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# A MODEL FOR A STABLE CORONAL LOOP

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## ABSTRACT

We present here a new plasma-physics model of a stable active-region arch which corresponds to the structure observed in the EUV. Pressure gradients are seen, so that the equilibrium magnetic field must depart from the force-free form valid in the surrounding corona. We take advantage of the data and of the approximate cylindrical symmetry to develop a modified form of the commonly assumed sheared-spiral structure. The dynamic MHD behavior of this new pressure/field model is then evaluated by the Newcomb criterion, taken from controlled-fusion physics, and the results show short-wavelength stability in a specific parameter range. Thus we demonstrate the possibility, for pressure profiles with widths of the order of the magnetic-field scale, that such arches can persist for reasonable periods. Finally, the spatial proportions and magnetic fields of a characteristic stable coronal loop are described.

*Subject headings:* hydromagnetics — Sun: corona

### I. INTRODUCTION

It is difficult, from a plasma-physics viewpoint, to understand the relatively long-lived, large-scale loops observed in the solar corona near active regions (Kahler, Krieger, and Vaiana 1975; Foukal 1975; Vorpahl *et al.* 1975). This is true because of the known tendency of cylindrical and/or toroidal plasma structures to be unstable on the very short magnetohydrodynamic (MHD) time scale (Newcomb 1960; Goedbloed 1971). This time is of order $S^{-1/2}$ ($S \equiv$ magnetic Reynolds number $> 10^9$ for coronal structures) smaller than the resistive flaring time (Van Hoven 1976), to say nothing of the observed quiescent period between flares.

The only sure influence for stabilization is the impression of a strong axial magnetic field, $B_\parallel \gtrsim (R/a)B_\perp \gg B_\perp$, where $R$ is the loop radius of curvature, $a$ the filament radius, and $B_\perp$ the field transverse to the axis, and the last inequality takes account of the large observed values of the (quasi-toroidal) *aspect ratio* $R/a$. Such a condition on the field components is difficult to justify in the case of a narrow free-standing solar arch since it requires the existence of large circulating currents in the external plasma. This does not correspond to the accepted picture of the structure of the surrounding coronal field, which must be nearly force-free.

A clue to the resolution of this problem comes from some new EUV observations of the temperature and density structure of these elongated active-region loops (Foukal 1975, 1976; Levine and Withbroe 1976). These data indicate the existence of a shell structure, with positive radial pressure gradients near the core. Gradients of this sign should provide a stabilizing influence over the uniform-pressure, force-free case, which is unstable for relevant wavelengths (Voslamber and Callebaut 1962; Goedbloed and Hagebeuk 1972).

In this paper we give preliminary results from our calculation of a new plasma model of such a loop, based particularly on the recent EUV data, which we briefly describe. We fit the observed pressure profile and determine the corresponding magnetic-field structure required for static equilibrium. We then draw on the plasma theory developed for the study of toroidal laboratory-fusion devices to treat the question of the dynamic MHD stability of this configuration with respect to the relevant global perturbations.

### II. OBSERVATIONS

Elongated coronal active-region arches have been observed in X-rays and the EUV, with vastly improved spatial resolution, from the Apollo telescope mount on *Skylab*. The observed loop structures can be unstable (Kahler *et al.* 1975; Vorpahl, Tandberg-Hanssen, and Smith 1975) after varying periods, or sometimes can be stable for days (Landecker 1975; Vorpahl *et al.* 1976). Their typical aspect ratios are of the order of 10, with widths apparently sometimes less than the limit of resolution ($\sim$2000 km) and lengths greater than $10^4$ km.

For our present purposes the superior temperature resolution (at somewhat poorer spatial and temporal resolution) of the ATM EUV observations provides the more useful plasma information. To the extent that the X-ray and EUV active-region arches are similar, we can obtain a clue to their relative stability, or metastability, from their observed pressure structure.

Foukal (1975) has determined that the visibility of these coronal loops in the softer EUV lines is the result of lower temperatures on their axes, by up to two orders of magnitude below those in the external corona. Less certain (more model-dependent) are spatially nonmonotonic density variations in concentric shells, by a factor of 5. Levine and Withbroe (1976) have corroborated these temperature-variation results for a particularly well observed loop. Recent results by Foukal (1976) have emphasized the stationary character of these loops and their uniformity along their axes.

It has been shown that the observed global structure of systems of clustered loops can be approximated by a potential field (Poletto *et al.* 1975) or a small-$\alpha$, force-free field (Vorpahl and Broussard 1977) built upon the measured photospheric sources. However, the present observations do not clearly distinguish between these possibilities, for reasons derivable from the work of Levine and Altschuler (1974). It is shown there that large currents, in relation to the fields and dimensions encountered, are required for significant twisting of fields of such large scale.

## III. FIELD STRUCTURE

Here we use the observed small-scale pressure gradients to specify a magnetic structure which is locally non–force-free, yet is imbedded in a global force-free field.

We take advantage of the large aspect ratio and observed longitudinal uniformity (Foukal 1976) to make the approximation of cylindrical symmetry, which has been used successfully in predictions of the stability of similarly shaped laboratory devices, such as the tokamak. In this case the allowable external force-free field, which remains so even in the presence of resistive diffusion, is that of Lundquist (1951; Jette and Sreenivasan 1969).

The observed radial pressure profile is simulated by a simple Fourier expansion, which preserves its essential features. We write, for $r \leq a$,

$$\frac{p(r)}{p_c} \approx P_1 - P_2 \cos \pi \frac{r}{a} - P_3 \cos 2\pi \frac{r}{a}, \quad (1)$$

which provides for fitting the values of the pressure minimum at $r = 0$, a possible pressure maximum (Foukal 1975) at $r \approx a/2$, and for a smooth connection to the uniform ambient coronal pressure $p_c$ at $r = a$, the edge of the loop.

Most of the stability calculations described here use the values $P_1 = 0.55$, $P_2 = 0.45$, and $P_3 = 0$, describing a pressure minimum at the origin $p(0) = 0.1p_c$ which is indicated by Foukal's recent (1976) data. Several nonmonotonic profiles, with pressure maxima $p_m > p_c$, have also been tested, as will be mentioned in the following section.

Inside such a loop profile the equilibrium fields are found from the static momentum-transfer equation (Goedbloed 1971),

$$0 = -\nabla p + \boldsymbol{J} \times \boldsymbol{B},$$

where $p$, $\boldsymbol{J}$, and $\boldsymbol{B}$ are the pressure, current density, and magnetic field. Separating the current density $\boldsymbol{J}$ into components parallel and perpendicular to $\boldsymbol{B}$, we see that only $\boldsymbol{J}_\perp$ enters this force-balance equation, so that we need another relation for $\boldsymbol{J}_\parallel$. Since $\boldsymbol{J}_\parallel$ can always be written as $\boldsymbol{J}_\parallel = \alpha\boldsymbol{B}$, where the constant of proportionality is in general a function of $r$ and $t$, we must only specify the dependence of $\alpha$ on these variables. In the following we will select the simplest case, namely $\alpha$ constant and uniform. This seems the most appropriate choice for problems which are stationary, at least on a certain time scale, since spatial and temporal variations of $\alpha$ are strongly interrelated (Jette and Sreenivasan 1969). Moreover, with a constant $\alpha$ the field maintains its shape during the slow decay introduced by finite-resistivity effects.

In Ampere's law we then have

$$\begin{aligned} \nabla \times \boldsymbol{B} &= \mu_0(\boldsymbol{J}_\parallel + \boldsymbol{J}_\perp) \\ &\approx \alpha\boldsymbol{B}(r) + \mu_0 \boldsymbol{B} \times \nabla p/B^2 . \end{aligned} \quad (2)$$

The $r$ component of this equation specifies $B_\theta(r)$ in terms of $B_z(r)$ and $p(r)$. The parallel component can be put in the form of a Riccati equation (Davis 1962), which provides the unexpected result that $B_\theta(r)/B_z(r)$ must be the same as that in the constant-pressure Lundquist (1951) field. [This is the Bessel-function solution of the equivalent force-free equation $(\nabla^2 + \alpha^2)\boldsymbol{B} = 0$ in cylindrical geometry.] Thus in the present case we obtain

$$B_z(r) = B_0(r)J_0(\alpha r)$$

and

$$B_\theta(r) = B_0(r)J_1(\alpha r), \quad (3)$$

where the common field amplitude $B_0(r)$ is given by

$$\frac{d}{dr}(B_0{}^2) = -\frac{2\mu_0}{J_0{}^2(\alpha r) + J_1{}^2(\alpha r)} \frac{dp}{dr}, \quad (4)$$

from the $r$ component of equation (2). The requirement that $B_0{}^2 \geq 0$ puts a weak constraint on the allowed values of the parameters $\gamma \equiv \alpha a$ and $\beta_a \equiv 2\mu_0 p_c/B_0{}^2(a)$, otherwise leaving considerable leeway in their choice. The continuity of $\boldsymbol{B}$ and $\boldsymbol{J}$, and of their radial derivatives, ensures that the fields given by equation (3) also hold for $r > a$, with the same value of $\alpha$ and $B_0(r) = B_0(a) =$ constant.

The equilibrium fields resulting from this construction differ significantly from the Lundquist field (in the pertinent regime in which $\beta_a$ is not infinitesimal) for $r < a$, and then connect smoothly to this limiting force-free form in the external uniform-pressure region.

## IV. MAGNETOHYDRODYNAMIC STABILITY

The next element of a plasma model is the determination of the short-time-scale stability ($\tau \approx a/v_A$, where $v_A$ is the Alfvén speed) of the static equilibrium.

In the absence of MHD stability, coronal arches would not persist long enough to be observed. The calculation of this condition must, in general, cover two kinds of perturbations, localized and global (in relation to the cross section of the loop).

### *a*) *Local Stability*

This requirement is relevant to parts of the profile where $dp/dr < 0$, such as that where $r \geqslant a/2$ in equation (1) in those cases where $P_3 \neq 0$. A stability criterion for localized perturbations of this kind has been given by Suydam (1958), and extended to toroidal geometry by Mercier (1960).

The Suydam criterion dictates the minimum field shear necessary to stabilize the effects of a local negative pressure gradient. Since these two aspects of the equilibrium are already connected by the static considerations of the last section, localized stability has the effect of limiting the allowable ranges of the amplitude and the radial scale of the magnetic field.

Our calculations of the result of this straightforward necessary condition show that it indeed puts rather tight bounds on the acceptable values of $\gamma(\approx 5)$ at noninfinitesimal pressures ($\beta_a > 0.01$), for those cases where $P_3 \neq 0$ in equation (1), thus greatly restricting the stability boundaries at short wavelengths. In addition, an absolute maximum limit is put on $\beta_a$ ($\leqslant 0.15$ at $p_m/p_c = 2$) which effectively prevents the attainment of *global* stability, since this separate requirement demands large $\beta_a$ values, as will be described in § IV*b*. These results effectively limit our consideration to positive radial pressure gradients, confirming the recent observations of Foukal (1976).

### *b*) *Global Stability*

Having now specified and restricted the magnetic fields capable of supporting a significant pressure gradient, we must evaluate the more stringent requirement of their stability to global perturbations, that is, those which extend over a significant range of radius. This is done, in the cylindrical approximation, by the method of Newcomb (1960), which is equivalent to a calculation of the condition of marginal stability (Goedbloed 1971) of the equilibrium. This is specified by the linearized momentum-transfer equation with $\partial/\partial t = 0$, along with the applicable boundary conditions.

In this symmetry the marginal equation of motion for the small-amplitude radial displacement $\xi_r(r) \times \exp(im\theta + ikz)$ of a compressible fluid element is well known (see eq. [18] of Goedbloed 1971). The solution is singular at the zeros of

$$F(r; k, m) = \boldsymbol{k}\cdot\boldsymbol{B} = kB_z + (m/r)B_\theta = B_0(r)[krJ_0(\alpha r) + mJ_1(\alpha r)]/r\,, \quad (5)$$

so that it is more convenient to use a dependent variable of the form

$$R_r(r) = \frac{B_0(r)}{F(r)}\,\xi_r(r)\,,$$

related to the radial component of the perturbation magnetic field. The "small" solution (Newcomb 1960) of this transformed equation is well behaved at the singular points $r_s(k, m)$, where $F(r_s; k, m) = 0$, if the Suydam criterion is satisfied (Goedbloed 1971).

The Newcomb (1960) necessary and sufficient stability criterion may then be stated as follows. (i) Such a compressible cylindrical-pinch plasma equilibrium is MHD stable for all perturbations if it is stable for the $m = 1$ and very long wavelength $(k \to 0)m = 0$ modes. (ii) Stability must be tested independently in each *interval* $\{(0, r_{s1}), (r_{s1}, r_{s2}), \ldots\}$ of $r$, where $r_{si}(k, m)$ is the $i$th singular point, counting from the origin. (iii) In each interval one integrates the equation of motion, starting from the end points with the small solution (Newcomb 1960) as an initial condition, toward some interior point $r_{0i}$; if $R_{r+}$ and $R_{r-}$ are the right and left solutions, they must not vanish before they reach $r_{0i}$ and, at this point,

$$\frac{d}{dr}(\log R_{r-}) > \frac{d}{dr}(\log R_{r+}) \quad (6)$$

must be satisfied.

We have tested this criterion by computer integration of the equation of motion for $R_r$, using the pressure and field profiles previously described. We find that stability is improved by the positive pressure gradient near the axis. That is, the plasma is stable for smaller values of $|k/\alpha|$ than in the case of the pure force-free field (Voslamber and Callebaut 1962).

The critical importance of stability down to low values of $|k/\alpha|$ may be seen as follows. The axial wavelength of allowable perturbations must be less than twice the length of the loop, $\lambda \equiv 2\pi/|k| < 2\pi R \approx 20\pi a$. This condition may be restated in the form $|k/\alpha| > 0.1/\gamma$, providing a lower wavenumber limit beyond which all short-wavelength perturbations must be stable so that the loop itself will be stable.

For nonmonotonic pressure gradients, the Suydam-determined local-stability restriction of $\gamma \approx 5$ (and $\beta_a \leqslant 0.15$) gives difficulty in this respect. We have tested pressure profiles with maxima $p_m \approx p(a/2) \geq 2p_c$ and have found global stability only for wavelengths $\lambda \leqslant 12a$. Stable coronal arches could exist under these conditions, but their radius of curvature $R$ is too small to be adequately treated by the cylindrical approximation used here.

We turn then to the specific case of a *monotonic* pressure gradient ($P_3 = 0$), with a central pressure $p(0) = 0.1p_c$. Since the force-free case is known to be completely stable for $m \neq \pm 1$ and $|k/\alpha| \geq 1$ (Voslamber and Callebaut 1962), and a positive pressure gradient only improves the situation, we need only consider $m = 1$ perturbations for $|k| < \alpha$ ($m \to -1$ only changes $k \to -k$).

Under these conditions, the central zone of the equilibrium $0 < r < r_{s1}$ is known to be the most unstable for the pure force-free field (Voslamber and Callebaut 1962), with critical wavenumber limit $|k| > 0.27\alpha$, and so it must be considered first.

Since it is an essential feature of our model that

TABLE 1

STABLE PARAMETERS FOR THE MONOTONIC PRESSURE CASE*

| $\gamma$ | $\beta_a$ 0.8 | 1.0 | 1.25 | 2.0 |
|---|---|---|---|---|
| 5.0.......... | $S_{0.1}$ | $S_{0.1}$ | $S_0$ | ... |
| 4.0.......... | $S_{0.1}$ | $S_0$ | ... | $S_0$ |
| 2.0.......... | $U$ | $U$ | $S_{0.1}$ | |
| 1.5.......... | $U$ | $U$ | $U$ | $S_{0.1}$ |

* $S_0$ denotes stability for all $k$ in the first singular zone. Since the second zone is unstable for $|k/\alpha| < 0.093$, the first need only have stability for $|k/\alpha| > 0.1$, which is denoted $S_{0.1}$. $U$ means unstable. The blank entries have not yet been tested.

there is a non-zero pressure gradient near the origin, the full non–force-free perturbation equation must be used, and the stability test of equation (6) made for each equilibrium configuration as specified by $\gamma$ and $\beta_a$. We have not yet, at this preliminary stage, made an exhaustive fine-scale scan of the $(\gamma, \beta_a)$-plane, since considerable computer time is involved. However, we have found the approximate parameter boundaries for stable perturbations in the crucial (most unstable) first radial interval.

The results of our preliminary calculations are detailed in Table 1. They show that the equilibrium structure described in § III is stable for a relatively wide range of $\gamma$ (the ratio of pressure scale width to field scale width) at larger values of $\beta_a$ [the ratio of gas pressure to magnetic pressure divided by $J_0^2(\gamma) + J_1^2(\gamma)$]. We find complete stability for *all* wavenumbers in the range $1 \leqslant \gamma \leqslant 6$ at $\beta_a = 10$. When $\beta_a \approx 1$, all-$k$ stability is attained only for $\gamma \approx 4$. At this value of $\beta_a$, $|k/\alpha| > 0.1$ stability exists for $3 \leqslant \gamma \leqslant 6$. This weaker condition is sufficient, however, because it subsumes the separately determined stability behavior of the external singular zones.

For $\gamma \leqslant 3.8$, the positive pressure gradient does not penetrate into the second singular interval in $r$. Thus, in this and succeeding intervals, the fields are purely force-free. We have, for the first time in our knowledge, now calculated the stability limits in these zones. (Previously, the dominant instability was that of the first interval [Voslamber and Callebaut 1962], so further tests were less important.) The second zone is stable for $|k/\alpha| > 0.093$, and succeeding intervals have smaller instability ranges.

As a result of these considerations, all radial zones of the coronal loop model, with $R = 10a$, are stable for $\gamma \leqslant 1.1$ when $\beta_a \geqslant 3.6$. This condition is obtained by equalizing the $|k/\alpha|$ instability boundaries in the first and second singular intervals, thereby bracketing all of the remaining ones.

For $\gamma \geqslant 4.2$, the positive pressure gradient extends into the second singular interval, allowing stability for longer wavelengths. This could allow a shorter loop, with aspect ratio $\sim 3.3$, to be stable up to $\gamma \approx 5$ and therefore over a range of $\beta_a$ values.

The end result of these calculations indicates a reasonable, but not large, spectrum of magnetic-field parameters which will confine the observed coronal-loop structures. A specific description of the physical characteristics of such a stable arch will be given in the Conclusions.

## V. CONCLUSIONS

We have described a complete plasma model for the pressure profile of a coronal loop, based specifically on EUV observations. We have derived the required equilibrium magnetic fields and have calculated the MHD stability of this configuration for a range of the relevant plasma parameters.

The long-wavelength stability limits of a positive-pressure-gradient model are rather narrow, indicating a unique shape for the arch (within a length-scale factor).

A loop with major/minor radius ratio ($R/a$) of 10, and with a central temperature (pressure) of one-tenth of that in the surrounding corona, is supported in equilibrium by fields of the form given in equation (3). This type of arch is completely MHD stable for values of $\gamma = \alpha a \approx 1$ and $B_0(a) \approx 0.45(\mu_0 p_c)^{1/2}$.

For this particular stable combination of pressure/field parameters, the magnetic field at the edge of the loop (computed from typical coronal active-region pressures) is of order 2 gauss and the axial field at the center is about 30 gauss. These are not felt to be unacceptably large values for the active-region corona.

A loop conforming to this model would persist for an indefinite time in the absence of the effects of finite resistivity (Van Hoven 1976). We plan a subsequent investigation of the flaring, or nonideal MHD, behavior of such coronal arches.

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