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## A MICROECONOMIC APPROACH TO INDUCED INNOVATION

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Induced innovation is an important and well documented concept at the micro-economic level and has many important implications for development theory and policy (Hayami and Ruttan 1972, Ruttan 1973, de Janvry 1973, Schuh 1974, Binswanger 1973, 1974). This is reason enough to develop its microeconomic foundations and to get away from the graphic kinds of arguments on which the microeconomic version of the hypothesis now rests. (Ahmad, 1966). For empirical work in this area a mathematical treatment of induced innovation at the micro level is also necessary.

Moreover, the way in which induced innovation is handled in the growth literature has led to more skepticism about the usefulness of induced innovation in such a context. Nordhaus (1973) develops new critical arguments and summarizes the older misgivings about the growth model versions of induced innovation. He points out that better micro economic foundations are needed before progress can be made with induced innovation growth models.

This paper goes a step in this direction by reformulating innovation possibilities on the basis of research processes, which have expected pay-off functions in terms of efficiency improvements, and by explicitly introducing research costs. The benefits of research occur over the lifetime of the project into which the research results are embodied. This leads to the specifications of research as an investment problem in which present value is maximized. The solution is an optimal mix of research processes which determines simultaneously the bias and rate of technical change. The model is presented entirely in comparative static terms.

The following section is devoted to the reformulation of innovation possibilities. This is the crucial problem of any induced innovation framework. In section three the reformulated innovation possibilities are built into a model. Its behavior is examined under different assumptions about research budget constraints: no constraint is imposed on research funds at first. Then a constraint is imposed on research funds alone and finally the budget constraint covers both the research budget and the physical investment budget of the firm. The different budget constraints substantially change the behavior of the model.

Section IV shows that Ahmad's (1966) model and Kennedy's (1964) Innovation Possibility Frontier (IPF) are special cases of the model developed here. Section IV also discusses the problem of dynamic extension of the model and some limitations implied by its assumptions.

A few of the major implications of the model may be summarized here: The reformulation of invention possibilities on the basis of research processes which have a cost, leads one to reject the existence of a technological frontier which could be observed, at least in the most advanced firms. No firm would ever carry research to a point at which research payoffs become zero, a point which one may call the scientific frontier. How close a firm will come to the scientific frontier depends on where marginal benefits from research equal marginal costs of it. Indeed, rates and biases of technical change are determined by four factors:

- i) The relative productivity of alternative research lines, i.e., the size and biases of invention possibilities and the exogenous changes occurring in them over time.
- ii) The price or cost of research.

- iii) The total present value of factor costs and not only relative factor prices or relative factor shares.
- iv) The constraints on the research or investment budget of the decision making unit.

A quite surprising result is that, when no budget constraint on research resources exist, one cannot necessarily predict that a rise in the present value of the cost of, say, labor will result in stronger labor-saving bias. It may indeed lead to a stronger labor-using bias than before the rise. This shows that the intuitive idea on which Hicks (1964) based his induced innovation idea and which was the basis of all future theorizing does hold only under certain conditions, which are spelled out in this paper. Another result is that a budget constraint which covers both research and physical investment will tend to bias research into a capital saving direction even if it were neutral without such a constraint.

Viewing Kennedy's (1964) IPF model as a special case of the model developed here one can show the exact conditions which must hold for such a frontier to exist and to be stable over time. These conditions are so restrictive that it is safe to conclude that a stable IPF cannot arise from research processes and that therefore the concept should be abandoned.

## II

### INVENTION POSSIBILITIES

In their paper "A Model of Technological Research", Evenson and Kislev (1971) treat research as a sampling process, using seed research as an example. They assume that there exists a probability distribution of potential yield increases which is determined by nature, the state of basic sciences and plant breeding techniques. Research is viewed as drawing

successive trials from this distribution. Given the number of trials  $m$ , the expected pay-off from the research is the first order statistic or the largest yield increase found in the sample. All other trials can then be discarded since only the plant with the highest yield will be used for the new variety. Given the distribution of potential yield increases one can define ex ante the expected pay-off from research as the expected first order statistic of a sample of size  $m$ , which is a function of the sample size.

$$E(\Delta Y_{1m}) = h(m) \quad (1)$$

where  $\Delta Y_{1m}$  is the largest yield increase in a sample of size  $m$ .  $E(\Delta Y_{1m})$  is an increasing function of  $m$  but the marginal pay-offs decline as the sample size increases<sup>2</sup>, i.e.,

$$\begin{aligned} \frac{\partial E(\Delta Y_{1m})}{\partial m} &\geq 0 \\ \frac{\partial^2 E(\Delta Y_{1m})}{\partial m^2} &< 0 \end{aligned} \quad (2)$$

A research administrator who maximizes expected returns from research will equate marginal expected pay-off with the marginal cost of research<sup>3</sup>.

There are two sources of uncertainty in this model: The distribution of potential yield increases may be well defined, but most likely the decision maker will not know it with certainty. He may have formed expectations about it from his knowledge of previous research and the state of the arts. It should therefore be viewed as a subjective probability distribution of potential yield increases, whose parameters have an expected mean and variance.

The other source of uncertainty comes from the variance of the expected first order statistic, which would exist even if the underlying distribution was known with certainty.

In what follows it will be assumed that the decision maker is risk neutral, i.e. maximizes expected return from investment without considering the variance of the expected return. This is a considerable simplification because the optimal decision is the same whether the distribution of possible yield increases is known with certainty or not, and it is unaffected by the variance of the first order statistic. All derivations can be done as if we were dealing with a certainty model<sup>4</sup>.

To adapt Evenson and Kislev's (1971) view of technological research to the induced innovation problem, we have to specify the implications of research processes for factor proportions. If we write a factor augmenting production function as

$$Y = f\left(\frac{K}{A}, \frac{L}{B}\right) \quad (3)$$

where the notation is defined in Table 1, one can make the reduction in A and B functions of a research line, say m. Mathematically it would be easiest to assume that the reduction in A is a function of one research process while the reduction of B is a function of another research process. A research decision would then be a decision to augment one factor of production. In the real world, however, decisions to increase efficiency are never decisions to augment a factor but decisions to pursue different lines of research which result in the embodiment of some new finding or quality in a physical factor of production<sup>5</sup>. And only by coincidence would the factor into which the new quality is embodied be the one and only one which is augmented. A capital



Table 1: Summary of notation

Y		output
v, V		profit or present value (expected)
S		rate of technical change (expected)
Q		bias of technical change (expected) (positive for capital-saving technical change)
t		time
T		lifetime of plant
Variable of coefficient related to		
<u>Capital</u>	<u>Labor</u>	
A	B	augmentation coefficients or input-output ratios
$A^* = \frac{A_0 - A_1}{A_0}$	$B^* = \frac{B_0 - B_1}{B_0}$	proportional reductions of A and B
K	L	capital <u>stock</u> and annual labor <u>flow</u>
R	W	capital price and wage rate
r		rate of interest
$c_K$	$c_L$	capital cost, labor cost
$C_K$	$C_L$	present value of capital and labor cost
m	n	amount of primarily capital-saving and primarily labor-saving research
$p^m$	$p^n$	prices per unit of m and n
$\mu(m)$	$\mu(n)$	scale functions
$\mu_m, \mu_{mm}$	$\mu_n, \mu_{nn}$	their first and second order derivatives
		productivity coefficients or research
$\alpha^m$	$\alpha^n$	to reduce A
$\beta^m$	$\beta^n$	to reduce B

embodied technical change usually augments all factors in various degrees. An experiment station embodies new qualified in a seed variety ( a capital item). The physical quantity of seeds needed to produce one unit of output may or may not decrease, but the amount of land and labor needed will most likely decrease at any set of factor proportions. If a research result from a research process is embodied in a new machine it may decrease labor and capital requirements in various proportions and increases in capital requirement are not excluded. Hence the case in which one research line augments only one factor is not very attractive for model purposes. (This special case may be called the orthogonal case). The model will therefore assume that each research line affects both the labor and capital augmentation coefficients. In a model of induced innovation, where factor proportions are endogenous, at least two such research processes are necessary, each one with different relative impact on the augmentation coefficients. In the most general case one would like to define the research pay-off functions as follows:

$$\begin{aligned} A^* &= A^* (m, n, \dots, k) \\ B^* &= B^* (m, n, \dots, k) \end{aligned} \tag{4}$$

where  $m$ ,  $n$  and  $k$  are research lines and  $A^* = (A_0 - A_1)/A_0$  and  $B^* = (B_0 - B_1)/B_0$ . The subscript zero refers to the coefficients before research while the subscript 1 refers to the coefficient after research. A technological advance corresponds to positive values of  $A^*$  and/or  $B^*$

Equations (3) and (4) would lead to a very general model. A variable proportion production function is combined with very general pay-off functions in which research activities can interact in complex ways, reinforcing each other or competing with each other. Such a formulation

proved to be quite intractable. The following simplifying assumptions are therefore introduced:

(a) The production function is of fixed proportions, i.e.,

$$Y = \min \left( \frac{K}{A}, \frac{L}{B} \right) \quad (5)$$

so that A and B are now simply input-output ratios.

(b) Research results are additive, i.e. the results from one research processes can be implemented independent of the research results from the other process.

(c) Research is subject to decreasing returns.

(d) Only two research processes are considered and they are subject to the same scale function.

(b), (c), and (d) combined lead to the following specification of invention possibilities<sup>6</sup>.

$$\begin{aligned} A^* &= \mu(m)\alpha^m + \mu(n)\alpha^n \\ B^* &= \mu(m)\beta^m + \mu(n)\beta^n \end{aligned} \quad (6)$$

where  $\mu, v \geq 0$

$$\mu_m, \mu_n \geq 0 \quad (7)$$

$$\mu_{mm}, \mu_{nn} < 0$$

and the  $\alpha$ 's and  $\beta$ 's are constraint in either of two ways:

i) In the pure technical change case

$$\begin{aligned} \alpha^m &\geq \beta^m \geq 0 \\ \beta^n &\geq \alpha^n \geq 0. \end{aligned} \quad (8a)$$

ii) In the substitution case

$$\begin{aligned} \alpha^m &\geq 0 \geq \beta^m \\ \beta^n &\geq 0 \geq \alpha^n. \end{aligned} \quad (8b)$$

Graphically the model looks as follows (figure 1).

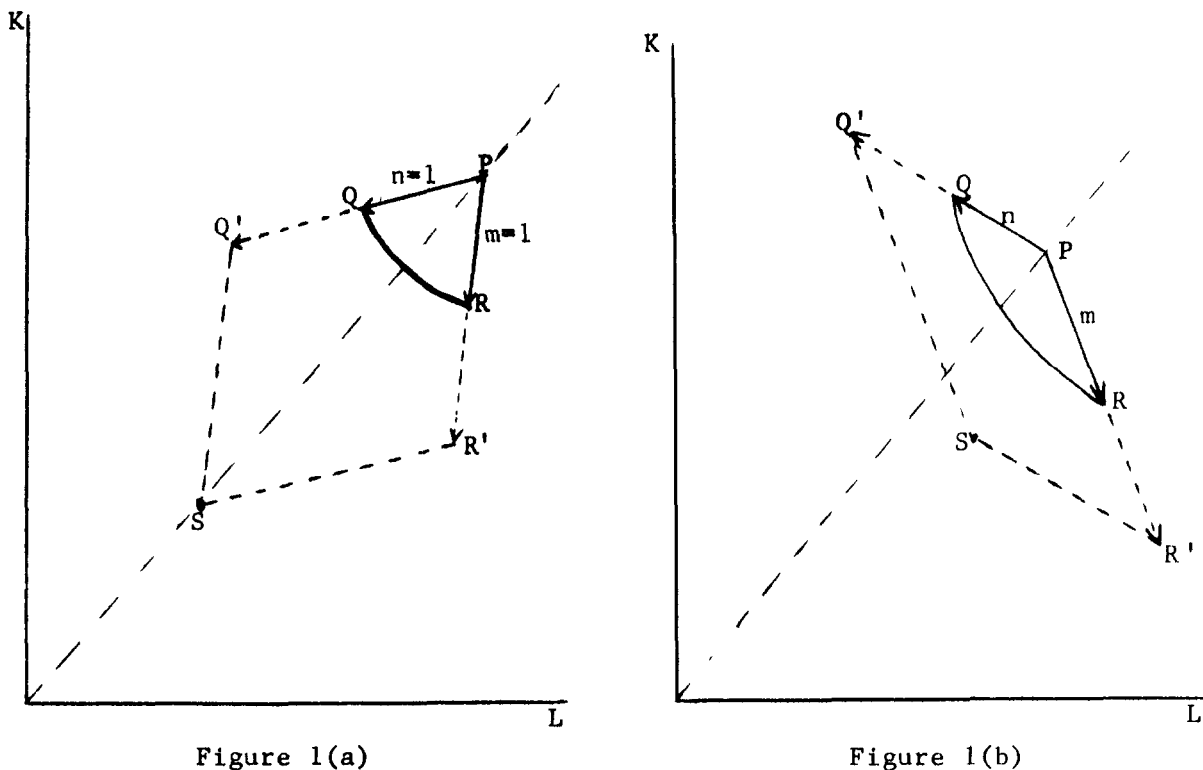


Figure 1 : Innovation Possibilities

Point  $P$  is  $(A_0, B_0)$ , the present input combination needed to produce one unit of output.

The arrow from  $P$  to  $Q$  indicates how much one unit of  $n$  alters this input combination while the arrow to  $R$  indicated how much the necessary input combination is altered by one unit of  $m$ . Under decreasing returns of both  $n$  and  $m$ , the line from  $Q$  to  $R$  is the "isoquant" of possible technologies which can be developed by a linear combination of one unit of  $n$  and one unit of  $m$ . If  $Q'$  and  $R'$  are the points where increases in  $n$  or  $m$  no longer yield any further productivity changes (i.e.  $\mu_m = 0, \mu_n = 0$ ), then the firm can achieve any input combination in the rectangle  $PQ'SR'$ . However, research pay-offs need not become zero anywhere in this model. Figure (1a) is the pure

technical change case of assumption (8a). Each line of research reduces both capital and labor requirements. In figure (1b), corresponding to the substitution case (8b), each line of research saves one factor at the expense of the other factor. The substitution case is more general since it includes factor substitution at a cost as a special case.<sup>7</sup>

Invention possibilities are neutral (as in figure (1a)) if

$$\begin{aligned} \alpha^m &= \beta^n \\ \alpha^m &= \beta^n . \end{aligned} \tag{9}$$

Neutrality would, however, only be a coincidence and is not assumed in the paper except in some special cases.

Additivity is quite restrictive. Results from one research cannot affect productivity of the other research. For ex post or objective research pay-off functions this would be too restrictive. Too many cases are documented in which research results from one research effort proved useful in other projects. But here we are dealing with subjective functions. The dramatic cases of pay-offs of one research line for other research lines were most often unexpected. If they had been expected, more resources would a priori have been devoted to the research lines. If ex ante the interactions are random, the choice of how to allocate a research budget among research lines will not be affected by interdependence and expected pay-off functions are independent.<sup>8</sup>

The assumption of decreasing returns to research (7) is necessary to define a well behaved problem. If the returns were constant, it would be possible to reduce the input requirements to zero. If the price of research also were constant, it would either not pay to pursue a research line at all

or alternatively pay to pursue it until at least one input requirement would be zero.<sup>9</sup>

Identical scale functions mean that returns to research decrease at the same rate with the number of  $m$  trials and the number of  $n$  trials. If they become zero (which is not necessary for the model) they become zero at the same number of trials. Since the research productivities of  $m$  and  $n$  can differ according to the productivity coefficients  $\alpha$  and  $\beta$  this is not a very restrictive assumption. It can also be relaxed easily, at the cost of a more complicated notation.

The reformulation of invention possibilities on the basis of actual research processes which have a cost leads directly to the rejection of any concept of an observable technological frontier on which at least the most advanced firms or countries would be producing. If returns to research never become zero such a frontier does not even exist conceptually. If they do become zero, as shown in figure one, the line  $Q'SR'$  could be termed the scientific frontier, i.e., the most efficient plant which can be built with a finite or infinite amount of research. But an economizing unit would not spend research resources up to the point where marginal expected benefits from research are zero but only the point where marginal benefits equal marginal cost (see equation (17) below).

The innovation possibilities described here are a comparative static concept, valid for one period only. As basic sciences advance and as information about the true underlying probability distribution of production coefficients accumulates, the expected pay-offs will change, i.e., the research productivity parameters will change from period to period. This problem will be discussed further in Section IV.

## III

## COMPARATIVE STATIC ANALYSIS

The production function (5) and the research possibilities described by (6) to (8) can be built into models of various complexities. The simplest case is developed first, a case in which the research costs and the research benefits occur in the same period and in which the benefits do not extend beyond one period. Extension to the case where the benefits accrue over a fixed number of years, i.e., the lifetime of the plant into which the results are embodied, is then achieved by a simple switch in notation, leaving the algebra unaffected. Extension to true multi-period optimization or growth models is not considered here, but the fundamental problems which such an extension faces are discussed in Section IV. The expected bias  $Q$  and the rate of technical change  $S$  for a fixed proportion production function are defined as follows:<sup>10</sup>

$$Q = A^* - B^* \begin{cases} > 0 \rightarrow \\ = 0 \\ < \end{cases} \begin{cases} \text{capital-saving} \\ \text{neutral} \\ \text{labor-saving} \end{cases} \quad (10)$$

Equation (10) can be rewritten in terms of the parameters of the research pay-off functions:

$$Q = \mu_m (\alpha^m - \beta^m) + \mu_n (\alpha^n - \beta^n) \quad (10a)$$

and its change as

$$dQ = \mu_m (\alpha^m - \beta^m) dm + \mu_n (\alpha^n - \beta^n) dn \quad (10b)$$

The expected rate of technical change is a function of the total value of research  $m + n$  (not single valued) and is written as

$$S = \frac{c_K A^* + c_L B^*}{c_K + c_L} \quad (11)$$

where  $c_K$  and  $c_L$  are capital and labor costs.<sup>11</sup>

Consider then a firm which wants to build a new plant. The firm has the option of buying a plant of existing design with the input-output ratios  $A_0$  and  $B_0$  and the fixed capacity  $Y$ . Alternatively the firm can do research to reduce the input-output ratios of the plant to be built according to the research functions (6) to (8). Since output is given, the only decision variables of the firm are  $m$  and  $n$ . Profits can be written as:

$$v = PY - RK_0 - WL_0 + \\ + RK_0 A^*(m,n) + WL_0 B^*(m,n) - mP^m - nP^n . \quad (12)$$

The first three terms are value of output, capital cost and labor costs of the plant of existing design. They are constant and will be collected into the constant term  $v_0 = PY - RK_0 - WL_0$ , i.e., the profits without any research

Writing

$$c_K = RK_0 = RYA_0 \\ c_L = WL_0 = WYB_0 . \quad (13)$$

We can rewrite (12) as follows:

$$v = v_0 + c_K A^*(m,n) + c_L B^*(m,n) - mP^m - nP^n \quad (14)$$

Substituting (6) into (14) and rearranging terms in  $m$  and  $n$  leads to

$$v = v_0 + \mu(m)(c_K \alpha^m + c_L \beta^m) + \mu(n)(c_K \alpha^n + c_L \beta^n) \\ - mP^m - nP^n \quad (15)$$

as the final form of the maximizing problem.<sup>12</sup>

#### No Budget Constraint

The behavior of the models is first examined without a constraint on the research budget. Investment into research will proceed up to the point where marginal research benefits are equal to marginal research costs, and not to the scientific frontier, unless research costs are zero. This is shown by the first order conditions.



$$\begin{aligned}\mu_m (c_K \alpha^m + c_L \beta^m) &= P^m & (=1) \\ \mu_n (c_K \alpha^n + c_L \beta^n) &= P^n & (=1) .\end{aligned}\tag{17}$$

The units of  $m$  and  $n$  are chosen such that their prices are equal to one.

To trace the behavior of the optimal solution, differentiate totally.

$$\begin{aligned}\mu_{mm} (c_K \alpha^m + c_L \beta^m) dm &= dP^m - \mu_m \alpha^m dc_K - \mu_m \beta^m dc_L \\ \mu_{nn} (c_K \alpha^n + c_L \beta^n) dn &= dP^n - \mu_n \alpha^n dc_K - \mu_n \beta^n dc_L .\end{aligned}\tag{18}$$

Multiplying the left hand side of the equations by  $\mu_m/\mu_m$  and  $\mu_n/\mu_n$  respectively and using the first order conditions; and multiplying all terms in  $dc_K$  and  $dc_L$  by  $c_K/c_K$  and  $c_L/c_L$  respectively leads to equations in proportional or logarithmic changes of  $c$  and  $P$ .

$$\begin{aligned}dm (\mu_{mm}/\mu_m) &= d \ln P^m - c_K \mu_m \alpha^m d \ln c_K - c_L \mu_m \beta^m d \ln c_L \\ dn (\mu_{nn}/\mu_n) &= d \ln P^n - c_K \mu_n \alpha^n d \ln c_K - c_L \mu_n \beta^n d \ln c_L\end{aligned}\tag{18a}$$

and solving for  $dm$  and  $dn$

$$\begin{aligned}dm &= \frac{d \ln P^m - c_K \mu_m \alpha^m d \ln c_K - c_L \mu_m \beta^m d \ln c_L}{\mu_{mm}/\mu_m} \\ dn &= \frac{d \ln P^n - c_K \mu_n \alpha^n d \ln c_K - c_L \mu_n \beta^n d \ln c_L}{\mu_{nn}/\mu_n} .\end{aligned}\tag{19}$$

Equations (19) lead to the first observation: It is neither factor prices alone, as in the Ahmad (1966) version of induced innovation, nor factor shares as in the Kennedy - Weizsäcker-Samuelson (1966) version of induced innovation which alone influence optimal research mix and hence rates and biases, but it is research costs and total factor costs which are important.

Considering factor prices alone neglects the importance of factor quantity in factor costs while factor shares alone neglect the impact of the scale of output on optimal research amounts.<sup>13</sup> Both approaches, of course, neglect research costs.

Given the signs of the derivatives of the functions in (7) one can show that research price has a negative effect on each line of research.

$$\frac{\partial m}{\partial \ln P^m} = \frac{\mu_m}{\mu_{mm}} \leq 0 \quad ; \quad \frac{\partial n}{\partial \ln P^n} = \frac{\mu_n}{\mu_{nn}} \leq 0 \quad . \quad (20)$$

The size of the negative effect depends on the curvature of the research functions. Since the amount of  $m$  research is independent of the price of  $n$  research (equation 20) it follows that total research and the rate of technical change decline if the price of either one or of both lines of research rises. If only the price of the more capital-saving research line  $m$  rises, technical change will be more labor-saving. This can be shown by substituting (20) into 10(b), holding  $P^n$  constant, and recalling the sign conventions 8(a) or 8(b).

$$\frac{\partial Q}{\partial \ln P^m} = \frac{\mu_m^2 (\alpha^m - \beta^m)}{\mu_{mm}} \leq 0 \quad . \quad (21a)$$

Conversely a rise in the price of  $n$  increases the capital-saving bias:

$$\frac{\partial Q}{\partial \ln P^n} = \frac{\mu_n^2 (\alpha^n - \beta^n)}{\mu_{nn}} \geq 0 \quad . \quad (21b)$$

Call the effect of the cost of a factor on the line of research which tends to save it more strongly the own-cost effect (e.g.,  $\partial m / \partial \ln c_K$ ) and call the

effect effect on the other line of research a cross-cost effect (e.g.,  $\partial n/\partial \ln c_K$ ). The own effects are positive.

$$\frac{\partial m}{\partial \ln c_K} = - \frac{c_K \mu_m^2 \alpha^m}{\mu_{mm}} \geq 0 \quad (22)$$

$$\frac{\partial n}{\partial \ln c_L} = - \frac{c_L \mu_n^2 \beta^n}{\mu_{nn}} \geq 0 .$$

The magnitudes of the own effects depend on the own costs, the own research productivity  $\alpha^n$  or  $\beta^m$  and on how far the process of research has already been carried (indicated by the ratio  $\mu_m^2/\mu_{mm}$ ). Since the logarithmic change of a product is the sum of the logarithmic changes of its components we can write  $d \ln c_K = d \ln R + d \ln K_0$  or  $d \ln c_K = d \ln R + d \ln Y + d \ln A_0$  and similarly for  $d \ln c_L$ . By the chain rule, therefore, the following equations hold:

$$\frac{\partial m}{\partial \ln c_K} = \frac{\partial m}{\partial \ln R} = \frac{\partial m}{\partial \ln K_0} = \frac{\partial m}{\partial \ln A_0} \geq 0 \quad (23)$$

$$\frac{\partial n}{\partial \ln c_L} = \frac{\partial n}{\partial \ln W} = \frac{\partial n}{\partial \ln L_0} = \frac{\partial n}{\partial \ln B_0} \geq 0 .$$

(The effect of  $Y$  is discussed later). Hence equiportional rises in any component of factor cost (factor price, input quantity, initial input-output coefficients) have an effect of equal sign and size on the research saving the particular factor. A higher input-output coefficient corresponds to a lower efficiency of the factors. Hence the less efficient a factor, the more research resources will be devoted to it. This decomposition of the factor

cost effects can always be done. Since all signs and magnitudes are identical it will be taken for granted in the remainder of the paper.<sup>14</sup>

The cross-cost effects are as follows:

$$\frac{\partial m}{\partial \ln c_L} = - \frac{c_L \mu_m^2 \beta^m}{\mu_{mm}} \quad (24)$$

$$\frac{\partial n}{\partial \ln c_K} = - \frac{c_K \mu_n^2 \alpha^n}{\mu_{nn}} .$$

The sign of these effects depends on whether we are in the pure technical change (equation 8a) case or in the substitution case (equation 8b). In the pure technical change case  $\alpha^n$  and  $\beta^m$  are positive, i.e., the capital-saving research line also saves labor, and vice versa. Therefore the cross effects will be positive, and a rise in labor costs will tend to increase research along the more capital-saving line of research.

In the substitution case, where  $\alpha^n$  and  $\beta^m$  are negative a rise in labor costs will tend to decrease research along the capital-saving line because that line uses labor; but the labor cost rise is a signal for more labor saving.

In the substitution case it is possible to get unique answers for the influence of factors costs on expected biases.

Rewriting (10b) in terms of  $\partial m / \partial \ln c_K$  and  $\partial n / \partial \ln c_K$  and using the chain rule leads to

$$\frac{\partial Q}{\partial \ln c_K} = \mu_m (\alpha^m - \beta^m) \frac{\partial m}{\partial \ln c_K} + \mu_n (\alpha^n - \beta^n) \frac{\partial n}{\partial \ln c_K} . \quad (25)$$

$(\alpha^m - \beta^m)$  is always positive and  $(\alpha^n - \beta^n)$  is always negative by equation (8a) or (8b). If  $\partial n / \partial \ln c_K$  is negative, then a rise in capital costs results in a rise in the capital-saving bias. The opposite result obtains for a rise in labor costs. This is the intuitive idea on which all induced innovation reasoning is based. But it can only be shown for the substitution case, not for the pure technical change case. Then  $\partial n / \partial \ln c_K$  is positive and the sign of (23) is undetermined. An example in which a continued rise in labor costs eventually leads to a reduction in the labor-saving bias will illustrate this point.

Let  $\mu(m) = m - 1/2m^2$  ,  $\mu(n) = n - 1/2n^2$   
 and  $\alpha^m = \beta^n = 1$  ,  $\alpha^n = \beta^m = 1/2$   
 and  $c_K = 1$  ,  $c_L = 1, 2, 4, 6$  respectively.

Table 2 shows the result for the optimal amounts of  $m$  and  $n$  and the bias.

Table 2: An example of the effect of labor costs on  
the bias

Factor Costs		Optimal research levels		Efficiency gains		Bias
$c_K$	$c_L$	$m$	$n$	$A^*$	$B^*$	$Q = A^* - B^*$
1	1	1/3	1/3	.4166	.4166	0
1	2	1/2	3/5	.5850	.6075	-.0225
1	4	2/3	7/9	.6821	.6975	-.0154
1	6	3/4	11/13	.7129	.7225	-.0096

The first increase of labor cost over capital cost leads to a labor-saving bias. But the size of this bias is reduced as labor costs continue to rise.

The reason for this behavior lies in the diminishing returns to research and the fact that both lines save some of both of the factors. As labor costs first rise, the more labor-saving line is expanded more rapidly than the capital-saving line. Hence, when labor costs continue to rise,  $n$  research is in a range with smaller marginal returns than  $m$  research. The absolute labor-saving achievable through expanding  $m$  may now be larger than the one achievable by  $n$  research despite the fact that, at equal  $m$  and  $n$ ,  $n$  is always more labor saving than  $m$ . Suppose labor costs were to rise to almost infinity, and capital costs were zero. Since both lines actually save labor it would be optimal to push both lines to the point where marginal pay-offs are zero. But if invention possibilities are neutral, the optimal point will be the points  $S$  in figure (1a), which corresponds to a neutral technical change.

To determine the effect of output on research levels, consider equation (19) and hold all prices and input coefficients constant,  $dlnc_K$  and  $dlnc_L$  can then simply be replaced by  $dlnY$  in (19). This leads to the following expressions:

$$\frac{\partial m}{\partial lnY} = - \frac{\mu_m}{\mu_{mm}} \mu_m (c_K \alpha^m + c_L \beta^m) = - \frac{\mu_m P^m}{\mu_{mm}} \geq 0$$

$$\frac{\partial n}{\partial lnY} = - \frac{\mu_n}{\mu_{nn}} \mu_n (c_K \alpha^n + c_L \beta^n) = - \frac{\mu_n P^n}{\mu_{nn}} \geq 0 \quad .$$

(26)

Since by (17) the terms in the brackets times the first derivatives of the scale functions are positive and equal to research prices, the signs of (26) are easily established. An increase in the capacity of the plant increases

both research levels and thus leads to a higher rate of technical change in both the substitution and the pure technical change cases. While the model has no economies of scale with respect to research it has such economics with respect to the output of the plant.

The effect of output on the bias is:

$$\frac{\partial Q}{\partial \ln Y} = - \frac{\mu_m^2}{\mu_{mm}} (\alpha^m - \beta^m) - \frac{\mu_n^2}{\mu_{nn}} (\alpha^n - \beta^n) . \quad (27)$$

(This derivation sets equation (26) into (10a) with the research prices equal to one.) This expression can be positive or negative in both the substitution and the pure technical change case. Hence scale effects need not be neutral even if research possibilities are neutral. This is again due to the assumption of diminishing returns to research.

The conclusions are briefly summarized here in terms of labor and the more labor-saving research line n. The conclusions for capital and m are analogous.

i) A rise in the price of n will result in a reduction of n research and turn the bias in a more labor-using direction.

ii) A rise in the price of n and/or m will result in a decrease of total research and hence in a smaller rate of technical change.

iii) An increase in the scale of output will increase both m and n research and the rate of technical change. The effect on the bias may not be neutral.

iv) Anything which changes factor costs changes the optimal research mix. A rise in the wage rate or the initial labor-output ratio tends to increase the amount of m research. Indeed, equiproportional rises in these two variables have equal effect on research quantities. Rises in labor costs

will lead to an increase in the more capital-saving research line  $m$  if we are in the pure technical change case, but to a decrease in the substitution case. A labor cost rise will always lead to a more labor-saving bias only in the substitution case. In the pure technical change case the effect on the bias is undetermined.

#### Benefits occurring over time

The model does not really change when we assume that the benefits from research occur over the total lifetime of the plant rather than only over one period. Since research results are embodied in the plant and hence have to be found before the plant is built, we still have a single period optimization, but it affects costs and benefits over the lifetime of the project. The firm now maximizes present value of the project and (12) is changed as follows:

$$\begin{aligned}
 V = & \int_0^T P(t)e^{-rt} dt - RK_0 - L_0 \int_0^T W(t)e^{-rt} dt \\
 & + RK_0 A^*(m,n) + B^*(m,n)L_0 \int_0^T W(t)e^{-rt} dt \\
 & - mP^m - nP^n .
 \end{aligned} \tag{28}$$

Remember that  $K_0$  is capital stock, not a flow while  $L_0$  is annual labor flow.  $P(t)$  and  $W(t)$  are the expected prices of output and the expected wage rate as a function of time. They may of course be constant. The firm has to pay for the total capital stock now and  $R$  is the purchase price per unit of capital. Maintenance costs of capital are neglected. The first three terms are again constant and are denoted by  $V_0$ .

Letting



$$\begin{aligned}
 C_K &= RK_0 && = \text{present value of capital cost} \\
 C_L &= L_0 \int_0^T W(t) e^{-rt} dt && = \text{present value of labor cost}
 \end{aligned}$$

We can rewrite (28) as

$$V = V_0 + C_K A^*(m,n) + C_L B^*(m,n) - mP^m - nP^n \quad . \quad (28a)$$

This equation has precisely the same form as (14) and can be transformed into an equation equivalent to (15) with capital C's replacing the lower-case c's. Hence all the conclusions of the previous section are identical if we replace factor costs  $c_K$  and  $c_L$  by present value of factor costs  $C_K$  and  $C_L$ . Not only current costs but the whole stream of future costs associated with the plant become important.<sup>15</sup>

The change is more than just a switch of notation. It can be used to illuminate the Fellner (1961) proposition on induced innovation. Salter (1960) stated that induced innovation was not a viable theory because the entrepreneur could not be induced simply by high wages to seek labor-saving innovations. He would be interested in saving costs regardless of whether it is labor costs or capital costs. The absolute level of the wage rate would not matter in that decision but only the relationship of it to the value of the marginal product. The previous section, as well as Ahmad's (1966) work have, of course, proven that this criticism is not valid. Fellner (1961) tried to get around the Salter criticism by asserting that it is not the level of the wage rate but the anticipation of a future wage rate rise which prompts the entrepreneur to seek out labor-saving inventions. But for the mechanisms of induced innovation it makes no difference whether the entrepreneur anticipates a rise in the wage rate or whether he believes that

$W(t)$  is a constant. Present value of labor costs will influence the research mix in both cases. Of course more research will be undertaken if  $W(t)$  is a rising function of time than if it is constant at its initial value. But that is a question of level, not of mechanism. Fellner's proposition is therefore misleading.

Reformulation in terms of present value also allows a consideration of the effect of the rate of interest (separately from the purchase price of capital  $R$ ) and of the lifetime of the project. Both occur only in labor costs. A rise in  $r$  decreases discounted labor costs by giving less weight to the cost of distant periods and a rise in the lifetime  $T$  raises labor costs by adding more periods, i.e.,

$$\frac{\partial \ln C_L}{\partial r} \leq 0 \quad ; \quad \frac{\partial \ln C_L}{\partial T} \geq 0 \quad ,$$

Insert these into (22) by the chain rule. Then

$$\frac{\partial n}{\partial r} \leq 0 \quad ; \quad \frac{\partial n}{\partial T} \geq 0 \quad . \quad (29)$$

Labor-saving research is reduced as the rate of interest (or the opportunity cost of capital) rises or as the lifetime of the project is reduced. In a similar way it is easily proved that an increase in  $r$  and a decrease in  $T$  will lead to an increase in  $m$  research in the substitution case and a decrease in the pure technical change case (equation 24). In the substitution case a rise in  $r$  and a reduction in  $T$  will lead to a more labor-using bias but in the pure technical change case the sign cannot be established. (equation 25).<sup>16</sup>

#### Research budget constraints: "The Kennedy Case"

This case is so-called, because, under very strict conditions discussed in section IV, it is possible to derive a Kennedy Innovation Possibility Frontier from equations (6) and a research budget constraint

$$mP^m + nP^n - F = 0 \quad (30)$$

where  $F$  is the research budget. Maximizing present value (equation 28a) subject to (30) leads to the following first-order conditions.

$$\begin{aligned} -mP^m - nP^n + F &= 0 \\ \mu_m(C_K\alpha^m + C_L\beta^m) &= (1 + \lambda)P^m \quad (= 1 + \lambda) \\ \mu_n(C_K\alpha^n + C_L\beta^n) &= (1 + \lambda)P^n \quad (= 1 + \lambda) \end{aligned} \quad (31)$$

Totally differentiating these equations, setting  $P^m = P^n = 1$ , and going through the same transformations to proportional changes used to go from (18) to (18a), the equations can be rewritten in the following matrix notation:

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & -g_{11} & 0 \\ 1 & 0 & -g_{22} \end{bmatrix} \begin{bmatrix} d\lambda^* \\ dm \\ dn \end{bmatrix} = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \end{bmatrix}$$

where  $d\lambda^* = d\lambda/(1 + \lambda)$

$$g_{11} = \mu_m/\mu_{mm} < 0 \quad ; \quad g_{22} = \mu_n/\mu_{nn} < 0$$

$$S_0 = dF - m \, d\ln P^m - n \, d\ln P^n$$

$$S_1 = \frac{C_K\mu_m\alpha^m}{1 + \lambda} \, d\ln C_K + \frac{C_L\mu_m\beta^m}{1 + \lambda} \, d\ln C_L - d\ln P^m$$

$$S_2 = \frac{C_K\mu_n\alpha^n}{1 + \lambda} \, d\ln C_K + \frac{C_L\mu_n\beta^n}{1 + \lambda} \, d\ln C_L - d\ln P^n$$

Hence, by inverting,

$$\begin{bmatrix} d\lambda^* \\ dm \\ dn \end{bmatrix} = \frac{1}{g_{11} + g_{22}} \begin{bmatrix} g_{11}g_{22} & g_{22} & g_{11} \\ g_{22} & -1 & 1 \\ g_{11} & 1 & -1 \end{bmatrix} \begin{bmatrix} s_0 \\ s_1 \\ s_2 \end{bmatrix} \quad (32)$$

and

$$dm = \frac{g_{22}s_0 - s_1 + s_2}{g_{11} + g_{22}} \quad (33)$$

$$dn = \frac{g_{11}s_0 + s_1 - s_2}{g_{11} + g_{22}} .$$

The denominator is always negative because both  $g_{11}$  and  $g_{22}$  are negative.

Assembling the terms in  $d\ln C_K$  from  $s_1$  and  $s_2$  we have:

$$\frac{\partial m}{\partial \ln C_K} = - \frac{C_K(\mu_m \alpha^m + \mu_n \alpha^n)}{(g_{11} + g_{22})(1 + \lambda)} \geq 0 \quad (34)$$

(34) is always positive if

$$\mu_m \alpha^m > \mu_n \alpha^n .$$

This is always the case when  $\alpha^n$  is negative (substitution case), but also holds, when  $\alpha^n$  is positive. The proof is as follows: setting  $p^m$  and  $p^n = 1$  we solve the first order conditions (31) for  $\mu_m$ , substitute into (35) and obtain the condition

$$\frac{\alpha^m}{C_K \alpha^m + C_L \beta^m} > \frac{\alpha^n}{C_K \alpha^n + C_L \beta^n} \quad (35)$$

$$\frac{1}{C_K + C_L (\beta^m / \alpha^m)} > \frac{1}{C_K + C_L (\beta^n / \alpha^n)}$$

Since under both conditions (8a) and (8b)  $\beta^m/\alpha^m < \beta^n/\alpha^n$  the inequality in (35) is satisfied regardless of the signs of  $\alpha^n$  and  $\beta^m$ . Similarly, it can be proved that the other own-cost effect is positive as well:

$$\frac{\partial n}{\partial \ln C_L} \geq 0 \quad (36)$$

Since, when the budget constraint is binding and research prices are equal,  $dm = -dn$ , it follows immediately that the cross-cost effects are equal to the own-cost effects.

$$\frac{\partial m}{\partial \ln C_K} = - \frac{\partial n}{\partial \ln C_K} \quad \text{and} \quad \frac{\partial n}{\partial \ln C_L} = - \frac{\partial m}{\partial \ln C_L} \quad (37)$$

In contrast to the unconstrained case this allows us to prove monotonic relationships between biases and factor costs.

Setting (37) in to (14):

$$\frac{\partial B}{\partial \ln C_K} = \mu_m(\alpha^m - \beta^m) \frac{\partial m}{\partial \ln C_K} - \mu_n(\alpha^n - \beta^n) \frac{\partial n}{\partial \ln C_K} \geq 0 \quad (38)$$

A rise in discounted capital (labor) costs will lead to a more capital-saving (labor-saving) bias. The expression  $d \ln C_K$  and  $d \ln C_L$  can, of course, be broken down into their components and each analyzed in turn. When the budget constraint is binding an increase in one line or research is only possible at the expense of the other line. This is why it is now possible, even in the pure technical change case where it was not possible before, to predict that a rise in discounted labor costs will result in a stronger labor-saving bias.

Furthermore, it can be shown that a rise in capital costs has an effect of equal size but opposite sign on research effort than an equiproportional rise in discounted labor costs when research prices are equal.

$$\frac{\partial m}{\partial \ln C_K} = - \frac{\partial m}{\partial \ln C_L} \quad (39)$$

From (34) and the equivalent equation for  $\partial m / \partial \ln C_L$  we can obtain the following condition which must hold for (39) to be satisfied.

$$- C_K (\mu_m \alpha^m + \mu_n \alpha^n) = + C_L (\mu_m \beta^m + \mu_n \beta^n)$$

or rearranging terms

$$\mu_n (C_K \alpha^n + C_L \beta^n) = \mu_m (C_K \alpha^m + C_L \beta^m) \quad .$$

Checking with the first order conditions (31), both sides are equal when research prices are equal, Q.E.D.

Since furthermore,  $dm = -dn$ , the scale of output  $Y$  can have no effect on the bias, because it affects  $C_K$  in the same proportion as  $C_L$ . Again in contrast to the previous unconstrained case, scale effects are now neutral.

The signs of the effects of research prices are identical to the unconstrained case. It cannot be proved that a change in the research budget has a neutral effect. Biases can result when one research activity is already so large that it encounters strongly diminishing returns. An increase in the research budget is then primarily spent on the previously neglected line of research.

The simple fact that a budget constraint exists allows one to derive sharper results than in the unconstrained case. It will be shown in Section IV that this case is the model which corresponds to Kennedy's Innovation Possibility Frontier or Ahmad's graphic model.

#### A Budget constraint on total investment resources

A budget constraint on research alone is useful to trace the allocation of research resources of a governmental research institute such

as an experiment station which indeed has a fixed research budget. But a firm can borrow to do research or reallocate resources from the physical investment program to research if it has a borrowing constraint but finds research more profitable than physical investment. Likewise a country can increase its savings rate or re-allocate resources to research from physical investment if it faces a fixed saving rate. Establishing a budget constraint for research and physical investment separately does not maximize returns from total investment because rates of returns of the two kinds of investments are not equated at the margin. Therefore, the previous model is only a narrow special case. And it turns out that a budget constraint on total investments alters the behavior of the model substantially.

The budget constraint is rewritten to allow the firm to use for research purposes what it saves in capital equipment.

$$mP^m + nP^n + C_K = F + C_K A^* \quad (40)$$

The sum of research and initial capital expenditures is equal to the total budget plus the reduction in capital costs made possible by the research.

Unfortunately, this budget constraint considerably complicates the problem. Therefore the specification of research possibilities is simplified such that  $m$  only affects  $A$  and  $n$  only  $B$ . This is the orthogonal case discussed before and  $\alpha^m$  and  $\beta^m$  are equal to zero in equation 6 and in 28(a).  $A^*$  and  $B^*$  respectively are functions of  $m$  and  $n$  alone. The first-order conditions of this problem, subject to (40), now become:

$$\begin{aligned} mP^m + nP^n + C_K &= F + C_K \mu(m) \alpha^m \\ C_K \mu_m \alpha^m &= P^m & (= 1) \\ C_L \mu_n \beta^n &= (1 + \lambda) P^n & (= 1 + \lambda) \end{aligned} \quad (41)$$

If research possibilities are completely neutral,  $\alpha = \beta$ , the existence of the budget constraint alone biases technical change in a capital saving direction. This can be proved as follows: If  $\mu$  is quadratic,<sup>17</sup> (i.e.  $\mu(m) = a_1 m - 1/2 a_2 m^2$ ) then  $\mu'_m = a_2(a_1/a_2 - m)$ . Since  $a_1/a_2$  is the level at which marginal returns to  $m$  become zero, say  $m^*$  we have  $\mu'_m = a_2(m^* - m)$ . Also, since  $m$  and  $n$  have the same returns function,  $m^* = n^*$ . Using this specialization and setting  $\alpha^m = \beta^n$  we can solve (41) for  $m$  and  $n$  explicitly.

$$\begin{aligned} m &= m^* - \frac{1}{C_K a_2 \alpha^m} \\ n &= m^* - \frac{1 + \lambda}{C_L a_2 \alpha^m} \end{aligned} \tag{42}$$

Even if capital and labor costs are equal ( $C_K = C_L$ ),  $m$  will be larger than  $n$ . When this does not hold, a rise in  $\lambda$ , the shadow price of the constraint, leaves  $m$  unaffected, but reduced  $n$ . In the orthogonal case a reduction in  $n$  is always a reduction in labor-saving bias or an increase in the capital-saving bias. Therefore, reducing the amount of total capital to the firm tends to lead to a capital-saving bias!

The amount of capital-saving research in this formulation is independent of the capital constraint, since it generates the capital cost saving needed to of the research. Going through similar procedure as in the "Kennedy" case we have:

$$\begin{bmatrix} d\lambda^* \\ dm \\ dn \end{bmatrix} = \begin{bmatrix} \varepsilon_{22} & 0 & 1 \\ 0 & \frac{-1}{\varepsilon_{11}} & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} q_0 \\ d\ln C_K - d\ln P^m \\ d\ln C_L - d\ln P^n \end{bmatrix} \tag{43}$$



where  $g_{11}$  and  $g_{22}$  are defined as before and

$$q_0 = dF - C_K(1 - \mu(m)\alpha^m)d\ln C_K - md\ln P^m - nd\ln P^n$$

Hence

$$dm = - \frac{1}{g_{11}} (d\ln C_K - d\ln P^m)$$

$$dn = q_0 \tag{44}$$

Note that capital requirements after research are positive, such that

$$C_K(1 - \mu(m)\alpha^m) = C_K(1 - A^*) > 0$$

From these equations it appears again that  $m$  is independent of labor costs and that changes in  $n$  (not  $n$  itself) are also independent of the labor costs. Changes in  $n$  occur every time some element of the budget constraint changes, and by the full amount of the change in the budget constraint.

Otherwise the conclusions are similar to the two cases discussed previously. A rise in  $C_K$  increases  $m$  and reduced  $n$  and therefore increase the capital saving bias. A rise in prices of research effort reduce the activity whose price has risen. But in this case a rise in the price of  $m$  also reduces  $n$ .

The purposes of the model using this budget constraint is to show how important the specifications of the budget constraint is for the model. Personally, I think that the model without budget constraint is by far the most interesting one.

## IV

## LIMITATIONS, EXTENSIONS AND EQUIVALENCES

Limitations

Relaxing the assumption of fixed proportions of the production function would probably not affect the conclusions of the model very much as long as the elasticity of substitution is small, but might have a substantial impact for large elasticities of substitution. Research is under way to assess this question.

The neglect of interdependence of research activities has been discussed in detail before and reasons have been given why this might not be a serious misspecification. It is hard to guess intuitively what the effect of the introduction of such interdependencies on the model might be.

Such a prediction is much easier in the case of increasing returns to research in the initial ranges of the research pay-off function. The behavior of the model would not be altered when no budget constraints exist because research would then take place in the range of diminishing returns of the research functions. If, however, a budget constraint was so narrow as to constrain the research functions into the range of increasing returns, specialization on one research line would become very likely. The choice of which research to undertake would still be determined by factor costs, research costs and research pay-offs.

Uncertainty has almost been assumed away by the assumption of risk neutrality. Risk aversion would have a strong impact on the model, because building the plant of existing design involves much less risk than doing

research. A risk averse decision maker would always do less research than a risk neutral one and he would favor that line of research which is relatively less risky.

Productivity growth of a firm or an economy arises from many kinds of investments. Research is one of them and is more or less scale independent, i.e., its implementation costs do most often not depend on the number of plants (or their scale) into which the research results are introduced. A similar scale independence may hold for certain management functions. But the costs of training of labor or higher quality of other production inputs or quality control, which are important sources of measured productivity growth, do depend on scale of inputs or outputs. The model presented here is restricted to efficiency investments whose costs are scale independent. An important generalization of the model would be one in which the costs of either  $m$  or  $n$  is made to depend on the scale of one of the inputs into which the results are embodied. Such a model should allow a theoretical investigation into the interdependence of scale-independent and scale-dependent efficiency investments, such as research and human capital formation.

#### Extension of the model into time

Evanson and Kislev (1971) have used a Markov process model to investigate the behavior of a model of research where yield improvements are possible in every time period without embodiment into capital investment and where the improvements are achieved by one sampling process of a Poisson distribution of potential yield increase.

Their work points out that a research pay-off function relating expected yield increases (or efficiency increases) to research effort cannot

be independent of time or independent of achieved yield levels (efficiency levels). Suppose the probability of potential yields is as in figure 2 and does not shift over time.

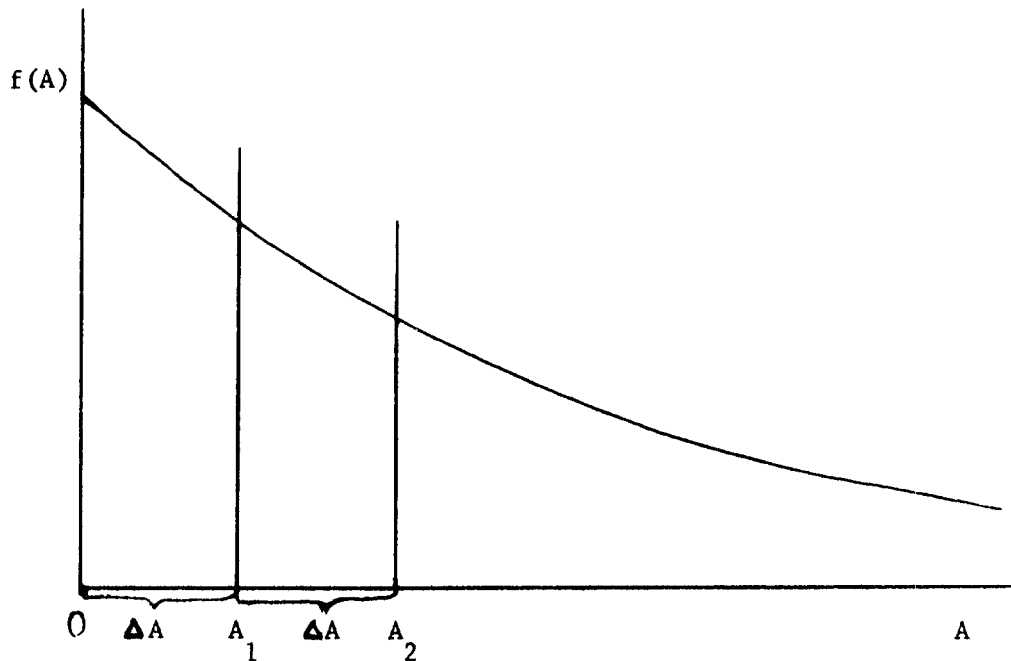


Figure 2

Suppose  $\Delta A$  has been achieved in period one putting the production point to  $A_1$ . To produce an increase of equal magnitude in period two will require a much larger sample because the required expected first order statistic is twice as large as in period one. Research uses up its own potential pay-offs. Only if the shift in the distribution of possible yield increases was precisely equal to achieved yield increase in each period would it make sense to have research pay-off functions independent of achieved yield levels (efficiency levels). But this would be a strange coincidence.

In addition it would also be unreasonable to assume that the probability distribution of potential efficiency increases remain unchanged over time since then profitable opportunities for research would be quickly exhausted. Research pay-offs are likely to change because of advances in basic sciences, results from supporting research and improvements in research methodology. The basic problem, which any extension into time therefore faces, is an a priori specification of how research pay-offs will change over time and how they are affected by achieved efficiency increases. Such a model would then trace how profitable opportunities for research arise in the first place and how they are exhausted subsequently and trace this interplay over time.

Unless an extension into time assumes that pay-off functions are known with certainty, it will also be necessary to specify formally the link between expected pay-offs which determine the research decisions and actual subsequent effects of the research which takes place. In the comparative static model discussed in this paper this problem was avoided, by having the comparative static results trace expected biases and expected rates of technical change, not actual ones. Of course, if expected pay-off functions do correspond to some extent to true pay-off functions, then the true biases and rates will correspond to some extent to the expected ones. But a dynamic model cannot get around this difficulty as easily.

#### The Kennedy Frontier:

The statements of induced innovation now available in the literature are either graphic (Ahmad 1966) or use Kennedy's (1964) Innovation Possibility Frontier in one way or another. Ahmad's graphic treatment is a nice approach

to show how factor prices affect innovation. His model can be regarded as a special case of the model presented here when there exists a fixed budget constraint on research resources alone and when his factor prices are reinterpreted as present value weights. Of course the idea of moving from insoquant to insoquant costlessly would have to be dropped.

Most other induced innovation models use Kennedy's Innovation Possibility Frontier. The IPF is defined as follows:

$$A^{**} = h(B^{**}) \quad (45)$$

where  $A^{**}$  and  $B^{**}$  are instantaneous rates of decrease of  $A$  and  $B$  (see footnot 10) and are the time continuous equivalents of our  $A^*$  and  $B^*$ . The amount of labor augmentation possible is a decreasing function of the amount of capital augmentation. Equation (45) is a production possibility curve of factor augmentation. Note that  $A^{**}$  is a function of  $B^{**}$  alone.

Nordhaus (1973), in his excellent critique of this approach, has shown that the assumption that  $A^{**}$  be a function of  $B^{**}$  alone, i.e., that achieved  $A$  and  $B$  levels do not enter the Innovation Possibility Frontier is a crucial assumption of the induced innovation theories which do use Kennedy's IPF. For if the achieved  $A$  and  $B$  levels are included in the IPF, a growth model will not be stable unless technical change is assumed to drift in a Harrod neutral way. Shares stability is then a result of assumption quite similar to a growth model without induced innovation at all. So such a model does not add to our understanding of technical change. Nordhaus also shows in his article that independence from achieved  $A$  and  $B$  levels implies that the innovation process has no memory at all.

By using the approach of research pay-off functions it is easy to strengthen Nordhaus' criticism and in fact to show that an IPF like (45) is

quite absurd as a specification of endogenous technical change. To include dynamic considerations, rewrite the two research pay-off functions in time continuous form and add the attained A and B levels, since it was shown in the previous section that such an addition is necessary when the model is extended dynamically, because research uses up its own pay-offs.

$$A^{**} = g_1(m, n, A)$$

$$B^{**} = g_2(m, n, B)$$

$$mP^m + nP^n = F$$

Under quite general assumptions it is possible to eliminate m and n from this system and solve for the maximum  $A^{**}$  achievable under the budget constraint as a declining function of  $B^{**}$ , i.e.,

$$A^{**} = H(B^{**}, A, B, P^m, P^n, F)$$

This equation will reduce to (45) if

- i) the levels of A and B do not enter the research pay-off functions,
- ii) research prices stay constant over time,
- iii) the research budget is constant over time,
- iv) the budget constraint covers only the research budget but not the total investment budget (proof left to the reader).

In deriving an IPF in this way we first see that it can only exist when there exists a budget constraint. But such a constraint prevents the economy from achieving an optimal amount of research in the first place. That amount is simply given by the budget constraint. One may of course think that some agency first sets an optimum research budget and the model then only traces its allocation. But the optimal amount of research would surely not stay constant over time in a growth model (condition iii), so this interpretation is not valid.

(Condition iv) implies that the budget constraint can not be interpreted as a savings constraint either, because total savings would cover both the research and physical investment budget. For an interpretation as a savings constraint one needs two such constraints, one on physical investment and one on research investment. But that implies that rates of return to the two investments will differ, except by coincidence, which again implies a misallocation of resources of the economy.

(Condition ii) that research costs stay constant over time implies that they cannot be tied to the wage rate of an economy. In a growth model where wages are variable one would then have to assume that the wages of scientists stay constant. This clearly makes no sense.

Most damaging to the IPF is condition (i) because it implies that research does not exhaust its own pay-offs as discussed in the previous section. Whenever research improves the efficiency of one factor it would have to shift the pay-off function by an amount which makes research pay-offs in the next period equal to the research pay-offs of the last period. Such a specification is non-sensical. This point gives added weight to Nordhaus's (1973) criticism discussed above.

In retrospect the IPF appears to be one of the most outstanding cases of implicit theorizing in the economic literature. The interesting problems posed by endogeneity of technical change, namely how to determine optimal amounts of research and how to trade it off against investments in physical capital is completely neglected in <sup>the</sup> theory.<sup>18</sup> It attempts to explain constancy of shares with biased technical change with an ingenuous device whose relationship to research processes was left in the dark long



after the implications of the device were explored in detail and became widely accepted wisdom. That the device cannot reasonably be generated by a real world research process did not matter.

## V

## CONCLUSIONS

Some of the direct implications of the model have been summarized in the introduction and on page 21. So only a few general remarks are added here.

The basis of this microeconomic approach is a reformulation of invention possibilities on the basis of research processes and the entrepreneurs perception of potential pay-offs of alternative research lines. This makes it relatively easy to check whether this approach is a description of the innovation process which has some basis in reality. It should also make it possible to use this model for empirical research because one can conceivably ask research decisionmakers what their expectations are of the pay-offs of various research lines. Descriptions of innovation possibilities like Kennedy's Innovation Possibility Frontier or Ahmad's Innovation Possibility Curve are so abstract that empirical measurements of them cannot even be attempted.

Starting from actual research processes also has the advantage that Innovation can be treated as an investment process in which the choice of the research investment portfolio depends on factor costs, research productivities and research costs and where the outcome of the choice process determines the direction and rate of technical change simultaneously.

Using the new specification of innovation possibilities made it possible to see precisely what the assumptions are which underlie the Kennedy approach to induced innovation and to show that this is a disguised approach of exogenous technical change which cannot lead to optimal investment resource allocation of an economy to physical capital and research. Furthermore, it certainly has nothing to do with research processes as they occur in the real world since it assumes research pay-offs as inexhaustible by the research process. The Kennedy approach should therefore be abandoned.

The description of innovation possibilities used here implies some simplifying assumptions which have been discussed at length. It may not be the best or only possible description, but it is hoped that it is a start in the right direction. In any case, the specification of innovation possibilities is the crucial problem and will probably occupy us in further research for quite some time to come.

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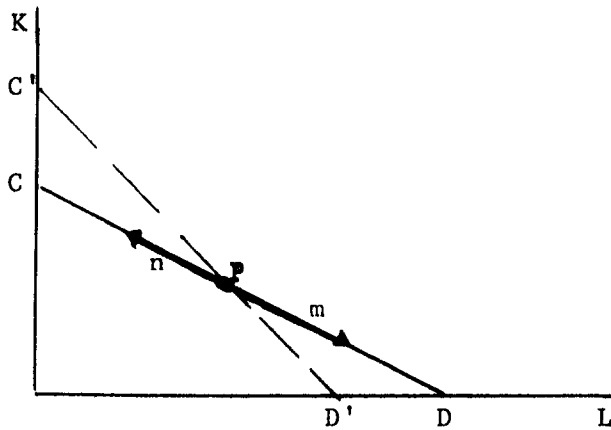
### Footnotes

1. I am very grateful to John Chipman, Robert Evenson, Leonid Hurvich, Vernon Ruttan and Paul Schultz for their corrections and helpful comments at earlier stages of this research. Of course all remaining errors are mine.
2. This holds for any distribution with finite variance.
3. Expected pay-off functions which behave like equation (1) and (2) can also be assumed for research problems which do not fit the sampling model of research very well, such as engineering processes. Then  $m$  can be interpreted as the amount of resources devoted to the research process rather than as sample size. The model developed in this paper covers both cases.
4. Some implications of risk aversion will be considered in section IV. The effect of the sampling variance on the research model with a single research process is explored in detail in Evenson and Kislev (1971).
5. Even organizational changes within the firm require that the employees of the firm learn the new procedures, which involves costs. If the employees later leave, this cost has to be incurred again. A good example of this is the implementation of new data processing systems in a firm.
6.  $A^*$  and  $B^*$  are treated as continuous functions of  $m$  and  $n$ . Of course, trials in a research process are discrete. The functions have therefore to be interpreted as continuous approximations of functions of discrete variables.

- 7 . A pure case of factor substitutability at a cost is reached when, after rescaling K and L such that their price ratio is 1,

$$\begin{aligned} |\beta^m| &\geq \alpha^m \\ |\alpha^n| &\geq \beta^n \end{aligned}$$

When the equalities hold, the n and m process move the point P along the budget line in opposite directions.



As soon as factor price change from C D to C' D' it may become profitable (depending on the price of n) to change factor proportions by the process n. But this version of the model has only corner solutions, i.e., either the level of m is zero or/and the level of n is zero.

8. Large research establishments with many research activities, would, however, tend to increase the overall research budget if they knew from experience that unexpected pay-offs occur frequently, but are random. The increase would be allocated in proportion to the research activities. Interaction of research lines which is known ex ante poses the same problem than interaction among projects poses in conventional cost-benefit analysis. No perfect solution to the problem has been found there. When interactions are very strong, projects are lumped together for

conventional cost-benefits analysis, and the interactions are neglected when they are weak. Each line of research used here could be considered to be a group of strongly interacting research projects such that interactions among  $n$  and  $m$  are minimized.

The model also neglects more basic or supportive research which firm or experiment stations often pursue and which is not aimed at directly yielding payoffs for a production process but rather at increasing the productivity of more applied lines of research. It could be viewed as aimed at altering the research productivity parameters and would have to be analyzed as a separate problem. In an agricultural experiment station context the lines considered here would correspond to breeding and agronomic research while cell biology and physiology corresponds to the supportive research. For empirical evidence on the productivity of such supportive research in agriculture see Evenson (1974).

9. It is sometimes argued that increasing returns to research are frequent because, after an initial period of investigation, a breakthrough sometimes occurs which substantially increases pay-off to that particular line of research. The true response curve exhibited increasing returns. But again, the ex ante subjective research function exhibited diminishing returns, since the breakthrough was not expected, or had a small probability associated with it. For had it been otherwise, the decision maker would have made a large research investment in this line right from the start. His small investment lead to the formulation of

a new expected research function with higher pay-offs. But this new research function will exhibit diminishing returns as well.

Of course, if a line of research requires an initial fixed investment before any trials can occur, the response curve will have an initial range of increasing returns. A research establishment without budget constraint will, however, not operate in this range, just as a competitive firm will not produce in such a range of an ordinary production function. In section IV this problem will be reexamined in the context of a budget constraint.

10. In the time continuous case this would become  $Q = (A^{**} - B^{**})$  where  $A^{**} = -d \ln A / dt$  and  $B^{**} = -d \ln B / dt$ .
11. This bias and rate of technical change are expected bias and rate, not actual one and the comparative static analysis traces influences on expected magnitudes.
12. Assumptions (7) and (8a) assure that the benefits from research (second and third term in (15)) are monotonically increasing functions of  $m$  and  $n$ , and that the second-order conditions for maximization hold. When (8b) replaces (8a), both monotonicity and the second-order conditions require that

$$\begin{aligned} |\alpha^m / \beta^m| &\geq c_L / c_K \\ |\beta^n / \alpha^n| &\geq c_K / c_L \end{aligned} \tag{16}$$

In each research line the absolute size of the factor saving effect must exceed the absolute size of the factor using effect by a fraction which depends on the relative share ratio. (16) is assumed to hold for the remainder of the paper when the substitution case (8b) is discussed.



13. This latter neglect is especially important when induced innovation is to be introduced into a many sector model as in Kennedy (1973), because research resources should also be allocated to sectors according to marginal benefits of research in each sector. Exogenously specifying Kennedy frontiers for each sector may optimize within-sector research resource allocation but not allocation among sectors.
14. At the micro level at which this model is developed initial efficiency is of course given. The relevant forces altering costs are factor prices and scale of output.
15. It is easy to introduce capital maintenance costs into the model by simply discounting them and adding them to  $C_K$ . Similarly for any other costs associated with labor in addition to wages.
- Also, if research takes time and delays the building of the plant by a number of years, lost output or the cost of continuing to operate with the old plant can be subtracted from  $V_0$ . The present value of the model with research has then to exceed the present value without research if research is to be undertaken at all.
16. Note that, if capital maintenance and operating costs were included in  $C_K$ ,  $r$  and  $T$  would influence research both through  $C_K$  and  $C_L$ . A method similar to the one used for  $Y$  then be needed to establish signs.
17. This assumption is not necessary, but the proof is not as simple otherwise.

18. Conslik (1969) has an interesting alternative to the IPF with which one could possibly address these questions. But in the model which he builds with it he again takes the decisions of how much resources allocated to research and how to allocate the research resources to increase augmented capital versus augmented labor as exogenous or depending on a mechanism for which he refuses to give an economic rationalization. Therefore, while his approach is interesting, it again represents a case of implicit theorizing.

## Abstract

A microeconomic approach to induced innovation, by Hans P. Binswanger.

Invention possibilities are reformulated using research processes which have a cost and different implications for rates and biases of technical change. In the comparative static model a firm has the choice to build a plant of existing design or to improve it by research. The firm maximizes present value over the lifetime of the plant. Research costs and present value of capital and labor costs influence research mix and rate and bias to technical change. Controversies in the literature of induced innovation are discussed in terms of the model. A rise in labor costs does not necessarily lead to a more labor saving bias.

Mathematical Supplement  
to  
"A Microeconomic Approach to Induced Innovation"

For Review Purposes only

The techniques used are all standard calculus. Up to equation (31) the details of the derivations are given in the paper. (31) is first differentiated totally and rearranged as follows:

$$(a) \quad -P^m dm - P^n dn = -dF + m dP^m + n dP^n$$

$$(b) \quad -P^m d\lambda + \mu_{mm} (C_K \alpha^m + C_L \beta^m) dm = -\mu_m \alpha^m dC_K - \mu_m \beta^m dC_L + (1 + \lambda) dP^m$$

$$(c) \quad -P^n d\lambda + \mu_{nn} (C_K \alpha^n + C_L \beta^n) dn = -\mu_n \alpha^n dC_K - \mu_n \beta^n dC_L + (1 + \lambda) dP^n$$

Now change the signs of all equations and set  $P^m = P^n = 1$  so that  $dP^m = d \ln P^m$  and  $dP^n = d \ln P^n$ . Then divide (b) and (c) by  $(1 + \lambda)$  and note that  $(C_K \alpha^m + C_L \beta^m)/(1 + \lambda) = 1/\mu_m$  and similarly for the bracketed expression in (c). This can be seen from 31. Also multiply all terms in  $dC_K$  by  $C_K/C_K$  and the terms in  $dC_L$  by  $C_L/C_L$ . This leaves us with

$$dm + dn = dF - m d \ln P^m - n d \ln P^n$$

$$P^m \frac{d\lambda}{1 + \lambda} - \frac{\mu_{mm}}{\mu_m} dm = \frac{C_K \mu_m \alpha^m}{1 + \lambda} d \ln C_K + \frac{C_L \mu_m \beta^m}{1 + \lambda} d \ln C_L - d \ln P^m$$

$$P^n \frac{d\lambda}{1 + \lambda} - \frac{\mu_{nn}}{\mu_n} dn = \frac{C_K \mu_n \alpha^n}{1 + \lambda} d \ln C_K + \frac{C_L \mu_n \beta^n}{1 + \lambda} d \ln C_L - d \ln P^n$$

This is the same thing than the matrix equation after (31) with the notation changed as in the definitions which follow that matrix equation. The matrix is then inverted to derive equation (32).

The procedure to go from (41) to (43) is the same one than to go from (31) to (32)