A mixed 0-1 integer programming for inventory model

A case study of TFT-LCD manufacturing company in Taiwan

Amy H.I. Lee

*Department of Industrial Engineering and System Management,
Chung Hua University, Hsinchu, Taiwan, and*

He-Yau Kang

*Department of Industrial Engineering and Management,
National Chin-Yi University of Technology, Taiping, Taiwan*

Abstract

Purpose – This paper seeks to construct a model for inventory management for multiple periods. The model considers not only the usual parameters, but also price quantity discount, storage and batch size constraints.

Design/methodology/approach – Mixed 0-1 integer programming is applied to solve the multi-period inventory problem and to determine an appropriate inventory level for each period. The total cost of materials in the system is minimized and the optimal purchase amount in each period is determined.

Findings – The proposed model is applied in colour filter inventory management in thin film transistor-liquid crystal display (TFT-LCD) manufacturing because colour filter replenishment has the characteristics of price quantity discount, large product size, batch-sized purchase and forbidden shortage in the plant. Sensitivity analysis of major parameters of the model is also performed to depict the effects of these parameters on the solutions.

Practical implications – The proposed model can be tailored and applied to other inventory management problems.

Originality/value – Although many mathematical models are available for inventory management, this study considers some special characteristics that might be present in real practice. TFT-LCD manufacturing is one of the most prosperous industries in Taiwan, and colour-filter inventory management is essential for TFT-LCD manufacturers for achieving competitive edge. The proposed model in this study can be applied to fulfil the goal.

Keywords Cybernetics, Inventory based ordering systems, Batch size, Discounts

Paper type Research paper

1. Introduction

A large number of mathematical models have been developed for inventory management, such as linear programming, nonlinear programming, dynamic programming, geometric programming, gradient-based nonlinear programming and fuzzy geometric programming. Mixed integer programming has also been adopted to

solve the inventory problem, and some recent researches are reviewed here. Kazan *et al.*
(2000) formulated a mixed integer linear programming model to identify less nervous
production schedules in a rolling horizon basis. In a production system that is not
flexible to changes in pre-determined production volume, the proposed model is
preferable to generate new schedules. Hsieh (2001) presented a 0-1 linear programming
approach for minimizing total average cycle stock with the constraints of a limited total
number of replenishments per unit time and a restricted set of possible intervals. The
relaxation of the 0-1 problem generated a more efficient and effective lower bound, and
the solution was used to propose a simple and efficient heuristic. Chang and Chang
(2001) developed a mixed integer optimization model for solving the inventory problem
with variable lead time, crashing cost and price-quantity discount, and derived a linear
programming relaxation based on piecewise linearization techniques. Tarim and
Kingsman (2004) proposed a mixed integer programming formulation to solve the
stochastic dynamic production/inventory lot-sizing problem with service-level
constraints. The optimal solution could minimize the total expected inventory
holding, ordering and direct item costs during the planning horizon. Tang *et al.* (2005)
developed a nonlinear 0-1 mixed integer programming model and a Lagrange relaxation
decomposition method for synchronized production and transportation planning in a
production distribution network. A synchronized schedule of production and
transportation could be determined, and the minimum total costs over the planning
horizon could be achieved. Wang and Sarker (2005) modeled an assembly – type supply
chain system as a mixed integer nonlinear programming problem. Branch-and-bound
method was used to solve small size problems, while a heuristic was developed to divide
a large size problem into several small size problems and each small problem was solved
individually. Wang and Sarker (2006) further developed a mixed integer nonlinear
programming model to solve a multi-stage supply chain system that operates under a
just-in-time delivery policy. Tarim and Kingsman (2006) further studied the single-item,
non-stationary stochastic demand inventory control problem under the non-stationary
(R, S) policy. The nonlinear cost function of the stochastic lot-sizing problem was first
solved by a piecewise linear approximation, and a certainty equivalent mixed integer
linear programming model was developed next for computing policy parameters. Chang
(2006) proposed an exact acquisition policy using mixed integer optimization
approaches for solving the single-item multi-supplier problems. Based on Chang and
Chang (2001), which did not consider the single item multi-supplier situations, Chang
et al. (2006) further developed a mixed integer approach for solving the single item
multi-supplier problem with variable lead-time, price quantity discount and resource
constraints. Kang (2006) developed a dynamic programming model and a mixed 0-1
linear programming model to solve a control wafers replenishment problem with
inventory deterioration. da Silva *et al.* (2006) introduced a multiple criteria mixed integer
linear programming model to solve the aggregate production planning problem with
three performance criteria: maximizing profit, minimizing late orders and minimizing
work force level changes. A decision support system based on the model was further
developed to facilitate the application in real practice.

Quantity discounts is an important issue in inventory management. With quantity
discounts, the purchase price from the suppliers is reduced if a large order is placed.
Usually, there are two major types of quantity discounts: all-units discount and
incremental discount (Cha and Moon, 2005). In the all-units discount, the discounted

price is applied to all units beginning with the first unit, if the quantity purchased belongs to a specified quantity level predetermined by the supplier. There are often a number of price breaks, and the unit discounted price decreases as the quantity level increases. In the incremental discount, the discounted price is only applied to those units inside the price break quantity. Therefore, different prices are applied to the units belonging to different price breaks. Quantity discounts have been considered in many inventory models, and some recent researches are Chang and Chang (2001), Papachristos and Skouri (2003), Yang (2004), Chen and Chen (2005), Wang (2005), Chang (2006), Chang *et al.* (2006) and Li and Liu (2006).

In previous researches of inventory problem, storage space and batch size were not often considered even though they are important issues that should not be ignored. Kanyalkar and Adil (2005) and Mandal *et al.* (2006) constructed inventory models with the consideration of constrained storage space. If the size of a piece of material is relatively large and storage capacity in a plant is limited, a limited quantity of materials can be stored in the plant. In addition, it is possible that the materials can only be purchased in a multiple of a fixed-sized batch; that is, a split batch is not allowed. For example, if the batch size is 100, the number of materials purchased each time must be an integer multiple of 100, e.g. 100, 200, and 300, etc. With these special characteristics, we will formulate the inventory replenishment problem with the constraints of quantity discounts, space and batch size by a mixed 0-1 integer programming model.

Because it is easier to solve a linear problem, the following proposition is required to transform a nonlinear integer problem into a mix integer problem (Chang, 2006).

Proposition 1. Assume that the Q_i is the purchase quantity, and B is the batch size. Q_i is an integer multiple of B, and the calculation is as follows:

$$\text{Min } Q_i = B \times F_i \tag{1}$$

$$\text{s.t. } F_i = \sum_{v=1}^{V} 2^{v-1} y_v \tag{2}$$

where, F_i is an integer variable, and y_v is a 0-1 variables.

Proof. The objective function in equation (1) can be represented as $B \times (y_0 + 2y_1 + 4y_2 + \cdots)$, where B is an integer variable and y_v ($v = 1, 2, \ldots V$) is a 0-1 variable. The equation is linearized by referring to Chang (2006). The linearization strategy can be examined by the term $B \times y_0$ as an example. The term $B \times y_0$ is linearized by the following inequalities constraints:

    (i) $(y_0 - 1)M + B \leq \tau \leq B,$

    (ii) $0 \leq \tau \leq y_0 M,$

where, M is a big value, τ is a continuous variable, y_0 is a binary variable, and B is an integer variable. The two cases are as follows:

    (1) If $y_0 = 0$, then $-M + B \leq \tau \leq B$ from (i), $0 \leq \tau \leq 0$ from (ii). Thus, $\tau = 0$.

    (2) If $y_0 = 1$, then $B \leq \tau \leq B$ from (i), $0 \leq \tau \leq M$ from (ii). Thus, $\tau = B$.

As a result, the inequalities constraints (i) and (ii) can ensure Q_i be an integer multiple of B.

The purpose of this research is to construct a mixed 0-1 integer programming model for inventory management under the production control of a pulling system and the consideration of quantity discount, storage and batch size constraints. The remaining of this paper is organized as follows. Section 2 describes the problem under consideration and the assumptions. Section 3 is the construction of the algorithms. In Section 4, a case study of colour filter inventory management in thin film transistor-liquid crystal display (TFT-LCD) manufacturing is presented, and sensitivity analysis of major parameters of the model is performed to depict the effects of these parameters on the solutions. Some conclusion remarks are made in the last section.

2. Problem description and assumptions

In order to simplify the complexity of the environment, we shall restrict the investigation with the following assumptions:

- The plant is make-to-stock (MTS), the demand rate for the material is reasonably constant in a period. However, it can be different in different periods.
- Each period can only place at most one order.
- The replenishment lead time is of known duration, and the entire order quantity is delivered at the same time in the beginning of a period.
- The order quantity must be a multiple of a fixed-sized batch. No split-batch is allowed. Shortages are not allowed.
- The price of each unit is dependent on the order quantity. All-units discount schedule is considered.
- Storage space is limited.
- The inventory holding cost for each unit is known and constant, independent of the price of each unit.
- Planning horizon is finite and known. In the planning horizon, there are n periods, and the duration of each period is the same.
- The initial inventory level (X_1) is zero.

All the required notations in this paper are defined as below.

2.1 Indices

i = planning period ($i = 1, 2, \ldots, n$).

k = price break ($k = 1, 2, \ldots, \kappa$).

v_i = integer number for calculating number of batches in period i ($v_i = 1, 2, \ldots, V_i$).

2.2 Parameters

B = batch size of materials, and the order quantity must be an integer multiple of B.

D_i = demand at period i.

H = inventory holding cost, per unit per period.

$I(t_{i-1})$ = beginning usable inventory level in period i at time t_{i-1}, and $I(t_{i-1}) = X_i + Z_i \times Q_i$.

M = a large number.

O = ordering cost per replenishment.

P = unit purchase cost.

p_k = unit purchase cost with price break k.

q_k = the upper bound quantity of price break k.

S = storage space (volume) available at the plant.

U_{ik} = a binary variable, set equal to 1 if materials are purchased with price break k in period i, and 0 if no purchase is made with price break k in period i.

y_{v_i} = a binary variable for calculating the number of batches in period i.

2.3 Decision variables

$P(Q_i)$ = purchase cost for one unit based on the discount schedule with the order quantity Q_i.

Q_i = purchase quantity in period i.

TC = total cost of materials in a planning horizon.

X_i = beginning inventory level in period i.

Z_i = a binary variable, set equal to 1 if a purchase is made in period i, and 0 if no purchase is made in period i.

Figure 1 is the graphical representation of multi-period inventory system. The beginning inventory level in period i (X_i) is equal to the beginning inventory level in period $i-1$ (X_{i-1}) plus the purchase amount in period $i-1$ ($Z_{i-1} \times Q_{i-1}$) and minus
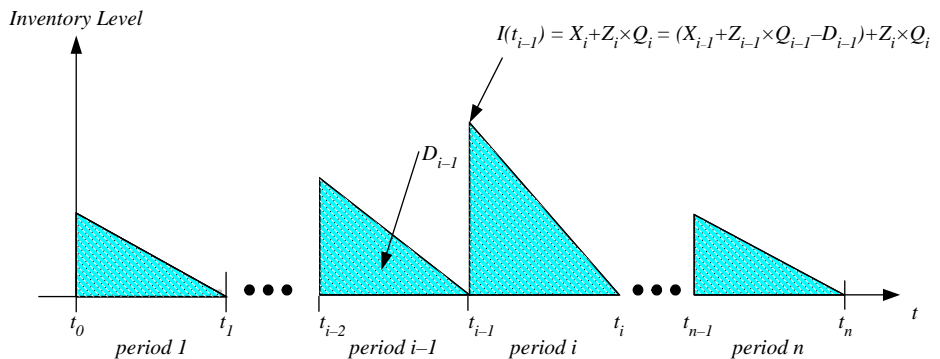

Figure 1.
Graphical representation
of inventory system

the demand in period $i - 1$ (D_{i-1}). The beginning usable amount in period i ($I(t_{i-1})$) is equal to the beginning inventory level in period i (X_i) plus the purchase amount in period i ($Z_i \times Q_i$), where Z_i represents whether a purchase is made in period i (1 if a purchase is made, and 0 if no purchase is made).

The objective of the proposed model is to minimize the total cost of materials in the system and to determine the optimal inventory level in each period. The objective function is to minimize the total cost, which includes ordering cost, holding cost and purchase cost in a planning horizon, and it is:

Total cost = total ordering cost + total holding cost + total purchase cost (3)

Equation (4) calculates the total ordering cost for the system, where O is the ordering cost per time and Z_i represents whether a purchase is made in period i (1 if a purchase is made, and 0 if no purchase is made).

$$\text{Total ordering cost} = O \times \sum_{i=1}^{n} Z_i \tag{4}$$

The beginning inventory in a period is equal to the beginning inventory level in the previous period plus the purchase quantity in the previous period minus the demand in the previous period as shown in equation (5). The holding cost for period i is equal to the holding cost of the inventory demanded for period i ($H/2 \times D_i$) plus the holding cost of the beginning inventory level in period $i + 1$ for period i. The total holding cost is the summation of the holding cost for each period, as in equation (6):

Beginning inventory in this period = beginning inventory in previous period

$$+ \text{purchase quantity in the previous period}$$

$$- \text{demand in the previous period}$$

(5)

$$\text{Total holding cost} = \sum_{i=1}^{n} \left(\frac{H}{2} \times D_i + H \times X_{i+1} \right) \tag{6}$$

The total purchase cost is obtained by equation (7), where $P(Q_i)$ is the unit purchase cost based on the discount schedule with the order quantity Q_i.

$$\text{Total purchase cost} = \sum_{i=1}^{n} (P(Q_i) \times Q_i \times Z_i) \tag{7}$$

3. Mixed 0-1 integer programming model for colour filter inventory problem in TFT-LCD manufacturing

In this section, we propose a mixed 0-1 integer programming model to solve the multi-period inventory problem and to determine an appropriate inventory level for each period. In this paper, we assume that a production planner's objective is to minimize the total cost of materials in the system and to determine the optimal purchase amount in each period. In each period, sufficient materials must be supplied for use in time, and shortages are not allowed.

The mixed 0-1 integer programming model can be formulated as follows:

$$\text{Minimize TC} = \sum_{i=1}^{n} \left(O \times Z_i + \frac{H}{2} \times D_i + H \times X_{i+1} + P(Q_i) \times Q_i \times Z_i \right) \quad (8)$$

$$\text{Subject to } X_{i+1} = X_i + Z_i \times Q_i - D_i, \quad i = 1, 2, \ldots, n \quad (9)$$

$$X_i + Z_i \times Q_i \leq S, \quad i = 1, 2, \ldots, n \quad (10)$$

$$Q_i = B \times \sum_{v_i=1}^{V_i} 2^{v_i - 1} y_{v_i}, \quad i = 1, 2, \ldots, n \quad (11)$$

$$q_{k-1} + M \times (U_{ik} - 1) \leq Q_i < q_k + M \times (1 - U_{ik}),$$
$$i = 1, 2, \ldots, n \text{ and } k = 1, 2, \ldots, \kappa \quad (12)$$

$$\sum_{k=1}^{\kappa} U_{ik} = 1, \quad i = 1, 2, \ldots, n \quad (13)$$

$$P(Q_i) = \sum_{k=1}^{\kappa} p_k \times U_{ik}, \quad i = 1, 2, \ldots, n \quad (14)$$

$$Z_i \in \{0, 1\}, \quad i = 1, 2, \ldots, n \quad (15)$$

$$y_{v_i} \in \{0, 1\}, \quad v_i = 1, 2, \ldots, V_i \quad (16)$$

$$U_{ik} \in \{0, 1\}, \quad i = 1, 2, \ldots, n \text{ and } k = 1, 2, \ldots, \kappa \quad (17)$$

$$\text{and all variables are nonnegative.} \quad (18)$$

The objective function, equation (8), is to minimize the total cost, which includes the ordering cost, holding cost and purchase cost in a planning horizon. These costs are explained before in equations (4), (6) and (7). The operative constraints are as follows. In constraint (9), the beginning inventory of a period, X_{i+1}, is equal to the beginning inventory level in the previous period, X_i, plus the purchase quantity in the previous period, $Z_i \times Q_i$, minus the demand in the previous period, D_i. Constraint (10) ensures that the beginning usable inventory must be less than or equal to the storage space S. Constraint (11) sets the purchase quantity to be an integer multiple of batch size B. Constraint (12) sets the purchase quantity between a lower bound quantity q_{k-1} and an upper bound quantity q_k in a price break k, where M is a large number. Constraint (13) makes sure that materials can only be purchased with one single price break k in each period i. Constraint (14) determines the purchase cost per unit, $P(Q_i)$, under the discount schedule based on the total quantity purchased in period i. Constraint (15) lets

a purchase is either made of not made in each period. Constraint (16) is a binary variable for calculating the number of batches in period i. Constraint (17) is a binary variable for determining the price break k applied to the purchase quantity in period i.

Special cases will be studied for the model, with combinations of limited or unlimited storage space, different batch sizes, regular or discounted purchase costs.

4. Numerical example

In order to illustrate the effectiveness of the proposed mixed 0-1 integer programming model, a case study of colour filter inventory management in a TFT-LCD manufacturer is presented. The software LINGO is used to implement the model.

4.1 TFT-LCD manufacturing and colour filter inventory management

The progress in high technology has led to a widely use of TFT-LCD. Because of their low weight, slender profile, low-power consumption, high resolution, high brightness and low-radiance advantages, TFT-LCDs have been used in a wide range from portable appliances to notebook and desktop monitors and even to large screen digital televisions. The size of TFT-LCD keeps increasing as the manufacturing technology of TFT-LCD evolves, and a larger TFT-LCD allows a larger display application and an improved productivity. However, as the size of TFT-LCD increases, the size of TFT-array substrates and colour filter substrates has to increase simultaneously, leading to a more complicated inventory problem of large-sized substrates.

There are five major processes in the manufacturing of TFT-LCD: TFT array fabrication, colour filter (BM) fabrication, colour filter (RGB) fabrication, cell assembly and module assembly. The TFT array fabrication process is very similar to that used to fabricate semi-conductor devices. The steps, including cleaning, deposition of thin films, photolithography, and wet and dry etching of the thin films, are all very similar. The glass substrate must be processed five to seven times through cleaning, inspection, film deposition, resist coating, exposure, developing, etching and resist strip (Lin *et al.*, 2004).

A LCD panel consists of an array substrate and a colour filter substrate joined together. While array substrates are manufactured in house, the majority of colour filter substrates must be acquired from colour filter manufacturers. There are two colour filter fabrication processes in a colour filter plant: colour filter (BM) fabrication and colour filter (RGB) fabrication.

In the cell assembly process, TFT-array substrates and colour filters are assembled. In the process, an LCD panel is made by assembling the two substrates together and filling the space between them with liquid crystal. The assembled substrates are scribed using a cutting wheel and separated into individual cells. Finally, each cell will pass through grinding, lamination and test, and the final product is called a LCD panel.

The last step in the production of TFT-LCD panels is the module assembly process, where TFT-LCD panels are finished by connecting additional components, such as backlight units, light polarizing films, driver ICs, and product cases. After the module production process, TFT-LCD panels can be sold to downstream manufacturers, where they are installed in LCD TVs, monitors, or notebook computers, etc.

The manufacturing cycle time of TFT-LCD in TFT-LCD manufacturers' part is about 9-13 days, which include 5-7 days for array process, 3-5 days for cell assembly process and one day for module assembly process (Wu *et al.*, 2006). Because the first two processes, the front-end processes, are highly automated, comprise the major

portion, e.g. 90 per cent, of the total investment, and have the longest cycle times, a good production planning and equipment utilization of the two processes are essential to reduce the cycle time, increase the throughput, and strengthen the competitive edge of the companies. As a result, TFT-LCD manufacturers usually adopt the MTS strategy for the first two processes and the assembly-to-order strategy for the module assembly process (Lin *et al.*, 2004).

Colour filter inventory management is very important in the fabrication of TFT-LCD panels. Colour filters substrates are one of the most expensive raw materials and are usually purchased from colour filter manufacturers. Thus, sufficient amount of colour filters must be available in the plant to maintain a smooth production flow. As the generation of TFT-LCD increases, the size and the unit cost of colour filters increases, and the storage of these large-sized colour filters becomes more difficult. To summarize, in order to reduce cost and to ensure product availability, the inventory management of colour filters is especially important in TFT-LCD manufacturing.

4.2 Basic input information

Actual data are taken from an anonymous TFT-LCD manufacturer located on the Science-Based Industrial Park in Hsinchu, Taiwan. The manufacturer has different plants for TFT array fabrication, cell assembly and module assembly. After the TFT array fabrication, TFT-array substrates are moved to the cell plant to assemble with colour filter substrates, which are purchased from a colour filter manufacturer. Therefore, adequate number of colour filters must be purchased and stored in the plant. The objective of the model is to minimize the total cost of colour filters in the system and to determine the optimal purchase amount of colour filters in each period.

Based on an interview with the management of the TFT-LCD fab, we define each planning horizon to be ten days and each period to be one day. Thus, each planning horizon contains ten periods. In addition, we set ordering cost per replenishment (O) to be \$120 and unit holding cost per period (H), which includes the handling cost, storage cost and capital cost, to be \$0.1. Table I shows the demand (D_i) at each period i. Table II shows the discount schedule under different purchased quantity. For instance, if the purchased quantity in a period is between 2,001 and 3,000 units, the price for each unit, starting from the first unit, is \$39.

Eight special cases are examined here, as shown in Table III. Each case may be varied in its storage space (limited or unlimited), batch size ($B = 1,100$ or 1,000) and purchase price (regular or discounted). For example, in Case 1, the maximum storage space is

Table I.
Demand of each period in a planning horizon

i	1	2	3	4	5	6	7	8	9	10
D_i	610	350	410	1,080	1,778	661	1,524	1,025	336	234

Table II.
Discount schedule

Price break (k)	Purchased quantity (Q)	Price (P)
1	0-1,000	40
2	1,001-2,000	39.5
3	2,001-3,000	39
4	3,001 or more	38.8

Case	Storage space (S)	Batch size (B)	Unit purchase cost (P)
1	3,000	1,000	Fixed at $40
2	M	1,000	Fixed at $40
3	M	100	Fixed at $40
4	M	1	Fixed at $40
5	3,000	1,000	By discount schedule
6	M	1,000	By discount schedule
7	M	100	By discount schedule
8	M	1	By discount schedule

3,000 units of colour filters. The firm can only purchase an order with an integer multiple of the batch size of 1,000, for example 1,000, 2,000, and 3,000, etc. The unit purchase price is fixed at $40 no matter how many units are purchased a time; that is, no discount is given. Use Case 7 as another example, there is no limit on the storage space since M is a large number. An order can be 100, 200, and 300, etc. Discounted price is given to all units based on the total quantity purchased at a time using the discount schedule in Table II. If 30 batches are purchased, the total number of units is 3,000. According to Table II, each unit costs $39, and the total purchase cost is $117,000 (3,000 × $39).

4.3 Experimental result and analysis
Based on the proposed mixed 0-1 integer programming model using LINGO, the results of the first four cases with a fixed price are obtained and summarized in Table IV, and the results of the other four cases with the prices determined by the discount schedule are shown in Table V:

- *Case 1.* The maximum storage space is 3,000 units. The units purchased in each order must be a multiple of the batch size, 1,000 units. The price is fixed at $40. The beginning inventory in period 1 (X_1) is zero, and a purchase is made for the period ($Z_1 = 1$) with a purchase quantity (Q_1) of 1,000 units, which are one batch.

	Cases			
	1	2	3	4
	S = 3,000	S = M[a]	S = M	S = M
	B = 1,000	B = 1,000	B = 100	B = 1
Period variables	P = 40	P = 40	P = 40	P = 40
(X_1, Z_1, Q_1)	(0, 1, 1,000)	(0, 1, 1,000)	(0, 1, 1,400)	(0, 1, 1,370)
(X_2, Z_2, Q_2)	(390, 0, 0)	(390, 0, 0)	(790, 0, 0)	(760, 0, 0)
(X_3, Z_3, Q_3)	(40, 1, 2,000)	(40, 1, 2,000)	(440, 0, 0)	(410, 0, 0)
(X_4, Z_4, Q_4)	(1,630, 0, 0)	(1,630, 0,0)	(30, 1, 1,100)	(0, 1, 1,080)
(X_5, Z_5, Q_5)	(550, 1, 2,000)	(550, 1, 2,000)	(50, 1, 2,400)	(0, 1, 2,439)
(X_6, Z_6, Q_6)	(772, 0, 0)	(772, 0, 0)	(672, 0, 0)	(661, 0, 0)
(X_7, Z_7, Q_7)	(111, 1, 2,000)	(111, 1, 3,000)	(11, 1, 1,600)	(0, 1, 1,524)
(X_8, Z_8, Q_8)	(587, 1, 1,000)	(1,587, 0, 0)	(87, 1, 1,600)	(0, 1, 1,595)
(X_9, Z_9, Q_9)	(562, 0, 0)	(562, 0, 0)	(662, 0, 0)	(570, 0, 0)
(X_{10}, Z_{10}, Q_{10})	(226, 1, 1,000)	(226, 1, 1,000)	(326, 0, 0)	(234, 0, 0)
TC ($)	361,706.4	361,686.4	325,316.4	321,583.9

Note: [a]M is a large number

Table IV.
Results of the four cases
with P fixed

Cases

Period variables	5 $S = 3{,}000$ $B = 1{,}000$ P: from Table II	6 $S = M$ $B = 1{,}000$ P: from Table II	7 $S = M$ $B = 100$ P: from Table II	8 $S = M$ $B = 1$ P: from Table II
(X_1, Z_1, Q_1)	(0, 1, 3,000)	(0, 1, 3,000)	(0, 1, 2,100)	(0, 1, 2,000)
(X_2, Z_2, Q_2)	(2,390, 0, 0)	(2,390, 0, 0)	(1,490, 0, 0)	(1,390, 0, 0)
(X_3, Z_3, Q_3)	(2,040, 0, 0)	(2,040, 0, 0)	(1,140, 0, 0)	(1,040, 0, 0)
(X_4, Z_4, Q_4)	(1,630, 0, 0)	(1,630, 0, 0)	(730, 1, 3,000)	(630, 1, 3,000)
(X_5, Z_5, Q_5)	(550, 1, 2,000)	(550, 1, 2,000)	(2,650, 0, 0)	(2,550, 0, 0)
(X_6, Z_6, Q_6)	(772, 0, 0)	(772, 0, 0)	(872, 0, 0)	(772, 0, 0)
(X_7, Z_7, Q_7)	(111, 1, 2,000)	(111, 1, 4,000)	(211, 1, 3,000)	(111, 1, 3,008)
(X_8, Z_8, Q_8)	(587, 1, 2,000)	(2,587, 0, 0)	(1,687, 0, 0)	(1,595, 0, 0)
(X_9, Z_9, Q_9)	(1,562, 0, 0)	(1,562, 0, 0)	(662, 0, 0)	(570, 0, 0)
(X_{10}, Z_{10}, Q_{10})	(1,226, 0, 0)	(1,226, 0, 0)	(326, 0, 0)	(234, 0, 0)
TC ($)	352,466.4	351,746.4	316,446.4	312,760

Table V.
Results of the four cases
with discounted P

Because the demand for period 1 (D_1) is 610 units, the beginning inventory in period 2 (X_2) becomes 390 units, which can meet the demand for period 2, 350 units. No purchase is necessary for period 2. However, a purchase is made for period 3 with a quantity of 2,000 units (Q_2), and the beginning usable inventory for period 3, $I(t_3)$, becomes 2,040 (390 − 350 + 2,000) units, an amount less than the maximum storage space of 3,000 units. Because the storage space is limited in this case, colour filters are purchased six times in a planning horizon, compared to five times each for Cases 2-4. The total cost for a planning horizon is $361,706.4.

- *Case 2.* The parameters for this case are very similar to those for Case 1, except that there is no limit for storage space ($S = M$). The replenishment decisions are identical for the first six periods and the last two periods under the two cases, and the differences between the two cases are in period 7 and 8. A purchase is made for period 7 under both cases; however, the purchase quantity (Q_7) is 3,000 for Case 2 but 2,000 for Case 1. The reason is because Case 2 has no storage space limit but Case 1 has a maximum storage space of 3,000 units. The beginning inventory in period 7 is 111 units, and only 2,000 units can be purchased under Case 1 to have a beginning usable inventory, $I(t_7)$, of 2,111 units. If 3,000 units are purchased, $I(t_7)$ will become 3,111 units, an amount exceeds the storage space. As a result, with no storage space constraint, the total cost for a planning horizon under Case 2, $361,686.4, is lower than that under Case 1.

- *Case 3.* The condition under Case 3 is better than that under Case 2. While Case 2 has a batch size of 1,000 units, Case 3 has a batch size of 100 units. Therefore, Case 3 has purchase quantities of 1,400, 1,100, 2,400 and 1,600 units, which are integer multiples of 100 units. With a looser constraint, Case 3 performs much better than Case 2, with a total cost of $325,316.4.

- *Case 4.* With a batch size of one unit only and an unlimited storage space, this case performs the best among the first four cases, and the total cost for a planning horizon is $321,583.9.

- *Case 5.* The parameters in this case are identical to those in Case 1, except that the purchase prices are determined by the discount schedule here. Unlike in Case 1 in which six purchases are made, Case 5 has only four purchases: 3,000 units in period 1 and 2,000 units each in period 5, 7 and 8. The reason for fewer purchases obviously is to gain purchase quantity discounts. With 3,000 units or 2,000 units for a single purchase, the unit discounted price is, respectively, $39 and $39.5, as shown in Table II. Because the unit holding cost is $0.1 per period, a small number in compared with the discount savings of $1 or 0.5 per unit, larger quantity orders are tend to be made. With purchase quantity discount, the total cost for a planning horizon is $352,466.4.

- *Case 6.* The replenishment decisions for the first six periods are the same as under Case 5. Because there is no storage limit here, 4,000 units are purchased in period 7, in contrast with 2,000 units each in period 7 and 8 under Case 5. As a result, only three purchases are made under Case 6, and the total cost for a planning horizon is $351,746.4.

- *Case 7.* With a looser batch size constraint than Cases 6 and 7 also makes three purchases but with different quantity and periods, and a lower total cost of $316,446.4 is achieved.
- *Case 8.* With a batch size of one unit, an unlimited storage space, and purchase quantity discounts, this case performs the best among all eight cases, and the total cost for a planning horizon is $312,760.

A sensitivity analysis is performed for Case 8 ($S = M$, $B = 1$, P is with discount). Consider the situation in which only O or H changes by a fixed proportion (0.1, 0.5, 1.5 and 3) of the parameter, while the other parameters remain unchanged. Sensitivity measures are calculated for $O = 360$, 180, 60, 12 and $H = 0.3$, 0.15, 0.05, 0.01, and the results are summarized in Table VI.

Based on the sensitivity analysis, we can infer the following:

- An increase in ordering cost (O) causes increases in total ordering cost and may lead to a decrease in number of orders. For $O = 12$, 60, 120 and 180, the replenishment decisions are the same; that is, three purchases are made in a planning horizon ($Q_1 = 2,000$, $Q_4 = 3,000$, $Q_7 = 3,008$). The increase in total cost is due to the increase in total ordering cost. For example, total cost for a planning horizon increases $180 (from TC = $312,580 to 312,760) when O increases from $60 to 120. This is purely due to the $60 increase in ordering cost per time ($60 increase in ordering cost × 3 purchases = $180 increase in total cost). However, when the ordering cost increases from $180 to 360, the number of purchases decreases from three times to two times. This is because discount savings from large-sized orders can be obtained and holding cost becomes relatively cheaper when ordering cost increases significantly.

- An increase in unit holding cost per period (H) causes increases in total holding cost. This may lead to an increase in number of orders in order to compensate the holding cost, and an increase in total ordering cost and total cost for a planning horizon are resulted. When $H = 0.01$ and 0.05, only two purchases are made ($Q_1 = 3,000$ and $Q_5 = 5,008$). When H increases to 0.1, three purchases are made ($Q_1 = 2,000$, $Q_4 = 3,000$, $Q_7 = 3,008$), and the same replenishment decisions are made for $H = 0.15$. When $H = 0.3$, three purchases are made at the same periods as when $H = 0.1$ and 0.15, but the purchase quantities are different ($Q_1 = 1,370$, $Q_4 = 3,519$, $Q_7 = 3,119$). A significant decrease in order quantities is seen from $Q_1 = 2,000$ for $H = 0.1$ and 0.15 to $Q_1 = 1,370$ for $H = 0.3$. This is because the holding cost keeps increasing and the discount savings from large-sized orders are not enough to compensate the holding cost. To summarize, an optimal solution is obtained with the consideration of the interactions among discounted purchase price, holding cost and ordering cost.

- The breakeven points for the changes in O and H are further obtained as in Table VII. When $O = 301.6$, the fab can make either two purchases ($Q_1 = 3,000$, $Q_5 = 5,008$) or three purchases ($Q_1 = 2,000$, $Q_4 = 3,000$, $Q_7 = 3,008$), and the total cost remain the same with $313,305. When $H = 0.07412$, either two purchases or three purchases can be made, and the replenishment decisions for two (three) purchases are exactly the same as the two (three) purchases when $O = 301.6$.

Parameters	Values	X_1, Z_1, Q_1	X_2, Z_2, Q_2	X_3, Z_3, Q_3	X_4, Z_4, Q_4	X_5, Z_5, Q_5	X_6, Z_6, Q_6	X_7, Z_7, Q_7	X_8, Z_8, Q_7	X_9, Z_9, Q_9	X_{10}, Z_{10}, Q_{10}	TC ($)
O	360	0, 1, 3,000	2,390, 0, 0	2,040, 0, 0	1,630, 0, 0	550, 1, 5,008	3,780, 0, 0	3,119, 0, 0	1,595, 0, 0	570, 0, 0	234, 0, 0	313,422
	180	0, 1, 2,000	1,390, 0, 0	1,040, 0, 0	630, 1, 3,000	2,550, 0, 0	772, 0, 0	111, 1, 3,008	1,595, 0, 0	570, 0, 0	234, 0, 0	312,940
	120	0, 1, 2,000	1,390, 0, 0	1,040, 0, 0	630, 1, 3,000	2,550, 0, 0	772, 0, 0	111, 1, 3,008	1,595, 0, 0	570, 0, 0	234, 0, 0	312,760
	60	0, 1, 2,000	1,390, 0, 0	1,040, 0, 0	630, 1, 3,000	2,550, 0, 0	772, 0, 0	111, 1, 3,008	1,595, 0, 0	570, 0, 0	234, 0, 0	312,580
	12	0, 1, 2,000	1,390, 0, 0	1,040, 0, 0	630, 1, 3,000	2,550, 0, 0	772, 0, 0	111, 1, 3,008	1,595, 0, 0	570, 0, 0	234, 0, 0	312,436
H	0.3	0, 1, 1,370	760, 0, 0	410, 0, 0	0, 1, 3,519	2,439, 0, 0	661, 0, 0	0, 1, 3,119	1,595, 0, 0	570, 0, 0	234, 0, 0	315,213
	0.15	0, 1, 2,000	1,390, 0, 0	1,040, 0, 0	630, 1, 3,000	2,550, 0, 0	772, 0, 0	111, 1, 3,008	1,595, 0, 0	570, 0, 0	234, 0, 0	313,405
	0.1	0, 1, 2,000	1,390, 0, 0	1,040, 0, 0	630, 1, 3,000	2,550, 0, 0	772, 0, 0	111, 1, 3,008	1,595, 0, 0	570, 0, 0	234, 0, 0	312,760
	0.05	0, 1, 3,000	2,390, 0, 0	2,040, 0, 0	1,630, 0, 0	550, 1, 5,008	3,780, 0, 0	3,119, 0, 0	1,595, 0, 0	570, 0, 0	234, 0, 0	311,946
	0.01	0, 1, 3,000	2,390, 0, 0	2,040, 0, 0	1,630, 0, 0	550, 1, 5,008	3,780, 0, 0	3,119, 0, 0	1,595, 0, 0	570, 0, 0	234, 0, 0	311,150

Table VI.
Effects of parameter
changes on the system
under Case 8

Parameters	Changes	X_1, Z_1, Q_1	X_2, Z_2, Q_2	X_3, Z_3, Q_3	X_4, Z_4, Q_4	X_5, Z_5, Q_5	X_6, Z_6, Q_6	X_7, Z_7, Q_7	X_8, Z_8, Q_7	X_9, Z_9, Q_9	X_{10}, Z_{10}, Q_{10}	TC ($)
O	301.6	0, 1, 3,000	2,390, 0, 0	2,040, 0, 0	1,630, 0, 0	550, 1, 5,008	3,780, 0, 0	3,119, 0, 0	1,595, 0, 0	570, 0, 0	234, 0, 0	313,305
	301.6	0, 1, 2,000	1,390, 0, 0	1,040, 0, 0	630, 1, 3,000	2,550, 0, 0	772, 0, 0	111, 1, 3,008	1,595, 0, 0	570, 0, 0	234, 0, 0	313,305
H	0.07412	0, 1, 2,000	1,390, 0, 0	1,040, 0, 0	630, 1, 3,000	2,550, 0, 0	772, 0, 0	111, 1, 3,008	1,595, 0, 0	570, 0, 0	234, 0, 0	312,426
	0.07412	0, 1, 3,000	2,390, 0, 0	2,040, 0, 0	1,630, 0, 0	550, 1, 5,008	3,780, 0, 0	3,119, 0, 0	1,595, 0, 0	570, 0, 0	234, 0, 0	312,426

Table VII.
Breakeven points for
O and H under Case 8

5. Conclusion
This paper proposes a mixed 0-1 integer programming model to determine the replenishment quantity of colour filters for multi-periods. The case study demonstrates the practicality of the proposed model in minimizing the total cost. The analysis provided in this study is very useful for managers in designing a replenishment policy for TFT-LCD manufacturers to deal with colour filters which have the characteristics of large-size, price quantity discount and batch-sized orders. For future research, we can consider a case with multiple suppliers and different discount schedules for the suppliers. A model that takes into account stochastic demand and lead time and different priority of orders can also be established.

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Corresponding author
He-Yau Kang can be contacted at: kanghy@ncut.edu.tw