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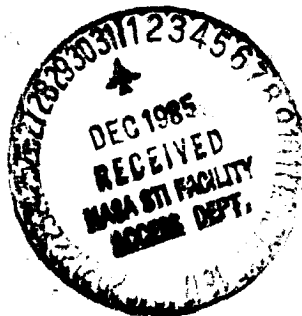
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A Model for Closing the Inviscid Form of the Average-Passage Equation System

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A MODEL FOR CLOSING THE INVISCID FORM OF THE AVERAGE-PASSAGE EQUATION SYSTEM

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ABSTRACT

A mathematical model is proposed for closing or mathematically completing the system of equations which describes the time-average flow field through the blade passages of multistage turbomachinery. These equations referred to as the average-passage equation system govern a conceptual model which has proven useful in turbomachinery aerodynamic design and analysis. The closure model is developed so as to insure a consistency between these equations and the axisymmetric through-flow equations. The closure model was incorporated into a computer code for use in simulating the flow field about a high-speed counter-rotating propeller and a high-speed fan stage. Results from these simulations are presented.

INTRODUCTION

Engineers have long recognized the difficulty associated with adopting a "First Principle" approach based on directly solving the Navier-Stokes equations for the purpose of designing (or analyzing) vehicles which operate in high Reynolds number turbulent flows. However, numerous examples exist, such as turbomachinery blading, aircraft wings and bodies, inlets and nozzles, which clearly show that models which describe an "averaged" flow state can be used to design aerodynamic vehicles and provide answers to many aerodynamic problems. In both external and internal aerodynamics, the "averaged" state most often modeled is one in which the flow appears steady. In general, the number of equations associated with this "averaged" flow representation does not equal the number of unknowns. The problem of mathematically completing this system of equations so that they may be solved is referred to as the closure problem. The flow models associated with the completed system of equations must be considered semi-empirical for they rely heavily on empirical correlations to introduce the effects of turbulent motion and, in the case of turbomachinery, the additional effects of unsteadiness and spatial nonuniformities into these "averaged" flow representations. For nonturbomachinery application, the equation governing such a flow is the familiar

Reynolds-averaged Navier-Stokes equation. In general, the length scales associated with this equation are sufficiently restricted so as to make them amenable to numerical simulation. Indeed there is considerable activity these days in the external aerodynamic community to develop numerical simulators based on these equations for flows over an entire aircraft.

For turbomachinery involving more than one blade row, the Reynolds-averaged form of the Navier-Stokes equations do not describe a flow which is steady in time. On the contrary, they describe a flow which is highly unsteady in which blade rows are moving relative to one another, generating disturbances whose time scales range from a fraction of wheel-speed to many times that of blade passing frequency and whose length scales range from the circumference of the machine to the thickness of the laminar sublayer region of the turbulent boundary layers. Simulation based on the Reynolds-averaged Navier-Stokes equations are well beyond the capabilities of today's computers for all but the simplest of multistage geometries. They also do not govern the conceptual flow model traditionally used to design multistage turbomachinery. As noted above, multistage designs are based on flow models in which the flow appears steady within each blade row. In addition, with respect to a given blade row, these models assume the flow to be spatially periodic from one blade passage to another. In Ref. 1, a mathematical derivation of the equations governing this flow was presented. These equations were referred to as the average-passage equation system. This derivation was carried out for arbitrary configurations and clearly showed the relationship between the Navier-Stokes equations, their Reynolds-averaged form, and their average-passage form. The closure problem associated with the average-passage form of the Navier-Stokes equations was also identified. This work put the average-passage model on a sound mathematical base equivalent to that of the Reynolds-averaged Navier-Stokes model. A brief summary of that work is presented in the next section. The purpose of the present work is to elaborate further on the issue of closure for the average-passage equation system and to propose a closure model for the inviscid form of this equation system. This

closure model was used to obtain the results presented in Ref. 2. Those results and the ones to be presented in this work show that the present model appears applicable to configurations in which the average-passage flow field is nearly irrotational between blade rows.

MODEL EQUATION HIERARCHY

In Fig. 1, a hierarchy of equations are shown which can be used to analyze turbomachinery flows. The Navier-Stokes equations appear at the upper left-hand corner of this figure. These equations are assumed to provide a complete description of the flow field, including a complete description of turbulent motions. To use these equations as a basis for simulating turbomachinery flows requires sufficient computer capacity to resolve all of the time and length scales associated with high Reynolds number flows. In addition, since turbomachinery flows are statistically nonstationary, a sufficient number of computations would have to be performed over a range of randomly chosen initial conditions to insure a statistical steady-state description of the flow. Such simulations are clearly beyond the capacity of today's most advanced computers. The next box (i.e., Fig. 1) contains the Reynolds-averaged form of the Navier-Stokes equations. They are derived by ensemble averaging the Navier-Stokes equations and hence govern a deterministic description of the flow field. An illustration of this description for a two-stage configuration in which the first and second rotors have five and four blades respectively while the first and second stage stators have four and five blades is presented in Fig. 2. The rotors rotate relative to the stators, and, therefore, the flow will be unsteady in either the rotor or stator frame of reference. As noted previously, the time scales associated with this unsteady flow are quite diverse, which makes simulation for all but the most simple of geometries beyond the capabilities of today's computers. The closure problem associated with these equations requires the modeling of the familiar Reynolds stress and energy correlations. It is by means of these correlations that the "average" effects of random fluctuations in momentum and energy of a fluid particle are introduced into the equations governing the deterministic flow field.

The third box from the left in Fig. 1 represents the time-averaged form of the Reynolds-averaged Navier-Stokes equations. These equations govern the time-averaged flow field as viewed by an observer whose frame of reference is fixed to a given blade row. An illustration of this description for the two-stage configuration used to illustrate the Reynolds-averaged flow model is also presented in Fig. 2. All rotating blade rows have a unique time-averaged flow field associated with them. In a similar fashion, all nonrotating blade rows have their own time-averaged flow field representation. These two flow fields are not the same. For both flow fields, the blade rows which rotate relative to the blade rows which are stationary (i.e., with respect to one another) appear smeared. Their physical appearance is very similar to what one observes when viewing a high-speed propeller. Within the context of the time-averaged flow description, these smeared blade rows are replaced by actuator ducts (i.e., actuator ducts of finite thickness). These ducts are represented by a body-force distribution which can add or extract energy from the flow. In addition, the time-average flow equations contain correlations

between time-varying flow variables. These correlations arise because the Reynolds-averaged Navier-Stokes equation is nonlinear. These correlations represent the time-average of the fluctuating density field and products of the fluctuating velocity field as well as the time-average of the fluctuating density, fluctuating velocity, and fluctuating total enthalpy field. It is through these correlations that the "averaged" effect of the relevant unsteady physical phenomena is introduced into the time-averaged representation. The modeling of the body forces and energy sources associated with the smeared blade rows and the temporal correlations, plus the modeling of the time-averaged Reynolds stresses, forms the closure problem associated with the time-averaged equations. Finally, it should be noted that, for a single-stage configuration, the time-averaged flow field associated with either blade row will be spatially periodic over the pitch of that blade row. Thus, if the closure issue associated with the time-averaged representation can be addressed without undue complexity, it should be feasible to conduct a simulation based on this flow model for a single stage.

For a multistage configuration in which the number of rotor blades differ from rotor to rotor, or for which the number of stator blades differs from stator to stator, the time-average flow field will not, in general, be spatially periodic over the pitch of any given blade row. An averaging-procedure may be introduced which transforms this spatially aperiodic flow field into one that is periodic over the pitch of a given blade row. The resulting flow field is referred to as the average-passage flow and appears in the fourth box from the left in Fig. 1. Each blade row in a multistage machine has associated with it an average-passage flow field. An illustration of this description is shown in Fig. 2. For the two-stage machine under consideration there exists four average-passage flow descriptions due to the number of blades assigned to each wheel. The geometry of neighboring blade rows (rotating and stationary) for which the blade count is not an integral multiple of the blade row of interest, and are stationary relative to this blade row, appear smeared in this flow description. Their appearance is similar to that of the rotating blade rows in the time-averaged flow description. It should be noted that all of the blade rows which rotate relative to the blade row of interest appear smeared, since the average-passage description is also a time-averaged description. The four average-passage flows illustrated in Fig. 2 are coupled to one another through a system of body forces, energy sources, and temporal and spatial correlations. The closure problem associated with this flow description consists of developing mathematical expressions for the spatial and temporal correlations in addition to the body forces and energy sources. These correlations introduce the transport on the "average" of momentum and energy between the time-averaged representation and the average-passage representation.

Many analyses currently used to analyze multi-blade row turbomachinery involve iterating between a meridional flow analysis and a blade-to-blade analysis. Within the context of the present discussion, these analyses may be viewed in one of two ways. They may be thought of as attempting to describe the average-passage flow field. If one gives these analyses this interpretation, then one immediately notes that their derivation lacks mathematical rigor. As a result, the closure problem associated with the average-passage representation is never addressed,

for it is completely overlooked. On the other hand, one may interpret these as axisymmetric analyses in which the blade-to-blade solution, along with some empirical correlations, is used to close the meridional flow equations. In this case, these analyses are rigorous because the closure problem that being the closure of the axisymmetric representation is generally clearly defined. However one wishes to interpret these analyses, one must be impressed with the degree of accuracy with which they predict the axisymmetric flow field in the neighborhood of design conditions. As one moves away from the neighborhood of the design point, however, the validity of these analyses appears to degenerate quickly. This disagreement is thought to be due to the inability of the blade-to-blade model to properly account for large spanwise migration of flow which occurs at these off-design conditions. To analyze such situations, a true three-dimensional analysis is needed. The average-passage model provides a framework for developing such an analysis, as illustrated by the work presented in Ref. 2. The accuracy of such simulation will, of course, depend upon the validity of the closure model used in the simulation.

The next box in Fig. 1 represents the axisymmetric flow model, which is the mainstay of many turbomachinery design systems. The field equations for this model can be derived by tangentially averaging the average-passage equation system. An illustration of the geometry associated with this representation is also provided in Fig. 2. Each average-passage flow model can be related to an axisymmetric model. The equations governing these four axisymmetric models must be equal to one another, for there can only be one axisymmetric or through-flow representation of the flow field within a multiblade row configuration. The average-passage equations thus define the three-dimensional passage flows having a common axisymmetric flow description. All of the blade rows within the axisymmetric description appear smeared and are mathematically replaced by actuator ducts. These ducts exert a force on the fluid which may add or extract energy from the flow. There may also be energy sources or sinks within the ducts which are associated with blade heat transfer. Over the years, numerous publications have appeared which dealt with modeling these forces and the energy sources. Quite often they are estimated from cascade or blade-to-blade analyses tempered by empirical correlations. In addition, the axisymmetric or through-flow equations contain correlations between temporal varying flow variables as well as correlations between spatial varying flow variables. These correlations introduce on the "average" the effect of radial transport of momentum and energy from the average-passage representation. Only very recently have models for these correlations appeared in the open literature. Sehra (3) was one of the first to attempt to incorporate these correlations into a through-flow code. His correlation model was based on data obtained from a high-speed isolated rotor test. Jennions (4) modeled these correlations using results from an inviscid blade-to-blade analysis. He was able to develop an iterative procedure for incorporating these correlations into a through-flow analysis. Finally, the Adkins and Smith (5) model for accounting for the effects of the spanwise mixing in multistage machinery, may be thought of as an attempt at modeling the correlations which appear in the axisymmetric model. The last box in Fig. 1 represents a quasi-one-dimensional equation system. These equations result from averaging the axisymmetric equation over the span of the flow annulus. This equation system is often used in engine stability

studies and in preliminary design to establish the flow properties along the pitch-line of a machine. Closure of this system of equations can be quite involved. It requires models for the blade forces, energy sources, spatial and temporal correlations associated with the blade-to-blade flow field, as well as a model for the force exerted by the casing on the flow.

The flow models identified in Fig. (1) are by no means complete, nor were they ever intended to be complete. The purpose of this figure was to illustrate symbolically the connection between a hierarchy of equations associated with turbomachinery aerodynamics. It is hoped that the rational derivation of the average-passage equation system will ultimately lead to the development of three-dimensional viscous computer codes for multistage configurations. Such codes will enhance our ability to analyze turbomachinery flows, especially at off-design conditions. For it is our inability to accurately predict off-design performance of multistage machinery which is often the major contributor to their high development costs and not problems associated with poor design performance. In the next section, the closure model associated with the inviscid form of the average-passage equation system will be developed.

THE CLOSURE PROBLEM

For simplicity we shall only address the closure problem associated with solving the inviscid form of the average-passage equation system as it pertains to a single stage. A solution to the corresponding multistage problem can be obtained by a direct extension of the analysis which follows. For a single-stage configuration, each blade row has associated with it an average-passage equation system. As noted in the previous section, the dependence of the flow through the first blade row upon that through the second is introduced by means of a body force, energy source, and time-average correlations between fluctuating flow variables. Likewise, a corresponding dependency exists between the flow through the second blade row and that through the first. In Ref. 1, the body force and energy source which appear in the inviscid form of the average-passage equation system were shown to depend upon the ensemble-averaged pressure. This "averaged" pressure was estimated from samples of the pressure field taken over a period of one revolution of the wheel recorded at the instant a blade passes an observer whose frame of reference is fixed to that of the blade row of interest. If one assumes the average-passage flows of the two blade rows to be nearly irrotational outside of the blade passage region, then this ensemble-averaged pressure is nearly equal to the average-passage pressure distribution on the surface of the neighboring blade row. Hence, from the solution for the first blade row, one can estimate the body force and energy source which appear in the equations for the second blade row. In a similar fashion, one may estimate the body force and energy source which appear in the equations for the first blade row from a solution to the corresponding equations for the second blade row.

The remaining terms which must be estimated are the temporal correlations associated with the time-varying flow field. The origin and nature of these correlations were discussed in the previous section. To develop a model for these correlations, we decompose the absolute velocity field, \mathbf{V} , according to the equation

$$\vec{v}(r, \theta, z, t) = \vec{v}^{(AX)}(r, z) + \left[\vec{v}^{(1)}(r, \theta - \Omega_1 t, z) - \vec{v}^{(AX)}(r, z) \right] + \left[\vec{v}^{(2)}(r, \theta - \Omega_2 t, z) - \vec{v}^{(AX)}(r, z) \right] + \vec{v}^{(3)}(r, \theta, z, t) \quad (1)$$

where $\vec{v}^{(AX)}$ represents the axisymmetric velocity component, $\vec{v}^{(1)}$, the time-averaged absolute velocity field as observed in a frame of reference fixed to the first blade row, $\vec{v}^{(2)}$, the corresponding velocity field observed in a frame of reference fixed to the second blade row, and $\vec{v}^{(3)}$, the component of velocity which is unsteady in either frame of reference. The remaining variables which appear in Eq. (1) are the cylindrical coordinates r, θ, z , time t , and the rotational speed of the first and second blade rows Ω_1, Ω_2 . In a similar fashion, the total enthalpy, H , measured in the absolute frame of reference can be decomposed according to the equation

$$H(r, \theta, z, t) = H^{(AX)}(r, z) + \left[H^{(1)}(r, \theta - \Omega_1 t, z) - H^{(AX)}(r, z) \right] + \left[H^{(2)}(r, \theta - \Omega_2 t, z) - H^{(AX)}(r, z) \right] + H^{(3)}(r, \theta, z, t) \quad (2)$$

Fixed in the frame of reference of the first blade row, the velocity field $\vec{v}^{(1)}$ will appear steady in time, while the components $\vec{v}^{(2)}$ and $\vec{v}^{(3)}$ will appear to be unsteady. If we define the velocity component $\vec{v}''(r, \theta, z, t)$ as

$$\vec{v}''(r, \theta, z, t) \equiv \vec{v}^{(2)}(r, \theta - \Omega_2 t, z) - \vec{v}^{(AX)}(r, z) \quad (3)$$

the correlations which appear in the average-passage momentum equations associated with the first blade row are obtained by forming the time-average of the product of the fluid density and $(\vec{v}'' + \vec{v}^{(3)})^2$, (1). The result is

$$R_{ij} = \overline{\rho v_i'' v_j''} + \overline{\rho v_i'' v_j^{(3)}} + \overline{\rho v_i^{(3)} v_j''} + \overline{\rho v_i^{(3)} v_j^{(3)}} \quad (4)$$

where the subscripts i, j take on the values of 1, 2, and 3. On the right-hand of Eq. (4), these subscripts are used to denote the axial, tangential, and radial velocity components respectively. The variable ρ is the fluid density, and the over-bar represents the time-average of the variables which appear beneath it. Thus for $i = 1$ and $j = 2$, R_{12} denotes the temporal correlation between the density ρ , and the product of the axial and tangential components of the fluctuating velocity field. For low Mach number flows in which the density may be assumed constant, the correlation $\overline{\rho v_i'' v_j''}$ will be independent of tangential position θ , since v_i'' is spatially periodic over the pitch of the second blade row. This correlation is thus associated with the transport on the "average" of momentum across the axisymmetric stream surfaces. The remaining correlations which appear in Eq. (4), however,

will be functions of θ if $v^{(3)}$ is spatially aperiodic over the pitch of the second blade row. In general this will be the case.

Based on the arguments used to derive Eq. (4) and the analysis presented in Ref. (1), the correlations which appear in the energy equation are obtained by forming the time-average of the product of ρ , $H'' + H^{(3)}$ and $v'' + v^{(3)}$. The result is

$$Q_i = \overline{\rho H'' v_i''} + \overline{\rho H'' v_i^{(3)}} + \overline{\rho H^{(3)} v_i''} + \overline{\rho H^{(3)} v_i^{(3)}} \quad (5)$$

where

$$H''(r, \theta, z, t) \equiv H^{(2)}(r, \theta - \Omega_2 t, z) - H^{(AX)}(r, z) \quad (6)$$

The first correlation in Eq. (5) is independent of θ if the fluid density is constant. This is the result of H'' and v_i'' being spatially periodic over the pitch of the second blade row. In general, the remaining correlations will not exhibit this behavior. As a result, the total enthalpy associated with the average-passage flow field for a multiblade row configuration will be nonuniform in the tangential direction. Kerrebrock and Mikolajczak (6) were the first to attempt to analyze the fluid mechanics associated with this phenomena. They attributed it to the transport of excess total temperature of a fluid particle in a rotor wake across the stator passage. Their analysis of this process was based on kinematics. Although the present work makes no attempt at developing an alternative model of this phenomena, it does suggest that it is associated with the dynamics of stator-blade rotor-wake interaction.

For an inviscid nearly irrotational flow, the magnitude of the unsteady component $\vec{v}^{(3)}$ will be comparable or less than the magnitude of \vec{v}'' , except for regions near blade leading edges. In particular, in regions where the body force and the energy source are finite, the correlation associated with \vec{v}'' (i.e., Eqs. (4) and (5)) will be significantly larger than those associated with $\vec{v}^{(3)}$. For this reason we assume that the correlations in Eqs. (4) and (5) associated with the unsteady velocity component $\vec{v}^{(3)}$ can be neglected. As a result, the correlation R_{ij} and the correlation Q_i can be directly evaluated from the average-passage solutions. For a stage, this implies that the flow field through both blade rows must be evaluated simultaneously.

To incorporate the suggested closure model into a numerical simulation, one may envision a two-tier iteration procedure as depicted in Fig. 3. In the inner loop, the body forces, energy sources and correlations are frozen. An average-passage flow field is evaluated based on the value of these quantities and the imposed boundary conditions. In the outer loop, the body forces, energy sources, and correlations are updated based on the converged inner loop solutions. We must update these terms, as previously noted, in a manner which yields a unique axisymmetric representation of the flow field through the machine. This will insure that the average-passage representation of the flow is consistent with the axisymmetric representation.

The equations to be solved in the outer loop may be derived starting from the equations of motion expressed in the vector form (2).

$$L(U_n) + \int K(U_n) dv + \int S_{n-1} dv = 0 \quad (7)$$

The operator $L(U_n)$ in this equation represents the net flux of mass (i.e., continuity of flow through a control volume), axial and radial momentum, angular momentum, and energy through a differential volume of fluid, while $K(U_n)$ represents the added contribution of the pressure field and centrifugal acceleration to the balance of radial momentum. The components of the vector U_n are density, axial and radial momentum, angular momentum, and total inertial energy. The symbol S_{n-1} represents the sum of the contribution of the body force, energy source, and temporal correlations to the momentum and energy equations, while the subscript n denotes the iteration index of the outer loop. The remaining symbol, dv , denotes the volume of a differential volume of fluid. Based on the discussion presented earlier, the temporal correlations which are embedded in S_{n-1} are simply a function of average-passage flow field associated with the neighboring blade row. Thus, for a single-stage configuration, the field equation for the first blade row can be written as

$$L(U_n^{(1)}) + \int K(U_n^{(1)}) dv + \int S_{n-1}^{(2)} dv = 0 \quad (8)$$

while the corresponding equation for the second blade row is

$$L(U_n^{(2)}) + \int K(U_n^{(2)}) dv + \int S_{n-1}^{(1)} dv = 0 \quad (9)$$

The superscript (1) and (2) respectively denote the variables associated with the first and second blade row passage flow fields. Next we multiply both Eqs. (8) and (9) by an operator A , which forms the axisymmetric average of its argument. This is equivalent to averaging the three-dimensional equations of motion (i.e., Eqs. (8) and (9)) over the tangential direction. For the first blade row, the axisymmetric average of the operator L is

$$AL(U_n^{(1)}) = L^{(AX)}(AU_n^{(1)}) + \int S_{n-1}^{(1)} Adv \quad (10)$$

while for the second row

$$AL(U_n^{(2)}) = L^{(AX)}(AU_n^{(2)}) + \int S_{n-1}^{(2)} Adv \quad (11)$$

In both of these expressions, the operator $L^{(AX)}$ denotes the axisymmetric counterpart of L . The axisymmetric average of the combined integrals which appear in Eq. (8) is

$$A \left[\int K(U_n^{(1)}) dv + \int S_{n-1}^{(2)} dv \right] = \int A \left[K(U_n^{(1)}) dv \right] + \int S_{n-1}^{(2)} Adv \quad (12)$$

Similarly, the axisymmetric average of the combined integrals in Eq. (9) is

$$A \left[\int K(U_n^{(2)}) dv + \int S_{n-1}^{(1)} dv \right] = \int A \left[K(U_n^{(2)}) dv \right] + \int S_{n-1}^{(1)} Adv \quad (13)$$

These last results follow because S is independent of tangential position. Based on the above equations, the axisymmetric average of Eqs. (8) and (9) may be expressed as

$$L^{(AX)}(AU_n^{(1)}) + \int A \left[K(U_n^{(1)}) dv \right] + \int \left[S_{n-1}^{(1)} + S_{n-1}^{(2)} \right] Adv = 0 \quad (14)$$

$$L^{(AX)}(AU_n^{(2)}) + \int A \left[K(U_n^{(2)}) dv \right] + \int \left[S_{n-1}^{(2)} + S_{n-1}^{(1)} \right] Adv = 0 \quad (15)$$

Upon convergence of the outer loop, Eqs. (14) and (15) yield identical solutions for the axisymmetric flow field. In addition, these equations provide a means of updating the variables $\int(U_n^{(1)})$ and $\int(U_n^{(2)})$ without evaluating the body forces, energy sources, and correlations directly. This becomes apparent as soon as one notes that the vectors $U_n^{(1)}$ and $U_n^{(2)}$ are known having been evaluated in the inner iteration loop, while the quantities $\int(U_{n-1}^{(1)})$ and $\int(U_{n-1}^{(2)})$ are known from the previous outer iteration loop.

This simple strategy for incorporating the closure model into a numerical simulation has been implemented into the computer code outlined in Ref. 2. That code has been used successfully to simulate the flow about high-speed counter-rotating propellers as illustrated by the results presented in Ref. 2. We shall present additional results from that simulation as well as that for a high-speed fan stage.

RESULTS

The model proposed for closing the inviscid form of the average-passage equation system was based on the assumption that, within the confined region of a blade row, the correlations associated with the blade row interaction velocity field are small relative to those associated with the steady aerodynamic blade loading. The justification for this assumption can be based on the argument that the unsteady airload, which is an indication of the magnitude of the velocity component associated with blade row interaction, is generally smaller than its time-averaged counterpart which serves as a measure of the magnitude of the nonaxisymmetric component of the average-passage velocity field. Data presented in a recent publication by Dring et al, (7), shows this to be the case in the midspan region of a turbine stage. Outside of the confines of a blade row, the magnitude of both of these velocity fields should be comparable; however, their magnitude is small compared to the magnitude of the axisymmetric velocity field. As an illustration that hardware does exist in which one may find regions in which such flows exist, we present the circulation as a function of radius at a number of axial locations generated by a high-speed counter-rotating propeller. These results are for a flight Mach number of 0.72 and an advance ratio for both propellers of 2.8. The circulation is defined as the integral over a blade pitch of the product of nondimensional radius and nondimensional tangential velocity. The tangential velocity is nondimensionalized by the far-field speed of sound, while the radius is rendered nondimensional by the tip diameter of the first propeller. The results presented in Figs. 4(a) to (d), are for an axial location

slightly forward of the first propeller, aft of the trailing edge of the first propeller, slightly forward of the second propeller, and aft of the trailing edge of the second propeller respectively. For each axial location, two plots are drawn. The first (solid line) represents the axisymmetric flow field obtained from the average-passage simulation of the first propeller. The second (dashed line) corresponds to the axisymmetric flow field obtained from the average-passage simulation of the second propeller. It is quite apparent that both results agree with each other to within plotting accuracy and hence are more than adequate for assessing blade row performance. Upstream of the first propeller the circulation must be zero since there is no swirl present in the incoming flow. At the trailing edge of the first propeller, the circulation is nearly constant over the inboard portion, decreasing in a smooth monotonic fashion towards zero as the tip is approached (i.e., $r = 0.5$). Thus the aerodynamic loading of the inboard region is nearly independent of radius, which implies a near-free vortex design. The flow between the two propellers would therefore be nearly irrotational. The reduction in circulation with radius in the outboard region produces a weak tip vortex which convects downstream. In the axisymmetric flow representation, this tip vortex is smeared into a ring vortex. At the leading edge of the second propeller, the circulation distribution is seen to be nearly identical to the distribution at the trailing edge of the first propeller. This result further substantiates that the flow field between the two propellers is nearly irrotational, for in an irrotational unsteady flow the time-averaged circulation (or angular momentum) is conserved along the axisymmetric stream lines. The slight redistribution of circulation that one observes in the outboard region is attributed to spanwise mixing of angular momentum due to the tip vortex. Figure 4(d) shows the distribution of circulation at the trailing edge of the second propeller. The second propeller appears to take out almost all of the swirl produced by the first propeller. The change in the swirl distribution across the second propeller implies that the spanwise aerodynamic loading is nearly uniform over the inboard region of the second propeller. The inboard region is behaving as a free-vortex design. The results shown in Fig. 4 strongly suggest that the closure model developed in this work should be applicable to this and similar high-speed counter-rotating propellers. This is confirmed by the comparison between the measured and predicted nacelle pressure distribution presented in Ref. 2. Further comparisons are planned as shown as experimental data becomes available.

An attempt was also made to predict the average-passage flow fields generated by a high-speed fan stage. The stage chosen was the first of a two-stage machine designed and tested at NASA Lewis (8). The computation was performed for an operating point near maximum efficiency of the first stage. This point was chosen to minimize the effect of viscosity on the measured flow variables. The rotor's rotational speed was 80 percent of design and the stage pressure ratio and adiabatic efficiency were 1.352 and 0.891 respectively. The inferred velocity field between the blade rows resembled that induced by a free-vortex design in the midspan region. The inlet boundary conditions in the computation were chosen to produce an inlet absolute Mach number and flow angle distribution which approximated the measured distributions. At the downstream boundary the nondimensional pressure at the hub at the exit of the stator was set equal to the measured value. The absolute Mach number distribution

at the inlet to the rotor is shown in Fig. 5, while the relative Mach number distribution across the rotor is shown in Fig. 6. The measured results at the inlet to the rotor agree very well with the predicted results, as they should, due to the choice of inlet conditions. The predicted exit relative Mach number distribution appears to be in good agreement with the measured results, especially in the midspan region. It should also be noted that the predicted relative Mach number is less than measured over most of the rotor span. This result is to be expected since the blade boundary layers restrict the flow area, thus reducing the diffusion capabilities of the rotor. The measured distribution also shows the existence of an end-wall casing boundary layer which obviously cannot be predicted by the present inviscid analysis.

The relative flow angle distribution at the leading and trailing edge of the rotor was also computed and is shown in Fig. 7 along with the measured distribution. This angle is defined as the angle between the relative circumferential velocity component and the meridional component. At both stations the predicted results appear to be in reasonable agreement with the measurements inboard of the tip region. The discrepancy in the tip end-wall region is caused by the inability of the present inviscid analysis to properly simulate the three-dimensional end-wall flow. Over the region inboard of the tip, the neglect of the influence of viscosity on the simulated axisymmetric flow field produces more turning of the flow than experimentally measured. By introducing the effect of viscosity into the current average-passage model (which includes the outlined closure model), the agreement between prediction and experiment should improve in the midspan region.

The next series of results are for the stator. The absolute Mach number entering and leaving the stator is plotted as a function of blade span in Fig. 8. The corresponding plots for the absolute flow angle is shown in Fig. 9. This angle is defined as the angle between the absolute tangential velocity component and the meridional component. The agreement between the predicted results and measurements appears to have deteriorated from that for the rotor. This illustrates the difficulty in predicting multiblade row flows. A small error in predicting the performance of the first blade row can escalate very quickly into a large error in predicted performance of later blade rows. This problem becomes particularly acute whenever there are appreciable regions of flow separation in the end-wall region, as appears to be the case in the stator hub region. The poor agreement in the stator tip region is attributed to the end-wall flow induced by the rotor. To analyze these flow regions requires a model which incorporates the proper end-wall flow physics. A step in this direction might be made by including the effects of viscosity into the current average-passage flow solver. An additional issue is the development of a closure model for the average-passage model applicable to highly rotational flows. Research in both of these areas is currently underway.

CONCLUSION

A model was formulated to close the inviscid form of the system of equations governing the average-passage flow fields for a stage. This model was developed so as to insure consistency between the average-passage equation system and the axisymmetric flow equations. This closure model was used successfully to simulate the average-passage flow fields

associated with a high-speed counter-rotating propeller. The model was also used in a simulation of a high-speed fan stage operating near measured peak efficiency. This simulation showed the rotor results to be in reasonable agreement with measurements outside of the end-wall region. For the stator the simulation yielded results which were only qualitatively correct. The lack of quantitative agreement was attributed to neglect of viscosity and the questionable applicability of the present closure model to the end-wall regions where the flow is known to be highly rotational. Research directed at overcoming these shortcomings is currently underway.

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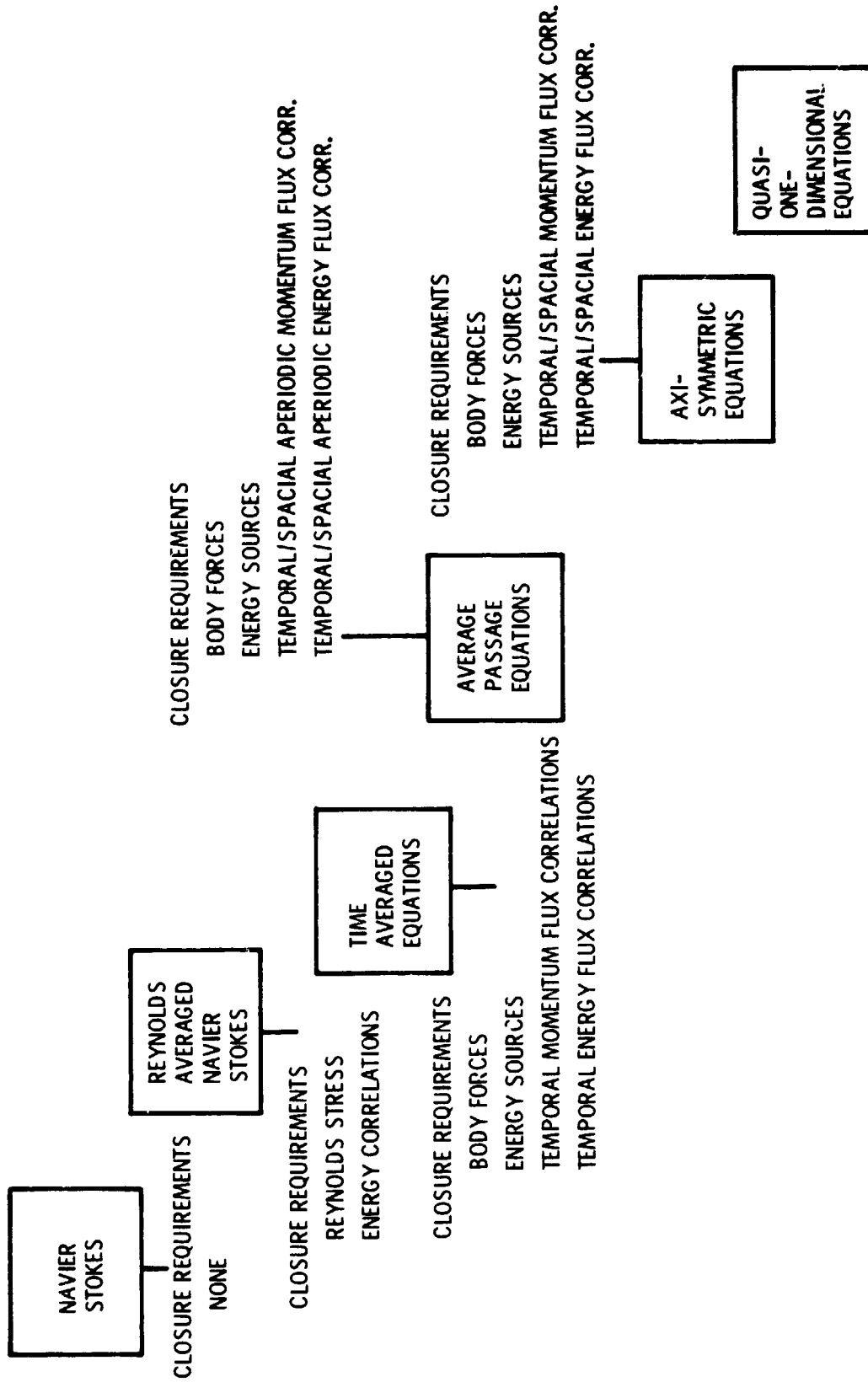


Figure 1. - Flow models for use in turbo machinery design/analysis.

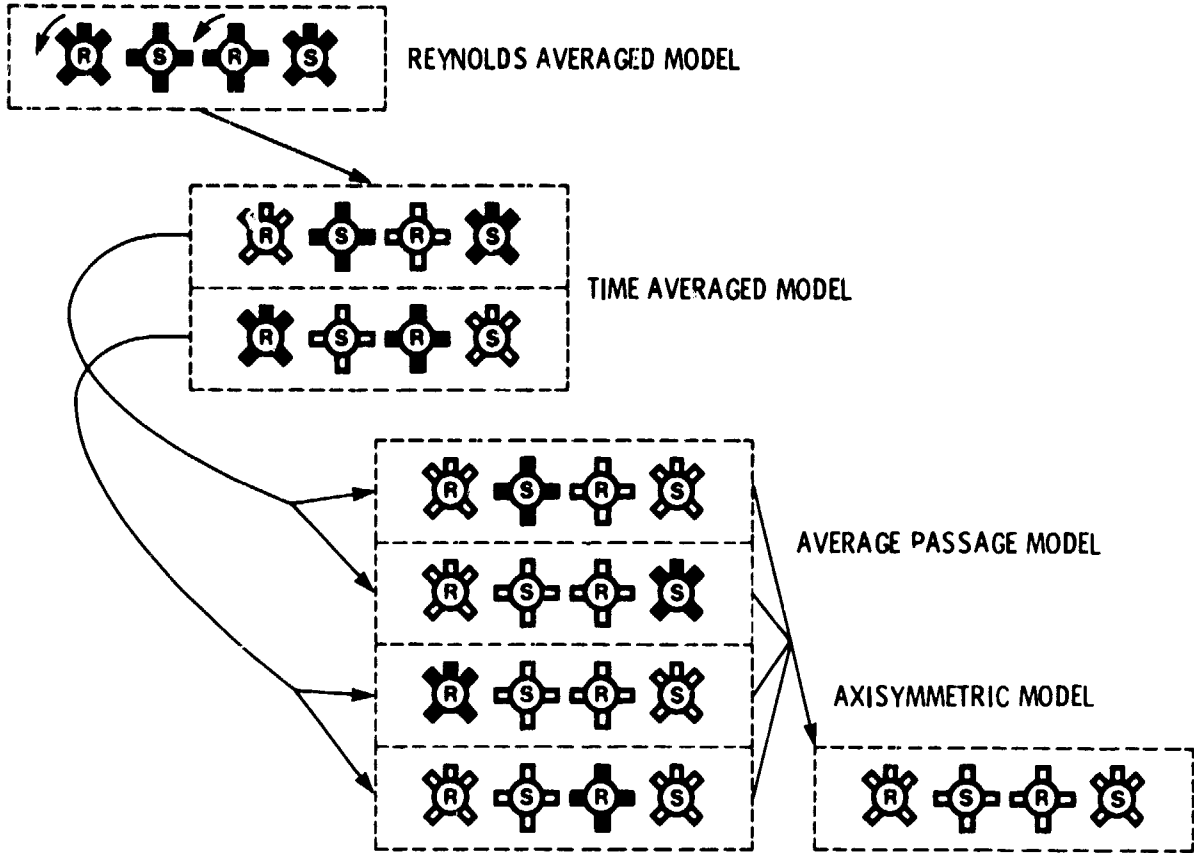


Figure 2. - Two stage flow models.

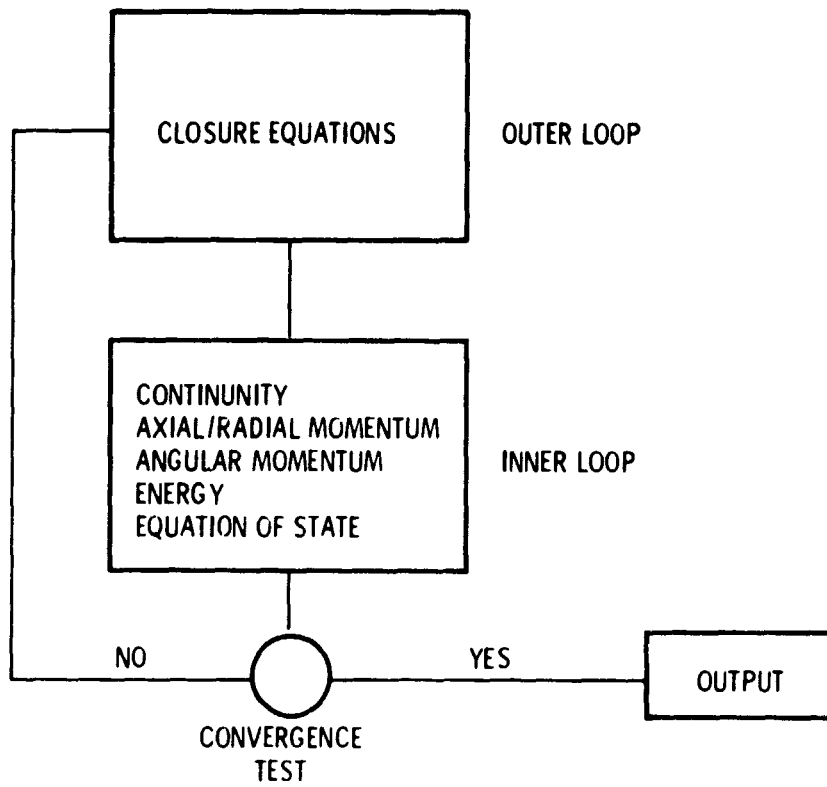


Figure 3. - Solution strategy.

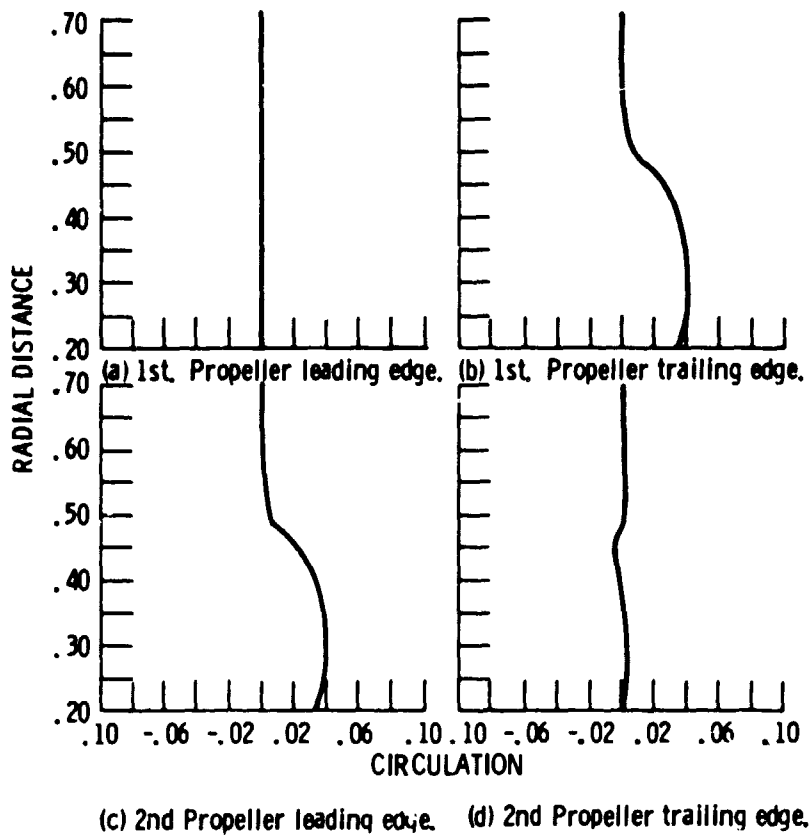


Figure 4. - Circulation distribution.

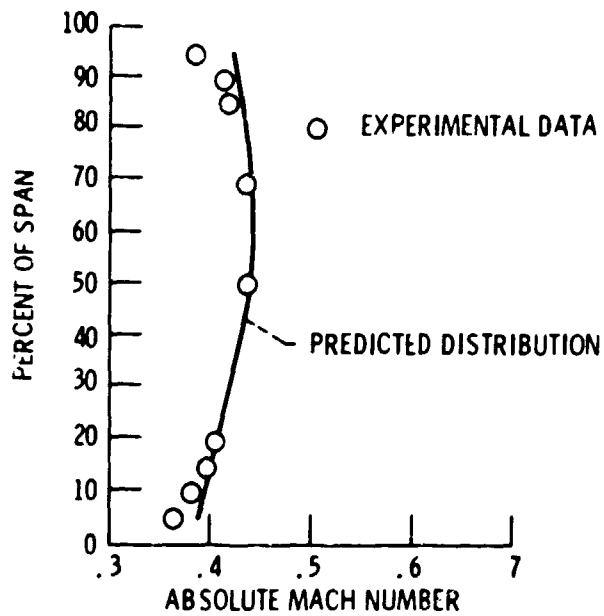


Figure 5. - Absolute Mach distribution (Rotor inlet).

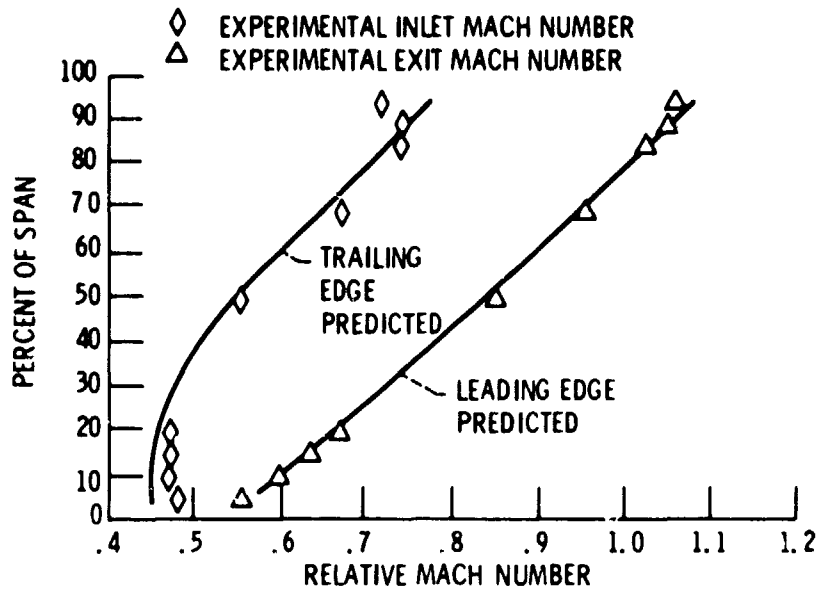


Figure 6. - Relative Mach number distribution (Rotor).

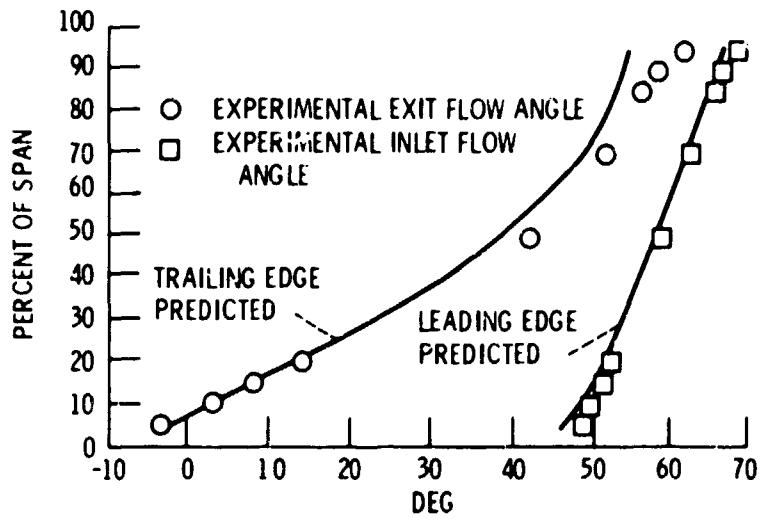


Figure 7. - Relative flow angle distribution (Rotor).

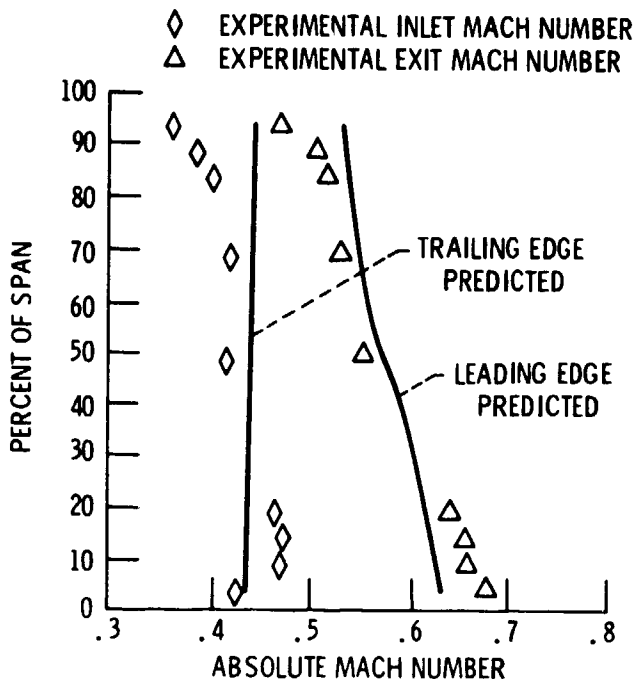


Figure 8. - Absolute Mach number distribution (Stator).

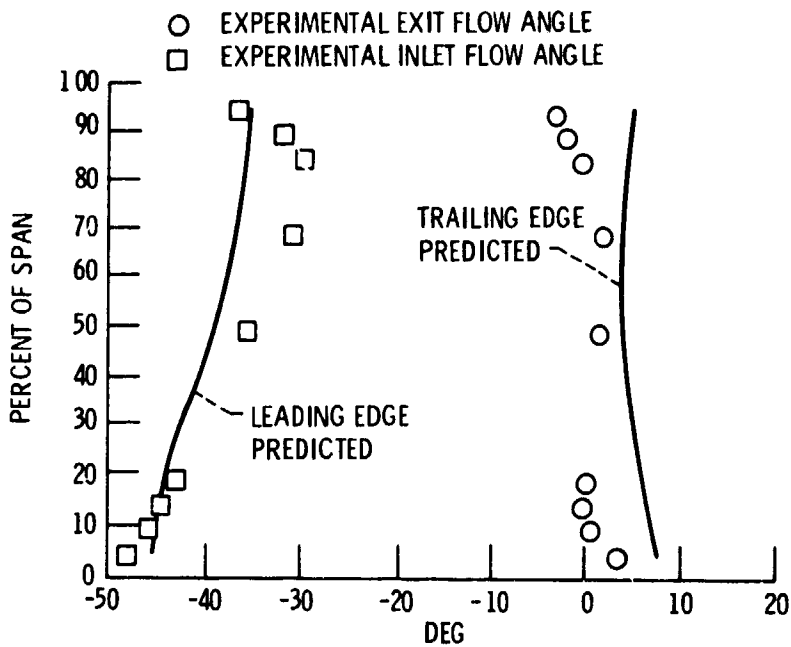


Figure 9. - Absolute flow angle distribution (Stator).

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16. Abstract A mathematical model is proposed for closing or mathematically completing the system of equations which describes the time-average flow field through the blade passages of multistage turbomachinery. These equations referred to as the average-passage equation system govern a conceptual model which has proven useful in turbomachinery aerodynamic design and analysis. The closure model is developed so as to insure a consistency between these equations and the axisymmetric through-flow equations. The closure model was incorporated into a computer code for use in simulating the flow field about a high-speed counter-rotating propeller and a high-speed fan stage. Results from these simulations are presented.					
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