

# A mixing layer theory for flow resistance in shallow streams

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[1] A variety of surface roughness characterizations have emerged from nineteenth and twentieth century studies of channel hydraulics. When the water depth  $h$  is much larger than the characteristic roughness height  $k_s$ , roughness formulations such as Manning's  $n$  and the friction factor  $f$  can be explicitly related to the momentum roughness height  $z_o$  in the log-law formulation for turbulent boundary layers, thereby unifying roughness definitions for a given surface. However, when  $h$  is comparable to (or even smaller than)  $k_s$ , the log-law need not be valid. Using a newly proposed mixing layer analogy for the inflectional velocity profile within and just above the roughness layer, a model for the flow resistance in shallow flows is developed. The key model parameter is the characteristic length scale describing the depth of the Kelvin-Helmholtz wave instability. It is shown that the new theory, originally developed for canopy turbulence, recovers much of the earlier roughness results for flume experiments and shallow gravel streams. This study is the first to provide such a unifying framework between canopy atmospheric turbulence and shallow gravel stream roughness characterization. The broader implication of this study is to support the merger of a wealth of surface roughness characterizations independently developed in nineteenth and twentieth century hydraulics and atmospheric sciences and to establish a connection between roughness formulations across traditionally distinct boundary layer types. *INDEX TERMS*: 1860 Hydrology: Runoff and streamflow; 1824 Hydrology: Geomorphology (1625); 3379 Meteorology and Atmospheric Dynamics: Turbulence; *KEYWORDS*: Manning's roughness, momentum roughness height, friction factor, mixing layer analogy, shallow gravel bed, canopy turbulence

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## 1. Introduction

[2] Equations for mean channel flow velocity, such as Manning's equation and various resistance equations, are widely used in hydraulic engineering and surface hydrology [e.g., French, 1985; Dooge, 1992; Hauser, 1996; Hornberger et al., 1998]. For channels in which shear velocity can be determined from measurements of depth and slope, the key parameter that must be specified is the absolute surface roughness, e.g., the momentum roughness height  $z_o$  or Manning's roughness  $n$ . The roughness measures can be linked analytically for simplified boundary layer flow types when the water depth  $h$  is much larger than the mean roughness height of the protruding elements  $D$  [e.g., Chen, 1991]. However, when  $h/D$  is small (say  $<10$ ), existing boundary layer theories (e.g., the log-law [Monin and Yaglom, 1971]) that describe the flow resistance may fail. This failure becomes evident when estimating the flow resistance for mountain rivers characterized by high gra-

dients and boulder beds [e.g., Bathurst, 1985]. While several empirical and semi-empirical models have been useful over this range of  $h/D$  [e.g., Hey, 1979; Colosimo et al., 1988; Leopold, 1994], a theoretical framework that describes the flow resistance has been lacking but is now receiving broad attention [e.g., Millar, 1999; Ferro and Pecoraro, 2000].

[3] In this paper we present a new theory that predicts the flow resistance from surface roughness measures and water depth using a mixing layer analogy rather than the standard rough-wall boundary layer theory. The mixing layer analogy provides analytical linkage between depth, roughness, and velocity for  $h/D < 2$ . The new theory is tested with measurements for natural gravel streams and laboratory flumes [Bathurst et al., 1981; Bathurst, 1985; Colosimo et al., 1988; Hey, 1979].

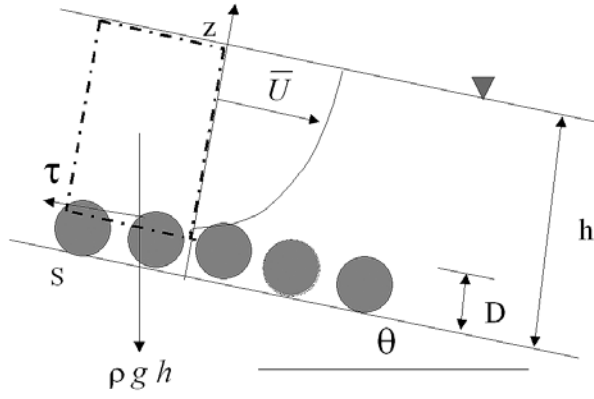
## 2. Theory

[4] Before describing the new theory, we present a brief review of the relationship between,  $z_o$ ,  $n$  and flow resistance for large  $h/D$ .

### 2.1. Deep Layer Formulation (Large $h/D$ )

[5] Consider the wide channel flow represented in Figure 1, subject to the conditions that (1)  $\theta$  is sufficiently

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**Figure 1.** Schematic of the simplified momentum balance for uniform flow in a wide rectangular stream.

small so that,  $\sin(\theta) \approx \tan(\theta) = S$ , where  $S$  is the bed slope, (2) the stream is rectangular and sufficiently wide so that the hydraulic radius  $R_h \approx h$ , (3) the depth  $h$  is much larger than  $D$ , (4) the flow is fully turbulent with statistics that are steady and planar uniform, and (5) the log-law is reasonable for the time-averaged velocity ( $\bar{u}$ ) over the profile depth.

[6] The force balance between frictional and gravitational forces results in

$$\tau_t = \rho g h \sin(\theta) = \rho g h S \quad (1)$$

where  $\tau_t = \rho u_*^2$  is the turbulent stress (i.e., total stress) acting on the streambed,  $g$  is the gravitational acceleration,  $\rho$  is the density, and  $u_*$  is the friction velocity. With these definitions (1) provides

$$u_* = \sqrt{g h S} \quad (2)$$

[7] From the Prandtl-Karman mixing length theory, the velocity profile is approximated by the “log-law”:

$$\bar{u} = \frac{u_*}{k} \ln\left(\frac{z}{z_0}\right) \quad (3)$$

where the overbar denotes time-averaging, and  $k(=0.4)$  is Von Karman’s constant [e.g., *Izakson*, 1937]. In (3), the zero-plane displacement is neglected as  $z \gg z_0$ . The depth-averaged mean velocity is given by:

$$U = \frac{u_*}{k(h-z_0)} \int_{z_0}^h \ln\left(\frac{z}{z_0}\right) dz = \frac{u_*}{k} \ln\left(\frac{h}{e z_0}\right) \quad (4)$$

where we made use of the assumption that  $h - z_0 \approx h$ . Rearranging (4) into the dimensionless form commonly used for flow resistance equations gives

$$\frac{U}{u_*} = \frac{1}{k} \ln\left(\frac{h}{e z_0}\right) \quad (5)$$

For comparison, Manning’s equation for a wide rectangular channel is

$$U = \frac{1}{n} h^{2/3} S^{1/2}. \quad (6)$$

where  $U$  and  $h$  are in  $m-k-s$  units thereby leading to an  $n$  that is not dimensionless. Combining (2) and (6) gives

$$\frac{U}{u_*} = \frac{h^{1/6}}{n\sqrt{g}}. \quad (7)$$

The Darcy-Weisbach equation is

$$\frac{U}{u_*} = \sqrt{\frac{8}{f}} \quad (8)$$

At first glance, (5), (7) and (8) appear to have very different forms. However, for large values of  $\left(\frac{h}{e z_0}\right)$  the logarithmic function is well approximated by the classic 1/7th power law [*Blasius*, 1913; *Hinze*, 1959, p. 479; *Brutsaert and Yeh*, 1970; *Reynolds*, 1974; *Chen*, 1991]

$$\ln\left(\frac{h}{e z_0}\right) \approx \frac{5}{2} \left(\frac{h}{e z_0}\right)^{1/7} \quad (9)$$

Such simple power law approximations have been recognized as early as 1880 and used extensively [*Stevenson*, 1880; *Davies*, 1972; *Brutsaert*, 1982]. Applying (9) to (5)

$$\frac{U}{u_*} \approx \frac{5}{2k} \left(\frac{h}{e z_0}\right)^{1/7} \quad (10)$$

Upon comparing (10) with (7) and (8) we obtain

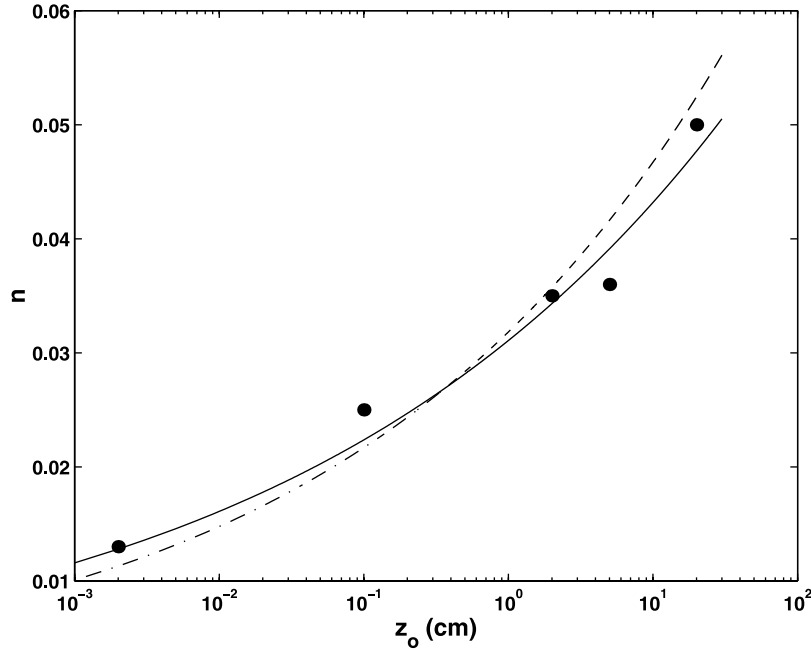
$$n = \left(\frac{2ke^{1/7}}{5g^{1/2}}\right) z_0^{1/7} \approx 0.06 z_0^{1/7} \quad (11)$$

and

$$\sqrt{\frac{f}{8}} = \left(\frac{2ke^{1/7}}{5}\right) \left(\frac{z_0}{h}\right)^{1/7} \approx 0.18 \left(\frac{z_0}{h}\right)^{1/7} \quad (12)$$

The relationship  $n = 0.06 z_0^{1/7}$  is consistent with *Chen’s* [1991] results for hydrodynamically rough flows with a 1/7th power law describing  $U$ . Other studies [e.g., *Raudkivi*, 1990; *Yen*, 1992] have used a 1/6th power law and yielded results that can be extended to arrive at  $n = 0.069 z_0^{1/6}$ .

[8] In Figure 2, we compare the predicted relationship between  $n$  and  $z_0$  with reported measurements of  $z_0$  and  $n$  for similar surfaces (reproduced in Table 1 from the original references) in which  $\frac{h}{D}$  is large. The  $z_0$  estimates in Figure 2 are from atmospheric surface layer experiments while the  $n$  estimates are for water flow over similar surfaces (floodplain data are used because  $\frac{h}{D}$  is likely to be large). The agreement between predictions and measurements is surprisingly good given the simplifying approximations and the crude surface characterizations for which measured  $n$  and  $z_0$  are presented. These types of relationships complement earlier work by *Powell* [1950] in which  $n$  was



**Figure 2.** Comparison between measured  $n$  (solid circles) and modeled  $n = 0.06 \left(\frac{z_o}{100}\right)^{1/7}$  using the deep layer formulation (solid line). For reference,  $n = 0.0389 \left(\frac{30z_o}{100}\right)^{1/6}$  is also shown (Dash-dotted line).

explicitly related to the surface friction factor ( $f$ ) of the Darcy-Weisbach equation.

## 2.2. Shallow Layer Formulation

[9] For shallow streams the log-law is not applicable, as a significant portion of the flow is within and near the roughness layer. Recent approaches adopt semi-empirical corrections to the log- or power law velocity distribution using a combination of divergence functions and incomplete self-similarity approaches [Ferro and Pecoraro, 2000] to reproduce velocity distribution exhibiting nonmonotonic variation. Here, we make use of the fact that such a flow regime is analogous to turbulent flow within and above rigid vegetation canopies. Recent advances [Raupach *et al.*, 1996] in canopy atmospheric turbulence suggest that the structure of the flow near extensive and porous roughness elements resembles a mixing layer with an inflection near the mean height of the roughness elements ( $D$ ) as shown in Figure 3. This inflectional profile connects the slow moving flow within the roughness space to the faster moving fluid above and has been observed in a wide range of canopy experiments including wind-tunnel strips and rods, corn, wheat, eucalyptus and pine forests [Garratt, 1992; Raupach *et al.*, 1996; Katul and Albertson, 1998; Finnigan, 2000].

Detailed  $\bar{u}$  profiles measurements by Marchand *et al.* [1984] suggest that shallow streams exhibit such inflectional profiles, analogous to canopy turbulence. Note that  $\bar{u}$  over rough-wall boundaries do not possess an inflection point.

[10] An approximate mean velocity profile that reproduces the key features of the inflectional profile, including the important flow instabilities, is given by

$$\frac{\bar{u}}{u_o} = 1 + \tanh\left(\frac{z-D}{L_s}\right) \quad (13)$$

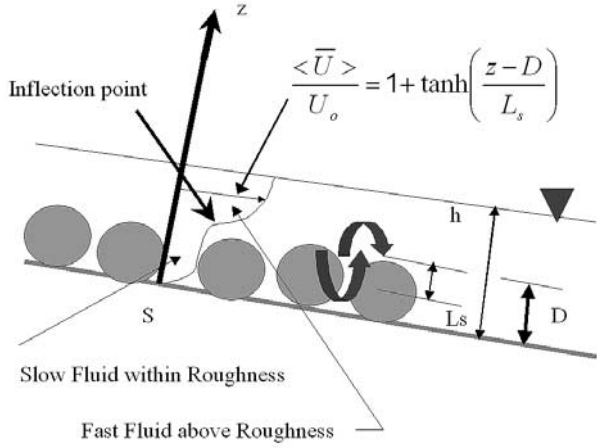
where  $L_s$  is a characteristic energetic eddy size (i.e., mixing length), typically produced by a Kelvin-Helmholtz type instability at  $z = D$  and  $u_o$  is the mean reference velocity at  $z = D$  [Michalke, 1964; Raupach *et al.*, 1996; Metais, 1996]. In this first order analysis we assume that  $\frac{Du_*}{\nu} > 10^5$  so that the mixing layer in Figure 3 is approximately inviscid [Tennekes and Lumley, 1972; Townsend, 1976] where  $L$  is the kinematic viscosity of water ( $\sim 10^{-6} \text{ m}^2 \text{ s}^{-1}$ ). We emphasize that a necessary condition to the generation of such Kelvin-Helmholtz type instabilities is that the mean velocity has a point of

**Table 1.** Published Atmospheric  $z_o$  (Four Decades of Variation) and Stream  $n$  (Fourfold Variation) for Comparable Surface Covers<sup>a</sup>

Surface Type	$z_o$ , cm	$n^b$
Smooth tarmac [Bradley, 1968]	0.002	0.013 (concrete)
Short Grass [Sutton, 1953]	0.1	0.025 (earth with grass)
Airport grass [Kondo, 1962]	2.3	0.035 (floodplain with grass)
Grass, 50 cm tall [Sutton, 1953]	5.0	0.040 (mature field crop)
1–2 m high vegetation [Fichtl and McVeil, 1970]	20	0.050 (brush with heavy tall weeds)

<sup>a</sup>For the  $n$  data of Chow [1959], only mean values are used for deep streams (e.g., floodplain).

<sup>b</sup>From Chow [1959].



**Figure 3.** Onset of free shear (vis-à-vis wall bounded) turbulent flows in shallow streams. The flow within the roughness elements is much slower than the flow in the fluid aloft resulting in a turbulence structure that resembles a free shear flow (e.g., a mixing layer) rather than a rough wall boundary layer.

inflection; Blasius type boundary layers do not admit such instability (i.e., Rayleigh's point of inflection theorem [Drazin and Reid, 1981; Panton, 1984]). The inflection in the mean velocity profile implies an elevated maximum in the mean vorticity profile, which feeds the turbulent vorticity field through these Kelvin-Helmholtz instabilities. Experimental and computational flow visualization of such vorticity production is given by Tritton [1988], Comte et al. [1989], Lesieur [1990], and Ragab and Sheen [1993].

[11] For an incompressible and inviscid mixing layer,  $L_s^{-1}$  represents the wave number of the fastest growing instability. Vertical velocity spectra collected over a wide range of vegetation types, including wind tunnel strips, agricultural crops, and forests, suggest  $L_s \sim \alpha D$  where  $\alpha \approx 0.5$  [see Raupach et al., 1996; Katul et al., 1998; Katul and Albertson, 1999; Brunet and Irvine, 2000; Scanlon and Albertson, 2001], though  $\alpha$  may be different for gravel bed streams. We note that if, in fact, the complex features of the flow within and above gravel bed roughness are characterized by three length-scales because of anisotropy and inhomogeneity, as suggested by Nikora et al. [1998], then arguably the 3-D modes of the Kelvin-Helmholtz instabilities must be considered. However, we can preserve simplicity on the basis of Squires' theorem for inviscid flows [Drazin and Reid, 1981; Monin and Yaglom, 1971], which holds that for any 3-D unstable mode, there exists a corresponding 2-D mode. Therefore  $L_s$  can be estimated from the 2-D rather than the 3-D instability problem. In essence, what is relevant to the present mixing layer theory is the vorticity depth (or  $L_s$ ) of the 2-D Kelvin-Helmholtz wave problem.

[12] With  $L_s \sim \alpha D$  the depth-averaged velocity is given by

$$\frac{U}{u_o} = \frac{1}{h} \int_0^h \left[ 1 + \tanh\left(\frac{z-D}{\alpha D}\right) \right] dz = 1 + \frac{\alpha D}{h} \ln \left( \frac{\cosh\left(\frac{1}{\alpha} - \frac{h}{\alpha D}\right)}{\cosh\left(\frac{1}{\alpha}\right)} \right) \quad (14)$$

By letting  $u_o = C_u u_*$ ,  $\xi = \frac{h}{D}$ ,  $f(\xi, \alpha) = 1 + \alpha \frac{1}{\xi} \ln \left( \frac{\cosh\left(\frac{1}{\alpha} - \frac{\xi}{\alpha}\right)}{\cosh\left(\frac{1}{\alpha}\right)} \right)$ ,

$$\frac{U}{u_*} = C_u f(\xi, \alpha) \quad (15)$$

where  $C_u$  is a similarity constant to be determined later. The result in (15) is highly dependent on the definition of  $D$ . In natural gravel bed streams, the assumption that  $D \sim D_{84}$  is commonly employed [Hey, 1979; Bray, 1979; Wiberg and Smith, 1991], where  $D_{84}$  is a characteristic bed-material size of which 84% of the bed-stream material is finer. Upon comparing equations (7) and (15), an explicit relationship between  $n$  and  $D_{84}$  can be derived and is given by

$$n = \frac{h^{1/6}}{\sqrt{g} C_u f(\xi, \alpha)}. \quad (16)$$

Analogous relationship between  $f$  and  $D_{84}$  can be determined from (8) and (15).

[13] While the use of  $D_{84}$  as a standard measure of bed roughness is common, other measures are equally popular. For example, in many engineering flows, it is common to express surface roughness in equivalent type sand-grain roughness ( $k_s$ ). This roughness measure is defined as the sand-grain size that would, in fully rough flows, reproduce the same friction coefficient as the original surface after Nikuradse's [1933] pioneering work. Interestingly, as discussed by Schlichting [1968], Reynolds [1974], and Yen [1992],  $z_o = k_s/30$  for rough-wall boundary flows which leads to  $k_s \sim 3D_{84}$ . Hence, in (15)–(16),  $D_{84}$  can be replaced by  $k_s/\beta$  if the latter roughness is known or available, where  $2.95 < \beta < 3.5$  [e.g., Hey, 1979; Whiting and Dietrich, 1990; Wiberg and Smith, 1991; Pitlick, 1992].

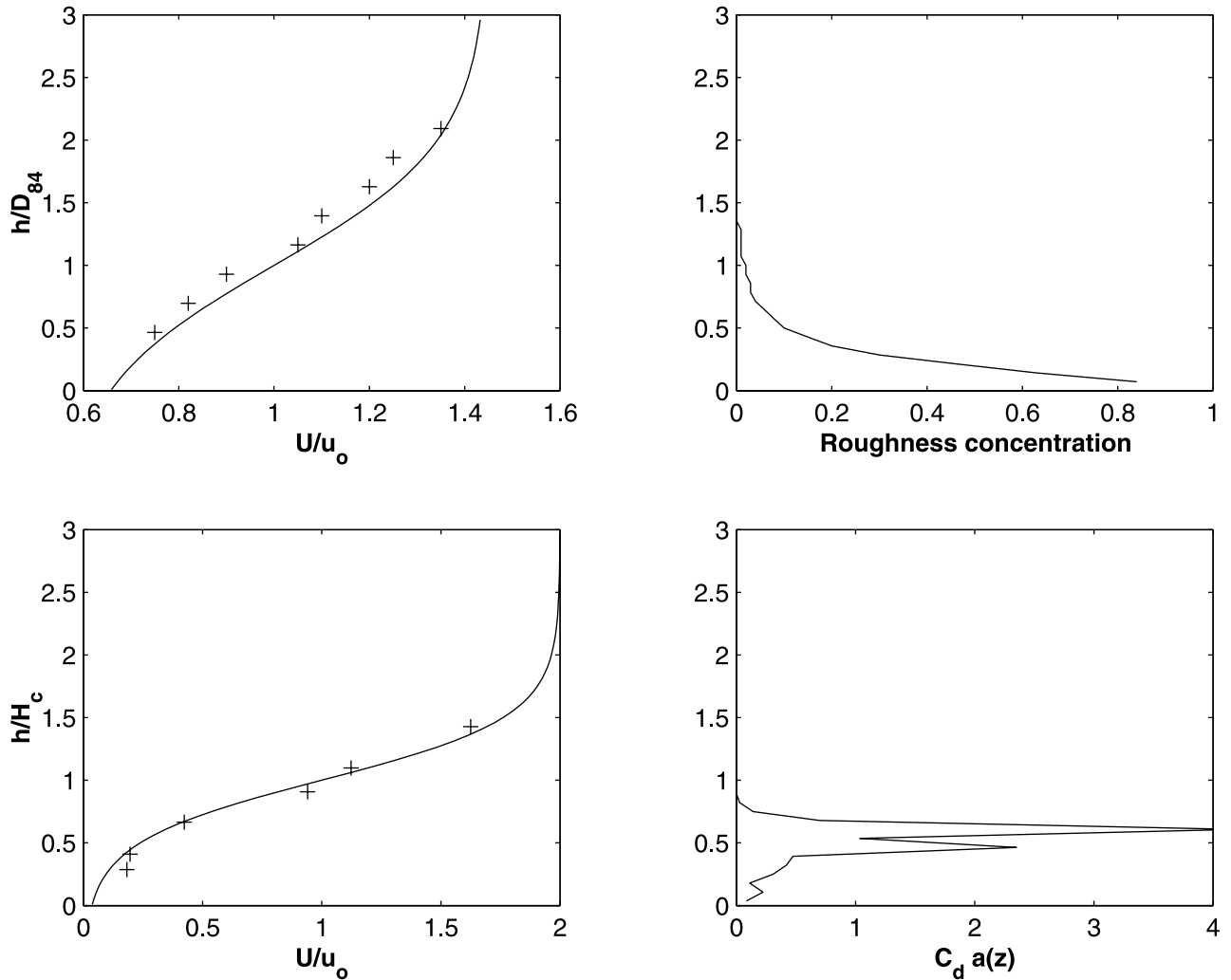
[14] Previous models for flow resistance or  $f$  commonly employ dimensional analysis in which  $f$  is assumed to depend on the following dimensionless groups [e.g., Colosimo et al., 1988]

$$\frac{1}{\sqrt{f}} = f_{sim} \left( \frac{h}{D}, \phi, \psi, R_e, \varsigma, F_r, Y \right) \quad (17)$$

where  $\phi$  and  $\psi$  represent the influence of the cross-sectional channel shape and the form of the grain-size curve, respectively,  $R_e$  is the Reynolds number,  $\varsigma$  is the flow sinuosity,  $F_r$  and  $Y$  are the Froude number and the sediment mobility parameter, respectively, and  $f_{sim}$  is a similarity function determined from experiments. Typically for wide gravel bed streams on steep slopes for which stream reaches can be approximated as straight section,  $R_e$ ,  $\varsigma$ ,  $\phi$ , and  $Y$  are of minor importance when compared to  $h/D$ . It is for this reason that several similarity formulations produce  $f$  that depends on  $h$  and  $D$ . The functional dependence from such studies is empirically derived [e.g., Hey, 1979; Bathurst, 1985]. In contrast, the proposed mixing layer analogy in (14) theoretically describes the form and parameters of  $f_{sim}$  provided  $L_s$  is a known fraction of  $D$ .

[15] The two unknown constants  $\alpha$  and  $C_u$  are required for the proposed theory. To estimate  $C_u$ , we note that the flow near a shallow gravel bed roughness is not identical to either canopy turbulence or a fully rough-wall boundary layer. This suggests that the numerical value of  $C_u$  is bounded by its representative values for these two flow





**Figure 4.** The similarity in inflectional velocity profiles for gravel streams (top) with  $\alpha = 1$  and canopy flows (bottom) for  $\alpha = 0.5$ . The gravel measurements are reported by *Wiberg and Smith* [1991] and the canopy flow measurements (both data sets are shown as pluses) are for a 14 m tall ( $= H_c$ ) pine forest described by *Katul and Albertson* [1998]. The roughness distribution for the gravel stream (top right) is from *Gomez* [1994]. The leaf area density, adjusted by the foliage drag coefficient ( $C_d$ ), is also shown for reference [*Katul and Albertson*, 1998].

types. For dense canopies,  $C_u \approx 3.3$  [*Raupach et al.*, 1996; *Katul et al.*, 1998]. For rough-wall boundary layer flows,  $C_u$  can be estimated from the log-law using  $C_u \approx \frac{1}{k} \ln(\frac{D_{84}}{z_o})$ . Assuming  $z_o \sim D_{84}/10$  [*Brutsaert*, 1982; *Wiberg and Smith*, 1991] results in  $C_u \approx 5.8$ . Hence, in a first order analysis, we choose  $C_u \approx 4.5$  or the mean of these two estimates. With regards to  $\alpha$ , we expect that the sizes of the instabilities defining  $L_s$  to be as large as the obstacle size  $D_{84}$  thereby leading to  $\alpha \sim 1$ . The ability of the fluid to fully penetrate the entire roughness element depth in gravel bed channels differs from the situation in dense canopies in which the instabilities originating at the canopy-atmosphere interface penetrate a limited portion of the canopy depth  $h_c$ . Figure 4 illustrates the effect of  $\alpha$  on two velocity profiles: one measured in a stream [*Marchard et al.*, 1984; *Wiberg and Smith*, 1991], while the other is measured in a pine forest [*Katul and Albertson*, 1998]. The contrast between mean velocity attenuation in streams and canopies is evident with severe attenuation for the canopy case ( $\alpha = 0.5$ ) and

less for gravel bed streams ( $\alpha = 1$ ). The severe attenuation in the case of canopies is perhaps best illustrated via the roughness concentrations, also shown in Figure 4. Notice that much of the roughness concentration is distributed within the lower 20% of  $D_{84}$  for the gravel bed stream while much of the roughness concentration is in the top 50% of the canopy height for the pine forest. Hence, for the pine forest, much of the fluid momentum is extracted within the upper 50% layers in clear contrast to the gravel stream, in which significant momentum is maintained in the top 60% of  $D_{84}$ . It is for this reason that instabilities can penetrate the entire  $D_{84}$  for gravel streams but are restricted to the upper 50% for dense canopies. Table 2 summarizes the key differences between rough wall boundary layer turbulence, canopy turbulence, and gravel bed streams turbulence. A necessary condition for the application of (15) and (16) is that  $\bar{u}(z)$  must satisfy, at some  $z$  along  $h$ , the condition  $\frac{d^2 \bar{u}}{dz^2} (\bar{u} - u_o) < 0$  (Fjortoft's theorem [see *Panton*, 1984]). In practice, this condition is almost always satisfied

**Table 2.** Characteristic Roughness, Velocity, and Mixing Length for Canopy Turbulence, Rough-Wall Boundary Layers, and Gravel Bed Streams<sup>a</sup>

Flow Type	Canopy Turbulence	Rough-Wall Turbulence	Gravel Bed Turbulence
Roughness measure	$h_c$	$z_0$	$D_{84}$
$C_u$	3.3 (neutral flows)	5.8	4.5 (expected value)
Mixing length	$\sim 0.5 h_c$	$\sim k z$	$\sim 1.0 D_{84}$

<sup>a</sup> Here,  $h_c$  is the canopy height,  $z_0$  is the momentum roughness height,  $D_{84}$  is the relevant bed material diameter,  $k = 0.4$  is the Von Karman constant, and  $z$  is the height from wall surface.

for the mean flow inside the roughness elements when  $\alpha \leq 1$ . When  $\alpha \gg 1$ . (e.g., as may occur in a mobile bed), the entire mixing layer analogy may be invalid.

[16] Finally, we note that this approach is consistent with the closure model calculations of *Wiberg and Smith* [1991], which relied on closure principles of the spatially averaged mean momentum equation. In the *Wiberg-Smith* approach, the mechanism by which the roughness elements extract momentum from the fluid is treated explicitly and locally. When the roughness elements are arranged densely, integration of the momentum balance results in a mean velocity profile possessing a strong inflection point analogous to the hyperbolic tangent profile of *Michalke* [1964]. In the proposed formulation, we assume a priori that the flow within the roughness elements is much slower than the flow aloft, thereby leading to a velocity profile exhibiting a strong inflection point (analogous to a mixing layer).

### 3. Results

[17] With  $C_u = 4.5$  and  $\alpha = 1$ , predictions from (15) and (16) can now be independently tested against the experiments summarized in Table 3. In Figure 5, predictions of  $n$  and  $U/u_*$  from the mixing layer formulation are compared with measurements reported for several shallow gravel streams by *Hey* [1979], *Bathurst* [1985], and *Colosimo et al.* [1988] as well as a laboratory flume by *Bathurst et al.* [1981]. The range of  $S$ ,  $D$ , and  $h$  encountered in these data sets are also presented in Table 3. For reference, we compared our results with the standard semi-empirical formulations proposed by *Hey* [1979] and *Leopold and Wolman* [1957] given by

$$\begin{aligned} \text{Hey} \quad \frac{1}{\sqrt{f}} &= 2.03 \log_{10} \left( 12 \frac{h}{D_{84}} \frac{1}{3.5} \right) \\ \text{Leopold} \quad \frac{1}{\sqrt{f}} &= 2 \log_{10} \left( \frac{h}{D_{84}} \right) + 1 \end{aligned} \quad (18)$$

Among these formulations, the Leopold equation is widely used for estimating river roughness in geomorphology

[*Limerinos*, 1970; *Dunne and Leopold*, 1978; *Leopold*, 1994]. In (18), we assumed that  $k_s = \beta D_{84}$  with  $\beta = 3.5$  [*Hey*, 1979]. We emphasize that the two equations in (18) were derived from regression analysis applied to a wide range of gravel bed streams in Europe and the United States.

[18] With  $\alpha = 1$  and  $C_u = 4.5$ , the mixing layer theory agreed well with the semi-empirical regression curves and all measurements (Figure 5) in the range of  $h/D_{84}$  between 0.2 and 7 (Figure 5). A practical advantage of the mixing layer formulation over the formulations in (18) is that for  $h/D_{84} < 0.5$ , the friction  $nD_{84}^{-1/6}$  remains nearly constant and comparable in magnitude to the larger  $h/D_{84}$ . Hence, when  $h/D_{84} < 1$ , the mixing layer formulation is well behaved in terms of reproducing realistic values of  $n$ . In contrast,  $nD_{84}^{-1/6}$  determined from logarithmic relationships for mean velocity display extreme variations when  $h/D_{84} < 1$ . For values of  $h/D_{84} > 7$ , the flow becomes sufficiently deep to resemble a rough-wall boundary layer (vis-à-vis a mixing layer) thereby permitting the use of the log-law.

[19] The mixing layer theory reproduces the measurements and regression models collected or calibrated for high stream roughness conditions (i.e.,  $h/D_{84} < 7$ ). Though not superior to the Leopold or Hey semi-empirical models, our approach is based on an entirely different view of shallow gravel stream turbulence. This suggests that the interactions between roughness elements and the fluid leading to drag and resistance are consistent with a mixing layer analogy.

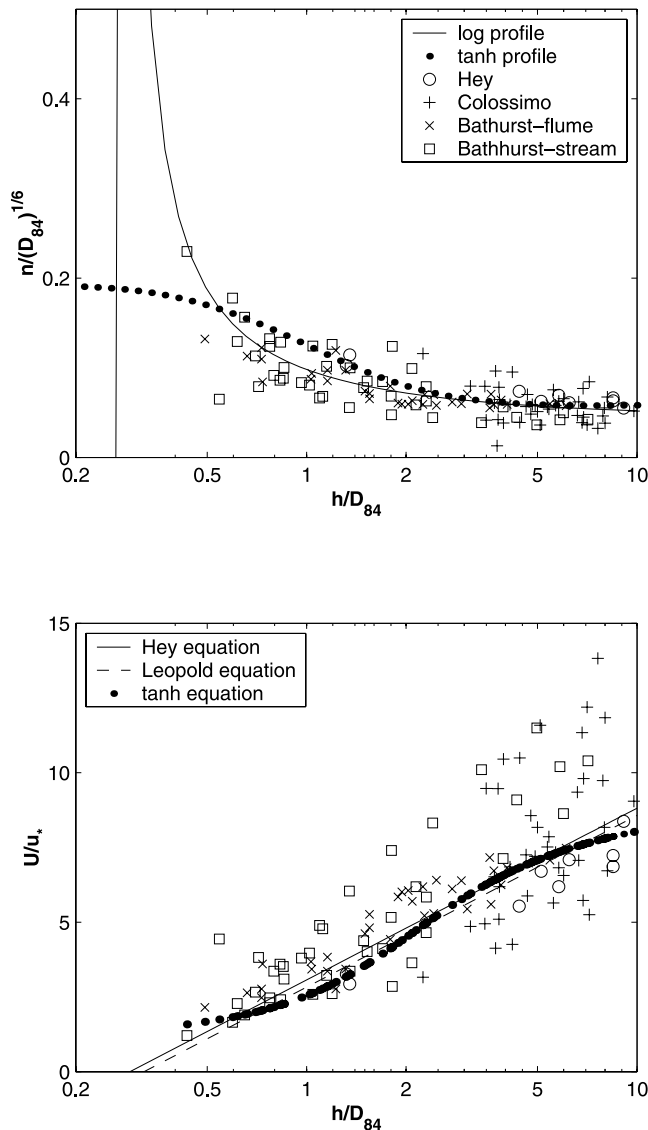
### 4. Conclusions

[20] We proposed a new theory applicable to a wide range of  $h/D$  ( $\in [0.2, 7]$ ), thereby permitting an objective estimate of the flow resistance from surface roughness properties. The new theory acknowledges that the mean velocity within the roughness elements is small, while above the roughness elements the mean velocity is large. The layer between the slow and fast moving fluids leads to free shear instabilities more consistent with a mixing layer than the classical boundary layer analogy. In many natural streams, it is likely that protrusions from the surface into the fluid are sufficiently dense to slow down the fluid but not so dense as to

**Table 3.** Range of Bed Slopes ( $S$ ), Water Depth ( $h$ ), and  $D_{84}$  From the Experiments of *Bathurst* [1985], *Colosimo et al.* [1988], and *Hey* [1979]<sup>a</sup>

Experiment	$N$	$h/D_{84}$	$S$	$h, m$	$D_{84}, m$
<i>Bathurst</i> [1985], rivers, 13 British mountain rivers	41	0.43–7.0	0.004–0.0373	0.14–1.3	0.24–0.50
<i>Bathurst et al.</i> [1981], flume measurements	33	1.1–7.0	0.020–0.080	0.008–0.042	0.007–0.09
<i>Colosimo et al.</i> [1988], river, Calabria, Italy	43	2.2–12	0.0026–0.019	0.26–0.58	0.046–0.12
<i>Hey</i> [1979], rivers, Wye, Dulas Severn, and Tweed, United Kingdom	17	1.3–26	0.0025–0.031	0.29–3.43	0.046–0.25

<sup>a</sup> The number of sections or stages ( $N$ ) for each study is also shown.



**Figure 5.** Predicted and measured  $n/D_{84}^{1/6}$  and  $U/u_*$  as a function of  $h/D_{84}$  for the data sets in Table 3. For reference, the Hey and Leopold formulations in (18) are shown.

impose a no-slip boundary condition on the fluid at a displacement height. This finite velocity within the roughness elements implies that monotonic power law velocity profiles are not always suitable for such problems. We showed that the new mixing layer theory with its inflectional profile yields mean flow velocities at high relative roughness that are consistent with many data sets and widely used semi empirical formulations for flow resistance. Hence, this study provides a unifying framework for much of the ongoing research in atmospheric canopy turbulence and shallow gravel streams, somewhat analogous to the relationship between the log-law and Manning's equation.

[21] With recent developments in laser altimeters, it may be possible to relate  $U$  (or flow rate) to depth, slope, and observable bed characteristics via the proposed theory. High-resolution laser altimeters from aircraft can now measure the three-dimensional structure of terrain (including obstacles and vegetation) and water surface position

[Weishampel et al., 2000; Dubayah, 2000] thereby providing critical information about streambed roughness needed to infer  $D_{84}$  (or  $k_s$ ) and water depth  $h$ . Alternative methods such as bed surface elevation profiles and vertical photographs can be used to estimate  $D_{84}$  or  $k_s$  [e.g., Gomez, 1993; Lane, 2000]. With such detailed measurements,  $n$  (or  $f$ ) and  $U$  may be inferred from a combination of remotely sensed observations and the newly proposed theory in (15) and (16).

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