A Minimax Polynomial Approximation Objective Function Approach for Optimal Design of Power System Stabilizer by Embedding Particle Swarm Optimization

Bhanu Pratap Soni^{*1}, Akash Saxena², Vikas Gupta¹

¹Department of Electrical Engineering, Malaviya National Institute of Technology ²Swami Keshvanand Institute of Technology, Jaipur, India -302017 *Corresponding author, e-mail: er.bpsoni2011@gmail.com

Abstract

The paper presents a novel approach based on Minimax approximation and evolutionary tool Particle Swarm Optimization (PSO) to fabricate the parameters of Power System Stabilizers (PSSs) for multi machine power systems. The proposed approach employs PSO algorithm for find the setting of PSS parameters. The worth mentioning feature of this work is the formulation of objective function with the help of swing curves interpolation. A novel transformation known as minimax approximation is used for converting the objective into the polynomials of degree one, two and three. To construct the objective function based on interpolation second order sensitivity analysis is performed. The performance of the PSSs is tested under different topological changes, operating conditions and system configurations. Nonlinear simulation reveals that proposed PSSs are effectively deal with local and interarea modes of oscillations. PSS design obtained through lower order polynomial expression of objective function is able to deal with the oscillatory modes efficiently.

Keywords: automatic voltage regulator (AVR), minimax approximation, multimachine power system, particle swarm optimization (PSO), power system stabilizer (PSS).

Copyright © 2015 Institute of Advanced Engineering and Science. All rights reserved.

1. Introduction

In the modern power system high performance Automatic Voltage Regulators (AVRs) (high gain and fast response) are equipped to ensure transient stability. AVRs of high gain introduce negative damping in the system. To provide a cost effective control PSSs are employed with AVRs. The requirement for modern excitation system exists in the fact that PSS and AVR both are dynamically interlinked [1]. The purpose of the parameter design is to make the PSSs provide proper damping for power system oscillations. PSS parameter estimation problem is an optimization problem, where aim of the optimization process is to maximize the damping of the power network. It is quite empirical to state that there is a tradeoff between synchronizing torque provided by the AVR and damping torque provided by the PSS [2]. Researchers have experimented with the different type of objective functions[]. The PSS parameter estimation problem has been addressed with different optimization algorithms [3-11]. The major contribution reported in literature revolves around overall systems dynamic response i.e. overshoot and settling time, convergence characteristics of proposed methodology, solution quality, time elapsed and comparison of the methodology with conventional techniques [8-11]. The traditional objective function reported in literature considered damping ratios, damping frequencies and weighted combination of these to solve the tuning parameters of PSSs. The inferior modes are shifted to D shaped and fan shaped regions. Sheng kuan wang proposed a new scale which drifted eigenvalues in fan shaped mode with the tip at damping ratio [7].

In this paper, PSO algorithm [12] is employed to solve PSS parameter estimation. The realisa-tion of objective function in 1^{st} , 2^{nd} and 3^{rd} order polynomials are done with the help of sensitivities of derivatives and MATLAB curve fitting tool. For testing results, the proposed approach is applied on two test cases of multi machine power systems. Assertiveness of proposed methodology has been tested on different type of disturbances, loading conditions, and system configurations.

2. Problem Statement

2.1. Power System Model

Formulation of the power system can be concluded and understand by following set of equations.

$$\dot{x} = f(x, u) \tag{1}$$

Where x is the state variables, u is the vector of input variable. In the PSS the power system is usually linearized and operating equilibrium as the study of the small disturbance comes in small signal stability. Equation (1) can further be transformed as:

$$\dot{x} = A\Delta x + B\Delta u \tag{2}$$

If *n* is the total no of machines size of A will be $4n \times 4n$, Δx is $4n \times 1$ state vector, while Δu vector is $n_{pss} \times 1$.



Figure 1. Structure of Power System Stabilizer

2.2. PSS Structure

Conventional lag lead structure of PSS is shown in Figure 1. The structure is used in this work which has transfer function (3). Further the modern excitation system with AVR and PSS is shown in Figure 2.

$$U_{i} = K_{stabi} \frac{sT_{w}}{1+sT_{w}} \cdot \frac{1+sT_{1i}}{1+sT_{2}} \cdot \frac{1+sT_{3i}}{1+sT_{4}} U$$
(3)

The objective function J is to be minimized with the constraints.

$$J = \min \left\{ \begin{pmatrix} t_1^1 \\ 2 \end{pmatrix} \Delta w_1^2 + \Delta w_2^2 + \dots + \Delta w_{10}^2 \end{pmatrix} + \begin{pmatrix} t_2^2 \\ 2 \end{pmatrix} \Delta w_1^2 + \Delta w_2^2 + \dots + \Delta w_{10}^2 \end{pmatrix} + \dots \\ \dots + \begin{pmatrix} \sum_{2}^{t_k} \Delta w_1^2 + \Delta w_2^2 + \dots + \Delta w_{10}^2 \end{pmatrix} \right\}$$
(4)

Subject to:

The objective of the optimization is to find the set of variables K_{stabi} , T_{1i} , T_{3i} , for i = 1, 2, ..., n to achieve adequate damping in the system. Here n is the size of network or total number of alternators in system. It is assumed that all alternators have incorporated with PSSs. In this work washout time constant $T_w = 10s$ and $T_2 \& T_4$ are considered as 0.05s. The left over over parameters K_{stab} , T_1 and T_3 are assumed to be the modifiable parameters; hence the number of the parameters for 3 machines will be 9 and for 10 machines system will be 30.

2.3. Construction of Objective Function

The speed deviation based objective function is employed here for the PSS parameter estimation problem. The steps for constructing the objective function are as following.

- Step 1. Initialize the optimization process, read system data, select the contingencies/operating conditions 'm' and simulation time steps 'k'.
- Step 2. Initialize the PSS data
- Step 3. Simulate the system and store the values of speed deviations of the generators for different faults.
- Step 4. Apply stopping criterion, if not satisfied then go to step 2.

Step 5. End



Figure 2. Modern excitation system with AVR & PSS

2.4. Embedding Objective Function in Interpolating Polynomial

The idea is to approximate objective function f(x) by a polynomial p(x) that gives a uniform accurate description in an interval [a,b]. Here the [a,b] is the interval of application of the certain disturbance. Let the function f(x) is an approximate continuous function on an interval [a,b]. This function with is realized the set of polynomials of degree at most n and let k bounded function defined [a,b], Minimax approximation algorithm suggests that maximum error is minimized [4]. The objective is to find a function k(x) to minimize Equation (6).

$$J = \max_{a \le x \le h} f(x) - k(x) \tag{6}$$

A sensitivity analysis is depicted in Table 1 to Table 9 and indices like first and second derivatives are calculated to ensure the truthfulness of interpolation fit. However it is worth mention here that Chebyshev expansion polynomial of first kind can closely approximate Minimax polynomial [4]. The proposed work transforms the traditional objective function into three types of polynomials of degree 1^{st} , 2^{nd} and 3^{rd} order. The parameter tuning is done while optimizing each polynomial with the help of PSO.

2.6. Sensitivity Analysis

To formulate the objective function on the basis of speed deviation, the sensitivity analysis is required. It is interesting to know that how the PSS parameter can be interpolated for constructing the objective function. The objective of the sensitivity problem is to compute the derivative of the function. Suppose a function:

f(x) = G(u(x), x)

Sensitivity of the function with respect to parameter *x* is given by the following equation:

$$\frac{\partial f}{\partial x} = \frac{\partial u}{\partial x}(u(x), x)\frac{\partial u}{\partial x} + \frac{\partial G}{\partial x}u(x)$$

In this work finite method of difference [4] is used to calculate the derivatives of the various parameters of PSS. The analysis is shown in the Table 1 to Table 9. The operating points between the simulation time steps are selected and the sensitivity of the speed deviation of all 3 generators are calculated. The values of different first order and second order sensitivities are shown in Table 1 to 9. Following points are worth mention here:

- a) For the high values of simulation time step i.e. when fault is generated after a long wait than the second derivative of generator 3 for T_1 is less sensitive. It is also observed that for lower values of the simulation time step the second order derivatives of generator 3 are very high. For T_3 parameter the second order derivative of generator 2 is most sensitive. In fact it achieves highest value. The same analogy is followed when the simulation step is delayed and the second order derivative attains higher values for generator 2.
- b) Table 1 to Table 9 shows the absolute values of sensitivities for K_{stab} . T_1 and T_3 . The sensitivity analysis is used to obtain the weights for combination of the effect of the PSS parameters and forming the objective functions linear, polynomial 2 and cubic polynomials.

Table 1. Interpolation Fit for T_1 of Generator 1 (3 Machine System)

x _i	0.6	3.54	6.48	9.42	12.36	15.3	18.2	21.1	24.1	27	30
$f(x_i)$	12.6727	-22.530817	' -320.65	-1166.1	-2843.2	-5636.4	-9830.1	-15708	-23556	-33658	-46297
$d'\!f(x_i)$	0.497	-40.567	-178.362	-412.888	-744.145	-1172.13	-1696.85	-2318.3	-3036.48	-3851.39	-4763.03
$d''\!f(x_i)$	2.48325	-30.418	-63.319	-96.221	-129.12	-162.02	-194.92	-227.82	-260.72	-293.63	-326.53

Table 2. Interpolation Fit for T_3 of Generator 1 (3 Machine System)

x _i	0.6	3.54	6.48	9.42	12.36	15.3	18.2	21.18	24.12	27.06	30
$f(x_i)$	12.260	3266.94	412028.	196771.	.8215973.	631109	.453655	85086.	2126879	180509	247451
$d'\!f(x_i)$	-2.268	259.18	331022.	612288.	.014055.3	96324.7	49096.0	612369.	416144.0	620421.9	925201.1
$d'' f(x_i)$	3.5594	174.29	99345.0	39515.7	78686.51	8857.25	71028	1198.7	41369.48	31540.22	21710.96

Table 3. Interpolation Fit for $K_{\rm stab}$ of Generator 1 (3 Machine System)

x _i	0.6	3.54	6.48	9.42	12.36	15.3	18.24	21.18	24.12	27.06	30
$f(x_i)$	13.2753	313.5417	13.713	13.8083	313.8464	13.8465	13.8275	13.808	313.808 ⁻	113.845	713.9402
$d'\!f(x_i)$	0.108	0.073369	90.044	0.0216	0.00542	2-0.004303	3-0.00757	-0.0043	0.0052	30.0214	0.043
$d'' f(x_i)$	-0.0131	-0.011	-0.0088	3-0.0066	-0.0044	-0.0022	-1.10E-0	50.0021	0.00438	30.00658	30.00877

Table 4. Interpolation Fit for T_1 of Generator 2 (3 Machine System)

<i>x</i> _i	0.6	3.5	6.4	9.4	12	15	18	21.1	24.1	27.06	30
$f(x_i)$	12.28	9.713	-26.72	-154.1	-429.7	-910.7	-1654	-2717	-4156.68	3-6030.2	3-8394.8
$d'\!f(x_i)$	-1.5938	-3.3955	-24.633	-65.309	-125.42	-204.96	-303.95	-422.36	-560.23	-717.52	9-894.26
$d'' f(x_i)$	2.69273	3-3.91839	9-10.5295	5-17.1406	6-23.7517	-30.3628	8-36.9739	9-43.585′	1-50.1962	2-56.807	3-63.4184

Table 5. Interpolation Fit for T_3 of Generator 2 (3 Machine System)

x _i	0.6	3.54	6.48	9.42	12.3	15.3	18.2	21.1	24.1	27	30
$f(x_i)$	17.271	719721.	2140499	455544	1000000)240000	0350000)550000	0820000	01600000	019000000
$d'\!f(x_i)$	78.8527	718608.	368836.4	4150763	264389	409713	586735	795456	140000	01300000	1600000
$d'' f(x_i)$	911.61	11693.	522475.3	333257.2	244039	54820.9	65602.7	76384.6	87166.4	97948.3	108730

Table 6. Interpolation Fit for K_{stab} of Generator 2 (3 Machine System)

x _i	0.6	3.54	6.48	9.42	12.3	15.3	18.2	21.1	24.1	27	30
$f(x_i)$	13.7935	13.7092	13.6415	13.5888	13.5495	13.5221	13.505	113.496	913.49	513.500	613.5095
$d'\!f(x_i)$	-0.0316	-0.0257	-0.0203	-0.0155	-0.0112	-0.0074	-0.0042	2 -0.0014	4 0.000	70.0024	0.00355
$d'' f(x_i)$	0.002092	20.00191;	30.00173 <i>′</i>	10.00155	50.001376	60.00119	71.02E-3	30.0008	30.000	60.0004	80.00030

Table 7. Interpolation Fit for T_1 of Generator 3 (3 Machine System)

x _i	0.6	3.54	6.48	9.42	12.36	15.3	18.24	21.18	24.1	27	30
$f(x_i)$	12.9689	-21.5696	6-415.283	8-1596.16	6-3992.19	9-8031.37	7-14141.7	7-22751.1	1-34287.7	7-49179.3	3-67854
$d'\!f(x_i)$	0.811412	2-48.5694	4-243.525	5-584.055	5-1070.16	6-1701.84	-2479.1	-3401.92	2-4470.33	3-5684.3 <i>′</i>	1-7043.86
$d'' f(x_i)$	7.96141	-41.5538	3-91.069	-140.584	1-190.099	9-239.615	5-289.13	-338.645	5-388.16	-437.676	6-487.191

Table 8. Interpolation Fit for T_3 of Generator 3 (3 Machine System)

x _i	0.6	3.54	6.48	9.42	12.36	15.3	18.24	21.18	24.12	27.06	30
$f(x_i)$	13.7248	-74.4346	693.471	1-2366.1	-5615.03	3-10963	-18932.	7-30046.8	3-44828.	1-63799.:	3-87483.1
$d'\!f(x_i)$	1.03427	-90.639	-360.106	6-807.368	3-1432.42	2-2235.27	7-3215.92	2-4374.3	6-5710.5	9-7224.6	1-8916.43
$d'' f(x_i)$	-0.94429	9-61.4185	5-121.893	3-182.367	7-242.841	1-303.315	5-363.78	9-424.264	4-484.73	8-545.21	2-605.686

Table 9. Interpolation Fit for K_{stab} of Generator 3 (3 Machine System)

x _i	0.6	3.54	6.48	9.42	12.36	15.3	18.24	21.18	24.12	27.06	30
$f(x_i)$	15.2778	14.9018	14.5859	14.3247	14.1131	13.946	13.8181	13.7243	13.6593	13.618	13.5951
$d'\!f(x_i)$	-0.13867	7-0.11738	-0.09785	-0.08011	-0.06410	-0.04987	-0.0374	-0.02671	-0.01778	-0.0106	-0.00521
$d'' f(x_i)$	0.00754	0.006940	0.006340	0.005740	0.005139	90.004539	3.94E-3	0.003338	30.002738	30.00213	30.001537

3. Case 1: Three Machine Power System

The case taken over here to understand the response of different polynomials is the 3 machine 9-bus system [13], the minute observations on the system shows that without installing PSSs on generating machine, system get unstable for various perturbations.

	Та	ble 10). Gen	erating		Table 11. Loading Conditions										
Base Case		Case-A		Case-B		Case-C			Base Case		Case-A		Case-B		Case-C	
Р	Q	Р	Q	Р	Q	Р	Q		Ρ	Q	Р	Q	Р	Q	Ρ	Q
0.716	0.270	1.527	0.249	1.3283	0.2393	0.5077	0.3029	-	1.25	0.50	0.75	0.39	1.50	0.90	0.65	0.55
1.63	0.0665	1.00	-0.003	1.00	-0.006	1.85	0.1134		0.90	0.30	0.90	0.30	1.20	0.80	0.45	0.35
0.85	-0.108	0.65	-0.117	0.850	-0.12	0.85	-0.094		1.00	0.35	1.00	0.35	1.00	0.50	0.50	0.25

A Minimax Polynomial Approximation Objective Function Approach for... (Bhanu Pratap Soni)

5

Table 10 and Table 11 give different operating conditions related with generation as well as load side. As stated in literature these operating conditions are considered as hard as far as system stability is concerned [7].

System response is judged with load patterns and following perturbations:

- a) A 6-cycle fault disturbance at bus 6 at the end of line 5-6 with Case A, Case B. The fault has been cleared without tripping
- b) A 6-cycle fault is cleared by tripping the line 5-6 with Case C.



Figure 4. Speed Deviation of Generator 2

Figure 5. Speed Deviation of Generator 1



Figure 6. Speed Deviation of Generator 3

4. Case 2: New England Power System

In this case, the 10 machine 39 bus system [14] is considered. The system is comparatively larger than 3 generator system and dynamic as interarea oscillation is considered.

4.1. Test System

The system is tested over different perturbations and configurations which is extremely hard for system stability [12]

4.2. PSS Design

To design the proposed PSS by using minimax approximation interpolation polynomials, three different operating conditions and critical line outages are considered which are the enormously rigid from the stability point of view. They can be considered as:

- a) Base Case; No outage of line
- b) Case A; outage of line 22-23
- c) Case B; outage of line 1-39

Speed deviation curve of generator 9 is shown to demonstrate the effectiveness of the proposed PSSs as it is the nearest with the fault location (line 14-15), another speed deviation curve of generator 3 is shown as the generator location is also a key derivative considering the above given conditions.





Figure 7. Speed Deviation of Generator 9

Figure 8. Speed Deviation of Generator 3



Figure 9. Speed Deviation of Generator 9

4.3. Discussions

1) It is observed from the speed deviation curves of different generators from Figure (3) to Figure (6) that PSS designed through linear polynomial objective function gives best solution as far as the overall system stability is concerned. However, it is worth mention here that system gets unstable while using the PSSs parameters obtained from either polynomial 2^{nd} order or polynomial 3^{rd} order.

2) On New England System, the critical operating condition (Case B) reveals the efficacy of the proposed linear PSS prominently. PSS designed from other than linear polynomial fit show a poor dynamic response in this operating condition.

3) While observing the speed deviations related with the polynomial fit of order 1^{st} , 2^{nd} and 3^{rd} ; it is observed that the maximum error term is minimized with using linear i.e. 1^{st} order fit.

5. Conclusion

Work presented in this paper is to transform the traditional objective function into minimax polynomials. Table 1 to 9 shows various interpolation statistics while optimizing the PSSs parameter i.e. time constants and PSSs Gains sensitivity with respect to the objective function. Higher values of 2^{nd} derivative shows that it presents a poor fit to the objective function; however values for 2^{nd} derivatives is zero in case of linear polynomials. The technique used for optimization is PSO. The response obtained under different operating conditions shows that linear fit is the most suitable fit for obtaining the PSSs parameters. By using linear objective function the small signal stability can be enhanced.

References

[1] Dudgeon GJW, et al. The Effective role of AVR and PSS in power systems: frequency Re-sponse Analysis. *IEEE Transaction on Power System.* 2007; 22(4): 1986-1994.

- [2] F P deMello, C Concordia. Concepts of Synchronous Machine Stability as Affected by Excita-tion Control. IEEE Trans. 1969; 88: 316-329.
- [3] Akash Saxena, Bhanu Pratap Soni, Vikas Gupta. A Chronological Review and Comparison of Four Evolutionary Based Algorithms. *European Journal of Advances in Engineering and Technology*. 2015; 2(1): 35-41.
- [4] Akash Saxena, Bhanu Pratap Soni, Vikas Gupta. *Minimax Approximation Synthesis in PSS Design by Embedding Gravitational Search Algorithm*. In 2014 IEEE International Conference on Advanced Communication Control and Computing Technologies (ICACCCT). 2014: 929-934.
- [5] Baibhav Biishal, Bhanu Pratap Soni, Akash Saxena, Vikas Gupta. Design of Damping Controller for Multimachine Power System by using Simplified Swarm Optimization. In 2014 IEEE Inter-national Conference on Computational Intelligence and Computing Research. 2014: 493-498.
- [6] MA Abido. Robust design of multi machine power system stabilizers using simulated anneal-ing. *IEEE Trans. Energy Conversion Power System.* 2000; 15(3): 297-304.
- [7] Sheng-KuanWang. A Novel Objective function and algorithm for optimal PSS parameter De-sign in a Multi Machine Power system. *IEEE Trans. Power Syst.* 2013; 28(1): 522-531.
- [8] YL Abdel-Magid, MA Abido. Optimal multiobjective design of robust power system sta-bilizers using genetic algorithms. *IEEE Trans. Power System.* 2003; 18(3):1125-1132.
- [9] M. A. Abido and Y. L. Abdel-Magid. Robust design of multi machine power system sta-bilisers using tabu search algorithm, Proc. Inst.Elect. Eng., Gen., Transmission, Distrib.. 2000; 147(6):387-394.
- [10] MA Abido. Optimal design of power-system stabilizers using particle swarm optimization. IEEE Trans. Energy Convers. 2002; 17(3): 406-413.
- [11] MA Abido, YL Abdel-Magid. Optimal design of power system stabilizers using evolutionary programming. *IEEE Trans. Energy Convers.* 2002; 17(4): 429-436.
- [12] Kennedy J, Eberhart RC. Particle swarm optimization. In Proc. IEEE int'l conf. on neural networks, IEEE service center. Piscataway, NJ. 1995; 4: 1942-1948.
- [13] PM Anderson, AA Fouad. Power System Control and Stability. Ames, IA, USA: Iowa State Univ. Press. 1977.
- [14] MA Pai. Energy Function Analysis for Power System Stability. Kluwer Academic Publishers. 1989.
- [15] A Bazanella, A Fischman, A Silva, J Dion, L Dugrad. *Coordinated robust controllers in power systems.* In IEEE Stockholm Power Tech Conference. 1995: 256-261.
- [16] Vivek Prakash, Bhanu Pratap Soni, Akash Saxena, Vikas Gupta. Comparative Study of Optimization Algorithms for Enhancement of Small Signal Stability by Designing PSS for Multi-machine Power System. In IEEE International Conference on Energy, Economics & Environment. 2015.
- [17] Cherif, Nacim, Allaoui Tayeb, Benasla Mokhtar. The Use of Multiband PSS to Improve Transient Stability of Multimachine Power System. *International Journal of Power Electronics and Drive Systems* (*IJPEDS*). 2013; 3(3): 298-303.
- [18] Xu Lin. Coordinated Control of SVC and PSS for Transient Stability Enhancement of Multi-Machine Power System. TELKOMNIKA Indonesian Journal of Electrical Engineering. 2012; 11(2): 1054-1062.