \\ 0.366 \cdot 10^{-4} & 0.302^{o} & -0.896 \\ 0.444 & -0.896 & 0.338 \end{vmatrix} \quad \begin{matrix} (\theta) \\ (\gamma) \\ (\alpha) \end{matrix} \qquad (9.14)$$

Now it turns out that the matrices (9.13) and (9.14) are in good agreement with (9.11) and (9.12) respectively. The standard deviation of γ in (9.14) has become very nearly equal to the standard deviation of γ in (9.12). In addition, however, the standard deviations of α in (9.12) and (9.14) have become nearly equal as well. Also the three elementary correlation coefficients in (9.12) and (9.14) show a very good correspondence.

These facts seem to justify the assumption made about the presence of a vertical velocity of the air during the flight tests. Furthermore from these data it is concluded that :

1. The error model on which the covariance matrix A has been based, is substantially correct.
2. It is admissible to compute the covariance matrix A for nonsteady flight on the basis of this error model.

9.3. Nonsteady flight.

For nonsteady manoeuvres the covariance matrix A can be determined as for steady flight, if the matrices S and B are known. Where for nonsteady flight S depends on t as well, a choice has to be made as to the time t for which A shall be computed. To make a comparison with A for steady flight possible, the characteristic t in the matrix S has been chosen equal to the characteristic time for steady flights: t = 30 sec.

9.3.1. The sensitivity matrix S for nonsteady flight.

In contrast with steady flight the variables θ and γ in the elements of S (7.14) vary during a manoeuvre. When computing the matrix S at t = 30 sec, data for one particular manoeuvre, i.e. 16036702, have been used. These data are :

$$V = 42.7 \text{ m/sec} \qquad \theta = 0.0816 \qquad \gamma = -0.0256$$

$$\int_0^{30} \sin \theta dt = 6.43$$

substitution in (7.14) yields the matrix S :

$$S = \begin{array}{c} \\ \\ \\ \\ \end{array} \begin{array}{cccccc} (\Delta C_o) & (\Delta A_{x_o}) & (\Delta A_{z_o}) & (\Delta\theta_o) & (\Delta q_o) & (w_w) \\ \end{array}$$

$$S = \left| \begin{array}{cccccc} 0 & 0.1 & 0 & 1 & 30 & 0 \\ 0.023 & 0.156 & -0.703 & 0.27 & 4.1 & 0.023 \\ -0.023 & -0.056 & +0.703 & 0.73 & 25.9 & -0.023 \end{array} \right| \begin{array}{c} (\theta) \\ (\gamma) \\ (\alpha) \end{array} \quad (9.15)$$

9.3.2. The covariance matrix B for nonsteady flight.

The covariance matrix B is built up from data from regression analyses based on (6.7) and (6.8). As in the case of steady flight, no covariances are found between ΔC_o, ΔA_{x_o} and ΔA_{z_o} on the one hand and $\Delta\theta_o$ and Δq_o on the other. Hence they must be assumed to be zero. The elements of the sensitivity matrix S, however, are corrected in such a way as to compensate for this effect. This correction is discussed in Appendix 3.

Regression analyses on the data of a nonsteady manoeuvre result in :

$$B = \left| \begin{array}{cccccc} 0.303\cdot10^{-4} & -0.134\cdot10^{-4} & -0.162\cdot10^{-5} & 0 & 0 & 0 \\ -0.134\cdot10^{-4} & 0.671\cdot10^{-5} & 0.869\cdot10^{-6} & 0 & 0 & 0 \\ -0.162\cdot10^{-5} & 0.869\cdot10^{-6} & 0.116\cdot10^{-10} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.132\cdot10^{-7} & -0.469\cdot10^{-9} & 0 \\ 0 & 0 & 0 & -0.469\cdot10^{-9} & 0.142\cdot10^{-10} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0484 \end{array} \right| \begin{array}{c} (\Delta C_o) \\ (\Delta A_{x_o}) \\ (\Delta A_{z_o}) \\ (\Delta\theta_o) \\ (\Delta q_o) \\ (w_w) \end{array}$$

(9.16)

and in modified form :

$$B^{*} = \begin{vmatrix} 0.505\cdot10^{-2} & -0.940 & -0.864 & 0 & 0 & 0 \\ -0.940 & 0.252\cdot10^{-2} & 0.983 & 0 & 0 & 0 \\ -0.864 & 0.983 & 0.341\cdot10^{-3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.115\cdot10^{-3} & -0.968 & 0 \\ 0 & 0 & 0 & -0.968 & 0.376\cdot10^{-5} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.22 \end{vmatrix} \begin{matrix} (\Delta C_o) \\ (\Delta A_{x_o}) \\ (\Delta A_{z_o}) \\ (\Delta\theta_o) \\ (\Delta q_o) \\ (w_W) \end{matrix}$$

(9.17)

The same value of $\sigma_{w_W}^{2}$ has been used, as obtained in 9.2.3. from the analysis of steady flight conditions.

9.3.3. The covariance matrix A for nonsteady flight.

The covariance matrix A follows with (7.3) from S in (9.15) and B in (9.16).

$$A = \begin{vmatrix} 0.693\cdot10^{-7} & 0.131\cdot10^{-7} & 0.565\cdot10^{-7} \\ 0.131\cdot10^{-7} & 0.256\cdot10^{-4} & -0.256\cdot10^{-4} \\ 0.565\cdot10^{-7} & -0.256\cdot10^{-4} & 0.257\cdot10^{-4} \end{vmatrix} \begin{matrix} (\theta) \\ (\gamma) \\ (\alpha) \end{matrix} \qquad (9.18)$$

and, in the modified form :

$$A^{*} = \begin{vmatrix} 0.015^{o} & 0.988\cdot10^{-2} & 0.0424 \\ 0.988\cdot10^{-2} & 0.290^{o} & -0.979 \\ 0.0424 & -0.979 & 0.290^{o} \end{vmatrix} \begin{matrix} (\theta) \\ (\gamma) \\ (\alpha) \end{matrix} \qquad (9.19)$$

In the latter matrix, θ, γ and α have again been expressed in degrees. Since ΔA_{x_o} can be determined in principle in the nonsteady flight, the remaining error in θ is much smaller than in the case of steady flight. The standard deviations of γ and α are very nearly equal to those found in steady flight. The main cause of these errors appears to be w_W.

If the influence of w_W could be greatly reduced, as might be the case for a high speed aircraft, the elements in S :

$$\frac{\partial \gamma}{\partial w_W} = \frac{1}{V} \quad \text{and} \quad \frac{\partial \alpha}{\partial w_W} = -\frac{1}{V}$$

- being equal to 0.023 and -0.023, respectively in (9.15) - could effectively be put equal to zero. Then the covariance matrix A in the modified form would change into :

$$A^{*}_{w_W=0} = \begin{vmatrix} 0.015^{o} & 0.936 & 0.996 \\ 0.936 & 0.0031^{o} & 0.902 \\ 0.996 & 0.902 & 0.0124^{o} \end{vmatrix} \quad \begin{matrix} (\theta) \\ (\gamma) \\ (\alpha) \end{matrix} \qquad (9.20)$$

The same result would have been obtained if w_W had been equal to zero in the present flight tests. The remaining small standard deviations on the main diagonal might be considered as the minimal values of the errors in θ, γ and α which could have been achieved by the method described in this report and with using the present instrumentation system under the most favourable (atmospheric) conditions.

9.4. The residual values of $\Delta\Delta h$ and ΔU_a.

The method described in this report has been based on the requirement that the computed trajectory of the aircraft should be as well as possible in accordance with the measured trajectory. In particular the various zeroshifts are determined in such a way as to minimise the variances of $\Delta\Delta h$ and ΔU_a, where :

$$\Delta\Delta h = \Delta h_{computed} - \Delta h_{measured}$$

$$\Delta U_a = U_{a_{computed}} - U_{a_{measured}}$$

The residual values of $\Delta\Delta h$ and ΔU_a must in principle be ascribed to the noise in the pressure measurements. In Fig.8 a typical example of the residual values of $\Delta\Delta h$ and ΔU_a have been plotted as functions of time for a nonsteady manoeuvre. In Table 2 the standard deviations of $\Delta\Delta h$ and ΔU_a are given for all steady and nonsteady flights. The mean values of $\sigma_{\Delta\Delta h}$ and $\sigma_{\Delta U_a}$ are 0.184 m and 0.122 m/sec respectively.

9.5. The zeroshifts.

The zeroshifts ΔA_{x_o}, ΔA_{z_o} and Δq_o computed for the steady and nonsteady flights are given in Table 2 as well. The mean values of the zeroshifts ΔA_{z_o} and Δq_o of the six testflights are different. The scatter in ΔA_{z_o} and Δq_o resulting from measurements during the same testflight are small. The standard deviations derived from Table 2 are 0.0066 m/sec^2 and 0.00010 rad/sec respectively. The zeroshift ΔA_{x_o}, determined only from nonsteady flight, shows a much greater scatter than ΔA_{z_o}. The standard deviation as derived from Table 2 is 0.025 m/sec^2.

9.6. Remarks.

9.6.1. The reliability of ΔA_{x_o}.

From the matrix B^* for nonsteady flight, see (9.17) in 9.3.2., it appeared that ΔA_{x_o} and ΔA_{z_o} are strongly related, the elementary correlation coefficient $r_{\Delta A_{x_o}, \Delta A_{z_o}}$ being 0.983. The standard deviation of ΔA_{x_o}, as derived from the matrix B^* (9.17) is considerably larger than that of ΔA_{z_o}. This corresponds with the scatter of ΔA_{x_o}, and ΔA_{z_o} for the 7 nonsteady manoeuvres in Table 2. The values of the zeroshifts ΔA_{x_o} and ΔA_{z_o} which are found from the flight tests can be considered to be built up from two parts. The first part is the result of the actual zeroshift during the measurements of the accelerometers and their active electronic filters. The second part of the computed error is the result of the linear dependency between ΔA_{x_o} and ΔA_{z_o}.

The scatter of ΔA_{x_o} for the nonsteady manoeuvres in Table 2 is larger than could reasonably be expected, considering the instrumentation system used. It might therefore be concluded that the lineair dependency between ΔA_{x_o} and ΔA_{z_o} has influenced the computed values of these two zeroshifts.

To compute the covariance matrix A for steady flights (par. 9.2.3.) this large scatter of ΔA_{x_o}, $\sigma_{\Delta A_{x_o}}$ =0.0252 m/sec^2, in the matrix B^* (9.8) was nevertheless necessary to obtain the same variance of Θ in (9.13) as from the results of the steady flights in (9.11). When computing the covariance matrix A, the variance of Θ is mainly determined by the variance of ΔA_{x_o} in the matrix B.

To reduce the remaining uncertainty of the computed zero-shifts it seems most appropriate to improve the stability of the active filters and to employ an accelerometer for A_x having a zero output fixed between more narrow limits. In this way ΔA_{x_o} might be neglected, thereby improving the accuracy with which the other zeroshifts can be determined, particularly ΔA_{z_o}

9.6.2. <u>The accuracy of the flightpath angle γ.</u>

In spite of the uncertainty in the zeroshifts just mentioned, the computed flight path of the aircraft appears to be in good correspondence with the measured trajectory. This can be concluded from the small values of the residual standard deviations of $\Delta\Delta h$ and ΔU_a given in Table 2. If γ is considered merely a variable of the reconstructed aircraft's motion relative to a geodetical system of axes, the covariance matrices $\bar{A}$ for steady and nonsteady flights at w_W=0,((9.10) in par.9.2.3. and (9.20) in par.9.3.2) yield a standard deviation of γ : $\sigma_\gamma = 0.0031^o$.

10. Conclusions.

In this report a method has been described to reconstruct the flight path of an aircraft during nominally symmetric steady or nonsteady manoeuvres, from measurements on board the aircraft.

It is a characteristic of this method, that the angles Θ, γ and α are determined by computation and not by direct measurements. In addition, systematic errors in the instrumentation system and in the estimates of the initial conditions of required integrations can be determined and corrected for with some accuracy.

The results of an error analysis, carried out in this report, are the covariance matrix B of the systematic errors and the covariance matrix A of the angles Θ, γ and α. These covariance matrices were obtained for steady as well as for nonsteady flight. On the basis of these covariance matrices the following conclusions are obtained :

1. In the reconstruction of the aircraft's flight path the following standard deviations were found :

change in altitude	: 0,184 m
horizontal speed	: 0,122 m/sec
angle of pitch	: 0,015 ^{o}
flight path angle	: 0,290 ^{o}
angle of attack	: 0,290 ^{o}

2. The influence of a constant vertical velocity of the atmosphere w_W can not be eliminated. It can be considerable in the speed region in which the method has been applied.

3. The accuracy with which the zeroshift of A_x can be determined using the present instrumentation system is insufficient. In future measurements the uncertainty in the zeroshift of A_x should to be eliminated by choosing a transducer having a sufficiently low zero uncertainty.

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APPENDIX I.

The acceleration of the centre of gravity of the aircraft.

1. The relation between the specific force A and the acceleration a.

The specific force A is defined as the external force F acting on a body, devided by the mass of the body :

$$\bar{A} = \frac{\bar{F}}{m}$$

where $\bar{A}$ and $\bar{F}$ are denoted as vectors.

The relation between the acceleration $\bar{a}$ of a body and the specific force $\bar{A}$ acting on the body in the attractive field of the earth can be derived from the following relation :

$$m.\bar{a} = \bar{F} + \bar{W}$$

and

$$\bar{a} = \frac{\bar{F}}{m} + \bar{g}$$

where $\bar{W}$ is the weight vector. Substitution of $\bar{A}$ leads to :

$$\bar{a} = \bar{A} + \bar{g} \qquad \text{(A1.1)}$$

2. The specific forces acting in the centre of gravity in the directions of the aircraft body axes.

The acceleration of the centre of gravity of the aircraft is computed from the specific forces measured with three accelerometers. In the tests described in this report these instruments were forcebalance accelerometers mounted orthogonally in a rigid box, see Ref. 6. This accelerometer-box has its own system of axes $O_A X_A Y_A Z_A$ as indicated in Fig. A1.

The calibration of the accelerometers is carried out with respect to the axes of the accelerometer-box. The origin O_A and the axes of the box do not concide with the aircraft's c.g. and body axes during the measurements.

Hence the measured specific forces A_{X_m}, A_{Y_m} and A_{Z_m} have to be transformed to the values which would have been measured at the aircraft's c.g. in the directions of the body axes. The position of the accelerometer-box relative to the c.g. is described by means of the aircraft measurement axes $O_mX_mY_mZ_m$, see Fig. A2. The accelerometer-box was mounted in the aircraft such that the Y_A-axis was parallel to the Y-axis of the aircraft.

The transformation takes place as follows.
Firstly the angle θ_A, between the X_A-axis and the X-axis is corrected for, according to Fig. A2.
Calling the specific forces, in O_A in the directions of the body ax s A'_{x_m}, A'_{y_m} and A'_{z_m} then :

$$A'_{x_m} = A_{x_m} \cdot \cos\theta_A + A_{z_m} \cdot \sin\theta_A$$

$$A'_{y_m} = A_{y_m}$$

$$A'_{z_m} = -A_{x_m} \cdot \sin\theta_D + A_{z_m} \cdot \cos\theta_D \qquad (A1.2)$$

Secondly these specific forces are transferred to the position of the aircraft's c.g. To this end it is recalled that the acceleration at any point P(x, y, z) in the system of measurement axes is :

$$a_x = a_{x_{c.g.}} - (x-x_{c.g.})(q^2+r^2) + (y-y_{c.g.})(pq-\dot{r}) + (z-z_{c.g.})(pr+\dot{q})$$

$$a_y = a_{y_{c.g.}} + (x-x_{c.g.})(pq+\dot{r}) - (y-y_{c.g.})(p^2+r^2) + (z-z_{c.g.})(qr-\dot{p}) \qquad (A1.3)$$

$$a_z = a_{z_{c.g.}} + (x-x_{c.g.})(pr-\dot{q}) + (y-y_{c.g.})(qr+\dot{p}) - (z-z_{c.g.})(q^2+p^2)$$

Due to (A1.1), simular relations hold for the specific forces A_x, A_y and A_z. The point P(x, y, z) is taken at the origin O_A of the accelerometer-box. The coordinates of O_A are x_A, y_A, z_A in the system of measurement axes, see Fig. A2, where y_A is assumed to be negligible.

The specific forces in the aircraft's c.g., taken in the directions of the aircraft's body axes now become with (A1.3) :

$$A_{x_{c.g.}} = A'_{x_m} + (x_A - x_{c.g.})q^2 - (z_A - z_{c.g})\dot{q}$$

$$A_{y_{c.g.}} = A_y$$

$$A_{z_{c.g.}} = A'_{z_m} - (x_A - x_{c.g.})\dot{q} - (z_A - z_{c.g.})q^2 \qquad \text{(A1.4)}$$

Using the relation (A1.1) the components of the acceleration $\bar{a}$ at the centre of gravity can be found from (A1.4).

3. <u>The accelerations at the aircraft's c.g. taken in the directions of the geodetical axes.</u>

The accelerations can be obtained from the specific forces by applying (A1.1). In order to transform the specific forces along the body axes of the aircraft to the specific forces along the geodetical axes, successive rotations through the opposites of the angles of roll, pitch and yaw ($-\varphi$, $-\theta$ and $-\psi$ respectively) are required, see Fig.A3. The specific forces $A_{x_{c.g.}}$, $A_{y_{c.g.}}$ and $A_{z_{c.g.}}$ acting in the centre of gravity are obtained according to (A1.4).

A rotation of the system of axes through the angle $-\varphi$ changes the unit vector along the axes as expressed by the matrix :

$$\left| -\varphi \right| = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi & \cos\varphi \end{vmatrix}$$

A rotation through the angle $-\theta$ changes the unit vector :

$$\left| -\theta \right| = \begin{vmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{vmatrix}$$

Finally a rotation through the angle $-\psi$ has the following effect on the unit vector :

$$|-\psi| = \begin{vmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Applying the three rotations in the order indicated, leads to the specific forces along the geodetical axes :

$$\begin{vmatrix} A_{hor,x} \\ A_{hor,y} \\ A_{vert} \end{vmatrix} = |-\psi| \circ |-\Theta| \circ |-\varphi| \begin{vmatrix} A_{x_{c.g.}} \\ A_{y_{c.g.}} \\ A_{z_{c.g.}} \end{vmatrix}$$

Since the gravity vector is vertical, the following relations based on (A1.1) hold :

$$a_{hor,x} = A_{hor,x}$$

$$a_{hor,y} = A_{hor,y}$$

$$a_{vert} = -A_{vert} - g$$

where a_{vert} is taken positive when pointing upwards.
After some elaboration the expression for $a_{hor,x}$, $a_{hor,y}$ and a_{vert} become :

$$a_{hor,x} = A_{x_{c.g.}} \cos\Theta \cos\psi + A_{y_{c.g.}} (\sin\Theta \sin\varphi \cos\psi - \cos\varphi \sin\psi) + A_{z_{c.g.}} (\sin\Theta \cos\varphi \cos\psi + \sin\varphi \sin\psi) \qquad (A1.5)$$

$$a_{hor,y} = A_{x_{c.g.}} \cos\Theta \sin\psi + A_{y_{c.g.}} (\sin\Theta \sin\varphi \sin\psi - \cos\varphi \cos\psi) + A_{z_{c.g.}} (\sin\Theta \cos\varphi \sin\psi - \sin\varphi \cos\psi) \qquad (A1.6)$$

$$a_{vert} = A_{x_{c.g.}} \sin\Theta - A_{y_{c.g.}} \cos\Theta \sin\varphi - A_{z_{c.g.}} \cos\Theta \cos\varphi - g \qquad (A1.7)$$

Since the nonsteady manoeuvre was nearly symmetric, it is assumed in the analysis of the systematic errors that $\psi = 0$. As a consequence the following expression for $a_{hor} = a_{hor,x}$ is used in Chapter 5 :

$$a_{hor} = A_{x_{c.g.}} \cos\Theta + A_{y_{c.g.}} \sin\Theta \sin\varphi + A_{z_{c.g.}} \sin\Theta \cos\varphi \qquad (A1.8)$$

In this report the well known expressions for rate of pitch $\dot{\Theta}$ and rate of yaw $\dot{\psi}$ are used :

$$\dot{\Theta} = q \cos\varphi - r \sin\varphi \qquad (A1.9)$$

$$\dot{\psi} = \frac{q \sin\varphi + r \cos\varphi}{\cos\Theta} \qquad (A1.10)$$

APPENDIX 2.

Summary of operations for the calculation of Θ, γ and α.

1. Introduction.

In the main body of this report the principle of the Θ, γ and α calculation and the determination of the systematic errors have been discussed. To round off this description, some additional expressions are derived in this Appendix.

The angle of pitch Θ

In Chapter 3 the following expression for Θ_o in steady symmetric flight is used :

$$\Theta_o = - \operatorname{arctg} \left(\frac{A_x}{A_z} \right)_{t_o} \qquad (3\text{-}5)$$

In actual flight the state of the aircraft at t_o may not be steady enough to allow this expression to be used. Using the equations of motion of a rigid aircraft in symmetric flight :

$$\begin{aligned} - g \sin \Theta + A_x &= \dot{u} + w\, q \\ g \cos \Theta + A_z &= \dot{w} - u\, q \end{aligned} \qquad (A2.1)$$

and assuming that :

$$w\, q = \dot{u} = \dot{w} = 0$$

an estimate of Θ_j at time t_j is then obtained from (A2.1) by substituting the velocity V for u :

$$\Theta_j = - \operatorname{arctg} \left\{ \frac{A_x}{A_z + V.q} \right\}_j$$

The scanning cycle of the instrumentation system used in the test described has a period of 2.4 sec. The first scanning period is used to obtain a better first estimate of Θ_o. To this end at $t_j = t_o$ an improved value of the angle of pitch θ_o is estimated :

$$\theta_o = \frac{1}{24} \sum_{t_j=t_o}^{t_j=t_o+2.3} \left\{ \theta_j - \int_{t_o}^{t_j} \dot{\theta} dt \right\} \tag{A2.2}$$

where :

$$t_{j+1} - t_j = 0.1 \text{ sec}$$

and :

$$\dot{\theta} = q \cos \varphi - r \sin \varphi \tag{A1.9}$$

The angle of pitch at any time during the manoeuvre follows from :

$$\theta_j = \theta_o + \int_{t_o}^{t_j} \dot{\theta} dt \tag{3.1}$$

The angle of yaw.

Angle of yaw is computed using an expression similar to (3.1) for θ_j :

$$\psi_j = \psi_o + \int_{t_o}^{t_j} \dot{\psi} dt \tag{A2.3}$$

All nonsteady manoeuvres are assumed to be started at an sideslip angle $\beta = 0$. The angle of yaw for $t_j=t_o$ is $\psi = 0$, since the X_g-axis of the geodetical system of axes is determined by the heading of the aircraft at t_o.
Substitution of $\dot{\psi}$ from Appendix 1 leads to :

$$\psi_j = \int_{t_o}^{t_j} \frac{q \sin \varphi - r \cos \varphi}{\cos \theta} dt \tag{A2.4}$$

The rate of climb C.

According to (3.8) the change in altitude at any time t_j can be written as :

$$\Delta h_j = \Delta h_o^{*} + \int_{t_o}^{t_j} C_o dt + \iint_{t_o}^{t_j} a_{vert} dt^2 \qquad (3.8)$$

The asterisk* is used in (3.8) to distinguish between the change in altitude determined by the pressure measurements Δh^* and the computed change in altitude Δh.
The expression (3.8) is used to find C_o by choosing $t_j = t_f$ at the end of the manoeuvre and using Δh_f^* obtained from pressure measurements instead of Δh_f in the left hand side of the equation.
The result is :

$$C_o = \frac{\Delta h_f^* - \Delta h_o^* - \iint_{t_o}^{t_f} a_{vert} dt^2}{t_f - t_o} \qquad (A2.5)$$

When doing so, however, ''improved'' values of Δh_o^* and Δh_f^* are first calculated in a similar way as for Θ_o :

$$\Delta h_o^* = \frac{1}{24} \sum_{t_j=t_o}^{t_j=t_o+2.3} \left\{ \Delta h_j^* - \int_{t_o}^{t_j} C_o dt - \iint_{t_o}^{t_j} a_{vert} dt^2 \right\}$$

or :

$$\Delta h_o^* = -1.15\, C_o + K \qquad (A2.6)$$

where :

$$K = \frac{1}{24} \sum_{t_j=t_o}^{t_j=t_o+2.3} \left\{ \Delta h_j^* - \iint_{t_o}^{t_j} a_{vert} dt^2 \right\} \qquad (A2.7)$$

And also for Δh_f^* :

$$\Delta h_f^{*} = \frac{1}{24} \sum_{t_j=t_f-2.3}^{t_j=t_f} \left\{ \Delta h_j^{*} + \int_{t_j}^{t_f} C_o dt + \int_{t_j}^{t_f} \left\{ \int_{0}^{\tau} a_{vert} dt \right\} d\tau \right\}$$

or :

$$\Delta h_f^{*} = 1.15\ C_o + \iint_{t_o}^{t_f} a_{vert} dt^2 + L \qquad \text{(A2.8)}$$

and :

$$L = \frac{1}{24} \sum_{t_j=t_f-2.3}^{t_j=t_f} \left\{ \Delta h_f^{*} - \iint_{t_o}^{t_f} a_{vert} dt^2 \right\}$$

Inserting (A2.6) and (A2.8) in (A2.5) leads to the following expression for C_o :

$$C_o = \frac{L - K}{t_f - t_o - 2.3} \qquad \text{(A2.10)}$$

Once C_o has been determined according to (A2.10) Δh_o^{*} can be found from (A2.6).

The horizontal component U_a of the velocity V.

The horizontal component of the velocity V has been denoted U_a. For $t_j = t_o$ an estimate of U_a is obtained in a similar manner as for θ_o:

$$U_{a_o}^{*} = \frac{1}{24} \sum_{t_j=t_o}^{t_j=t_o+2.3} \left\{ \sqrt{(U_{a_j}^{*})^2 - (\int_{t_o}^{t_j} a_{hor,y} dt)^2} - \int_{t_o}^{t_j} a_{hor,x} dt \right\} \qquad \text{(A2.11)}$$

where :

$$U_{a_j}^{*} = \sqrt{V_j^{*2} - C_j^{\,2}} \qquad \text{(A2.12)}$$

The value of U_{a_j} for any time t_j is computed with :

$$U_{a_j} = \sqrt{(U_{a_o}^{*} + \int_{t_o}^{t_j} a_{hor,x} dt)^2 + (\int_{t_o}^{t_j} a_{hor,y} dt)^2} \qquad (A2.13)$$

2. Summary of operations.

In this paragraph the operations of the Θ, γ, α calculation are summarized. A flow diagram of the programme is given in Fig. A.4. The numbers given to the operations in that Figure correspond with the numbers given below.

1. $\dot{\Theta}_j = (q \cos \varphi - r \sin \varphi)_j$

2. $\Theta_o = \frac{1}{24} \sum_{t_j=t_o}^{t_j=t_o+2.3} \left\{ - \text{arctg} \left(\frac{A_x}{A_z+Vq} \right)_j - \int_{t_o}^{t_j} \dot{\Theta} dt \right\}$

3. $\Theta_j = \Theta_o + \int_{t_o}^{t_j} \dot{\Theta} dt$

$$\dot{\psi}_j = \left(\frac{q \sin \varphi - r \cos \varphi}{\cos \Theta} \right)_j$$

$$\psi_j = \int_{t_o}^{t_j} \dot{\psi} \, dt$$

4. $a_{hor,x} = A_x \cos \Theta \cos \psi + A_y (\sin \Theta \sin \varphi \cos \psi - \cos \varphi \sin \psi)$
$\qquad + A_z (\sin \Theta \cos \varphi \cos \psi + \sin \varphi \sin \psi)$

$a_{hor,y} = A_x \cos \Theta \sin \psi + A_y (\sin \Theta \sin \varphi \sin \psi - \cos \varphi \cos \psi)$
$\qquad + A_z (\sin \Theta \cos \varphi \sin \psi - \sin \varphi \cos \psi)$

$$a_{vert} = A_x \sin \Theta - A_y \cos \Theta \sin \varphi - A_z \cos \Theta \cos \varphi - g$$

5. $$K = \frac{1}{24} \sum_{t_j=t_o}^{t_j=t_o+2.3} \left\{ \Delta h_j^* - \iint_{t_o}^{t_j} a_{vert} dt^2 \right\}$$

$$L = \frac{1}{24} \sum_{t_j=t_f-2.3}^{t_j=t_f} \left\{ \Delta h_j^* - \iint_{t_o}^{t_j} a_{vert} dt^2 \right\}$$

$$C_o = \frac{L - K}{t_f - t_o - 2.3}$$

6. $$C_j = C_o + \int_{t_o}^{t_j} a_{vert} dt$$

7. $$\Delta h_o^* = K - 1.15 \, C_o$$

$$U_{a_j}^* = \sqrt{V_j^{*2} - C_j^{\,2}}$$

$$U_{a_o}^* = \frac{1}{24} \sum_{t_j=t_o}^{t_j=t_o+2.3} \left\{ \sqrt{(U_{a_y}^*)^2 - (\int_{t_o}^{t_j} a_{hor,y} dt)^2} - \int_{t_o}^{t_j} a_{hor,x} dt \right\}$$

8. $$\Delta\Delta h_j = \Delta h_o^* + \int_{t_o}^{t_j} C_j dt - \Delta h_j^*$$

$$\Delta U_{a_j} = \sqrt{(U_{a_o}^* + \int_{t_o}^{t_j} a_{hor,x} dt)^2 + (\int_{t_o}^{t_j} a_{hor,y} dt)^2} - U_{a_j}^*$$

9. $$a_{hor} = A_x \cos\Theta + A_y \sin\varphi \sin\Theta + A_z \sin\Theta \cos\varphi$$

$$X_{1_j} = t_j$$

$$X_{2_j} = \iint_{t_o}^{t_j} \sin\Theta \, dt^2$$

$$X_{3_j} = -\iint_{t_o}^{t_j} \cos\Theta \cos\varphi \, dt^2$$

$$X_{4_j} = \iint_{t_o}^{t_j} a_{hor} dt^2$$

$$X_{5_j} = \iint_{t_o}^{t_j} a_{hor} . t . dt^2$$

$$X_{6_j} = \int_{t_o}^{t_j} \cos\Theta \, dt$$

$$X_{7_j} = \int_{t_o}^{t_j} \sin\Theta \cos\varphi \, dt$$

$$X_{8_j} = -\int_{t_o}^{t_j} (a_{vert}+g) . dt$$

$$X_{9_j} = -\int_{t_o}^{t_j} (a_{vert}+g).t.dt$$

10. Regression I.

$$\Delta\Delta h_j = a_o + a_1 X_{1_j} + a_2 X_{2_j}$$

11. $\Delta A'_{z_o} = a_2$

$$\Delta U^{*}_{a_j} = \Delta U_{a_j} - \Delta A'_{z_o} \cdot X_{6_j}$$

12. Regression II.

$$\Delta\Delta U^{*}_{a_j} = a_o + a_1 X_{8_j} + a_2 X_{9_j}$$

13. $\Delta\theta'_o = a_1, \quad \Delta q'_o = a_2$

$$\Delta\Delta h^{*}_j = \Delta\Delta h_j - \Delta\theta'_o . X_{4_j} - \Delta q'_o . X_{5_j}$$

14. Regression III.

$$\Delta\Delta h^{*}_j = a_o + a_1 . X_{1_j} + a_2 . X_{2_j} + a_3 . X_{3_j}$$

15. $\Delta\Delta h_o = a_o \quad \Delta C_o = a_1$

$$\Delta A_{x_o} = a_2 \quad \Delta A_{z_o} = a_3$$

16. Test $\left|\Delta A_{x_o}\right| \geqslant k.\sigma_{\Delta A_{x_o}}$

17. $\Delta U^{**}_{a_j} = \Delta U_{a_j} - \Delta A_{x_o} . X_{5_j} - \Delta A_{z_o} . X_{6_j}$

18. Regression IV.

$$\Delta U_{a_j}^{**} = a_o + a_1 . X_{8_j} + a_2 . X_{9_j}$$

19. $\Delta U_{a_o} = a_o$ $\quad$ $\Delta q_o = a_2$

$\Delta \Theta_o = a_1$

20. Regression V.

$$\Delta\Delta h_j^{*} = a_o + a_1 . X_{1_j} + a_2 . X_{3_j}$$

21. $\Delta\Delta h_o = a_o$ $\quad$ $\Delta A_{z_o} = a_2$

$\Delta C_o = a_1$

$$\Delta U_{a_j}^{**} = \Delta U_{a_j} - \Delta A_{z_o} . X_{6_j}$$

22. Regression VI.

$$\Delta U_{a_j}^{*} = a_o + a_1 . X_{8_j} + a_2 . X_{9_j}$$

23. $U_{a_o} = a_o$ $\quad$ $\Delta \Theta_o = a_1$

$\Delta q_o = a_2$

24. $$C_j = C_o + \int_{t_o}^{t_j} a_{vert} dt$$

$$\Delta h_j = \Delta h_o + \int_{t_o}^{t_j} C_j . dt$$

$$U_{a_j} = \sqrt{\left(U_{a_o} + \int_{t_o}^{t_j} a_{hor,x} dt\right)^2 + \left(\int_{t_o}^{t_j} a_{hor,y} dt\right)^2}$$

$$V_j = \sqrt{U_{a_j}^{\;2} + C_j^{\;2}}$$

$$U_{a_j}^{*} = \sqrt{V_j^{*2} \quad C_j^{\;2}}$$

$$\Delta U_{a_j} = U_{a_j} - U_{a_j}^{*}$$

$$\sigma_{\Delta\Delta h} = \sqrt{\frac{\sum_{j=1}^{j=n} \Delta\Delta h_j^{\;2}}{n-1}}$$

$$\sigma_{\Delta U_a} = \sqrt{\frac{\sum_{j=1}^{j=n} \Delta U_{a_j}^{\;2}}{n-1}}$$

25. $$\gamma_j = \text{arc sin} \frac{C_j}{V_j}$$

$$\alpha_j = \theta_j - \gamma_j$$

APPENDIX 3.

The elements of the sensitivity matrix S.

1. Introduction.

As stated in paragraph 7.2 the elements of the sensitivity matrix S are the partial derivatives of θ, γ and α with respect to the systematic errors and w_w. The equations from which the derivatives were obtained have been given in paragraph 7.2. In this Appendix the derivatives themselves are determined.

It has been mentioned in paragraph 9.2.2 that the covariances in the covariance matrix B between ΔC_o, ΔA_{x_o}, ΔA_{z_o} and $\Delta\theta_o$, Δq_o are unknown and have to be assumed to be zero. The elements of the sensitivity matrix S, however, have to be corrected in such a way as to compensate for this effect. A preliminary remark can be made here.

In the θ, γ, α calculation the regression analyses are performed in a fixed order. Firstly the regression analysis on the $\Delta\Delta h$ equation (6.7) is performed. Secondly the regression analysis on the ΔU_a equation (6.8) is carried out. From (6.7) an estimate of ΔC_o, ΔA_{x_o} and ΔA_{z_o} is obtained. It has been argued in paragraph 6.2 that ΔA_{x_o} has to be equated to zero in case of steady flights. In case of steady flights the regression analysis based on (6.7) will give only estimates of ΔC_o and ΔA_{z_o}. From the regression analysis on the ΔU_a equation (6.8) estimates of $\Delta\theta_o$ and Δq_o are obtained. Errors in the estimates of ΔA_{x_o} and ΔA_{z_o} will influence ΔU_a^{**} and therefore $\Delta\theta_o$ and Δq_o. Due to the order in which the regression analyses are performed, errors in the estimates of $\Delta\theta_o$ and Δq_o cannot influence the estimates of ΔA_{x_o} and ΔA_{z_o} which have been determined before.

2. Steady symmetric flight.

The equation used for the determination of the elements of S for steady flight are :

$$\Delta\Delta h = \Delta\Delta h_o + \Delta C_o . t + \Delta A_{x_o} . \tfrac{1}{2}t^2 \sin\theta - \Delta A_{z_o} . \tfrac{1}{2}t^2 \cos\theta \qquad (7.4)$$

$$\Delta U_a = \Delta U_{a_o} + \Delta A_{x_o} . t \cos\theta + \Delta A_{z_o} . t \sin\theta - \Delta\theta_o g . t - \Delta q_o . g . \tfrac{1}{2}t^2 \qquad (7.5)$$

$$\Delta C = \Delta C_o + \Delta A_{x_o} . t \sin\theta - \Delta A_{z_o} . t . \cos\theta \qquad (7.6)$$

When x is any of the systematic errors or w_w :

$$\frac{\partial\theta}{\partial x} = \frac{\partial\theta_o}{\partial x} + t . \frac{\partial q_o}{\partial x} \qquad (7.8)$$

$$\frac{\partial\gamma}{\partial x} = \frac{U_a . \frac{\partial C}{\partial x} - C . \frac{\partial U_a}{\partial x}}{U_a^2} \qquad (7.9)$$

$$\frac{\partial\alpha}{\partial x} = \frac{\partial\theta}{\partial x} - \frac{\partial\gamma}{\partial x} \qquad (7.10)$$

The derivatives are further simplified by assuming that $U_a = V$.

a. The derivatives with respect to ΔC_o.

ΔC is the error in the computed rate of climb C. The derivative of C with respect to a systematic error is equal to the derivative of ΔC with respect to the systematic error. Therefore with (7.6) it follows :

$$\frac{\partial C}{\partial \Delta C_o} = \frac{\partial \Delta C}{\partial \Delta C_o} = 1$$

ΔC is not related with θ and U_a, therefore :

$$\frac{\partial\theta}{\partial \Delta C_o} = \frac{\partial U_a}{\partial \Delta C_o} = 0$$

With (7.9) and (7.10) the derivatives of γ and α are obtained :

$$\frac{\partial \gamma}{\partial \Delta C_o} = \frac{1}{V} \quad , \quad \frac{\partial \alpha}{\partial \Delta C_o} = -\frac{1}{V}$$

b. The derivatives with respect to ΔA_{x_o}.

The $\Delta\Delta h$ equation for steady flight is :

$$\Delta\Delta h = \Delta\Delta h_o + \Delta C_o.t + \Delta A_{x_o}.\tfrac{1}{2}t^2 \sin\Theta - \Delta A_{z_o}.\tfrac{1}{2}t^2 \cos\Theta \qquad (7.4)$$

It has been argued in paragraph 6.2 that ΔA_{x_o} has to be equated to zero. Since Θ is constant in steady flight, $\frac{1}{2}t^2 \sin\Theta$ and $\frac{1}{2}t^2 \cos\Theta$ are linearly dependent. Therefore (7.4) can be rewritten :

$$\Delta\Delta h = \Delta\Delta h_o + \Delta C_o.t - \Delta A_{z_o}^{*}.\tfrac{1}{2}t^2 \cos\Theta \qquad (A3.1)$$

where :

$$\Delta A_{z_o}^{*} = -\Delta A_{x_o}.\mathrm{tg}\,\Theta + \Delta A_{z_o}$$

and ΔA_{x_o} is the actual value of ΔA_{x_o} during the measurements.
In the Θ, γ, α calculation the regression analysis is, in the case of steady flight, based on (A3.1).

The zeroshifts $\Delta\Theta_o$ and Δq_o are estimated by the regression analysis on (6.8)

$$\Delta\Delta U_a^{**} = \Delta U_{a_o} - \Delta\Theta_o g.t - \Delta q_o.g\tfrac{1}{2}t^2$$

$$= \Delta U_a - \Delta A_{x_o}.t.\cos\Theta - \Delta A_{z_o}.t.\sin\Theta$$

Actually for steady flight ΔU_a^{**} has to be calculated by :

$$\Delta U_a^{**} = \Delta U_a - \Delta A_{z_o}^{*}.t.\sin\Theta$$

and (6.8) has to be rewritten :

$$\Delta U_a^{**} = \Delta U_{a_o} - \Delta\theta_o^{*}.g.t - \Delta q_o.g.\tfrac{1}{2}t^2 \qquad (A3.2)$$

In (A3.2) $\Delta\theta_o^{*}$ is :

$$\Delta\theta_o^{*} = \Delta\theta_o + \Delta A_{x_o}.\frac{1}{g}(\cos\theta + \mathrm{tg}\,\theta\sin\theta)$$

In this expression ΔA_{x_o} is the actual value of the zeroshift during the measurements.

The error in the estimate of $\Delta\theta_o$ is :

$$\Delta\theta_o^{*} - \Delta\theta_o = \frac{\Delta A_{x_o}}{g}(\cos\theta + \mathrm{tg}\,\theta\sin\theta)$$

Neglecting tg θ sin θ with respect to cos θ the derivative of θ with respect to ΔA_{x_o} is :

$$\frac{\partial\theta}{\partial\Delta A_{x_o}} = \frac{\cos\theta}{g}$$

Because ΔA_{x_o} has been completely compensated by ΔA_{z_o} and $\Delta\theta_o$, ΔC and ΔU_a are independent of ΔA_{x_o} and :

$$\frac{\partial C}{\partial\Delta A_{x_o}} = \frac{\partial U_a}{\partial\Delta A_{x_o}} = \frac{\partial\gamma}{\partial\Delta A_{x_o}} = 0 \qquad \text{and} \qquad \frac{\partial\alpha}{\partial\Delta A_{x_o}} = \frac{\cos\theta}{g}$$

c. The derivatives with respect to ΔA_{z_o} .

Since in (7.5) t.sin θ and g.t are linearly dependent (θ is constant), a remaining error in ΔA_{z_o} will be compensated by $\Delta\theta_o$, such that ΔU_a is independent of ΔA_{z_o} . The derivative of θ with respect to ΔA_{z_o} is :

$$\frac{\partial\theta}{\partial\Delta A_{z_o}} = \frac{\sin\theta}{g}$$

From (7.6) follows :

$$\frac{\partial C}{\partial \Delta A_{z_o}} = -t \cos \theta$$

In (7.5) ΔU_a is independent of an error in ΔA_{z_o}, therefore :

$$\frac{\partial U_a}{\partial \Delta A_{z_o}} = 0$$

With the equations (7.9) and (7.10) the derivatives of γ and α become :

$$\frac{\partial \gamma}{\partial \Delta A_{z_o}} = - \frac{t \cos \theta}{V} \quad , \quad \frac{\partial \alpha}{\partial \Delta A_{z_o}} = \frac{\sin \theta}{g} + \frac{t \cos \theta}{V}$$

d. The derivatives with respect to $\Delta\theta_o$.

From the equations (7.5) and (7.8) the following derivatives are obtained :

$$\frac{\partial \theta}{\partial \Delta\theta_o} = 1 \quad , \quad \frac{\partial C}{\partial \Delta\theta_o} = 0 \quad , \quad \frac{\partial U_a}{\partial \Delta\theta_o} = - g.t$$

With (7.9) and (7.10) follows :

$$\frac{\partial \gamma}{\partial \Delta\theta_o} = \frac{\gamma g t}{V} \quad , \quad \frac{\partial \alpha}{\partial \Delta\theta_o} = 1 - \frac{\gamma g t}{V}$$

e. The derivatives with respect to Δq_o.

The derivatives with respect to Δq_o are obtained from the same equations as those for $\Delta\theta_o$:

$$\frac{\partial \theta}{\partial \Delta q_o} = t \quad , \quad \frac{\partial \gamma}{\partial \Delta q_o} = \frac{+\gamma g t^2}{2V} \quad , \quad \frac{\partial \alpha}{\partial \Delta q_o} = t - \frac{\gamma g t^2}{2V}$$

Since the partial derivatives of θ, γ and α with respect to w_w have been given in (7.11) the sensitivity matrix S for steady symmetric flights can now be written as :

$$S = \begin{array}{c} \\ \\ \\ \end{array}\begin{array}{cccccc} (\Delta C_o) & (\Delta A_{x_o}) & (\Delta A_{z_o}) & (\Delta\theta_o) & (\Delta q_o) & (w_w) \\ \end{array}$$

$$S = \left|\begin{array}{cccccc} 0 & \frac{\cos\theta}{g} & \frac{\sin\theta}{g} & 1 & t & 0 \\ \frac{1}{V} & 0 & -\frac{t\cos\theta}{V} & \frac{\gamma g t}{V} & \frac{\gamma g t^2}{2V} & \frac{1}{V} \\ -\frac{1}{V} & \frac{\cos\theta}{g} & \frac{\sin\theta}{g}+\frac{t}{V}\cos\theta & 1-\frac{\gamma g t}{V} & t-\frac{\gamma g t^2}{2V} & -\frac{1}{V} \end{array}\right| \begin{array}{c} (\theta) \\ (\gamma) \\ (\alpha) \end{array}$$

2. Nonsteady symmetric flight.

The equations used for the determination of the elements of the sensitivity matrix S for nonsteady flight are :

$$\Delta\Delta h = \Delta\Delta h_o + \Delta C_o . t + \Delta A_{x_o} \iint_{t_o}^{t} \sin\theta \, dt^2 - \Delta A_{z_o} \iint_{t_o}^{t} \cos\theta \, dt^2$$

$$+ \Delta\theta_o \iint_{t_o}^{t} a_{hor} dt^2 + \Delta q_o \iint_{t_o}^{t} a_{hor} . t . dt^2 \qquad (5.8)$$

$$\Delta U_a = \Delta U_{a_o} + \Delta A_{x_o} \int_{t_o}^{t} \cos\theta \, dt + \Delta A_{z_o} \int_{t_o}^{t} \sin\theta \, dt$$

$$- \Delta\theta_o \int_{t_o}^{t} (a_{vert}+g) \, dt - \Delta q_o \int_{t_o}^{t} (a_{vert}+g) . t . dt \qquad (5.13)$$

$$\Delta C = \Delta C_o + \Delta A_{x_o} \int_{t_o}^{t} \sin\theta \, dt - \Delta A_{z_o} \int_{t_o}^{t} \cos\theta \, dt$$

$$+ \Delta\theta_o \int_{t_o}^{t} a_{hor} dt + \Delta q_o \int_{t_o}^{t} a_{hor} . t . dt^2 \qquad (7.13)$$

In addition the equations (7.8), (7.9) and (7.10) are used. As already mentioned in paragraph 7.2 a_{hor} during the nonsteady manoeuvre is assumed to be 0.5 m/sec^2 and a_{vert} is assumed to be equal to zero.

a. The derivatives with respect to C_o.

The derivatives for nonsteady flight turn out to be equal to those for steady flight.

b. The derivatives with respect to ΔA_{x_o}.

For nonsteady flight ΔA_{x_o} is obtained from the regression analysis. The remaining error in ΔA_{x_o}, however, will influence the other systematic errors. An error in ΔA_{x_o} can influence $\Delta\theta_o$ and Δq_o only, if in (5.13) the variables in the terms containing these zeroshifts are linearly dependent. Assuming that during the nonsteady manoeuvres cos $\theta \approx 1$ and $a_{vert} \approx 0$, (5.13) can be rewritten as :

$$\Delta U_a = \Delta U_{a_o} + \Delta A_{x_o}.t + \Delta A_{z_o}\int_o^t \sin\theta\, dt - \Delta\theta_o g.t - \Delta q_o.\tfrac{1}{2}g.t^2$$

In this equation the variables in the terms containing ΔA_{x_o} and $\Delta\theta_o$ are linearly dependent. Therefore it is assumed that in the θ, γ, α calculation for a nonsteady manoeuvre an error in ΔA_{x_o} will influence the estimate of $\Delta\theta_o$, so :

$$\frac{\partial\theta}{\partial\Delta A_{x_o}} = \frac{1}{g}$$

and

$$\frac{\partial U_a}{\partial\Delta A_{x_o}} = 0$$

From equation (7.13) the derivative of C is obtained :

$$\frac{\partial C}{\partial \Delta A_{x_o}} = \int_{t_o}^{t} \sin \theta \, dt$$

With (7.9) and (7.10) the derivatives of γ and α are determined :

$$\frac{\partial \gamma}{\partial \Delta A_{x_o}} = \frac{1}{V} \int_{t_o}^{t} \sin \theta \, dt$$

$$\frac{\partial \alpha}{\partial \Delta A_{x_o}} = \frac{1}{g} - \frac{1}{V} \int_{t_o}^{t} \sin \theta \, dt$$

c. The derivatives with respect to ΔA_{z_o} .

In contrast with ΔA_{x_o} , an error in ΔA_{z_o} will not influence $\Delta\theta_o$ in (5.13) because the variables of these systematic errors in (5.13) are not linearly dependent. During the nonsteady manoeuvre θ remains relatively small, varying continuously with time from a positive to a negative value.

Therefore :

$$\frac{\partial \theta}{\partial \Delta A_{z_o}} = 0$$

With the equations (5.13), (7.13), (7.9) and (7.10) the derivatives of U_a, C, γ and α become :

$$\frac{\partial U_a}{\partial \Delta A_{z_o}} = \int_{t_o}^{t} \sin \theta \, dt \quad , \quad \frac{\partial C}{\partial \Delta A_{z_o}} = - \int_{t_o}^{t} \cos \theta \, dt \approx -t$$

and :

$$\frac{\partial \gamma}{\partial \Delta A_{z_o}} = - \frac{t}{V} - \frac{\gamma}{V} \int_{t_o}^{t} \sin \theta \, dt$$

$$\frac{\partial \alpha}{\partial \Delta A_{z_o}} = \frac{t}{V} + \frac{\gamma}{V} \circ \int_{t_o}^{t} \sin \theta \, dt$$

d. The derivatives with respect to $\Delta\theta_o$.

The derivatives of θ, γ and α are determined from equations (7.8), (7.13), (5.13), (7.9) and (7.10) :

$$\frac{\partial \theta}{\partial \Delta\theta_o} = 1 \qquad , \qquad \frac{\partial \gamma}{\partial \Delta\theta_o} = \frac{t}{V} (\tfrac{1}{2} + \gamma g)$$

$$\frac{\partial \alpha}{\partial \Delta\theta_o} = 1 - \frac{t}{V} (\tfrac{1}{2} + \gamma g)$$

e. The derivatives with respect to Δq_o.

The derivatives of θ, γ and α with respect to Δq_o are determined from the same equations as $\Delta\theta_o$:

$$\frac{\partial \theta}{\partial \Delta q_o} = t \qquad , \qquad \frac{\partial \gamma}{\partial \Delta q_o} = \frac{t^2}{2V} \left\{ \tfrac{1}{2} + \gamma g \right\}$$

$$\frac{\partial \alpha}{\partial \Delta q_o} = t - \frac{t^2}{2V} \left\{ \tfrac{1}{2} + \gamma g \right\}$$

Since the derivatives of θ, γ and α with respect to w_w have been given in (7.11), the sensitivity matrix S for nonsteady flights can now be written as :

$$
\begin{array}{ccccccc}
 & (\Delta C_o) & (\Delta A_{x_o}) & (\Delta A_{z_o}) & (\Delta \theta_o) & (\Delta q_o) & (w_w) \\
\end{array}
$$

$$
S = \left| \begin{array}{cccccc}
0 & \frac{1}{g} & 0 & 1 & t & 0 \\[2ex]
\frac{1}{V} & \frac{1}{V}\cdot\int_{t_o}^{t} \sin\theta dt & -\frac{t}{V}-\frac{\gamma}{V}\int_{t_o}^{t} \sin.t.dt & \frac{t}{V}(\tfrac{1}{2}+\gamma g) & \frac{t^2}{2V}(\tfrac{1}{2}+\gamma g) & \frac{1}{V} \\[2ex]
-\frac{1}{V} & \frac{1}{g}-\frac{1}{V}\int_{t_o}^{t} \sin\theta dt & \frac{t}{V}+\frac{\gamma}{V}\int_{t_o}^{t} \sin\theta \, dt & 1-\frac{t}{V}(\tfrac{1}{2}+\gamma g) & t-\frac{t^2}{2V}(\tfrac{1}{2}+\gamma g) & -\frac{1}{V}
\end{array} \right|
\begin{array}{c}
(\theta) \\[2ex]
(\gamma) \\[2ex]
(\alpha)
\end{array}
$$

APPENDIX 4.

Correction of the static pressure during the nonsteady manoeuvre.

From Fig.A5 it can be seen that the remaining value of $\Delta\Delta h$ after the Θ, γ and α calculation contains remarkeble deviations during the oscillations of the aircraft in the nonsteady manoeuvre. In Fig.A6 the computed value of the change in altitude Δh, the measured value Δh^{*} and the difference $\Delta\Delta h$ between Δh and Δh^{*} during one oscillation are plotted as functions of time. This latter Figure shows two remarkable features :

1. the time-history of the computed Δh is smooth.
2. $\Delta\Delta h$ as a function of time indicates that the additional values of the vertical acceleration required to fit Δh to Δh^{*}, are larger than the actual accelerations during the nonsteady manoeuvre.

Several possible explanations of the remaining $\Delta\Delta h$ might be thought of. The laboratory aircraft, a DHC-2 Beaver, is equipped with a swiveling static tube mounted on the starboard wing tip, see Fig.A7. The position error for the aircraft with this static tube has been determined during steady flights in the same way as described in Ref. 7. Results of the steady state position error correction are given in Fig. A8.

The change in altitude Δh^{*} as given in Fig.A6, is obtained from the flight tests by comparing the static pressure from the static tube, corrected as just described, to a reference pressure in a thermos flask. The measured static pressure is also corrected for the pressure lag in the instrumentation system and for the influence of accelerations on the airmass in the tube between the static tube at the wingtip and the transducer in the fuselage.

Since none of these corrections succesfully eliminates the oscillations in $\Delta\Delta h$ apparent in Fig.A5, it is assumed that the measured static pressure during the oscillations of the aircraft is in error due to the nonsteady state of the airflow around the aircraft and due to the curvature of the streamlines, caused by the rate of pitch. On the basis of this assumption an attempt has been made to find a correction for the error in the static pressure.

Defining :

p = static pressure

p^* = static pressure in error due to the nonsteady airflow and streamline curvature.

Δp = $p - p^*$

and using (4.2) results in :

$$\Delta p = - \rho g \, \Delta\Delta h \qquad \text{(A4.1)}$$

Since q and $\dot{\alpha}$ were held responsible for the systematic variations of $\Delta\Delta h$ and consequently Δp during the oscillations, a regression analysis was carried out using :

$$\frac{\Delta p}{\frac{1}{2}\rho V^2} = - \frac{2g.\Delta\Delta h}{V^2} = a_o + a_1.\frac{q\bar{c}}{V} + a_2.\frac{\dot{\alpha}c}{V} \qquad \text{(A4.2)}$$

This equation yielded rather consistent results for the coefficients a_1 and a_2, a_o being essentially zero.

In the computer program described in Appendix 2 and Fig.A4, the static pressure has to be corrected for the effect expressed by (A4.2), before starting the θ, γ, α calculation, although values of $\frac{\dot{\alpha}c}{V}$ are unknown at that stage. Therefore, equation (A4.2) is somewhat modified, using the equation of motion along the Z-axis of the aircraft :

$$W \cos \theta \cos \varphi + Z = m \, (\dot{w} + pv - uq)$$

Deviding by m and neglecting the asymmetric terms, leads to :

$$g \cos \theta + A_z = \dot{w} - uq$$

Using the expressions :

$$\frac{\dot{\alpha}\bar{c}}{V} = \frac{\dot{w}\bar{c}}{V^2} \quad , \quad C_Z = \frac{A_z \cdot m}{\frac{1}{2}\rho V^2 S} \quad , \quad \mu_c = \frac{m}{\rho S\bar{c}} \quad ,$$

$$\cos\Theta = 1$$

results after some elaborations in :

$$\frac{\dot{\alpha}\bar{c}}{V} = -\frac{g}{V} - \frac{C_Z}{2\mu_c} + \frac{q\bar{c}}{V} \tag{A4.3}$$

Substitution of (A4.3) in (A4.1) leads to an expression for the ''nonsteady position error correction''.

$$\frac{\Delta p}{\frac{1}{2}\rho V^2} = c_o + \frac{c_1}{V^2} + c_2 . C_Z + c_3 . \frac{q\bar{c}}{V} \tag{A4.4}$$

This equation was used in a regression analysis on six nonsteady manoeuvres. The resulting coefficients are given in Table 4.

In Fig.A9 the remaining $\Delta\Delta h$ is given after the application of the ''nonsteady position error correction'' of the static pressure for one manoeuvre.

Table 1.

Some data on the De Havilland DHC-2 "Beaver" aircraft.

Wing span	14.63 m (48 ft)
Length	9.26 m (30.3 ft)
Wingsurface	23.23 m^2 (250 sq.ft)
Max weight	2315 kg (5100 lb)
Engine type	Pratt and Witney Wasp Jr. R-958.
Max Power	450 hp at 750 m (2500 ft) altitude.
Propeller	Hamilton Standard, two-blade constant speed.
Diameter	2.59 m (8.5 ft).

Table 2.

ΔA_{x_o}, ΔA_{z_o}, Δq_o, $\sigma_{\Delta\Delta h}$ and $\sigma_{\Delta U_a}$ from the steady and nonsteady flight.

Date and No.	ΔA_{x_o} m/sec^2	ΔA_{z_o} m/sec^2	Δq_o rad/sec	$\sigma_{\Delta\Delta h}$ m	$\sigma_{\Delta U_a}$ m/sec
24.2.'67	Steady symmetric flight				
1	-	+ 0.0800	- 0.00029	0.164	0.132
2	-	+ 0.0896	- 0.00058	0.167	0.096
4	-	+ 0.0884	- 0.00058	0.186	0.199
5	-	+ 0.0790	- 0.00042	0.185	0.210
6	-	+ 0.0823	- 0.00046	0.156	0.143
7	-	+ 0.0885	- 0.00043	0.200	0.160
9	-	+ 0.0755	- 0.00022	0.136	0.204
10	-	+ 0.0713	- 0.00040	0.193	0.104
11	-	+ 0.0782	- 0.00030	0.196	0.158
12	-	+ 0.0782	- 0.00016	0.092	0.131
13	-	+ 0.0813	- 0.00021	0.126	0.110
14	-	+ 0.0707	- 0.00016	0.157	0.069
15	-	+ 0.0746	- 0.00034	0.125	0.080
16	-	+ 0.0786	- 0.00031	0.126	0.154
17	-	+ 0.0792	- 0.00038	0.109	0.087
18	-	+ 0.0806	- 0.00025	0.167	0.271
19	-	+ 0.1166	- 0.00115	0.084	0.038
20	-	+ 0.0963	- 0.00059	0.098	0.163
21	-	+ 0.0982	- 0.00068	0.108	0.133
22	-	+ 0.0807	- 0.00028	0.126	0.178

Table 2 (continued)

Date and No.	ΔA_{x_o} m/sec^2	ΔA_{z_o} m/sec^2	Δq_o rad/sec	$\sigma_{\Delta\Delta h}$ m	$\sigma_{\Delta U_a}$ m/sec
16.3.'67	Steady symmetric flight				
13	-	- 0.0140	- 0.00017	0.175	0.063
14	-	- 0.0164	- 0.00017	0.153	0.102
15	-	- 0.0224	- 0.00018	0.167	0.066
16	-	- 0.0152	- 0.00020	0.144	0.078
17	-	- 0.0190	- 0.00020	0.145	0.094
18	-	- 0.0188	- 0.00026	0.185	0.071
19	-	- 0.0177	- 0.00031	0.156	0.186
20	-	- 0.0156	- 0.00024	0.370	0.107
16.3.'67	Nonsteady symmetric flight				
1	- 0.0547	- 0.0132	- 0.00051	0.183	0.223
2	- 0.0323	- 0.0072	- 0.00033	0.198	0.113
3	- 0.0739	- 0.0184	- 0.00018	0.282	0.070
4	- 0.0110	+ 0.0003	- 0.00028	0.229	0.091
5	- 0.0683	- 0.0146	- 0.00022	0.314	0.055
6	- 0.0533	- 0.0123	- 0.00026	0.214	0.074
7	- 0.0753	- 0.0221	- 0.00013	0.613	0.066

Table 3.

The values of C_Z, θ, γ, α, $\Delta\theta$, $\Delta\gamma$, $\Delta\alpha$ of the 28 steady symmetric flights.

No.	C_Z	θ	γ	α	$\Delta\theta$	$\Delta\gamma$	$\Delta\alpha$
1	-1.348	0.274	0.059	0.215	-0.0064	-0.0048	-0.0017
2	-0.982	0.206	0.058	0.148	-0.0004	-0.0008	0.0004
3	-0.781	0.158	0.045	0.114	-0.0033	-0.0030	-0.0003
4	-1.643	0.333	0.048	0.286	-0.0046	0.0020	-0.0066
5	-1.241	0.259	0.074	0.185	-0.0013	0.0097	-0.0110
6	-1.405	0.287	0.067	0.220	-0.0051	0.0032	-0.0083
7	-0.375	0.014	-0.025	0.039	-0.0033	-0.0011	-0.0021
8	-1.508	0.311	0.063	0.248	-0.0005	0.0032	-0.0037
9	-0.888	0.188	0.062	0.125	0.0014	0.0074	-0.0060
10	-1.087	0.233	0.059	0.173	0.0038	-0.0024	0.0062
11	-1.192	0.252	0.065	0.187	0.0014	0.0010	0.0005
12	-1.181	0.250	0.057	0.192	0.0014	-0.0061	0.0075
13	-0.756	0.154	0.045	0.110	-0.0013	-0.0010	-0.0003
14	-1.249	0.261	0.057	0.204	-0.0004	-0.0069	0.0065
15	-0.396	0.031	-0.016	0.047	0.0023	0.0023	0.0000
16	-1.498	0.311	0.058	0.253	0.0017	-0.0027	0.0043
17	-1.480	0.307	0.066	0.241	0.0010	0.0049	-0.0039
18	-0.538	0.089	0.021	0.068	-0.0027	0.0070	-0.0097
19	-0.540	0.091	0.010	0.081	-0.0016	-0.0050	0.0034
20	-1.336	0.278	0.065	0.212	-0.0009	0.0011	-0.0020
21	-0.433	0.047	-0.009	0.056	-0.0006	0.0001	-0.0006
22	-1.625	0.338	0.051	0.287	0.0034	0.0028	0.0006
23	-1.422	0.299	0.072	0.227	0.0043	0.0093	-0.0050
24	-1.388	0.290	0.061	0.229	0.0015	-0.0028	0.0043
25	-0.473	0.071	-0.002	0.073	0.0049	-0.0028	0.0077
26	-0.621	0.120	0.028	0.092	0.0004	-0.0008	0.0012
27	-0.615	0.119	0.029	0.091	0.0017	0.0005	0.0012
28	-1.553	0.322	0.042	0.028	0.0022	-0.0145	0.0167

Table 4.

The results of regression analyses on six nonsteady manoeuvres to determine the "nonsteady position error correction".

No.	c_o	c_1	c_2	c_3
1	$+ 0.859 \cdot 10^{-3}$	+ 56.98	+ 0.0336	+ 1.33
2	$- 0.150 \cdot 10^{-2}$	+ 54.45	+ 0.0306	+ 1.17
3	$+ 0.148 \cdot 10^{-2}$	+ 46.42	+ 0.0282	+ 1.22
4.	$- 0.208 \cdot 10^{-2}$	+ 58.95	+ 0.0331	+ 1.29
5	$+ 0.900 \cdot 10^{-3}$	+ 53.84	+ 0.0321	+ 1.25
6	$- 0.613 \cdot 10^{-2}$	+ 52.23	+ 0.0265	+ 1.21
mean value	$+ 0.108 \cdot 10^{-2}$	+ 53.82	+ 0.0307	+ 1.24

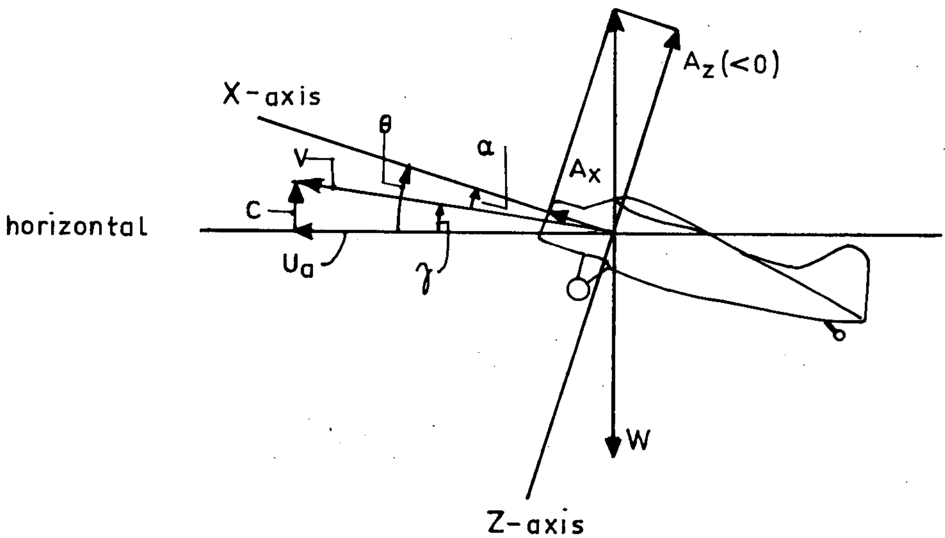


Fig. 1: Angle of pitch, flight path angle and angle of attack in symmetric flight.

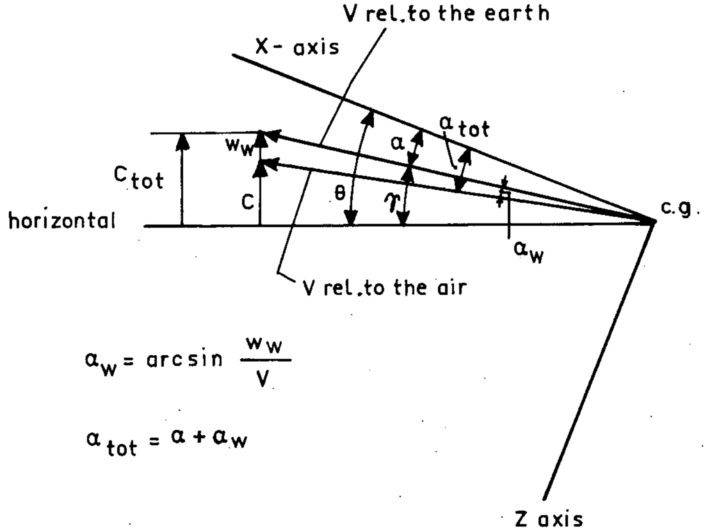


Fig. 2 b : Steady climb , $w_W \neq 0$

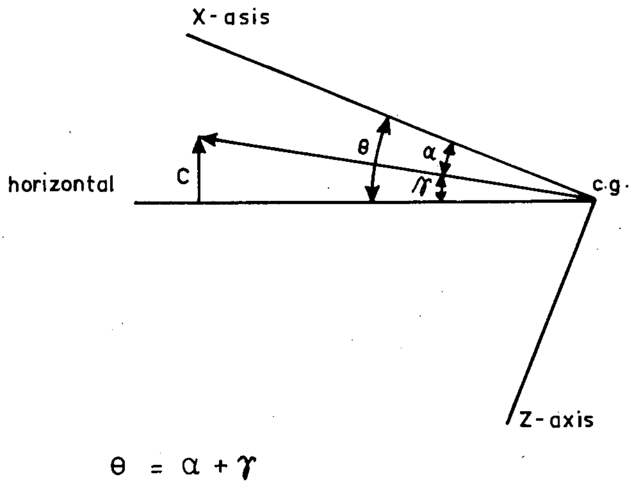


Fig. 2. a : Steady climb , $w_W = 0$

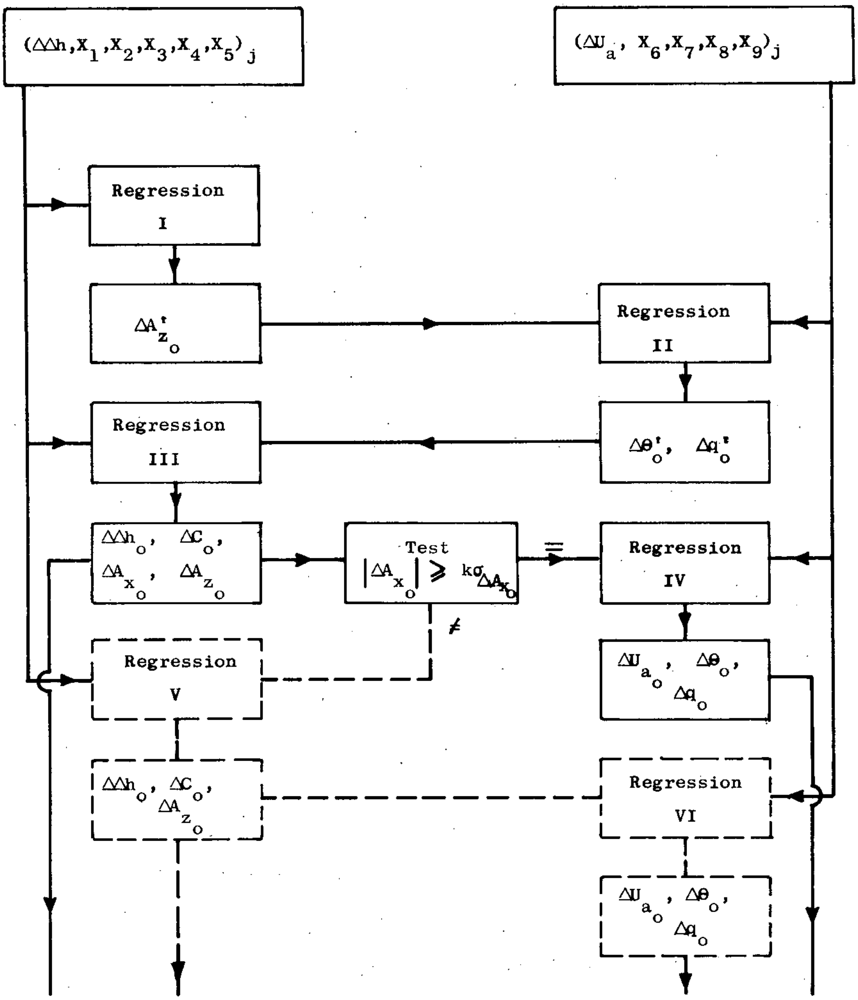


Fig.3 Regression analysis in the Θ, γ, α calculation.

Regression I : $\Delta\Delta h_j = \Delta\Delta h'_o + \Delta C'_o X_{1_j} + \Delta A'_{z_o} X_{3_j}$

Regression II : $\Delta U^*_{a_j} = \Delta U_{a_j} - \Delta A'_{z_o} X_{7_j} = \Delta U'_{a_o} + \Delta\theta'_o X_{8_j} + \Delta q'_o X_{9_j}$

Regression III : $\Delta\Delta h^*_j = \Delta\Delta h_j - \Delta\theta'_o X_{4_j} - \Delta q'_o X_{5_j} = \Delta\Delta h_o + \Delta C_o X_{1_j} + \Delta A_{x_o} X_{2_j} + \Delta A_{z_o} X_{3_j}$

Regression IV : $\Delta U^*_{a_j} = \Delta U_{a_j} - \Delta A_{x_o} X_{6_j} - \Delta A_{z_o} X_{7_j} = \Delta U_{a_o} + \Delta\theta_o X_{8_j} + \Delta q_o X_{9_j}$

Regression V : $\Delta\Delta h^*_j = \Delta\Delta h_j - \Delta\theta'_o X_{4_j} - \Delta q'_o X_{5_j} = \Delta\Delta h_o + \Delta C_o X_{1_j} + \Delta A_{z_o} X_{3_j}$

Regression VI : $\Delta U^*_{a_j} = \Delta U_{a_j} - \Delta A_{z_o} X_{7_j} = \Delta U_{a_o} + \Delta\theta_o X_{8_j} + \Delta q_o X_{9_j}$

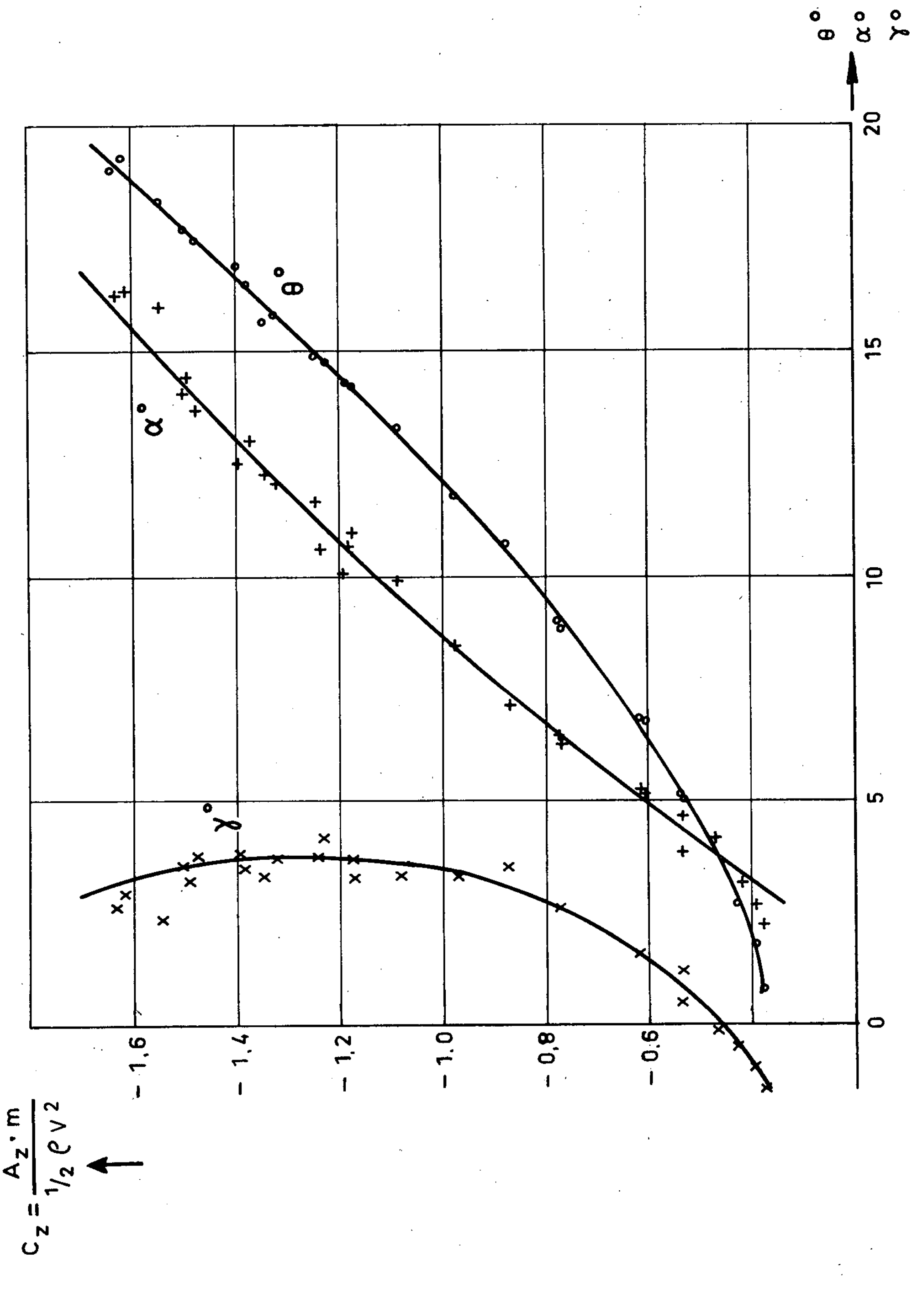


Fig. 4: θ, γ and α as function of C_z

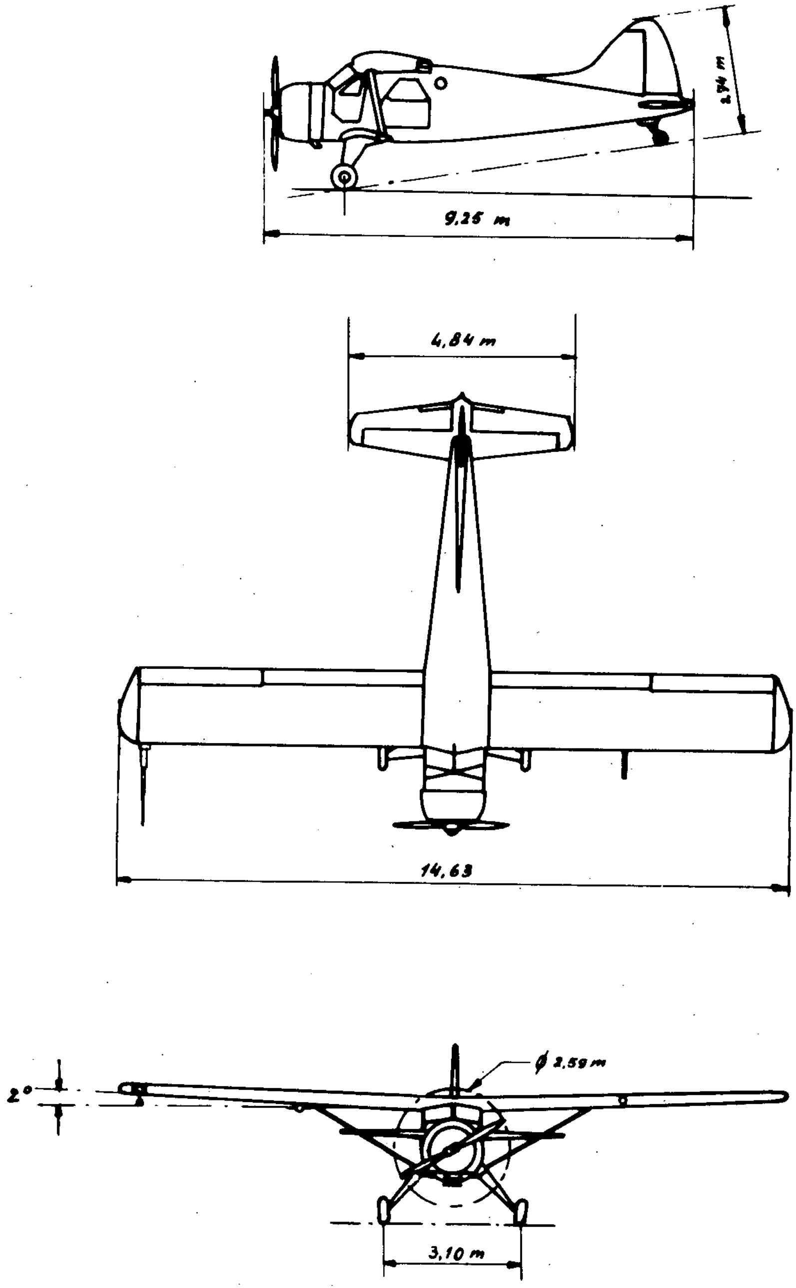


Fig. 5: General arangement of the DHC-2 Beaver.

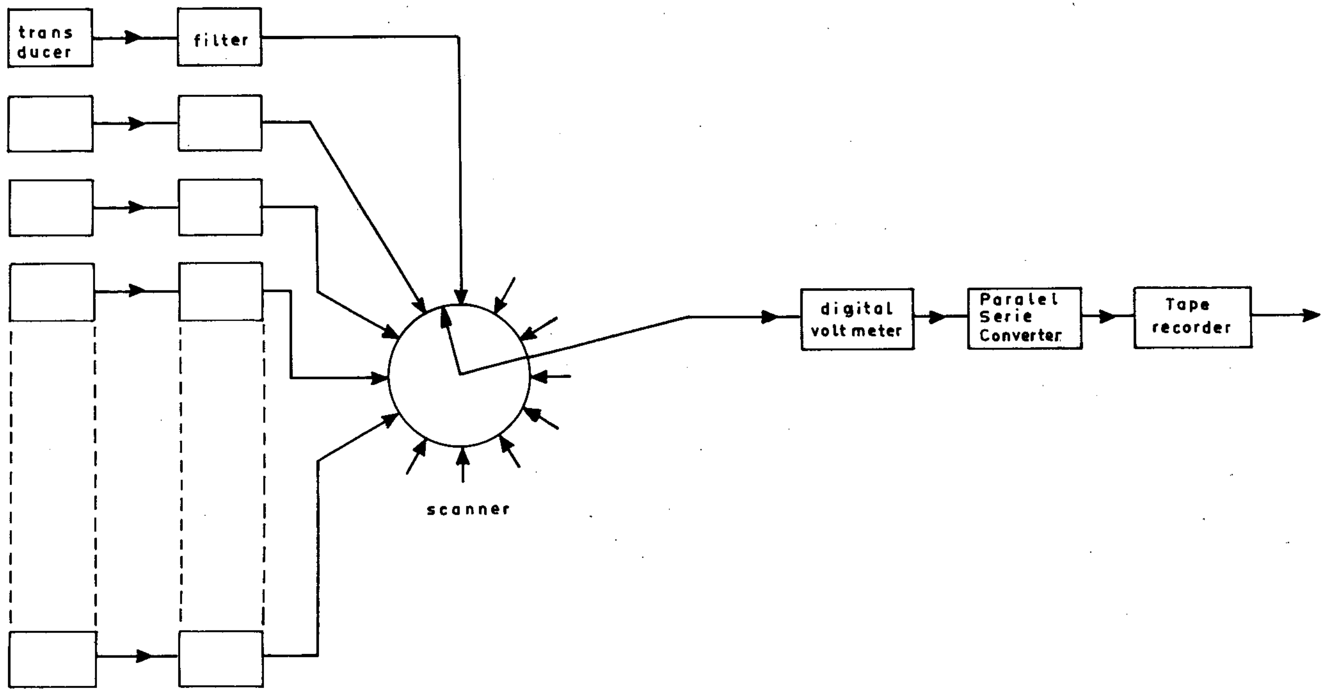


Fig. 6: Simplified block diagram of the instrumentation system.

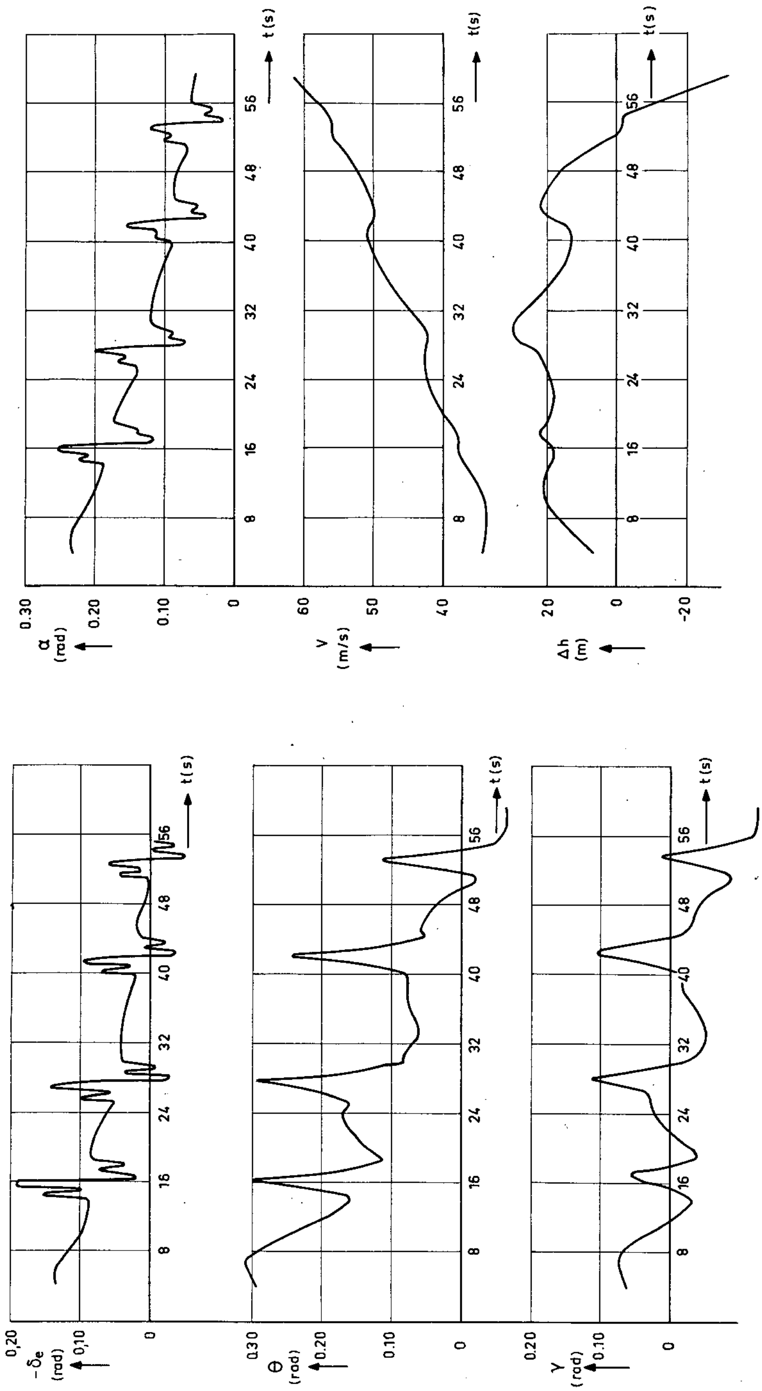


Fig. 7 : δ_e, Θ, γ, α, V and Δh during a nonsteady manoeuvre.

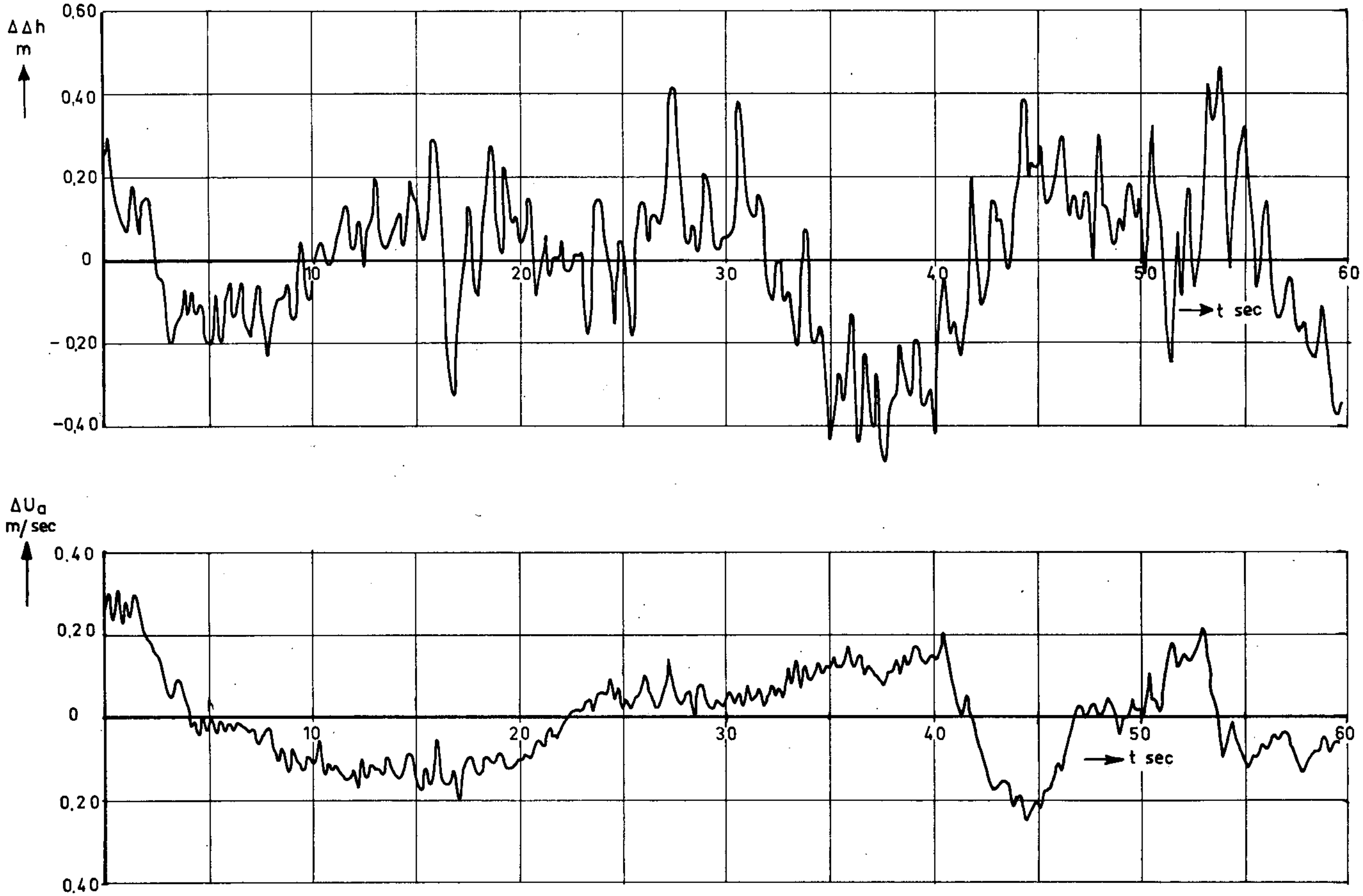


Fig. 8: ΔΔ h and ΔUa during a nonsteady manoeuvre.

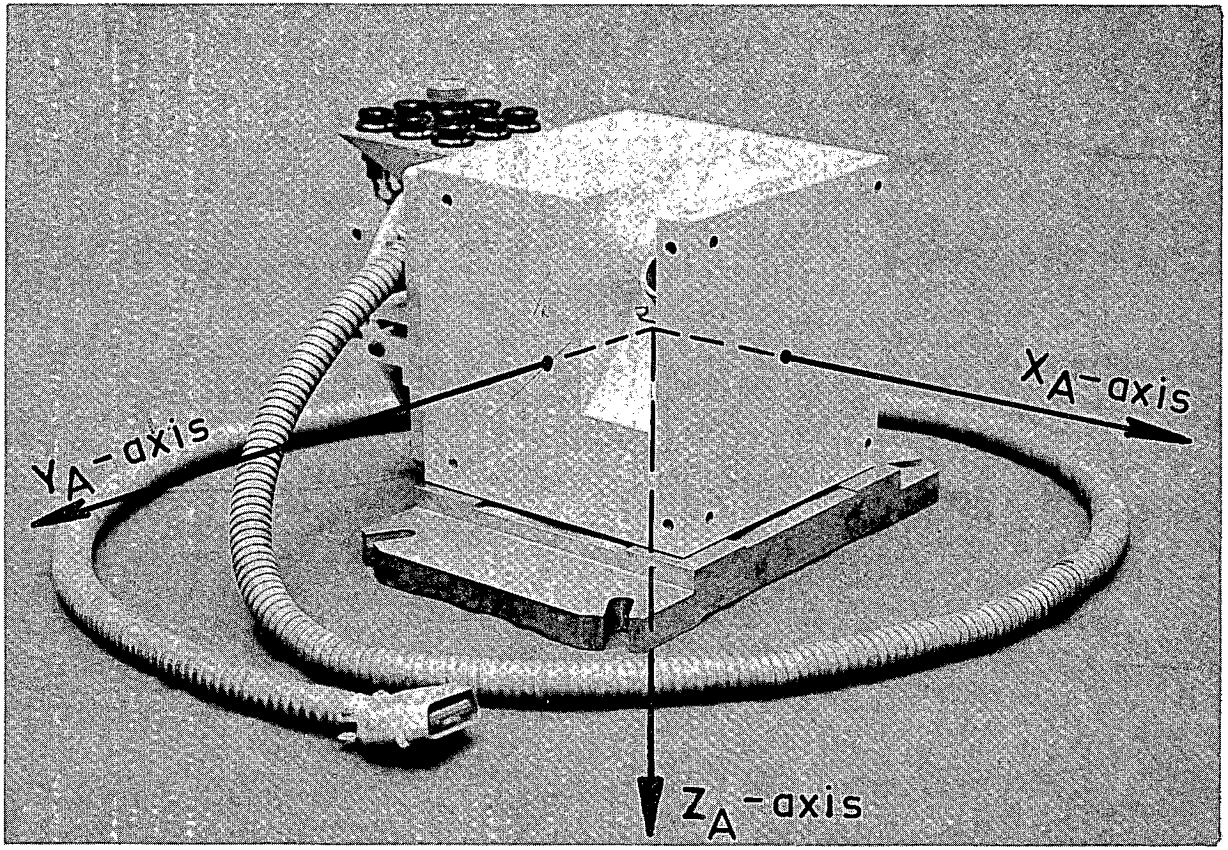


Fig. A I : Box, containing 3 accelerometers

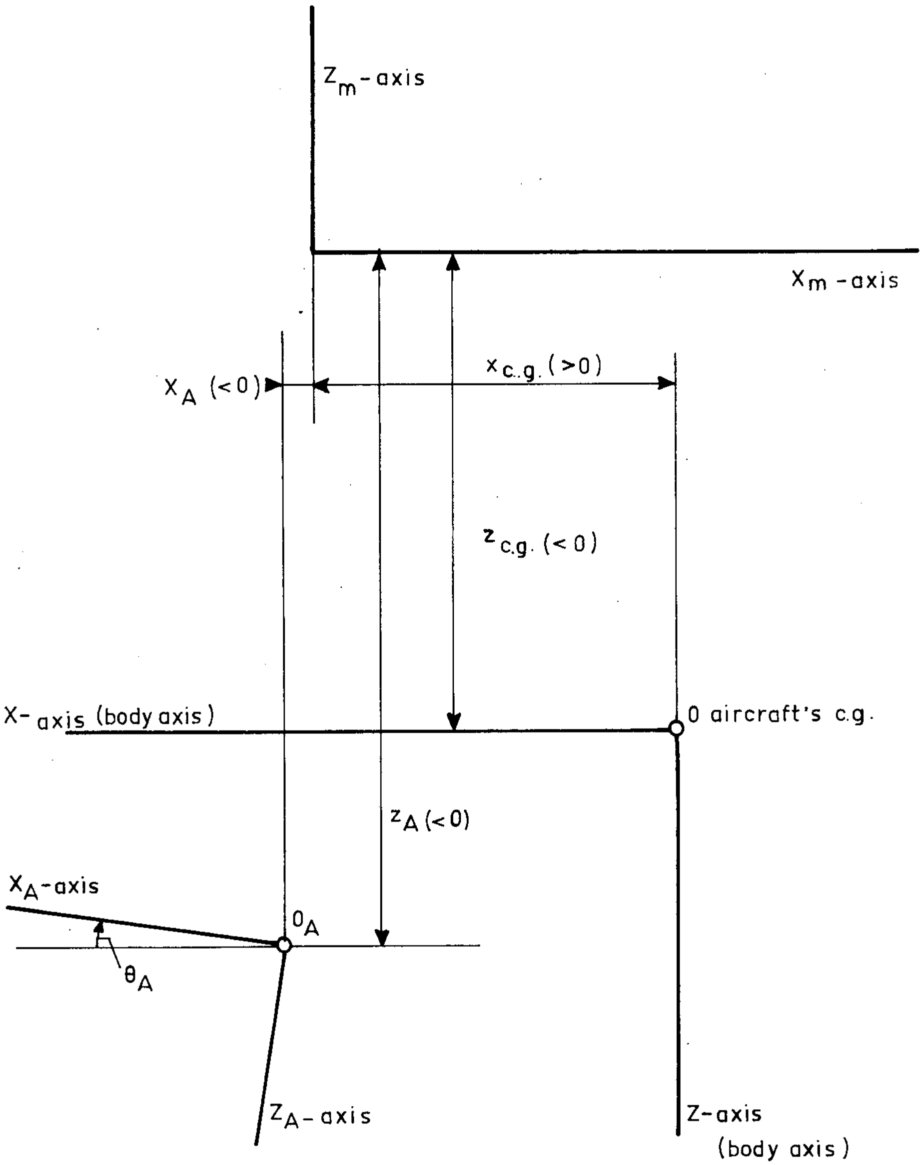


Fig. A 2 : Position of accelerometerbox in body - and measurement axes.

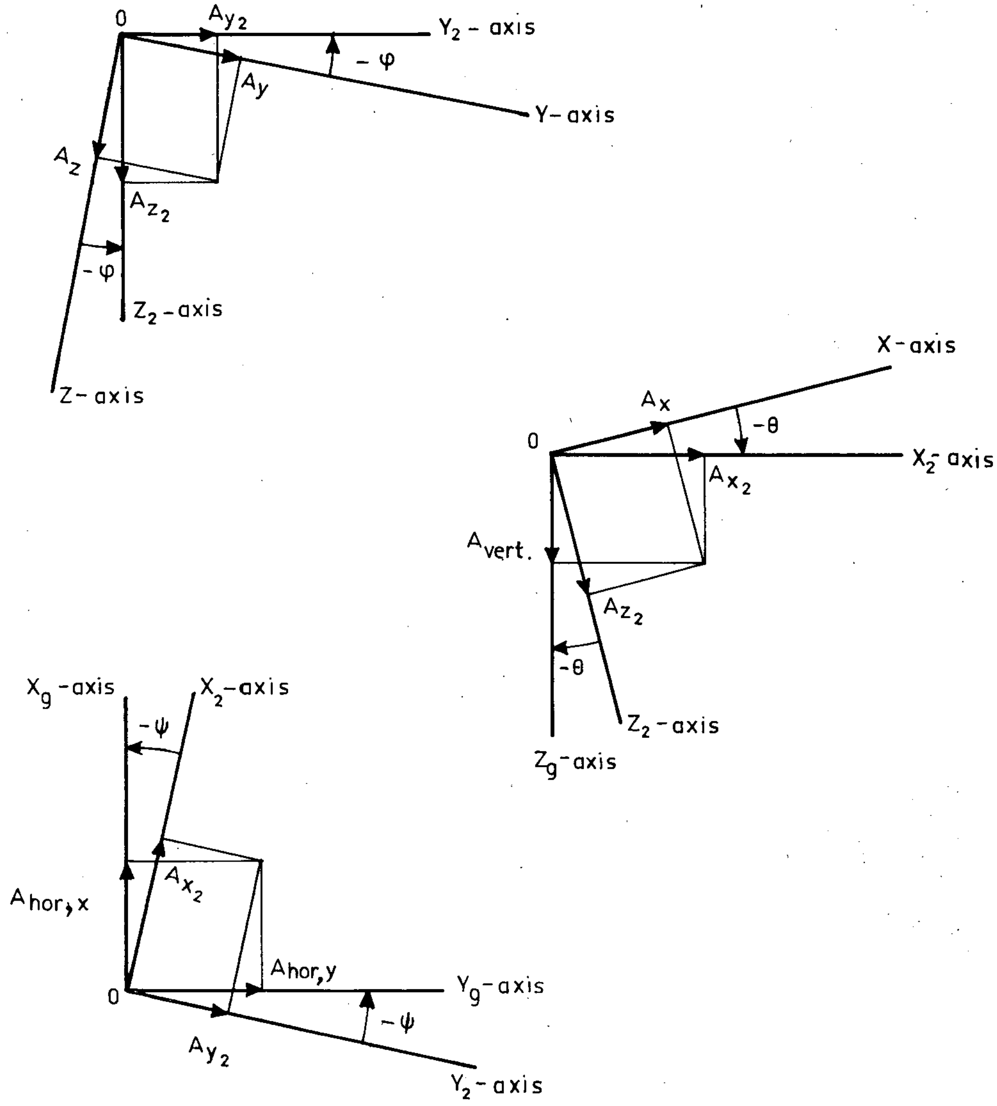


Fig. A 3 : Rotation through - φ, - θ and - ψ.

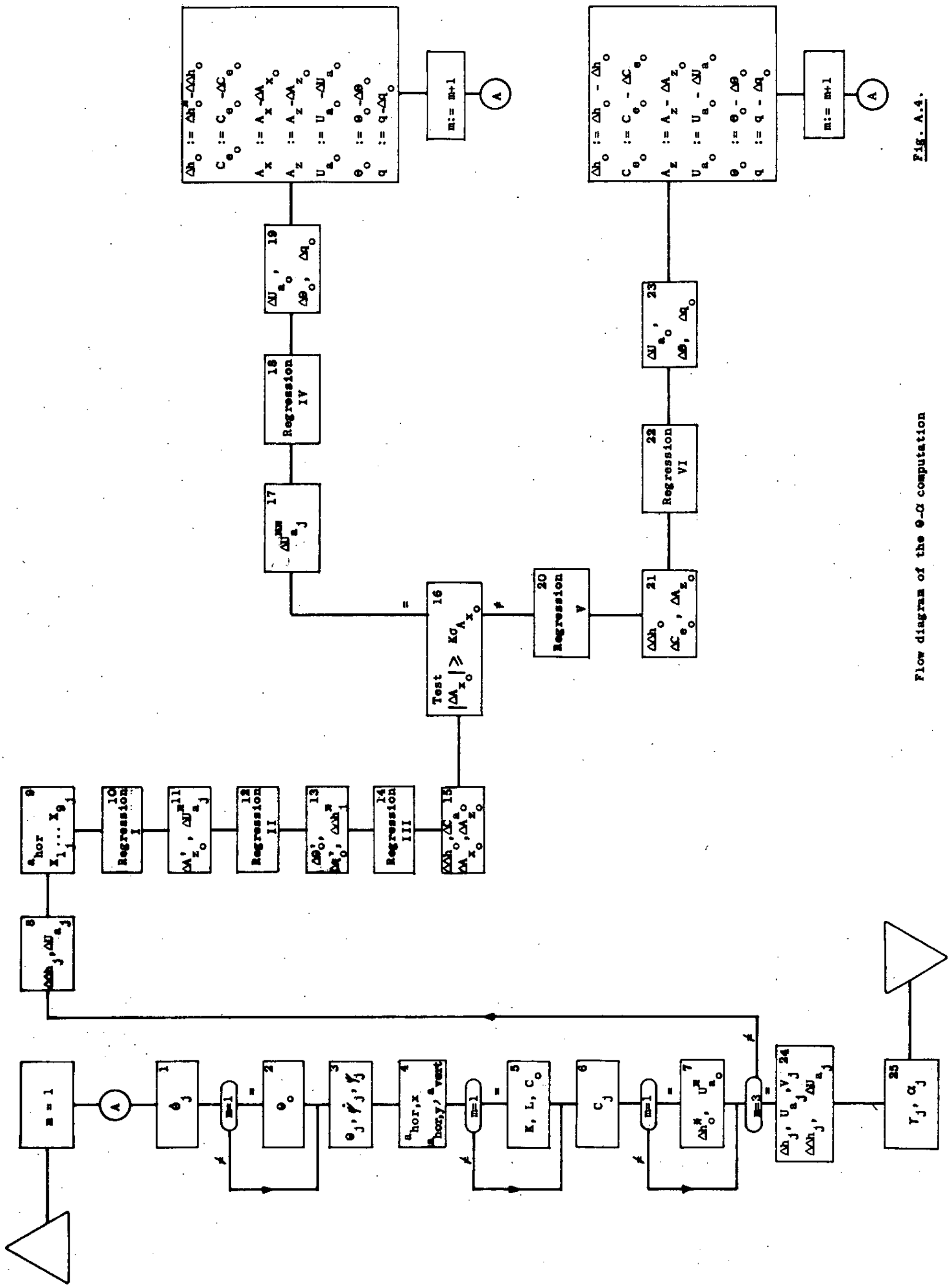


Fig. A.4.

Flow diagram of the Θ–α computation

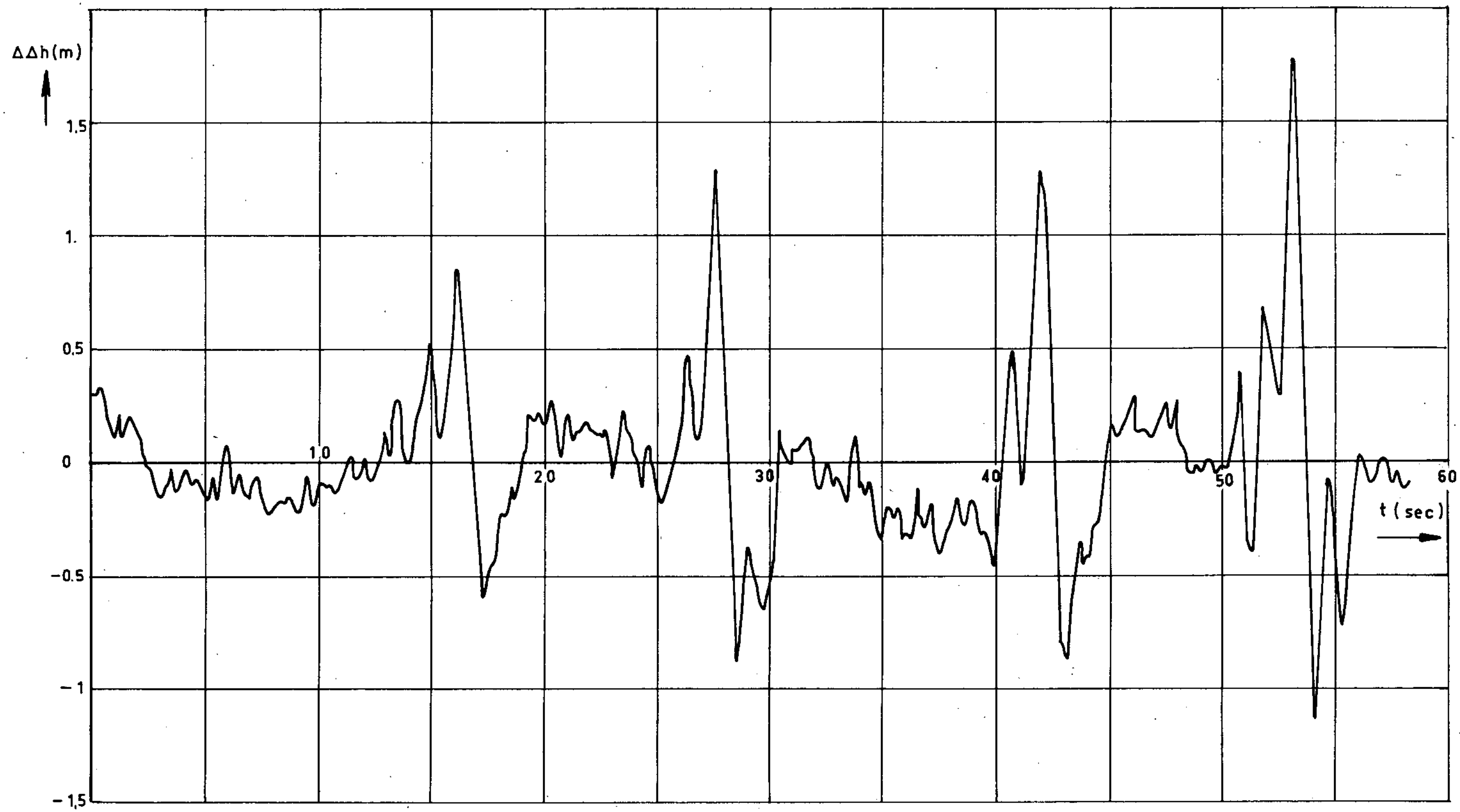


Fig. A5 : ΔΔh during a non steady manoeuvre without correction for the non steady position error.

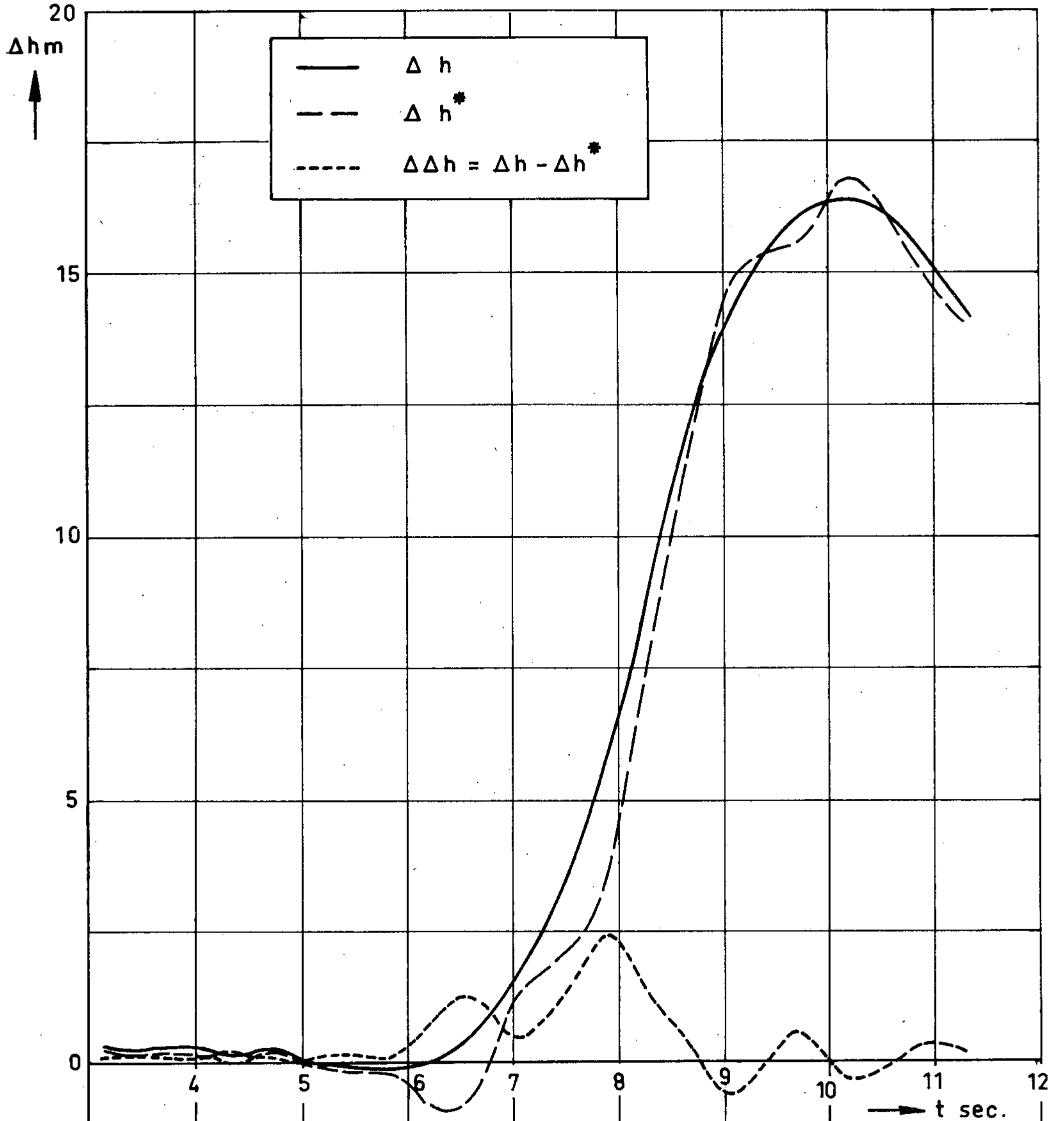
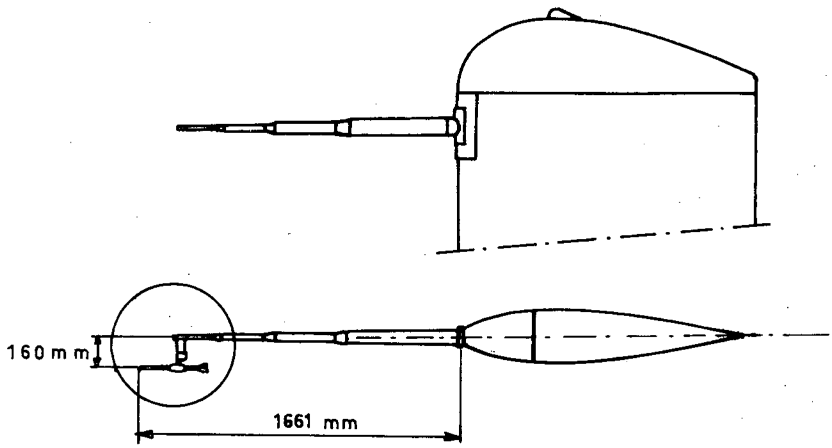


Fig. A6: The altitude change during an oscillation.



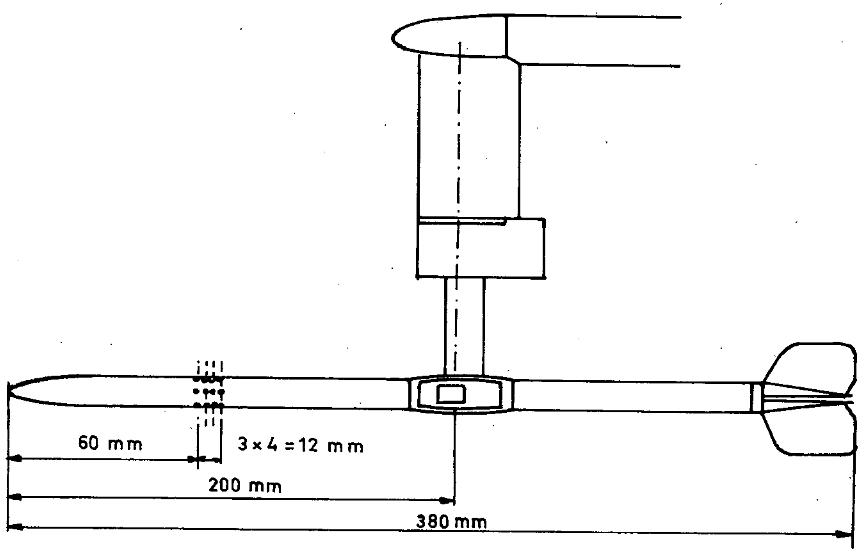


Fig. A7 : The swiveling static tube

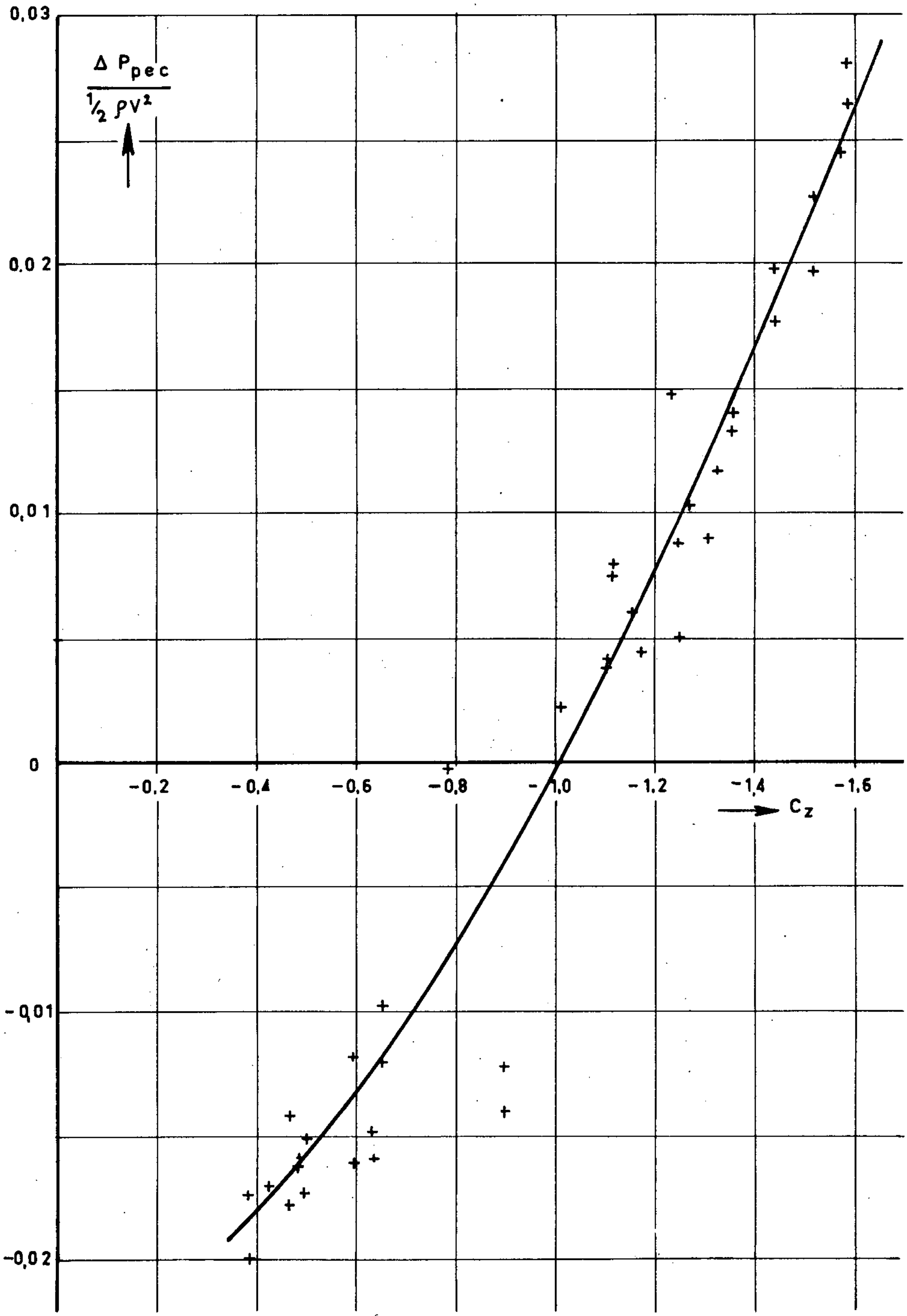


Fig. A 8 : Position error correction of the swiveling static tube.

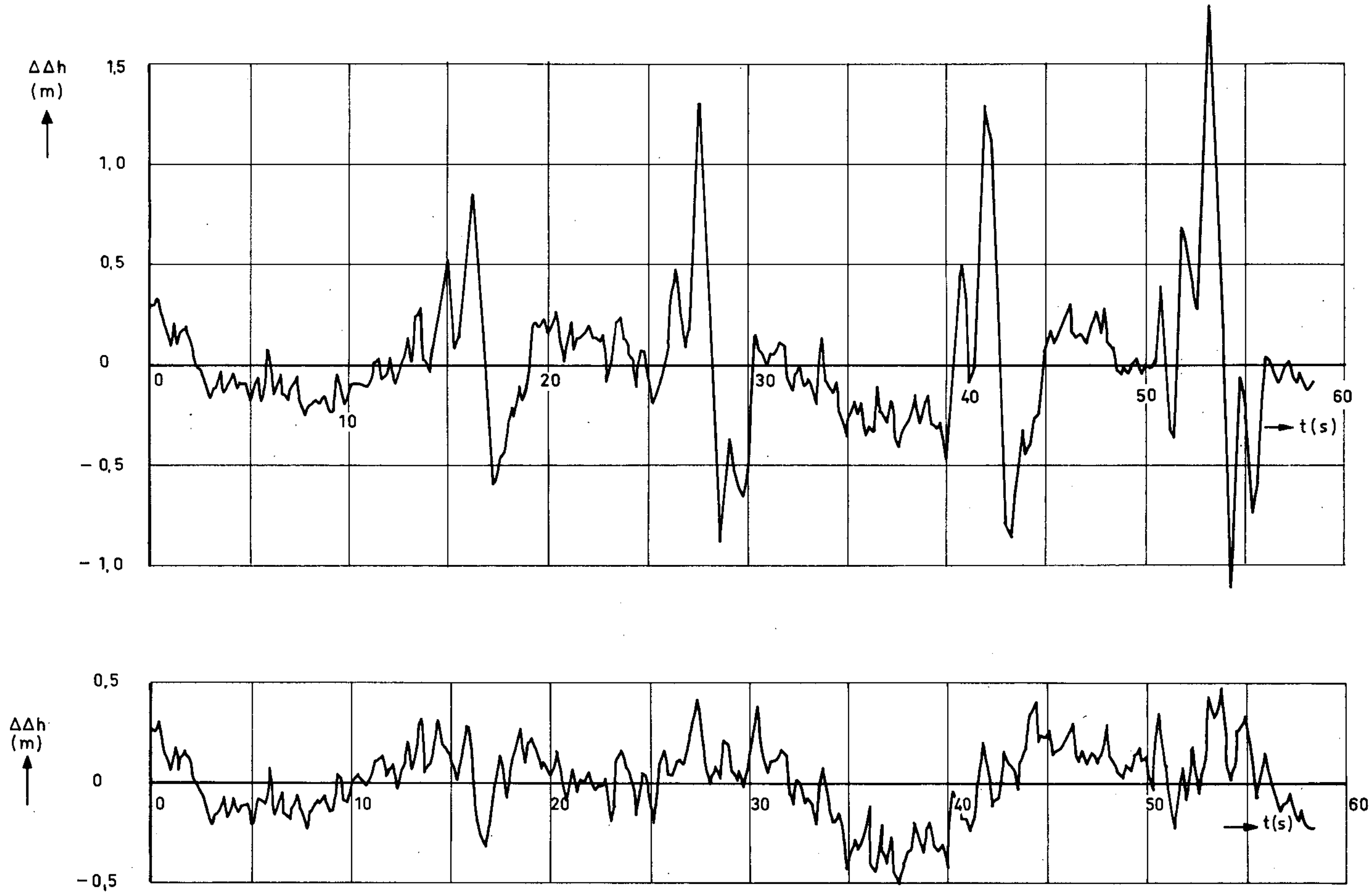


Fig. A 9: ΔΔh during a nonsteady manoeuvre before and after correction for the nonsteady possition error.