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# Chansombat S, Pongcharoen P, Hicks C. A mixed-integer linear programming model for integrated production and preventive maintenance scheduling in the capital goods industry. International Journal of Production Research (2018) 

## DOI link

https://doi.org/10.1080/00207543.2018.1459923
ePrints link
http://eprint.ncl.ac.uk/247018

Date deposited
18/04/2018
Embargo release date
16/04/2019
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https://doi.org/10.1080/00207543.2018.1459923

# A mixed-integer linear programming model for integrated production and preventive maintenance scheduling in the capital goods industry 

Sirikarn Chansombat ${ }^{\mathbf{1}}$, Pupong Pongcharoen ${ }^{1, *}$, and Christian Hicks ${ }^{\mathbf{2}}$<br>${ }^{1}$ Centre of Operations Research and Industrial Applications (CORIA), Faculty of Engineering, Naresuan University, Phitsanulok 65000, Thailand. Email: sirikarnc53@email.nu.ac.th ORCID ID: 0000-0003-4873-0478

Email: pupongp@ nu.ac.th and Scopus author ID: 17435722900

${ }^{2}$ Newcastle University Business School, 5 Barrack Road, Newcastle upon Tyne, NE1 7RU, UK. Email: chris.hicks@ncl.ac.uk and Scopus author ID: 7102667331

* Corresponding author: pupongp@nu.ac.th


#### Abstract

The scheduling literature is extensive, but much of this work is theoretical and does not capture the complexity of real world systems. Capital goods companies produce products with deep and complex product structures, each of which requires the coordination of jobbing, batch, flow and assembly processes. Many components require numerous operations on multiple machines. Integrated scheduling problems simultaneously consider two or more simultaneous decisions. Previous production scheduling research in the capital goods industry has neglected maintenance scheduling and used metaheuristics with stochastic search that cannot guarantee an optimal solution. This paper presents a novel mixed integer linear programming (MILP) model for simultaneously solving the integrated production and preventive maintenance scheduling problem in the capital goods industry, which was tested using data from a collaborating company. The objective was to minimise total costs including: tardiness and earliness penalty costs; component and assembly holding costs; preventive maintenance costs; and setup, production, transfer and production idle time costs. Thus, the objective function and problem formulation were more


extensive than previous research. The tool was successfully tested using data obtained from a collaborating company. It was found that the company's total cost could be reduced by up to $63.5 \%$.

Keywords: manufacturing and maintenance; integrated scheduling; capital goods; mixed integer linear programming

## 1 Introduction

Capital goods refers to "the stock of physical assets created in the past for current and future production", "capital goods are not produced to satisfy consumption needs directly, but to increase the eventual output of consumer goods and services" (Acha et al. 2004, p.507). Suppliers of capital goods are an important sector of the world economy that increases productivity and supports the diffusion of superior technologies (Fauceglia 2015). The main business activities of capital goods companies are the design, manufacture and construction of plant. Typical products include large steam turbines, offshore production facilities, cranes and ships. Individual products may be highly customised to meet individual customer requirements. Normally, the companies also produce spare parts and undertake subcontracting work for other companies using shared manufacturing resources (Hicks, Earl, and McGovern 2000; Hicks 1998).

Scheduling is concerned with "the allocation of limited resources to tasks over time, with the basic aim to ensure the efficient and effective use of the available resources. A classical problem area is the scheduling of manufacturing systems, in which machines (the resources) have to be allocated to jobs (the tasks) in the best possible way (minimising or maximising some objective function)" (Branke et al. 2016, p.110). Scheduling is one of the most popular research topics in production and operations management (Chaudhry and Luo 2005). However, despite the extensive literature on production scheduling, most of it is theoretical and does not model many of the complexities experienced in practice (Fuchigami and Rangel 2017). Relatively few papers have considered multiple level assembly processes (Na and Park 2014).

Production scheduling in the capital goods industry is particularly difficult because the products are customised to meet individual customer requirements and are supplied in low volume on a make-to-order (MTO) or engineer-to-order (ETO) basis. The main products have deep and complex product structures which gives rise to many levels of assembly process that need to be co-ordinated with component supply. The products contain a diversity of components, some of which are manufactured in low volume, whereas others are produced in medium or large quantities. This leads to a mix of jobbing, batch, flow and assembly processes. Many components have complex geometry and require numerous operations which leads to long process routings. Production scheduling must take into account manufacturing and assembly precedence relationships and finite capacity (Hicks and Braiden 2000; Hicks 1998). The characteristics of consumer and capital are summarised in Table 1. The capital goods production scheduling problem has been addressed by research that has used a variety of metaheuristics, which have aimed to minimise the total penalty cost (the combination of earliness and tardiness costs for assemblies and components); there has been a lack of research that has used enumerative search methods such as mixed integer linear programming to solve production scheduling problems in the capital goods industry. Previous research has assumed that manufacturing resources are continuously available with no breakdowns or preventative maintenance activities. There is no report in the international databases (ISI and SCOPUS) of research that has integrated production and preventive maintenance scheduling in the capital goods industry.

Table 1. The general characteristics of consumer products and capital goods.

| Characteristics | Consumer products | Capital goods |
| :--- | :--- | :--- |
| Product examples | Mobile phone, computer and <br> electrical machines | Large steam turbines, offshore <br> production facilities and cranes |
| Manufacturing quantity | Mass production | Low volume |


| Characteristics | Consumer products | Capital goods |
| :--- | :--- | :--- |
| Shop floor | Flow shop | Job shop |
| Size of product | Small or medium size | Large or extra-large size |
| Production strategies | Usually make-to-stock (MTS) or | Usually make-to-order (MTO) or |
|  | assemble-to-order (ATO) | engineer-to-order (ETO) <br> Usage |
| Machining time | Personal usage | Used to produce products or services |
| Assembly operations | Fhort | Long |
| Weight | Light weight | Numerous operations |
| Final assembly | Usually in manufacturing plant | Heavy weight |
| Price | Relatively low | Relatively high |
| Production period | Days-weeks | Months-years |

Integrated scheduling problems consider two or more simultaneous decisions, such as lot sizing and production scheduling, or production and maintenance scheduling (Fuchigami and Rangel 2017). Several authors have considered integrated scheduling including: Anwar and Nagi (1997) who considered integrated production scheduling and lot sizing; and Anwar and Nagi (1998) who investigated production scheduling and material handling; whilst Seidgar, Zandieh, and Mahdavi $(2017,2016)$ and Jung and Kim (2016) considered production and maintenance scheduling in two-stage assembly shops.

Production and maintenance scheduling are usually treated independently, with separate models for each function, which leads to suboptimal solutions, as the functions are interrelated (Hadidi, Al-Turki, and Rahim 2012a). Scheduling production and maintenance activities separately may cause conflicts between production and maintenance. The literature considers the integration of production and maintenance in two ways: i) interrelated models that comprise a model for one function that takes into account the other; and ii) integrated approaches that simultaneously model different elements of the production system (Hadidi, Al-Turki, and Rahim 2012a).

This paper presents a novel integrated production and maintenance scheduling tool for the capital goods industry that utilises a mixed integer linear programming (MILP) model that includes many of the complexities experienced by collaborating capital goods companies, which addresses a gap in the literature. The objective function minimises total production and
maintenance costs, which comprises earliness and tardiness costs; holding costs for components and assemblies; setup, production, transfer and idle time costs; and maintenance costs. Thus, the objective function considers more criteria than previous work. The proposed model was tested using datasets obtained from a collaborating capital goods company.

The remainder of the paper is organised as follows. Section 2 presents a comprehensive literature review. Section 3 describes the problem and model development including the modelling assumptions (section 3.1), notation (section 3.2) and MILP model formulation (section 3.3). Section 4 presents industrial case studies obtained from a collaborating capital goods company. This is followed by the computational results and discussions in section 5 . Section 6 provides conclusions and suggestions for future research.

## 2 Literature review

A scheduling problem may be described by a triplet $\alpha|\beta| \gamma$, where $\alpha$ describes the machine environment, $\beta$ provides details of the processing characteristics and constraints and $\gamma$ describes the objective to be minimised (Pinedo 2016, p.14). The machine environment may be : a single machine (1); identical machines in parallel $\left(P_{m}\right)$; identical machines in parallel with different speeds $\left(Q_{m}\right)$; unrelated machines in parallel $\left(R_{m}\right)$; a flow shop ( $F_{m}$, with machines in series); a flexible flow shop ( $F F_{c}$, with $c$ stages in series with a number of identical machines in parallel at each stage); a job shop ( $J_{m}$, with $m$ machines where each job has a predetermined route); a flexible job shop ( $F J_{c}$, where there are $c$ workstations, each with a number of identical machines in parallel, where each job has its own routing); or an open shop ( $O_{m}$, where each job has to be processed on all $m$ machines). Open shops operate on a make-to-order basis with requirements generated by customer orders; whereas closed shops operate on make-to-stock basis, with requirements determined by inventory replenishment decisions. The $\beta$ field may contain: release dates $\left(R_{j}\right)$; preemptions (prmp, when it is necessary to keep a job on a machine until it is finished); precedence constraints (prec); sequence dependent setup times ( $s_{j k}$ ); job
families (fmls); batch processing (batch(b), where a machine can process a batch of $b$ jobs simultaneously); breakdowns (brkdwn); machine eligibility restrictions $\left(M_{j}\right)$; a permutation ( $р$ rmu, the order of jobs going through the first machine is maintained throughout the system); blocking (block, which occurs when an upstream machine is unable to release a completed item due to a buffer being full); no-wait ( $n w t$, where jobs are not permitted to wait between two successive machines); and recirculation (rcrc, where a job may visit a machine or work centre more than once) (Pinedo 2016, p.14). There have been many comprehensive reviews of the scheduling literature (see for example, Graves 1981; Rodammer and White 1988; Blazewicz, Domschke, and Pesch 1996; Brucker and Brucker 2007; Pinedo 2016).

Na and Park (2014) commented that relatively few previous studies related to job shop scheduling problems have considered multi-level job structures. Further, much of this work has made many assumptions and used simplified representations. Studies that obtained optimal solutions by applying analytical methods have focused on scheduling problems relating to specific and limited cases (Na and Park 2014). More recently Lu et al. (2016) considered the assembly job shop as a generalised job shop, which includes both sequential and assembly operations with tree-structured precedence constraints. They also commented that the assembly job shop had received relatively little attention in the literature.

Previous research on production scheduling for assembly environments has used: dispatching rules (Huang 1984; Goodwin and Weeks 1986; Adam, Bertrand, and Surkis 1987; Fry, Philipoom, and Markland 1988; Mohanasundaram et al. 2003; Jang et al. 2003); heuristics (Anwar and Nagi 1998; Bhongade and Khodke 2012; Anwar and Nagi 1997); metaheuristics (Jung-Ug Kim and Yeong-Dae Kim 1996; Jang et al. 2003; Fattahi and Fallahi 2010; Costantino et al. 2014; Na and Park 2014; Lu et al. 2016); and mixed-integer linear programming (MILP) (Kolisch 2000; Yan et al. 2003; Chen and Ji 2007; Fattahi, Jolai, and Arkat 2009). However, this previous research was focused on a limited range of objectives, such as makespan, tardiness and
lead-time. There have been very few papers which have considered the minimisation of costs: Anwar and Nagi (1997) aimed to minimise makespan, setup and holding costs; Yan, Wang, and Jiao (2003) minimised over/under production costs, setup and idle time costs; and Chen and Ji (2007) minimised the cost of production idle time and earliness and tardiness penalties.

Work conducted by Pongcharoen (2001) and reported by Pongcharoen, Hicks, and Braiden (2004) developed a Genetic Algorithm (GA) for the finite capacity scheduling of complex capital goods with multiple levels of product structure, which was tested using data from a capital goods company. Pongcharoen et al. (2001) developed an efficient design of experiments approach to identify the best combinations of GA parameters and operators that produced solutions with minimum total cost. Pansuwan et al. (2010) designed an Artificial Bee Colony (ABC) algorithm to solve the same problem. The results indicated that the algorithm's performance could be improved dramatically by adopting a design of experiments approach to optimising the parameter settings. Chansombat et al. (2013) developed a conventional Bat Algorithm (BA) and a modified version, which increased the amount of local search. The modified BA outperformed the conventional BA, especially for the large and extra-large sized problems. More recently, Poungyeam et al. (2014) applied the conventional Krill Herd (KH) algorithm and developed a modified version that increased the amount of local search. The computational results indicated that the modified KH algorithm performed significantly better than the conventional KH algorithm. All metaheuristics involve stochastic random search and cannot be guaranteed to find an optimum solution (Blum et al. 2011). However, these near optimal solutions often produce results that are superior to those produced by typical planning heuristics. In contrast, enumerative search techniques, such as mixed-integer linear programming are guaranteed to find an optimum solution (Fister et al. 2015). These techniques have therefore been widely applied to solve scheduling problems, but not to the capital goods scheduling problem because of their complexity.

Maintenance activities are important operations for maintaining or restoring equipment to a specific state and guarantee a given service level (Ruiz, Carlos Garcia-Diaz, and Maroto 2007). Maintenance can also be categorised into two main classes: Corrective Maintenance (CM) and Preventive Maintenance (PM) (Wang 2002; Ahmad and Kamaruddin 2012). CM is unscheduled maintenance or repair required to return items/equipment to a defined state, which is carried out because of perceived deficiencies or failures. PM is carried out on a planned, periodic and specific schedule to keep equipment in a working condition (Sharma, Yadava, and Deshmukh 2011). There have been many reports of PM, particularly in transportation-related businesses, such as the airline industry (Ben Ahmed et al. 2017; Al-Thani, Ben Ahmed, and Haouari 2016); railways (Su et al. 2017; Baldi et al. 2016); and bus transit systems (Zhou et al. 2004; Haghani and Shafahi 2002).

The integrated scheduling of production and maintenance activities is very important for manufacturing operations and has therefore received considerable attention in both industry and academia. The problem is concerned with the allocation of limited resources over time to perform a series of manufacturing and maintenance operations, so that the requirements of all production and maintenance services are fulfilled whilst optimising some objective function(s).

Integrated production and maintenance scheduling using MILP has been considered for various manufacturing environments including: a single machine (Cheng et al. 2017; Cui and Lu 2017; Hnaien et al. 2016; Hadidi, Al-Turki, and Rahim 2012b; Beheshti-Fakher, Nourelfath, and Gendreau 2016); flow shops (Seif, Yu, and Rahmanniyay 2017; Bajestani and Beck 2015; Ramezanian, Saidi-Mehrabad, and Fattahi 2013), parallel machines (Berrichi, Yalaoui, and Yalaoui 2016; He, Li, and Xu 2016; Yoo and Lee 2016); job shops (Ye and Ma 2015; Li and Pan 2012; Wang and Yu 2010); open shops (Naboureh and Safari 2016; Azadeh et al. 2015); and two stage assembly shops (Seidgar, Zandieh, and Mahdavi 2017, 2016; Jung and Kim 2016). This literature has considered various objective functions including: minimising the makespan
(Von Hoyningen-Huene and Kiesmüller 2015; Qi, Wan, and Yan 2015); minimising the total weighted completion time ( $\mathrm{Nie}, \mathrm{Xu}$, and Tu 2015 ); minimising the total weighted tardiness and the number of tardiness tasks (Hedjazi 2015); minimising the combination of holding and maintenance costs (Hadidi, Al-Turki, and Rahim 2015); minimising the weighted sum of maximum earliness and maximum tardiness costs (Benmansour et al. 2014); minimising the total weighted tardiness and earliness costs (Haddad 2014); minimising the sum of production, holding and setup costs (Ramezanian, Saidi-Mehrabad, and Fattahi 2013); minimising maintenance cost (Rebai, Kacem, and Adjallah 2012); minimising the weighted completion time of jobs (Yalaoui, Chaabi, and Yalaoui 2014); minimising production cost (Erfanian and Pirayesh 2016); minimising the combination of production and maintenance costs (Beheshti-Fakher, Nourelfath, and Gendreau 2016); minimising the sum of inventory, setup, penalty and maintenance costs (Ghobadian et al. 2007); minimising the sum of production, inventory, setup and maintenance costs (Shamsaei and Van Vyve 2017); minimising the total cost of operations, setup, inventory carrying, maintenance, backordering and overtime (Purohit and Kumar Lad 2016); minimising the combination of stock-out and inventory costs (Leng et al. 2016); minimising the penalty cost due to earliness and tardiness (Yu and Seif 2016); minimising the operating cost, maintenance cost, overhaul cost and salvage value (Wu, Zhang, and Cheng 2017); and minimising the total maintenance cost and the total tardiness of jobs (Seif, Yu, and Rahmanniyay 2017).

Table 2 summarises a comprehensive literature review of previous research that has used MILP for solving integrated production and maintenance scheduling (IPMS) problems, categorised in terms of problem characteristics and the objective functions (performance measure). It can be seen that there is no previous research that has solved IPMS problems in the capital goods industry. In this work, cost-based scheduling performance was considered in terms of nine sub-costs: tardiness penalty costs; earliness penalty costs; holding costs relating to
component items; holding costs due to assembly items; setup costs; production costs for machining and assembly operations; transfer costs; costs due to production idle time; and preventive maintenance costs.

## 3 Problem description and model development

Production and preventive maintenance scheduling for the capital goods industry is difficult because of the complex characteristics of the products and processes. The product structures of capital goods are usually deep and complex, with many levels of assembly. Many components have long and complicated process routings (Bhamu and Sangwan 2014). There is finite resource capacity and there are many assembly and operation precedence constraints. A simplified example of product structure representation is shown in Figure 1. The root node represents the final product $\left(\mathrm{F}_{1}\right)$, which comprises assemblies $\left(\mathrm{A}_{1}, \mathrm{~A}_{2}\right.$, and $\left.\mathrm{A}_{3}\right)$; subassemblies $\left(\mathrm{S}_{1}, \mathrm{~S}_{2}, \mathrm{~S}_{3}\right.$, and $\left.\mathrm{S}_{4}\right)$; and components $\left(\mathrm{C}_{1}, \mathrm{C}_{2}, \mathrm{C}_{3}, \mathrm{C}_{4}\right.$, and $\left.\mathrm{C}_{5}\right)$ as the leaf nodes. All the nodes in the product structure will have a sequence of machining operations $\mathrm{O}_{1}, \mathrm{O}_{2} \ldots \mathrm{O}_{\mathrm{n}}$, which need to be completed sequentially. If the component $\mathrm{C}_{1}$ has three operations $\mathrm{O}_{1}, \mathrm{O}_{2}$ and $\mathrm{O}_{3}, \mathrm{C}_{1}$ can be represented as three intermediate items $\mathrm{C}_{1} \mathrm{O}_{1}, \mathrm{C}_{1} \mathrm{O}_{2}$ and $\mathrm{C}_{1} \mathrm{O}_{3}$ where $\mathrm{C}_{1} \mathrm{O}_{3}$ is the completed $\mathrm{C}_{1}$, since it has three operations. Capital goods typically have up to ten levels of assembly and many thousands of components. This gives rise to many assembly precedence constraints i.e. the subassemblies/components must be available for an assembly process to take place.

Table 2. A review summary of the integrated production and maintenance scheduling literature.


To reduce the probability of machine failure PM operations should take place on a routine basis with a predefined PM interval. If the planning horizon is longer than the PM intervals, each machine would require at least one preventive maintenance (PM) operation. The duration of PM activities on a particular machine is assumed to be deterministic and known in advance. Each PM operation must occur between the earliest starting time $E_{p m}$ and the latest finishing time $L_{p m}$. Each PM operation may delay the start and completion times of successive operations by up to the duration of the PM operation.

t


Components

t -










Final assembly operation


Figure 1. An example of product structure representation.

Figure 2 is an example of integrated production and maintenance schedule for the example shown in Figure 1. The schedule illustrates the allocation of three machines (MC1, MC2 and MC3) over time to perform all the machining, assembly and maintenance operations required to manufacture the final product $\left(\mathrm{F}_{1}\right)$. All the machining and assembly operations needed to be synchronised according to the product structure. In order to keep machine MC1 in a good condition, the PM operations were conducted every 50 time units (e.g. days). After 50 days
operation, the PM activity could be carried out between $\mathrm{Epm}_{11}$ and $\mathrm{Lpm}_{11}$ (day 50 to day 55). In this example, MC1 completed the second operation of the third component $\left(\mathrm{C}_{3} \mathrm{O}_{2}\right)$ on day 53 , a one-day PM activity on MC1 was then performed on day 54 . Likewise, the interval for PM operations on MC2 and MC3 were 60 and 100 time units, respectively. The first PM activities on MC2 and MC3 could be conducted during $\mathrm{Epm}_{21}-\mathrm{Lpm}_{21}$ and $\mathrm{Epm}_{31}-\mathrm{Lpm}_{31}$, respectively.


PMinterval
$\mathbf{E p m}_{31} \mathbf{L p m}_{31}$


| $\boxtimes 叉$ Machining operations | $\square$ | PM operations |
| :--- | :--- | :--- |
| $\square$ | Assembly operations | $\square$ |
| Final assembly oper ation |  |  |



Figure 2. An example of integrated production and maintenance schedule.

The objective of this work was to find an optimal schedule which minimised total costs (including penalty costs caused by tardiness and earliness, holding costs due to work-in-process and costs due to assembly, setup, production, transfer, production idle time and maintenance).

### 3.1 Assumptions

The integrated production and preventive maintenance scheduling model was based upon the following assumptions:

- The scheduling of production and maintenance are considered simultaneously;
- External demand is known and deterministic;
- Many products can be manufactured in each period;
- Each component may have multiple operations that take place in a specified order on designated work centres;
- An assembly process cannot start until all the components/subassemblies are available;
- Each machine is initially idle at the beginning of the scheduling period and can execute at most a single operation at a time;
- No pre-emptive priorities are assigned, i.e. once the processing of an operation on a machine has started, it has to be completed before another operation can be started on that machine;
- The duration of PM operations are deterministic and known in advance and can be performed after any operation has been completed;
- When a PM operation is performed on a machine, no any operation can be processed on that machine at the same time;
- Periodic preventive maintenance keeps machines 'as good as new'. The probability of machine breakdown is close to zero;
- PM operations can be performed within time periods. In other words, finishing time of PM operation occurs in the specified time period.


### 3.2 Notation

## Indices

$i \quad$ index of orders $i=1, \ldots, n$
$p, q \quad$ index of operations $p, q=1, \ldots, h, b$
$k \quad$ index of machines $k=1, \ldots, m$
$l \quad$ index of preventive maintenance operations $l=1, \ldots, s$

## Parameters

$n \quad$ number of orders
$h \quad$ number of machining operations
$b \quad$ number of total operations (machining + assembly operations)
$m$ number of machines
$s \quad$ number of preventive maintenance operations
$C t_{i} \quad$ tardiness penalty cost of order $i(£ /$ day $)$
$C e_{i} \quad$ earliness penalty cost of order $i(£ /$ day $)$
Ch holding cost of component (£/day)
Ca holding cost of assembly (£/day)
Cpm preventive maintenance cost on machine $k$ ( $£ /$ occurrence)
Cs setup cost (£/hour)
Cp production cost (f/hour)
Cf transfer cost (f/hour)
$I \quad$ production idle time cost ( $£ /$ hour)
$D_{i} \quad$ due date of order $i$
$A_{k} \quad$ ready time of machine $k$
$S T_{p k} \quad$ setup time of operation $p$ on machine $k$ (hours)
$P T_{p k} \quad$ processing time of operation $p$ on machine $k$ (hours)
$T T_{p k} \quad$ transfer time of operation $p$ on machine $k$ (hours)
$S T_{q k} \quad$ setup time of operation $q$ on machine $k$ (hours)
$P T_{q k} \quad$ processing time of operation $q$ on machine $k$ (hours)
$T T_{q k} \quad$ transfer time of operation $q$ on machine $k$ (hours)
$\mathrm{Npm}_{k} \quad$ number of preventive maintenance operations on machine $k$
$T p m_{k l}$ preventive maintenance time on machine $k$ in preventive maintenance operation $l$
$E p m_{k l}$ earliest starting time of PM on machine $k$ in PM operation $l$
$L p m_{k l}$ latest finishing time of PM on machine $k$ in PM operation $l$
Sh number of shifts per day
$R \quad$ set of immediate predecessor-successor pairs of operation $(p, q)$ such that operation $p$ must be processed immediately before operation $q$
$P M A_{k}$ set of preventive maintenance operations on machine $k$
$F_{p} \quad$ set of machine capable of processing operation $p$
$F_{q} \quad$ set of machine capable of processing operation $q$
M a large positive number

## Variables

$T_{i}^{I} \quad$ number of tardy days (integer) for order $i$
$T_{i} \quad$ number of tardy days (real number) for order $i$
$E_{i}^{I} \quad$ number of early days (integer) for order $i$
$E_{i} \quad$ number of early days (real number) for order $i$
$H_{p}^{I} \quad$ number of holding days (integer) for operation $p$
$H_{p} \quad$ holding time of operation $p$ (hours)
$H_{q} \quad$ holding time of operation $q$ (hours)
$C_{i} \quad$ completion time of order $i$
$C I_{p} \quad$ completion time of operation $p$
$C_{\max }$ production makespan
$S_{p} \quad$ production start time of operation $p$
$S_{q} \quad$ production start time of operation $q$
$S_{p_{i}} \quad$ production start time of operation $p$ for order $i$
$F p m_{k l}$ finishing time of preventive maintenance operation $l$ on machine $k$
$S p m_{k l} \quad$ start time for preventive maintenance operation $l$ on machine $k$
$Z_{p k}=\left\{\begin{array}{c}1 \text { if operation } p \text { is assigned to machine } k, \\ 0 \text { otherwise },\end{array}\right.$
$Y_{p q k}=\left\{\begin{array}{c}1 \text { if operation } p \text { precedes operation } q \text { on machine } k, \\ 0 \text { otherwise } .\end{array}\right.$
$P M_{p k l}=\left\{\begin{array}{c}1 \text { if operation } p \text { precedes } P M \text { operation l on machine } k, \\ 0 \text { otherwise } .\end{array}\right.$

### 3.3 Problem formulation

This section presents the MILP model formulated to represent the production and PM scheduling problem for complex manufacturing systems, with multiple products, multi-level product structures and multiple machines, which are common characteristics of the capital goods industry. The proposed MILP model is specified by the following equations:

Minimise: $\quad \sum_{i=1}^{n} C t_{i} T_{i}^{I}+\sum_{i=1}^{n} C e_{i} E_{i}^{I}+\sum_{p=1}^{h} C h H_{P}^{I}+\sum_{p=1}^{b-h} C a H_{P}^{I}+\sum_{p=1}^{b} \sum_{k=1}^{m} C s S T_{p k}+\sum_{p=1}^{b} \sum_{k=1}^{m} C p P T_{p k}+$

$$
\sum_{p=1}^{b} \sum_{k=1}^{m} C f T T_{p k}+\left(I \times\left(m \times C_{\max }-\sum_{p=1}^{b} \sum_{k \in F_{p}}\left(S T_{p k}+P T_{p k}+T T_{p k}\right) \times Z_{p k}\right.\right.
$$

$$
\begin{equation*}
\left.\left.-\sum_{k=1}^{m} A_{k}-\sum_{p=1}^{b} H_{p}\right)\right)+\sum_{k=1}^{m} \operatorname{CpmNpm}_{k} \tag{1}
\end{equation*}
$$

Subject to:
$C_{i} \leq C_{\max } \forall i$,
$S_{p} \geq \sum_{k \in F_{p}} A_{k} Z_{p k} \forall p$,
$S_{q} \geq S_{p}+\sum_{k \in F_{p}}\left(S T_{p k}+P T_{p k}+T T_{p k}\right) \times Z_{p k}(p, q) \in R$,
$C_{i}=S_{p_{i}}+\sum_{k \in F_{p_{i}}}\left(S T_{p_{i} k}+P T_{p_{i} k}+T T_{p_{i} k}\right) \times Z_{p_{i} k} \quad \forall_{i}$,

$$
\begin{align*}
& C I_{p}=S_{p}+\sum_{k \in F_{p}}\left(S T_{p k}+P T_{p k}+T T_{p k}\right) \times Z_{p k} \quad \forall_{p},  \tag{6}\\
& H_{q}=S_{q}-\left(S_{p}+\sum_{k \in F_{p}}\left(S T_{p k}+P T_{p k}+T T_{p k}\right) \times Z_{p k}\right) \quad \forall_{q},  \tag{7}\\
& S_{q} \geq S_{p}+S T_{p k} Z_{p k}+P T_{p k} Z_{p k}+T T_{p k} Z_{p k}-M\left(1-Y_{p q k}\right) \forall_{p, q, k \mid k \in F_{q} \cap F_{p}, p<q},  \tag{8}\\
& S_{p} \geq S_{q}+S T_{q k} Z_{q k}+P T_{q k} Z_{q k}+T T_{q k} Z_{q k}-M\left(1-Y_{q p k}\right) \forall_{p, q, k \mid k \in F_{q} \cap F_{p}, p<q},  \tag{9}\\
& Z_{p k}+Z_{q k} \geq 2\left(Y_{p q k}+Y_{q p k}\right) \forall_{p, q, k \mid k \in F_{q} \cap F_{p}, p<q},  \tag{10}\\
& Z_{p k}+Z_{q k} \leq Y_{p q k}+Y_{q p k}+1 \forall_{p, q, k \mid k \in F_{q} \cap F_{p}, p<q},  \tag{11}\\
& H_{p}^{I} \geq \frac{H_{p}}{\operatorname{Sh} \times 8} \quad \forall_{p \mid p \in W},  \tag{12}\\
& T_{i} \geq \frac{C_{i}-D_{i}}{S h \times 8} \quad \forall_{i},  \tag{13}\\
& E_{i} \geq \frac{D_{i}-C_{i}}{S h \times 8} \quad \forall_{i},  \tag{14}\\
& T_{i}^{I} \geq T_{i} \quad \forall_{i},  \tag{15}\\
& E_{i}^{I} \geq E_{i} \quad \forall_{i},  \tag{16}\\
& \sum_{k \in F_{p}} Z_{p k}=1 \quad \forall_{p},  \tag{17}\\
& \sum_{k \notin F_{p}} Z_{p k} \leq 0 \quad \forall_{p},  \tag{18}\\
& S_{p}-F p m_{k l}+M \times P M_{p k l} \geq 0 \quad \forall_{p, k \in F_{p, l \in P M A_{k}}},  \tag{19}\\
& F p m_{k l}-T p m_{k l}-\left(S_{p}+\sum_{k \in F_{p}}\left(S T_{p k}+P T_{p k}+T T_{p k}\right) \times Z_{p k}\right)+M \times\left(1-P M_{p k l}\right) \geq 0 \\
& \forall_{p, k \in F_{p}, l \in P M A_{k}}, \tag{20}
\end{align*}
$$

$E p m_{k l} \leq F p m_{k l} \leq L p m_{k l} \quad \forall_{k, l \in P M A_{k}}$,
$\operatorname{Spm}_{k l}=\operatorname{Fpm}_{k l}-\operatorname{Tpm}_{k l} \quad \forall_{k, l \in P M A_{k}}$,
$C_{\text {max }} \geq 0$,
$H_{p} \geq 0 \quad \forall_{p}$,
$E_{i}, T_{i} \geq 0 \quad \forall_{i}$,
$\mathrm{Fpm}_{k l}$, Spm $_{k l} \geq 0$ and integer $\forall_{k, l}$,

$$
\begin{array}{ll}
C_{i} \geq 0 \text { and integer } & \forall_{i}, \\
S_{p}, C I_{p} \geq 0 \text { and integer } & \forall_{i}, \\
E_{i}^{I}, T_{i}^{I} \geq 0 \text { and integer } & \forall_{i}, \\
H_{p}^{I} \geq 0 \text { and integer } & \forall_{p}, \\
Y_{p q k}, Z_{p k} \in\{0,1\} & \forall_{p, q, k}, \\
P M_{p k l} \in\{0,1\} & \forall_{p, k, l} \tag{32}
\end{array}
$$

The objective function, equation (1), considers the minimisation of the sum of the total cost that comprises nine sub-costs: the sum of tardiness penalty costs; earliness penalty costs; components' holding costs; assemblies' holding costs; setup costs; production costs; transfer costs; production idle time costs; and finally the costs of PM activities. Constraint (2) shows that the completion time of any order has to be less than or equal to production makespan $\left(C_{\max }\right)$. Constraint (3) ensures that the production start time of any operation must be equal to or greater than the ready time of the machine it is assigned to. Constraint (4) guarantees that an operation starts after its predecessor operations are processed. Constraint (5) defines the completion time of an order. Constraint (6) defines the completion time of an operation. Constraint (7) defines the holding time of an operation. Constraints (8)-(11) are disjunctive constraints, which determine that no two operations can be processed on the same machine simultaneously if both operations are assigned to the same machine. For constraint (8), if operation $p$ is scheduled before operation $q$ on machine $k,\left(Y_{p q k}=1\right)$, the starting time of operation $q$ must be greater than or equal to the completion time of operation $p$. Constraint (9) represents the complementary disjunctive constraints (8). In constraint (10), if operation $p$ and $q$ are scheduled on machine $k$, both operations must have been assigned to that machine. In constraint (11), if operations $p$ and $q$ are assigned to the same machine, one of them must be scheduled before the other. Equation (12) calculates the holding time before each operation. Equations (13) and (14) calculate the
tardiness and earliness of orders, respectively. Constraints (12)-(14) transform hours into days. Equations (15) and (16) convert the value of tardiness and earliness to an integer number of time periods by rounding up. Constraint (17) ensures that each operation is assigned to only one machine in its eligible machine set. Constraint (18) prevents the assignment of any operation to non-eligible machines. Constraints (19) and (20) show that if an operation is processed before a PM operation then the finishing time of that operation must be less than the starting time of the PM operation. In other words, if an operation is processed after a PM operation then the starting time of that operation must be greater than the finishing time of PM operation. Constraint (21) ensures that a PM operation is performed in each time period. Constraints (23)-(25) define the non-negative variables. Constraints (26)-(30) define the non-negative integer variables. Constraints (31) and (32) define the binary variables.

## 4 Industrial case studies

The proposed MILP model was tested using four case studies which were obtained from a collaborating capital goods company (Pongcharoen 2001; Hicks 1998). The characteristics of the problems are summarised in Figure 3. The existing layout of the manufacturing facilities in terms of a block plan is shown in Figure 4. The product structures for the small and extra-large problems are shown in Figure 5(a) and Figure 5(b). Table 3 and 4 summarise the characteristics of production scheduling problems for each problem size and the characteristics of the PM operations for each machine respectively. The working time was assumed to be 8 hours per shift, with three shifts per day. It was assumed at the beginning of the planning horizon that all the machines were ready for processing. The research assumed that the penalty cost for tardiness in delivery of the final product was $£ 1,000$ per day and the penalty cost for earliness was $£ 500$ per day. The holding cost for components was assumed to be $£ 250$ per day and for assemblies $£ 500$ per day. The preventative maintenance costs were assumed to be $£ 500$ per occurrence. Setup,
processing and transfer were assumed to be $£ 10$ per hour. Machine idle time was assumed to cost $£ 1$ per hour.


Figure 3. The characteristics of the four problems.


Figure 4. Initial layout of manufacturing facilities (Hicks 2004).


Figure 5(a). Small problem product structures.


Figure 5(b). Extra-large problem product structure.

Table 3. The characteristics of the scheduling problems.

| Problem sizes | Items | Machine number | Operations | $\begin{gathered} \text { Setup } \\ \text { time (hrs) } \end{gathered}$ | Processing time (hrs) | Transfer time (hrs) | $\begin{gathered} \text { Due date } \\ \text { (hrs) } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Small | $\mathrm{F}_{245}$ | M1000 | Assembly | 2 | 5 | 1 | $737.30^{\text {th }}$ |
|  | $\mathrm{A}_{246}$ | M1000 | Assembly | 2 | 5 | 1 | - |
|  | $\mathrm{A}_{247}$ | M1000 | Assembly | 2 | 5 | 1 | - |
|  | $\mathrm{C}_{244} \mathrm{O}_{1}$ | M1222 | Machining | 2 | 5 | 1 | - |
|  | $\mathrm{C}_{244} \mathrm{O}_{2}$ | M1113 | Machining | 2 | 5.25 | 1 | - |
|  | $\mathrm{C}_{244} \mathrm{O}_{3}$ | M1222 | Machining | 2 | 5.25 | 1 | - |
|  | $\mathrm{C}_{244} \mathrm{O}_{4}$ | M1315 | Machining | 2 | 5.75 | 1 | - |
|  | $\mathrm{C}_{244} \mathrm{O}_{5}$ | M1222 | Machining | 2 | 14.50 | 1 | - |
|  | $\mathrm{C}_{244} \mathrm{O}_{6}$ | M1226 | Machining | 2 | 28.50 | 1 | - |
|  | $\mathrm{C}_{244} \mathrm{O}_{7}$ | M1226 | Machining | 2 | 43.25 | 1 | - |
|  | $\mathrm{C}_{244} \mathrm{O}_{8}$ | M1125 | Machining | 2 | 46.75 | 1 | - |
|  | $\mathrm{C}_{244} \mathrm{O}_{9}$ | M1411 | Machining | 2 | 248.75 | 1 | - |
|  | $\mathrm{C}_{248} \mathrm{O}_{1}$ | M1411 | Machining | 2 | 5 | 1 | - |
|  | $\mathrm{C}_{248} \mathrm{O}_{2}$ | M1222 | Machining | 2 | 5 | 1 | - |
|  | $\mathrm{C}_{248} \mathrm{O}_{3}$ | M1222 | Machining | 2 | 5.25 | 1 | - |
|  | $\mathrm{C}_{248} \mathrm{O}_{4}$ | M1113 | Machining | 2 | 6 | 1 | - |
|  | $\mathrm{C}_{248} \mathrm{O}_{5}$ | M1222 | Machining | 2 | 21.75 | 1 | - |
|  | $\mathrm{C}_{248} \mathrm{O}_{6}$ | M1222 | Machining | 2 | 41.75 | 1 | - |
|  | $\mathrm{C}_{248} \mathrm{O}_{7}$ | M1125 | Machining | 2 | 45.75 | 1 | - |
|  | $\mathrm{C}_{248} \mathrm{O}_{8}$ | M1125 | Machining | 2 | 56.50 | 1 | - |
|  | $\mathrm{F}_{451}$ | M1000 | Assembly | 2 | 5 | 1 | $552^{\text {th }}$ |
|  | $\mathrm{A}_{452}$ | M1000 | Assembly | 2 | 5 | 1 | - |
|  | $\mathrm{A}_{453}$ | M1000 | Assembly | 2 | 5 | 1 | - |
|  | A 454 | M1000 | Assembly | 2 | 5 | 1 | - |
|  | $\mathrm{A}_{457}$ | M1000 | Assembly | 2 | 5 | 1 | - |
|  | A458 | M1000 | Assembly | 2 | 5 | 1 | - |
|  | $\mathrm{C}_{447} \mathrm{O}_{1}$ | M1312 | Machining | 2 | 50.50 | 1 | - |
|  | $\mathrm{C}_{447} \mathrm{O}_{2}$ | M1312 | Machining | 2 | 50.50 | 1 | - |
|  | $\mathrm{C}_{455} \mathrm{O}_{1}$ | M1312 | Machining | 2 | 50.50 | 1 | - |
|  | $\mathrm{C}_{455} \mathrm{O}_{2}$ | M1312 | Machining | 2 | 50.50 | 1 | - |
|  | $\mathrm{C}_{456} \mathrm{O}_{1}$ | M1312 | Machining | 2 | 50.50 | 1 | - |
|  | $\mathrm{C}_{456} \mathrm{O}_{2}$ | M1312 | Machining | 2 | 50.50 | 1 | - |
|  | $\mathrm{C}_{459} \mathrm{O}_{1}$ | M1312 | Machining | 2 | 50.50 | 1 | - |
|  | $\mathrm{C}_{459} \mathrm{O}_{2}$ | M1312 | Machining | 2 | 50.50 | 1 | - |
| Medium | $\mathrm{F}_{229}$ | M1000 | Assembly | 2 | 5 | 1 | $904{ }^{\text {th }}$ |
|  | $\mathrm{F}_{451}$ | M1000 | Assembly | 2 | 5 | 1 | $552^{\text {th }}$ |
|  | $\mathrm{A}_{230}$ | M1000 | Assembly | 2 | 5 | 1 | - |
|  | $\mathrm{C}_{226} \mathrm{O}_{1}$ $\vdots$ | M1211 | Machining | 2 | 5 | 1 | $\bar{\square}$ |
|  | $\mathrm{C}_{235} \mathrm{O}_{10}$ | M1211 | Machining | 2 | 149 | 1 | - |
| Large | $\mathrm{F}_{228}$ | M1000 | Assembly | 2 | 5 | 1 | $912^{\text {th }}$ |
|  | $\mathrm{F}_{4}$ | M1000 | Assembly | 2 | 5 | 1 | $1,136^{\text {th }}$ |
|  | $\mathrm{A}_{237}$ | M1000 | Assembly | 2 | 5 | 1 | - |
|  | $\mathrm{C}_{236} \mathrm{O}_{1}$ | M1211 | Machining | 2 | 5 | 1 | - |
|  |  |  |  | : | 交 | $\vdots$ | $\vdots$ |
|  | $\mathrm{C}_{241} \mathrm{O}_{7}$ | M1115 | Machining | 2 | 178.75 | 1 | - |
| Extra-large | $\mathrm{F}_{227}$ | M1000 | Assembly | 0 | 672 | 48 | $12,400.50^{\text {th }}$ |
|  | $\mathrm{A}_{250}$ | M1000 | Assembly | 0 | 672 | 48 | , |
|  | $\mathrm{A}_{298}$ | M1000 | Assembly | 0 | 672 | 48 | - |
|  | $\mathrm{C}_{249} \mathrm{O}_{1}$ | M1211 | Machining | 0 | 10.60 | 48 | - |
|  | $\stackrel{\vdots}{\mathrm{C}_{233} \mathrm{O}_{12}}$ | M1224 | Machining | 0 | 149 | 48 | : |

Table 4. The characteristics of preventive maintenance operations.

| Problem sizes | Machine number | PM time (hrs/time) | Number of PMs (times) | PM operations | Earliest PM time (hrs) | Latest PM time (hrs) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Small | M1000 | 1 | 2 | 1 | 300 | 310 |
|  |  |  |  | 2 | 600 | 610 |
|  | M1113 | 1 | 1 | 1 | 500 | 520 |
|  | M1125 | 1 | 1 | 1 | 400 | 430 |
|  | M1222 | 1 | 1 | 1 | 100 | 120 |
|  | M1226 | 1 | 1 | 1 | 500 | 550 |
|  | M1312 | 1 | 2 | 1 | 200 | 220 |
|  |  |  |  | 2 | 400 | 420 |
|  | M1315 | 1 | 1 | 1 | 100 | 200 |
|  | M1411 | 1 | 1 | 1 | 600 | 700 |
| Medium | M1000 | 1 | 2 | 1 | 250 | 333 |
|  |  |  |  | 2 | 500 | 583 |
|  | M1312 | 1 | 2 | 1 | 333 | 417 |
|  |  |  |  | 2 | 667 | 750 |
|  | $\vdots$ | $\vdots$ | $\vdots$ | ! | ! | : |
|  | M1129 | 1 | 1 | 1 | 533 | 583 |
| Large | M1222 | 1 | 2 | 1 | 333 | 417 |
|  |  |  |  | 2 | 667 | 750 |
|  | M1113 | 1 | 1 | 1 | 667 | 750 |
|  | M1115 | 1 | 1 | 1 | 1,167 | 1,250 |
|  |  |  |  | . | : | , |
|  | M1511 | 1 | 1 | 1 | 1,250 | 1,333 |
| Extra-large | M1000 | 1 | 3 | 1 | 4,000 | 7,000 |
|  |  |  |  | 2 | 15,000 | 18,000 |
|  |  |  |  | 3 | 23,000 | 26,000 |
|  | M1222 | 1 | 2 | 1 | 2,000 | 7,000 |
|  |  |  |  | 2 | 12,000 | 17,000 |
|  | $\begin{gathered} \text { M1129 } \\ \vdots \\ \text { M1212 } \\ \hline \end{gathered}$ | 1 | 1 | 1 | 5,000 | 15,000 |
|  |  | $\vdots$ | $\vdots$ | . | 引 | $\vdots$ |
|  |  | 1 | 1 | 1 | 5,000 | 10,000 |

## 5

## Computational results and discussions

This section presents the computational results obtained by the proposed MILP model using the software Gurobi solver (http://www.gurobi.com/) run on a personal computer with a Core I7, 3.50 GHz CPU and 6 GB RAM. Table 5 shows the details of the total costs associated with the optimal schedules for each problem size, which includes a comparison with the original planning of the company. It can be seen that the minimum total costs associated with the optimal schedules obtained from the proposed MILP model were far lower than the total costs associated with the Company's schedules. For example, the minimum total cost obtained from the proposed method for the extra-large problem was lower than the Company's total cost by $63.5 \%$. These
results give an indication of the potential improvements that could be achieved by the proposed model, however, the particular results are case specific. The bottom of Table 5 shows the CPU time, the numbers of variables and constraints for each problem.

Table 5. The optimal solutions obtained from the proposed method.

| Lists | Problem sizes |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Small |  | Medium |  | Large |  | Extra-large |  |
|  | Company | MILP | Company | MILP | Company | MILP | Company | MILP |
| Tardiness (days) | 0 | 0 | 17 | 1 | 27 | 27 | 861 | 674 |
| Earliness (days) | 12 | 0 | 1 | 0 | 23 | 0 | 0 | 0 |
| Component holding (days) | 7 | 8 | 61 | 40 | 224 | 148 | 3,304 | 309 |
| Assembly holding (days) | 35 | 19 | 94 | 22 | 197 | 13 | 10,913 | 2,184 |
| Setup time (hours) | 68 | 68 | 134 | 134 | 270 | 270 | 0 | 0 |
| Production time (hours) | 1,039 | 1,039 | 1,565.25 | 1,565.25 | 3,455.25 | 3,455.25 | 38,261.18 | 38,261.18 |
| Transfer time (hours) | 34 | 34 | 67 | 67 | 135 | 135 | 9,456 | 9,456 |
| Production idle time (hours) | 2,191 | 4,109.67 | 1,141 | 3,156.58 | 12,215.75 | 18,545 | 430,002.50 | 606,769.43 |
| Number of PM (times) | 10 | 10 | 13 | 13 | 23 | 23 | 37 | 37 |
| Penalty cost of tardiness (£) | 0 | 0 | 17,000 | 1,000 | 27,000 | 27,000 | 861,000 | 674,000 |
| Penalty cost of earliness (£) | 6,000 | 0 | 500 | 0 | 11,500 | 0 | 0 | 0 |
| Holding cost of components (£) | 1,750 | 2,000 | 15,250 | 10,000 | 56,000 | 37,000 | 826,000 | 77,250 |
| Holding cost of assemblies (£) | 17,500 | 9,500 | 47,000 | 11,000 | 98,500 | 6,500 | 5,456,500 | 1,092,000 |
| Setup cost (£) | 680 | 680 | 1,340 | 1,340 | 2,700 | 2,700 | 0 | 0 |
| Production cost (£) | 10,390 | 10,390 | 15,652.50 | 15,652.50 | 34,552.50 | 34,552.50 | 382,611.80 | 382,611.80 |
| Transfer cost (£) | 340 | 340 | 670 | 670 | 1,350 | 1,350 | 94,560 | 94,560 |
| Production idle time cost (£) | 2,191 | 4,109.67 | 1,141 | 3,156.58 | 12,215.75 | 18,545 | 430,002.5 | 606,769.43 |
| PM cost (£) | 5,000 | 5,000 | 6,500 | 6,500 | 11,500 | 11,500 | 18,500 | 18,500 |
| Total cost (£) | 43,851 | 32,019.67 | 105,053.50 | 49,319.08 | 255,318.25 | 139,147.50 | 8,069,174.30 | 2,945,691.23 |
| CPU time (seconds) |  | 13.987 |  | 1,058.98 |  | 1,868.40 |  | 3,337.50 |
| Total number of variables |  | 387 |  | 2,252 |  | 5,529 |  | 11,986 |
| - Binary |  | 230 |  | 1,961 |  | 4,954 |  | 10,924 |
| - Integer |  | 122 |  | 225 |  | 445 |  | 835 |
| - Linear |  | 35 |  | 66 |  | 130 |  | 227 |
| Number of constraints |  | 976 |  | 7,660 |  | 19,619 |  | 43,019 |

Figure 6 shows the percentage cost breakdown for each size of the problem. It can be seen that the tardiness penalty cost increased with increasing problem size. The other costs, such as production and PM costs decreased with increasing problem size. The tardiness penalty, production and PM costs varied significantly.


Figure 6. The percentage of cost breakdown associated with the optimal solutions.

Figures 7(a) and 7(b) show the optimal solutions for the small-size problem (eight machines) and extra-large-size problem (twenty five machines) as Gantt Charts.
Machines


[^0]Due date: $\mathrm{F}_{245}$
Due date: $\mathbf{F}_{227}$

四 Machining operations $\square$ PM operations
$\square$ Assembly operations $\square$ Final assembly operation

## 6 Conclusions and future work

Effective scheduling can help improve resource utilisation and delivery performance, which improves competitiveness. There is a vast literature on scheduling, yet much of it is theoretical and many of the models developed do not capture the complexity of practical environments. There is no research that has considered integrated production and assembly planning in complex assembly environments. Meta-heuristics have been used for solving capital goods scheduling problems, because they find near optimal solutions within acceptable computational time. This paper has presented an integrated mixed integer linear programming model for capital goods companies that simultaneously schedules production and maintenance. This approach involves enumerative search, therefore an optimal solution is guaranteed. The model includes the key characteristics of capital goods companies including multiple products, multiple machines, complex routings and deep and complex product structures that lead to complex assembly relationships. Further, the objective function included more costs that the models previously presented in the literature. The problem formulation was based upon the literature, but includes more terms to reflect the complexities of the capital goods industry. The experimental results obtained using datasets obtained from a capital goods company demonstrated the optimality and effectiveness of the proposed model. Costs could be reduced by up to $63.5 \%$ compared to the Company's schedule.

The integrated scheduling and maintenance problem is strongly NP-hard, therefore the execution time increases rapidly with increasing problem size. Meta-heuristics can be used to solve these problems more efficiently. The MILP scheduling approach could be used to test the quality of the solutions obtained by meta-heuristics based upon stochastic search. Future research directions may focus on the application of metaheuristics to solve integrated production and maintenance scheduling in capital goods industry or investigate other issues related to the integrated scheduling problems (e.g., lot sizing, or uncertainty issues in manufacturing environment).

## Acknowledgements

The first author would like to acknowledge funding from the Thailand Research Fund Royal Golden Jubilee Ph.D. Scholarships Program - Grant Number PHD/0135/2552.

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[^0]:    Machining operations $\square$ PM operations

