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A Model for Interference and Forgetting

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A new model for interference and forgetting is presented. The model is based on the Raaijmakers and Shiffrin search of associative memory (SAM) theory for retrieval from long-term memory. It includes a contextual fluctuation process that enables it to handle time-dependent changes in retrieval strengths. That is, the contextual retrieval strength is assumed to be proportional to the overlap between the contextual elements encoded in the memory trace and the elements active at the time of testing. It is shown that the model predicts a large number of phenomena from the classical interference literature. These include the basic results concerning retroactive inhibition, proactive inhibition, spontaneous recovery, independence of List 1 and List 2 recall, Osgood's transfer and retroaction surface, simple forgetting functions, the use of recognition measures, and the relation between response accuracy and response latency. It is shown that these results can be explained by a model that does not incorporate an "unlearning" assumption, thus avoiding many of the difficulties that have plagued the traditional interference theories.

In recent years, a number of memory models have been presented that successfully predict the major results concerning recall and recognition. Unfortunately, however, many of those models have not been applied in a systematic manner to the phenomena of interference and forgetting. This is especially regrettable since there exists a wealth of data, accumulated in the years when these topics were the main focus of memory research, that should not be disregarded by contemporary memory theories.

In this article, we present a model intended to explain the basic findings concerning interference and forgetting, findings that have been shown in many experiments to be relatively robust and reliable. The model is based on the general search of associative memory (SAM) theory (Raaijmakers & Shiffrin, 1981a) but incorporates a new model describing contextual fluctuation processes (Mensink & Raaijmakers, 1988).

The SAM theory is a probabilistic cue-dependent search theory that describes retrieval processes in long-term memory. Retrieval is assumed to be mediated by retrieval cues, such as category names, words from a to-be-remembered list, contextual cues, etc. Sampling and recovery of sampled images (or memory traces) are the mechanisms that constitute the central features of the theory. As has been documented in previous papers (Gillund & Shiffrin, 1984; Raaijmakers & Shiffrin, 1980, 1981a, 1981b), the SAM theory predicts a considerable number of memory phenomena, such as serial position effects, response latency, output interference, list-length effects, cued and noncued recall of categorized lists, and the relation between recognition and recall. However, as yet the theory has not been applied to interference phenomena. Since interference is generally considered as reflecting the most fundamental process of forgetting, a general theory of memory such as SAM should be able to handle such phenomena.

Although previously published versions of SAM (e.g., Raaijmakers & Shiffrin, 1981b) would be able to explain some of the interference phenomena, the theory is not yet equipped with a time-dependent mechanism that enables it to handle phenomena such as proactive inhibition (PI) and spontaneous recovery. For the explanation of such phenomena some aspect of SAM has to be turned into a time-dependent variable. In order to accomplish this, we started from the assumption that contextual fluctuation causes the interdependence between memory performance and retention time (Bower, 1972; Estes, 1955; Raaijmakers & Shiffrin, 1981a). On the basis of the assumption that the influence of context on recall is a function of time, we have derived a model for context that determines the contextual strength, the strength of the context cue at test to the stored memory images. A full account of the mathematical details of this development is presented by Mensink and Raaijmakers (1988).

It is assumed that the associative strength of the contextual cue at the time of testing to a particular memory image (which is related to the probability of retrieving that image; see below) is determined by the overlap between the context at the time of storage and the test context. As discussed by Raaijmakers and Shiffrin (1981a), there are two basic factors in the general SAM theory that may be used to explain forgetting (the observation of a lower probability of retrieval of a given image at Time B than at an earlier Time A). First, the cues used at Time A may be more strongly associated to that image than those used at Time B. Second, the strength and number of other images associated to the cues (even if the cues are the same) may be greater at Time B than at Time A. The contextual fluctuation process

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assumed in the present model can be viewed as an elaboration of this analysis. Contextual fluctuation may lead to a reduction in the contextual associative strengths because of a decreased overlap in contexts. At the same time, context changes may also lead to an increase in the strengths of other images, namely, those images that have a greater overlap with the current context cue.

Before we discuss the present model and its applications, we shall first give a brief review of the classical interference theories in order to introduce the major issues that are important for any theory of interference phenomena. The first serious attempt to explain these phenomena was a three-factor theory proposed by McGeoch (1932). In this theory, it is assumed that responses learned within the same situation compete with each other for recall at the time of test, thus causing retroactive inhibition (RI) as well as proactive inhibition (PI). The outcome of this competition depends on the associative strengths of the responses to-be-recalled. An important assumption on which this theory is based is the so-called independence postulate. This postulate states that the responses are acquired independently of each other. That is, the learning of a competing response has no effect on the strength of the originally learned response. We return to this assumption in our discussion of the applications of our model.

Another important, although often neglected, role is played by the situation in which the responses are learned and tested. According to McGeoch, this situation is not constant but changes as time progresses. It acts in fact as a stimulus for the retrieval of the responses. Perfect recall can be realized only when the situation at test is completely congruent with the one at the time of storage. However, situations change and therefore a decline in recall becomes inevitable.

The third factor, inadequate set at the time of testing, is similar to modern concepts like retrieval strategies and control processes. Although such a factor will undoubtedly be required for a full account of retrieval processes, there is little relevant empirical information available that would be useful for theory testing. This factor therefore is not further discussed in this article. As we shall show, our model may be viewed as a formalization of this verbally stated theory.

In 1940, Melton and Irwin revised McGeoch's theory by postulating a so-called unlearning mechanism in addition to response competition. The reason for this was that according to these investigators, response competition alone was not capable of explaining all interference data. A number of experiments were conducted in order to test the unlearning hypothesis, and eventually most memory researchers believed that the unlearning concept was properly established by an experiment using the modified modified free recall method (MMFR) as devised by Barnes and Underwood (1959). In this method, subjects are asked to recall both responses associated with a stimulus. This was supposed to eliminate response competition.

Eleven years earlier, Underwood (1948) had interpreted the unlearning concept as the analogue of the concept of extinction in classical conditioning theory. By doing so, he immediately provided the theory with the adopted concept of spontaneous recovery. This led to the fortunate result that the relation between PI and the length of the retention interval could now be easily explained: PI increases with the length of the retention interval because the List 1 responses spontaneously recover and because this exerts an increasingly interfering effect (because of response competition) on the List 2 responses. Interpreted in this way, indirect evidence for spontaneous recovery was obtained in experiments by Underwood (1948) and Briggs (1954). Thus, at the end of the 1950s, interference theory finds itself in excellent shape.

Unfortunately, it was soon shown that PI could also be observed on MMFR tests (Birnbaum, 1965; Houston, 1967; Koppenaal, 1963; Slamecka, 1966), a test that was supposed to eliminate response competition. This result cannot be explained by the unlearning/response-competition theory. Either the MMFR testing procedure is sensitive to some sort of response competition or there is some other factor (in addition to unlearning and response competition) involved in PI.

Moreover, additional problems arose in connection with the concept of spontaneous recovery. Koppenaal (1963) concluded that spontaneous recovery could not be demonstrated when MMFR testing was used. In a review of the spontaneous recovery phenomenon, Postman, Stark, and Fraser (1968) showed that it occurred only in specific circumstances. Thus, the evidence for the concept of spontaneous recovery seemed rather shaky.

This result, together with the observation that PI was observed in tests supposed to be free of response competition, eventually resulted in a revision of the theory. The unlearning hypothesis was replaced by the mechanism of response-set suppression (Postman et al., 1968; Postman & Underwood, 1973). In this new model, the observation of PI in MMFR tests was still troublesome, although it could be explained by assuming a process of output interference. Not surprisingly, Postman and Underwood (1973) were not quite satisfied with this ad hoc mechanism. On the other hand, a number of spontaneous recovery effects obtained by Postman et al. (1968) could be explained by this model.

So far, we have presented a brief review of interference theories and some of the data on which they have been based. In the next sections, we present a mathematical model for interference and forgetting and compare the predictions of this model with relevant data taken from the literature. We use one set of parameter values to generate all the predictions. As a result, we are interested not in fitting the data in a quantitative sense, but only in the qualitative trends predicted by the model.

Description of the Model

The SAM theory (Raaijmakers & Shiffrin, 1981a) can be described as a cue-dependent probabilistic search theory of retrieval. Its goal is to account for data observed in a variety of memory experiments using a variety of retrieval measures (free recall, cued recall, and recognition). Our goal in this article is not to describe the general SAM theory; rather, we restrict ourselves to a presentation of its concepts in relation to paired-associate modeling (the most common design in interference experiments). Following this, we discuss the contextual fluctuation model and the way it is used to determine the contextual cue strengths. For a full description of the basic concepts in SAM, see Raaijmakers and Shiffrin (1981a).

Storage

Consider the presentation of a paired associate (a–b). This pair enters the short-term store (STS). The amount of information stored in memory is determined by the nature of the processing operations carried out on the information in STS (elaborative rehearsal). It is assumed that the amount of elaborative rehearsal will be proportional to the length of time that an item is studied (rehearsed) in STS. This rehearsal process is modeled by a limited capacity buffer. Items that are simultaneously present in this buffer build up interitem associative strength. In free recall, an item will be associated to previously presented items, still present in the buffer. In such studies, all items in STS are assumed to be part of the rehearsal buffer. There is, however, evidence (Raaijmakers & Shiffrin, 1981b) that the rehearsal process is somewhat different in paired-associate paradigms. In this case, the buffer and STS do not coincide. That is, the two members of a pair are associated only to each other and not to members of other pairs, still present in STS. This is demonstrated by the absence of a primacy effect, indicating the absence of cumulative rehearsal. However, since previous items may still be in STS (although not actively rehearsed), a recency effect may still be observed (Murdock, 1974). Hence, it is assumed that at any one time the buffer is occupied only by a single paired associate and that the next pair always replaces the previous one (Raaijmakers & Shiffrin, 1981b).

In SAM, it is assumed that during the stay in the buffer, information about the items to be learned is transferred from STS to long-term store (LTS). This information is stored in what is called a memory image. An image (or episode) may be considered as the unit of episodic memory, the memory trace corresponding to a specific event in a particular spatio-temporal context. In paired-associate paradigms, the images are assumed to consist of information corresponding to the presented pairs. Hence, a single image includes both stimulus, response, and contextual information as well as associative information.[1] It should perhaps be mentioned that the assumption that a pair constitutes a single image differs somewhat from previous applications of SAM to free recall paradigms where the images corresponded to individual words. However, if we think of what is stored as an episodic event consisting of a (single) set of features (a quite common assumption in current theories), then the above assumption makes perfect sense. Moreover, it may be shown that the choice between the two ways of representing a pair of words is in fact a matter of preference, since the two versions make equivalent predictions (see Raaijmakers & Shiffrin, 1981b).

It is assumed that the amount of information stored (i.e., the number of encoded features) is proportional to the length of stay in the buffer. Since each paired associate is in fact always replaced immediately by the following pair, the length of stay in the buffer will be equal to the presentation time. As discussed in previous papers (see Raaijmakers & Shiffrin, 1981a), the memory structure is represented by a retrieval structure that gives the associative strengths between possible retrieval cues and the stored episodic images. These associative strengths are a function of the overlap between the set of features corresponding to the cue and the set of features corresponding to the image.

In the present application, two types of retrieval cues are used: item cues corresponding to the stimulus member of a pair and context cues. Although the encoding of the stimulus may be variable (stochastic), it is assumed that this is not a function of time. That is, the stimulus does not have to be encoded in exactly the same way on two occasions A and B, but the similarity of these two encodings does not depend on the temporal distance between A and B. Hence, the associative strength of the stimulus item to the stored image (henceforth called the interitem strength) is assumed not to depend on the length of the retention interval. Denoting the interitem strength of stimulus S_i to the corresponding image I_i by $S(I_i, S_i)$, the above assumption may be succinctly written as:

$$S(I_i, S_i) = bt_i,$$

where t_i equals the presentation time in seconds and b denotes the amount of associative information transferred per second (the parameter definitions follow the conventions adhered to in previous publications; see Raaijmakers & Shiffrin, 1981a). The parameter b is of course not a fixed constant but depends on such factors as the preexperimental associative strength, the imageability, and the encoding strategy. For the experiments discussed in this article, these factors are assumed to be held constant. The strength of the stimulus S_i to all other, unrelated, images is assumed to be equal to a small, residual value, d.

Note that in applications to interference paradigms, there will in fact be two interitem associative strength parameters (which in this article will be set equal to each other): b_1 for storage during List 1 learning and b_2 for storage during List 2 learning. Thus, if the a–b pair has been studied t_1 seconds and the a–c pair t_2 seconds, the item cue S_a will have a strength of b_1t_1 to the a–b image and b_2t_2 to the a–c image. In addition, S_a will be residually associated to all other images from the two lists (with strength d).

Next, we have to discuss the contextual retrieval strengths. It is this aspect that represents the major change with respect to previous applications of the SAM theory. This contextual retrieval strength is assumed to depend on the overlap between the context at the time of storage and the context at the time of testing. Hence, this strength should be a function not only of the number of presentations and the presentation time, but also of the retention interval and the interpresentation interval. In order to accomplish this, we have developed a contextual fluctuation model based on stimulus sampling theory. A full account is given by Mensink and Raaijmakers (1988).

It is assumed that context may be represented by a set of elements ($N + n$ in total). From these elements only n are in the active state at any time. These active elements constitute the current context. All other elements (N) are in the inactive state.

[1] Note that the assumption that item and associative information are both stored in the same image does not imply that item information may not be retrieved separately. This would only be true if recovery was assumed to be an all-or-none process (which it is not).

During a small time interval dt, with probability c, an interchange occurs between one element of each subset. This defines a fluctuation process: During a certain time interval active elements may become inactive while inactive elements may become active. At study time, only active elements can be stored in the episodic image.

We will assume that the number of active elements that is stored during the presentation of a paired associate is an exponential function of the presentation time t, with a rate parameter denoted by α:

$$C(t) = C(0)e^{-\alpha t} + n(1 - e^{-\alpha t}),$$

where $C(t)$ denotes the expected number of stored elements following t seconds of study. This may be simplified by noting that the probability that an active element that has not already been encoded in the image is stored during a study trial of t seconds is equal to:

$$w = \frac{C(t) - C(0)}{n - C(0)} = 1 - e^{-\alpha t}$$

As mentioned above, the retrieval strength of the test context to the stored image is assumed to be proportional to the overlap between the contextual elements stored in the image and the set of contextual elements active at the time of testing. Since the present model will be applied to interference studies, we will make use of a classification of the contextual elements that is based on the standard AB–AC interference design. Consider a particular a–b pair and its interfering counterpart a–c. At any time, the set of contextual elements may be partitioned in four subsets:

x_1 elements: elements stored only in the a–b image,
x_2 elements: elements stored only in the a–c image,
x_o elements: elements stored in both images,[2]
y elements: elements not stored in either image.

The contextual fluctuation model enables us to calculate at each time the expected number of elements of each type. This is based on the following formula which gives the expected number of elements of a certain class v ($v = x_1, x_2, x_o$, or y) that are active following t seconds of fluctuation, given that the state at time $t = 0$ is known (see Mensink & Raaijmakers, 1988). Let $A(t)$ represent the number of active elements at time t:[3]

$$A(t) = A(0)e^{-(\gamma+\beta)t} + K\left(\frac{\gamma}{\gamma+\beta}\right)[1 - e^{-(\gamma+\beta)t}] \quad (1)$$

In this equation, K denotes the total number of elements of that type (active plus inactive), and β and γ are two parameters of the fluctuation process. β equals the rate at which an active element becomes inactive, and γ equals the rate at which an inactive element becomes active.

Using these two equations it becomes possible to calculate at each moment in time the expected number of active elements of each type. The Appendix gives a brief summary of the relevant difference equations. On each trial during List 1 learning, a proportion w of the elements not yet stored in the a–b image will be encoded in that image. For reasons of simplicity, it is assumed that fluctuation occurs only between consecutive study trials and not within a study trial. Thus, the fluctuation process operates only in the interval between two presentations of a list, in the interval between the presentation of the AB list and the AC list, and in the interval between the AC list and the final test. This simplification will not affect the qualitative nature of the predictions since we are not interested in within-list phenomena (e.g., serial-position effects and output interference effects), but only in the average number of list items recalled.

During the interval between two study trials of List 1, some of the active conditioned (i.e., encoded) x_1 elements will be replaced by unconditioned y elements. Hence, during List 1 learning, both the number of active and the number of inactive x_1 elements increases. During List 2 learning, some of the x_1 elements will be encoded in the a–c image (x_o or overlap elements). In addition, as learning progresses, more and more of the y elements will also become encoded (x_2 elements). Thus, at the end of the List 2 learning phase, there will be eight types of elements: active x_1, x_2, x_o, and y elements and inactive x_1, x_2, x_o, and y elements.

During the retention interval (the interval between List 2 learning and the final testing) there will be some fluctuation between the active and the inactive sets. The overlap between the test context and the a–b image is given by the number of x_1 and x_o elements that are active at the time of testing. Similarly, the overlap between the test context and the a–c image is given by the sum of the active x_2 and x_o elements. The contextual associative strengths are proportional to these overlaps. Hence, the contextual associative strength at test to image I_i is given by

$$S(I_i, C) = \begin{cases} (A_1 + A_o)a \text{ for List 1 images} \\ (A_2 + A_o)a \text{ for List 2 images,} \end{cases} \quad (2)$$

where A_i denotes the expected number of elements of a particular type (x_1, x_2, or x_o) active at the time of testing and a is the constant of proportionality. Since all A_is are proportional to n, the number of elements active at any time, the contextual strengths are proportional to both n and a. Hence, these parameters are not separately identifiable. Because of this, n will be set equal to 1, in which case A_i refers to the proportion of active elements that are of a given type.

Note that as a result of our fluctuation model the contextual retrieval strengths will depend not only on the number of presentations but also on the interpresentation and retention intervals. This will enable us to predict a number of time-dependent interference effects.

Retrieval

As in previous applications of SAM, the retrieval process is assumed to consist of a number of elementary retrieval cycles,

[2] In most applications, the introduction of x_o elements would not be necessary. One could simply work with $x'_1 = x_1 + x_o$ and $x'_2 = x_2 + x_o$ (the number of elements encoded in the a–b and a–c image, respectively). In more complicated analyses (e.g., list discrimination), the present distinction is useful. Hence, it is maintained here for reasons of generality.

[3] We have used a simple reparametrization of the model that is somewhat easier to work with. The new parameters β and γ are related to the old parameters N and c in the following way: $\beta = c/n$ and $\gamma = c/N$.

each cycle composed of a sampling and a recovery phase. Retrieval is cue dependent, that is, what is elicited from memory is determined by the retrieval cues used at that moment. A number of images will be activated by the cues, in different degrees, depending on how strongly they are associated to the probe cues. During each cycle of the memory search, one image is sampled from the activated set. The information in the sampled image (or part of it) is then accessed and evaluated. This process is called recovery.

After a subject has learned n paired associates, there will be n corresponding images in LTS. At test, the subject is given the stimulus as a cue and is asked for the response. It is assumed that the search for the correct image is governed by the following rules. On every sampling attempt two cues are used: the stimulus cue, S_i, and the context cue, C, the set of context elements presently active. The probability of sampling image I is then given by Equation 3:

$$P_S(I_i \mid C, S_i) = \frac{S(I_i, C)S(I_i, S_i)}{\sum_{j=1}^{n} S(I_j, C)S(I_j, S_i) + Z} \qquad (3)$$

The additive constant Z in the denominator of this sampling equation, represents the interfering effect of all extraexperimental associations. It has the effect of reducing the sampling probabilities. If all contextual associative strengths are reduced (e.g., because of a long retention interval), the parameter Z assures that the probability of sampling an image from the experimental list becomes arbitrarily small. Without such an additive constant, the sampling probabilities would not decrease to zero, but to an intermediate value (since then these probabilities would be determined only by the relative strengths, the absolute values would not be important). Note that Z does not have to be a constant: The number of extraexperimental associations might increase as a function of the delay between study and test (such an assumption was made by Gillund & Shiffrin, 1984). In the present analysis, however, Z will be assumed constant, a somewhat idealized situation representing "pure delay."

As mentioned above, sampling of an image is not enough for recall. The information should also be successfully recovered. That is, with the aid of the information activated from the image, the subject tries to reconstruct the item (in this case the name of the response). This decoding or recovery process is fallible. It is assumed that the probability of successful recovery is a positive function of the associative strengths. The SAM theory proposes the following equation for the probability of successful recovery:

$$P_R(I_i \mid C, S_i) = 1 - \exp\,[-S(I_i, C) - S(I_i, S_i)].$$

Thus, the probability of recovery is a function of the sum of the retrieval strengths to be probe cues. This probability applies, however, only to the first time an image is sampled during the search with a particular set of probe cues. If recovery does not succeed on the first sampling attempt using a certain set of cues, then it will (within the same test trial) never succeed with this particular set of cues.

According to SAM, a number of retrieval (or sampling) attempts are made. The search process may end in two ways. The first possibility is that the subject retrieves and recalls a response. The second way is when the limit on the allowed number of samples has been reached. This stopping criterion is denoted by L_{max}.

In the general SAM theory, we also have the option of incrementing the strengths to the probe cues upon successful recall (learning on test trials). This assumption has been used previously to predict certain results in the free-recall paradigm. However, it is not necessary to use incrementing in our model: the nature of most of the predictions that we will consider does not change whether incrementing is used or not. Using Monte Carlo simulation (see the section *Retroactive Inhibition*), it may be shown that the qualitative pattern of the results does not depend on this assumption although the general level of recall is affected by incrementing (but this may be rectified by changing some of the other parameters). Hence, the value of the increment parameter will be set to 0, except where otherwise mentioned.

There would of course be no reason to do so if there were no advantages to it. However, not using the incrementing option enables us to present a number of analytically derived predictions. Thus, we do not necessarily have to use the Monte Carlo simulation technique as in previous applications of the SAM theory. This not only saves computer time and eliminates variability in the predicted results, but also enables us to fit the theory quantitatively to experimental data using standard minimization procedures and to analyze potential identifiability problems with respect to the parameters.

Applications of the Model to Interference Studies

In this section, we shall present a large number of predictions of the present model. We pay particular attention to those phenomena that have been the source of controversy and debate and that have led to modifications of the traditional interference theory (e.g., the findings that led to the postulation of the unlearning process and the eventual abandonment of this assumption). In addition, we shall examine a number of basic findings that are part of the body of fundamental experimental results and that are presented in most textbooks on memory (e.g., Osgood's transfer and retroaction surface and McGovern's analysis of different interference paradigms).

We will not be interested in "fitting" the model to these results in the usual sense, but only in the qualitative patterns. For this reason, we will use the same set of parameter values for all predictions (except where otherwise noted), even though the referenced experiments differ in terms of materials, procedure, and subjects. Had we been interested in presenting a more or less quantitative fit, we would have had to use a different set of parameter values for each simulated experiment. We would then have been vulnerable to the obvious criticism that the predicted effects might be parameter dependent. Instead, we wish to emphasize that the predictions are based on inherent properties of the model and not on the specific parameter values used. Thus, unless otherwise specified, the qualitative pattern of the predictions does not depend on the parameter values used. These parameter values are presented in Table 1. These values were selected so as to keep the predictions in the right ballpark.

Table 1
Parameters and Their Values Used in the Simulations

Parameter	Description	Value
α	Conditioning rate for contextual elements	.6
β	Rate at which active elements become inactive	.0035
γ	Rate at which inactive elements become active	.0001
a	Scale parameter for context strengths	.3
b_1	Interitem strength for List 1 items	.05
b_2	Interitem strength for List 2 items	.05
d	Residual associative strength	.01
L_{max}	Number of allowed samples	4
Z	Interference from irrelevant preexperimental associations	.001
t_1	Length of intertrial interval (default)	5
t_2	Length of interlist interval (default)	10

However, there are probably a large number of other combinations that also work quite well. In each analysis of a published experiment, all of the experimental parameters (such as list length, presentation time, and number of list presentations) have been set to the particular values used in that experiment. If such details were not available, the default values given in Table 1 were used. In addition, the analyses followed such procedural details as the use of learning to criterion or a fixed number of study trials on a given list.

Retroactive Inhibition

We shall first examine a number of basic findings on retroactive inhibition. To this end, the predictions of the model for the classic experiments of Barnes and Underwood (1959), Thune and Underwood (1943), and McGovern (1964) are presented.

Barnes and Underwood (1959) performed an experiment using the AB–AC paradigm. The purpose of this experiment was to show that RI could be observed even if response competition is eliminated. Such results were regarded as evidence against McGeoch's theory and as supporting the unlearning assumption (see Crowder, 1976).

In this experiment, the lists consisted of eight paired associates. List 1 was learned to a criterion of one perfect anticipation trial. List 2 was given either 1, 5, 10, or 20 study trials. A MMFR test (supposedly eliminating response competition) was administered following the last List 2 trial. Their results indicated that List 1 recall decreases as a function of the number of second list presentations, whereas List 2 recall increases. To arrive at a qualitative fit, we performed a Monte Carlo simulation of the model as applied to MMFR recall (simulation is necessary because of the learning-to-criterion aspect of the experimental design). The algorithm used to model the recall process is shown in Figure 1.

The interpretation of this recall algorithm is as follows. The counter i denotes the stimulus to be presented to the statistical "subject." Using the stimulus and context cues, the subject searches memory for the R_1 (or a–b) and the R_2 (a–c) images. The recovery process takes place as soon as one of these two images has been sampled. This may lead to two different outcomes. If recovery succeeds, the strength between the stimulus and the response may be incremented by an amount denoted by INC. For generality the incrementing process is included in Figure 1, but as mentioned earlier, we will not make use of incrementing. Thus, we set $INC = 0$. The retrieval of a response may or may not be followed by a list membership decision. (A box is drawn around this part of the flowchart because it is only needed for the Thune and Underwood predictions discussed below.) However, if recovery or sampling fails, the counter L is incremented and another sampling attempt is made as long as L is less than L_{max}. After a response is produced, the search may be continued for the other response ("ONE LEFT") if testing is

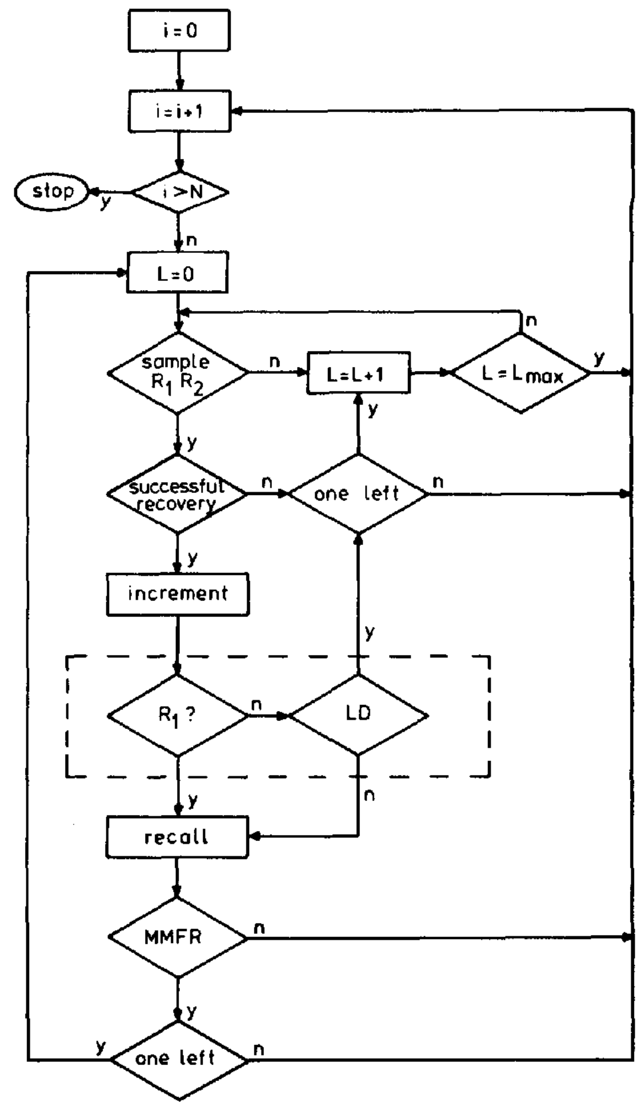

Figure 1. The recall algorithm as used for the computer simulation. (In case of modified modified free recall, the blocked region [representing list discrimination] of the flowchart should be disregarded. This part is only used in case of the Thune and Underwood [1943] simulation. For a more detailed explanation, see text.)

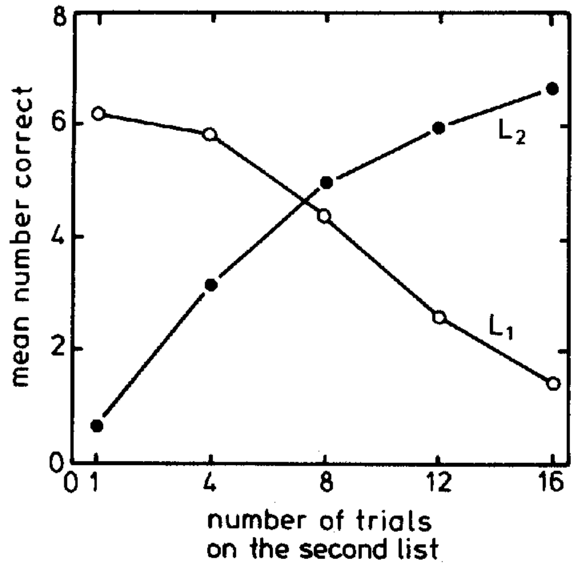

Figure 2. Predictions generated for the Barnes and Underwood (1959) study. (L_1 denotes the List 1 response curve; L_2 denotes the List 2 curve.)

by the MMFR method (MMFR = "Y"). Note that the unpaced character of the MMFR test is represented in the simulation program by the assumption that L is reset to zero when one of the two responses has been recalled, leaving an additional number of L_{max} sampling attempts for the other response. In case both responses have already been recalled or if L_{max} has been reached, the next stimulus is presented by incrementing the counter i. This continues until all stimuli have been tested. The results produced by this algorithm using the parameter values as given in Table 1 and disregarding the list-membership decision part, are presented in Figure 2.

It may be concluded that the model fits the pattern of data obtained by Barnes and Underwood. This is due to the changing value of the associative strength between the stimulus cue and the second list image. This strength increases as a function of the number of List 2 presentations, which causes a decrease in the sampling probability of a List 1 image and an increase in both the sampling and recovery probability of the second list images. Note that the model predicts retroactive interference on a MMFR test despite the fact that no unlearning is assumed.

Such deteriorations in the recall of List 1 responses may also be observed in the AB–AC design using the List 1 recall method (i.e., when the subject is asked to give only the List 1 response). However, besides response competition, results with this method are also partly influenced by list discrimination failures. Relevant data were presented by Thune and Underwood (1943). Using the AB–AC paradigm, they gave 5 acquisition trials on List 1 and either 2, 5, 10, or 20 trials on List 2. Furthermore, a control group was run that was not given a second list. Both lists consisted of 10 paired associates. Relearning of the first list always started approximately 20 min after the final List 1 study trial. The results showed that the measure of RI, that is, control group recall minus the recall score obtained from the interference group, increased as a function of the number of List 2 presentations, whereas the number of intrusions (due to list discrimination failures) appeared to be an inverted U-shaped function of this variable. Similar results were presented by Melton and Irwin (1940).

To simulate these results we used the algorithm shown in Figure 1. However, a small adaptation has to be made. It is assumed that the search process terminates as soon as one response, either correct or incorrect, has been recalled (MMFR = "N").

List discrimination must now be included in the algorithm. To accommodate list discrimination, we have made a number of simplifying assumptions. It is assumed that the first recalled response will be produced unless it is positively identified as belonging to the interfering list, that is, unless the subject is reasonably sure that the response is wrong. We believe that such list discrimination decisions have to be based on the contextual information encoded in the retrieved image. Each contextual element encoded in the image is supposed to give some information regarding list membership. Each time such an element is encoded during List 1 learning, there is a probability p_c that it may be conditioned to a list code. Hence, after learning of both lists, some contextual elements will be conditioned to both list codes, some only to the List 1 or the List 2 code, and some to neither code.

Since the contextual information encoded in a List 1 image will seldom lead to a positive List 2 membership identification, it is assumed that such a response will always be produced. The list discrimination decision with respect to retrieved List 2 images is assumed to be a function of the number of retrieved contextual elements conditioned to the List 1 code (denote this by v_1) and the number conditioned to the List 2 code (v_2). Thus, the probability that the subject (incorrectly) identifies a List 2 image as belonging to List 1 (i.e., fails to identify the response as a List 2 response) is assumed to be equal to

$$p(L_1) = \frac{v_1}{v_1 + \delta(t)v_2},$$

where $\delta(t)$ is a (time-dependent) bias parameter. Although this parameter is not used in the present applications, it is included here for generality, since it seems likely that $\delta(t)$ approaches 0 with very long delays. This would correspond to a strategy to emit every recalled response when retrieval becomes very difficult (which will be the case with long delays).

We wish to emphasize that the details of this model for list discrimination are not very important for our purposes. Undoubtedly, there are a variety of other ways in which this decision process might be modeled. In addition, there are other factors that will affect the difficulty of list discrimination, such as the within- versus between-list similarity. Notwithstanding these reservations, the present model does capture a number of significant aspects. First, list discrimination is based on retrieved contextual information. Second, it makes the not unreasonable prediction that a long interlist interval (or, more precisely, a relatively large context change between lists) makes list discrimination easier since this leads to a decrease in v_1 (fewer elements encoded in the List 2 image will be conditioned to the List 1 code). Finally, and most importantly for the present

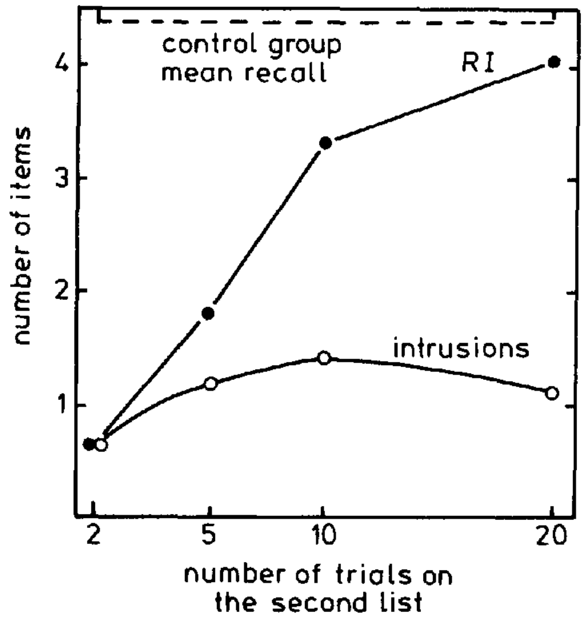

Figure 3. Predictions for the Thune and Underwood (1943) study. (The ordinate corresponds both to the retroactive inhibition [RI] curve as well as to the intrusion function. The RI measure equals the difference in recall between the control and interference condition.)

application, the probability of an incorrect decision decreases with the number of study trials on the interfering list (this increases v_2).

Given the parameter values of Table 1 and assuming p_c = 0.01, this model yields (for Thune & Underwood's, 1943, design) the following probabilities of correctly identifying a List 2 response: 0.46, 0.66, 0.78, and 0.87 in case of 2, 5, 10, and 20 List 2 trials, respectively. Figure 3 gives the predictions of the SAM model for experiments of this type. Again, the pattern of the predicted results agrees quite well with the observed data. Retroactive inhibition is shown to be a monotonically increasing function of the number of trials on the second, interfering, list, while the predicted number of intrusions is an inverted U-shaped function.

It must be mentioned that in our experience, the shape of the intrusion function depends on the parameter values used; that is, the inverted U-shaped function is not a parameter-free prediction. According to the model, a monotonically decreasing function may also be observed. This can be explained as follows. The number of intrusions depends on two factors. The first is list discrimination, the second is the probability of recalling a second list response. Both factors account for the inverted U-shaped function. However, these variables may interact in such a way that a monotonically decreasing function is observed. In particular, this will occur when the list discrimination probabilities happen to increase rapidly as a function of the number of List 2 trials. As a consequence, few List 2 responses are recalled in case of, say, two study trials, but these are given incorrectly as a response with a relatively high probability. The reverse is true when List 2 has been given many study trials. In such a situation, a monotonically decreasing intrusion function will be observed.

Finally, we will apply the model to the results obtained by McGovern (1964). McGovern compared the four major interference paradigms (C–D, C–B, A–C, and A–Br). We will consider the List 1 recall scores presented in her article. In this experiment, List 1 (consisting of eight pairs) was learned to a criterion of one errorless trial, requiring approximately 9 presentations. The second list was presented for 15 trials followed by a List 1 recall test. For our analysis it is important to note that on the final test the subject received a list of all stimulus terms and was instructed to guess if unsure about a particular association. Furthermore, two scoring methods were used, one stringent and one more liberal. With the stringent scoring method, a response is counted as correct only if it was given to the appropriate stimulus. In the liberal method, all responses recalled by the subject were scored as correct regardless of placement. With the stringent scoring method, the ordering of the conditions with respect to List 1 recall was C–D = C–B > A–C = A–Br. The liberal method, however, produced quite different results: C–B > A–Br > C–D > A–C.

Before we present the predictions for this type of experiment, we shall first discuss the (analytical) method that was used to obtain these predictions. According to our model, the probability of recalling the target response (R_1), given a maximum of $m = L_{max}$ sampling attempts, is given by

$$P(R_1) = [1 - (1 - P_s)^m][1 - \exp(-k_1 b_1 t - s_1)], \quad (4)$$

where P_s denotes the probability of sampling the appropriate List 1 image. For example, in case of k_1 study trials on List 1 (of t seconds each) and k_2 study trials on List 2 in the AB–AC paradigm,

$$P_s = \frac{k_1 b_1 t s_1}{k_1 b_1 t s_1 + k_2 b_2 t s_2 + (n-1)d(s_1 + s_2) + Z}; \quad (5)$$

S_i (i = 1 or 2) denotes the List i context strength as obtained from the fluctuation model. The first part of Equation 4 gives the probability of sampling the image at least once; the second part gives the probability of recovery. Note that this equation is only valid if there is no incrementing. To facilitate the derivation of the sampling probabilities for each of the interference paradigms used in this experiment, Figure 4 shows the various interfering relations in a pictorial way.

For example, in the A–C design, the stimulus a is shown to be strongly connected to the images (ab) and (ac). In addition, it is residually associated to (n − 1) List 1 images and to another (n − 1) List 2 image. Using the contextual strengths defined in Equation 2, we obtain the sampling probability given in Equation 5.

The difference between the four interference paradigms is reflected in the denominator of P_s. For the other cases this denominator is given as follows:

C–D: $k_1 b_1 t s_1 + (n-1)ds_1 + nds_2 + Z;$

C–B: $k_1 b_1 t s_1 + (n-1)ds_1 + nds_2 + Z;$

A–Br: $k_1 b_1 t s_1 + k_2 b_2 t s_2 + (n-1)d(s_1 + s_2) + Z.$

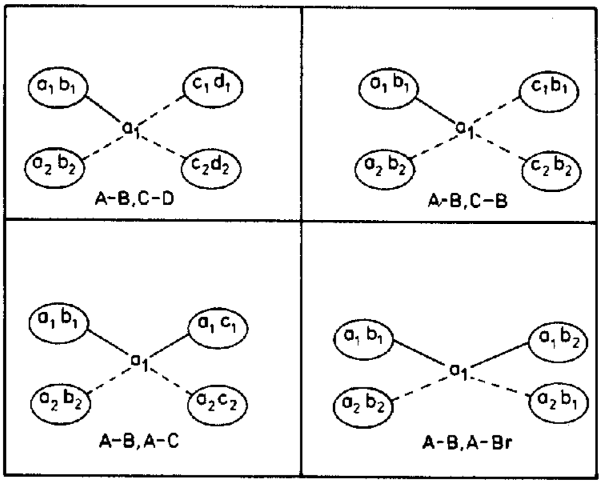

Figure 4. The retrieval structures for different interference paradigms. (Images are denoted by encircled alphabetical characters corresponding to the stimulus and the response. The stimulus cue is not encircled. Unbroken lines between the cue and the images represent strong associative connections; broken lines denote residual associative connections.)

Note that the equations for the C–D and C–B designs are identical, and similarly for the A–C and A–Br designs. Hence, this immediately yields the (parameter-free) prediction that these designs should lead to identical recall results. The predicted results for the stringent scoring method, obtained upon insertion of the parameter values of Table 1, are presented in Table 2. The results are in close agreement with the data presented by McGovern (1964). From Figure 4 it may be concluded that the difference between recall for the C–D and C–B paradigms on the one hand and the A–C and A–Br paradigms on the other, is due to the strong interfering association of the List 2 response in the latter two cases.

We also derived recall predictions for the liberal scoring method. It may be assumed that this method solely measures response availability. According to the model, this scoring procedure may have a remarkable effect in the C–B and A–Br paradigms. In both cases, each response is part of two images, one from each list.

In order to show how the equations used to generate the predicted results for the liberal scoring method were obtained, we will derive such an equation for the C–B paradigm. We will assume that the response belonging to a given stimulus may be found in the $(a\text{–}b)$ image or, via the residual association, in the $(c\text{–}b)$ image. Furthermore, as shown in Figure 4, these images may also be sampled using the other, residually associated, stimulus terms. This analysis is supposed to mimic the effects of the guessing instructions used by McGovern. If we denote the sampling and recovery probabilities of the image $(a\text{–}b)$ by x and r respectively, the probabilities connected with the $(c\text{–}b)$ image by y and s, and the sum of the sampling probabilities of any other image by w, then the probability of recalling a particular response b using its associated stimulus is given by

$$p(b) = 1 - \sum_{i=0}^{m} \sum_{j=0}^{m-i} \frac{m!}{i!j!(m-i-j)!} x^i (1-r)^{d_1} y^j (1-s)^{d_2} w^{m-i-j},$$

where $d_1 = 0$ for $i = 0$, else $d_2 = 1$. Similarly for d_2: $d_2 = 0$ for $j = 0$, else $d_2 = 1$. m is a shorthand for L_{max}. In this equation, i gives the number of times (out of m) that the $(a\text{–}b)$ image is sampled, and j gives the number of times the image $(c\text{–}b)$ is sampled.

The b response may also be recalled when memory is searched with one of the $(n - 1)$ residually associated stimuli. The probability that the response is recalled using such a stimulus may also be derived by the above reasoning. Denoting this probability by $p'(b)$, we arrive at the overall probability, $p_t(b)$, of recalling the response:

$$p_t(b) = 1 - [1 - p(b)][1 - p'(b)]^{n-1}.$$

In case of the other paradigms, such probabilities may be derived in a similar way. These equations lead to the results presented in the second column of Table 2. As mentioned above, the predicted increase in A–Br recall reflects the fact that two different stimuli may produce the same response. Thus, each response has a higher probability of being recalled than in the C–D paradigm, in spite of the interfering relations. This is also true for the C–B design. However, predicted C–B recall does not surpass A–Br recall, as in McGovern's data. In spite of the latter result, which indicates that the model (given the present set of parameter values) does not predict the increase in C–B recall as strongly as observed, we may conclude that the model predicts the differences between stringent and liberal scoring reasonably well. Moreover, it does so using fairly simple assumptions.

These results lead to the conclusion that the model is able to handle the major RI findings. We shall now turn to proactive inhibition.

Proactive Inhibition

The results of many experiments indicate that PI is a function of the degree of prior learning and of the retention interval (Houston, 1967; Koppenaal, 1963; Underwood, 1949). When List 2 learning is immediately followed by a retention test, PI will not be observed. As the length of the retention interval increases, a monotonic increase in PI is observed that eventually reaches an asymptotic level. Recall that the occurrence of PI in MMFR tests poses a complicated problem for the unlearning/response-competition theory. However, it shall be shown that

Table 2
Predicted Results for McGovern's (1964) Experiment

	Scoring method	
Design	Stringent	Lenient
A–B, C–D	5.03	5.05
A–B, C–B	5.03	5.66
A–B, A–C	2.11	2.12
A–B, A–Br	2.11	6.29

our model does not have any problems with this phenomenon. This will be confirmed by applying the model to the experiments of Koppenaal (1963) and Underwood (1949).

Koppenaal used the MMFR method for the final recall test (A–B, A–C paradigm). Before presenting the model predictions, we will first derive the analytical equation used to obtain these predictions. Sticking to the representation of the MMFR method as used in the computer simulations (see Figure 1), we may derive the probability of, for example, R_2 recall in the following way. Either R_2 is recalled as the first response in m ($=L_{max}$) sampling trials, or the other response, R_1, is recalled within the first m sampling trials and there remain m trials for R_2 to be recalled. Starting with the first possibility, we have to derive the probability that R_2 is recalled before R_1. (This derivation will also be needed for the prediction of Briggs's data; see below).

After both lists are learned, the stimulus is associated strongly to R_1 and R_2 (more precisely, the corresponding images), whereas it is residually associated to the other responses (R_o). The event that R_2 is recalled before R_1 may be accomplished in different ways. First, irrelevant responses (denoted by R_o) may be sampled before sampling and recovering R_2. Second, R_1 may be sampled before R_2 but recovery fails. If recovery of R_1 does not succeed when it is first sampled, then it will also fail on subsequent samplings since the same probe set is used. This rule also applies to R_2. Hence, R_2 will only be recalled if recovery is successful when the corresponding image is sampled for the first time. From these considerations, we may derive the probability of the following event: On the first n sampling trials neither R_1 nor R_2 is recalled, but on the $(n+1)$-th trial R_2 is sampled for the first time and successfully recovered.

$$f(n) = \Big[\sum_{i=0}^{n} \binom{n}{i} p(R_1)^i p(R_o)^{n-i}(1-r)^{\delta}\Big] p(R_2)s,$$

where $\delta = 0$ if $i = 0$, else $\delta = 1$; $p(R_i)$ denotes the sampling probability of R_i; and r and s correspond to the recovery probabilities associated with R_1 and R_2, respectively. Since we have a maximum of m sampling attempts, the combined probability of recalling R_2 prior to R_1 is equal to

$$p(R_2 \text{ before } R_1) = \sum_{n=0}^{m-1} f(n).$$

After simplification of this sum we finally arrive at

$$p(R_2 \text{ before } R_1) = rsp(R_2)\left[\frac{1 - p(R_o)^m}{1 - p(R_o)}\right] + s(1-r)[1 - (1 - p(R_2))^m].$$

The probability of recalling R_2 after R_1 has been recalled within the first m sampling trials equals

$$p(R_2 \text{ after } R_1) = \sum_{i=0}^{m-1} p(R_o)^i p(R_1) r\{1 - [1 - p(R_2)]^m\}s. \quad (6)$$

This equation is based on the fact that on the i trials prior to recall of R_1, R_o responses have to be sampled. The total probability of recalling R_2 on the MMFR test is then given by

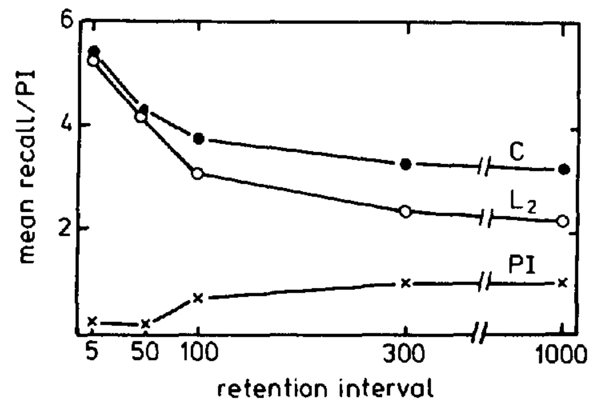

Figure 5. Predicted patterns of Koppenaal's (1963) study. (Control group [C] recall as well as modified modified free recall List 2 recall [L_2] are depicted. The difference is given by the proactive inhibition [PI] curve.)

$$p(R_2) = \{rsp(R_2) + p(R_1)r[1 - p(R_2)]^m s\}\left[\frac{1 - p(R_o)^m}{1 - p(R_o)}\right] + s(1-r)\{1 - [1 - p(R_2)]^m\}. \quad (7)$$

Using this equation and the parameter values from Table 1 we generated PI predictions for Koppenaal's experiment. The lists consisted of 10 pairs. List 1 was given 10 study trials and List 2, 5 study trials (of 2 s each). The results are shown in Figure 5 for various retention interval lengths (arbitrary time units).

In this figure, recall decreases in both the control condition (C) and the PI condition. However, relative to the control condition, second list recall in the PI condition (L_2) decreases faster, leading to an increase in the standard difference score for PI ($C - L_2$). In order to see more clearly how the model explains this effect, we will first rewrite the sampling probability P_s (Equation 3) for the List 2 image in the PI condition:

$$P_s = \frac{b_2 t}{b_2 t + b_1 t(s_1/s_2) + w},$$

where w equals the sum of all residual strengths divided by s_2. This equation clearly shows that the sampling probability is a function of the ratio of the contextual strengths s_1 and s_2. It can easily be shown that the ratio s_1/s_2 is an increasing function of the retention interval.

The consequence of this is that PI is induced, since the probability of sampling List 2 images deteriorates with increasing interval length, which is due not only to simple forgetting (as in the control condition) but also to a relative increase in the probability of sampling the List 1 image. It should be noted that PI will of course decrease again for very large retention intervals. This is evident from the fact that recall for both the control as well as for the PI condition eventually reaches zero.

Consider next the results obtained by Underwood (1949). Underwood manipulated both the amount of prior learning and the length of the retention interval. In this experiment, subjects were run using the A–B, A–C paradigm. The second list was learned until six (out of eight) responses were anticipated cor-

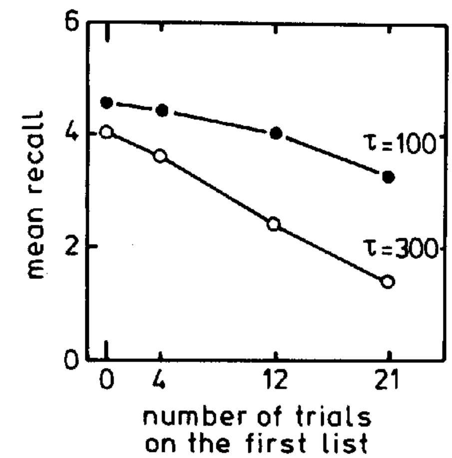

Figure 6. Predicted List 2 recall patterns in case of the Underwood (1949) proactive inhibition study. (Retention intervals are given by τ [arbitrary units].)

rectly. The first list was learned to a criterion of three responses correct, eight responses correct, or 100% correct plus five additional (over) learning trials. The final retention test was administered following an interval of either 20 or 75 min. The results indicated that forgetting of List 2 increased as a function of the degree of prior learning. The length of the retention interval seemed to have no effect on PI.

Our model accounts for these patterns in the following way. Increasing the number of study trials on the first list increases the interfering associative strengths of the List 1 images. However, the model also predicts an increase in PI as a function of the retention interval because of the ratio s_1/s_2. As noted above, this is not observed in Underwood's data. However, if we look at the data corresponding to the longest interval, we may conclude that the lack of a further decrease in recall as compared with the 20 min interval condition is probably due to floor effects in the recall scores. Figure 6 shows the predicted patterns based on a List 2 version of Equation 4 (this equation is used since Underwood made use of the relearning method, which is paced). Again, the model behaves satisfactorily. If we had used lower interitem strengths, we could have shown a lack of increase in PI due to floor effects.

An interesting finding concerning PI was presented by Anderson (1983a; see also Postman, Stark, & Burns, 1974). Usually, the negative interfering effects of prior learning are much stronger in the A–C design than in the C–D design. However, if the List 2 recall probabilities are made empirically equivalent for both designs by giving the A–C list (which suffers from negative transfer) more study trials and testing PI with an unpaced MMFR test, then A–C recall will surpass C–D recall. This counterintuitive phenomenon is predicted by Anderson's ACT* model (see Anderson, 1983a).

It can be shown that our model leads to the same prediction. In the present model (as well as in the ACT* model), the probability of recall is based on both the relative and the absolute strength of an image. The probability of sampling depends on the relative strength, whereas the recovery probability is based on the absolute strength. Following Anderson (1981), let $f(R)$ be the sampling probability for the image corresponding to response R and let $g(R)$ be the recovery probability. If the List 2 recall probabilities are equalized for the C–D (R_{cd}) and A–C (R_{ac}) conditions, then, according to the model, it must be the case that

$$f(R_{cd})g(R_{cd}) = f(R_{ac})g(R_{ac}).$$

Since the A–C condition suffers from negative transfer, more trials will be required in this condition in order to end up equally. Hence, the A–C images will have a higher absolute strength at the end of List 2 learning than the C–D images. Thus,

$$g(R_{cd}) < g(R_{ac}),$$

since the probability of recovery is a monotonic function of the absolute strength. However, since the recall probabilities are equal, it must also be true that

$$f(R_{cd}) > f(R_{ac}).$$

Consider now what happens if List 2 learning is followed by an unpaced retention test. In our model, this means that the parameter L_{max} must be set at a relatively high value. As a consequence,

$$f(R_{cd}) \simeq f(R_{ac}).$$

However, $g(\ )$ does not depend on L_{max}. Hence,

$$f(R_{cd})g(R_{cd}) < f(R_{ac})g(R_{ac}),$$

which demonstrates the prediction.

In order to check this theoretical analysis, we simulated this phenomenon as follows. List 1 was presented for five trials, and List 2 was learned to a criterion of 70% correct in both conditions. A recall test followed the retention interval. Both conditions were given L_{max} sampling attempts in order to keep them fully comparable. The predicted results are shown in Figure 7 where C–D and A–C recall are plotted as a function of L_{max}, which may be interpreted as the degree of "unpacedness" of the retention test.

The results indeed show that as a function of L_{max}, A–C recall eventually exceeds C–D recall (for related observations about interference and test unpacedness, see Adams, Marshall, & Bray, 1971). A general corollary that follows from this result is that statements such as "the response strengths are equalized" are theory dependent and must be interpreted quite cautiously. That is, equal recall probabilities do not necessarily imply equal response strengths. Thus, according to our model, if the recovery probabilities for the C–D and the A–C designs are equalized, this implies unequal sampling probabilities, and vice versa. We shall return to this issue when we examine the relation between response latency and response accuracy.

Spontaneous Recovery

As mentioned previously, our model produces (relative) spontaneous recovery (i.e., a relative increase in List 1 recall), and this factor influences the prediction of PI. It was demon-

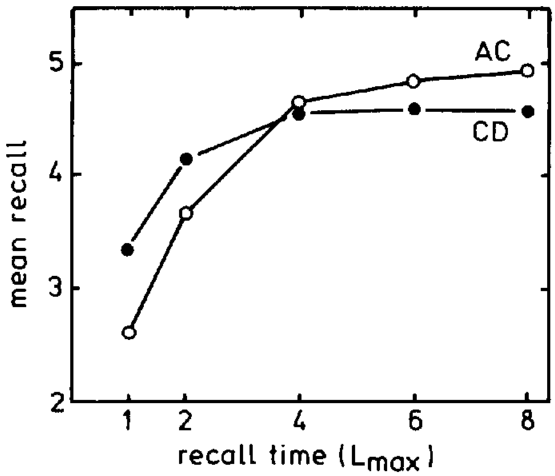

Figure 7. Application of the model to a proactive inhibition design, where the C–D and the A–C conditions are brought up to an equal recall level. (Recall is depicted as a function of L_{max}, the degree of unpacedness of the retention test.)

strated previously that PI depends on the ratio s_1/s_2, which is a function of the retention interval. It will now be shown that this same ratio is responsible for the prediction of spontaneous recovery (relative as well as absolute). We shall first show the model's predictions for Briggs's MFR data (1954) where spontaneous recovery is observed. Using the A–B, A–C design, Briggs's subjects were interrupted at certain recall levels during learning. Immediately after such an interruption, list stimuli were shown and subjects had to produce the first response that came to mind (MFR method). The most interesting result for the moment is the increase in List 1 responses produced as a function of the length of the retention interval. The same pattern of results may be analytically derived from our model. Before considering the outcome, we shall first discuss the method used to generate the predictions.

Because of the use of the MFR method, we have to derive the probability of recalling, say, R_1 before R_2, where both are associated to the same stimulus term. In Briggs's study, recall terminated as soon as R_1, R_2, or R_0 (another, irrelevant, response) is recalled. For our predictions, recall of R_0 is irrelevant, since we are only interested in the relative probabilities of recall of R_1 and R_2. Therefore, we will assume that recall only terminates upon recall of R_1 or R_2. This reduces the problem to deriving the probability of recalling, say, R_1 before R_2. Since we have already derived this probability (Equation 6), we may substitute the experimental design parameters (for example, presentation time, list length) used in Briggs's experiment into that equation. Solving the resulting equations, we obtain the predictions shown in Figure 8. We may conclude that the predictions are in good agreement with Briggs's data: Absolute recovery of List 1 responses is indeed predicted by the model.

The model may also be extended to the A–B, A–C, A–D paradigm where MMFR recall is used to measure forgetting. Such data were presented by Postman et al. (1968). In this experiment, the retention interval was either 2 or 20 min. In order to equalize the degrees of List 1 and List 3 learning, the first list was presented for six trials and the second and third lists for four trials each (see Postman et al., 1968). The results showed A–D recall to be a decreasing function, A–C recall remained approximately equal, whereas A–B recall increased, thus showing absolute spontaneous recovery. Using an extension of Equation 7, we generated corresponding predictions of our model. From the left panel of Figure 9 it may be verified that the predicted recall patterns are quite similar to those obtained by Postman et al. (1968).

However, this result is not as simple as it appears. When we generated these predictions, five different retention intervals were used. As shown in the right-hand panel of Figure 9, List 1 recall first shows a small decrease followed by a much larger increase (mainly because of the higher stored strength). The panel on the left depicts only two of these intervals and they have of course been chosen in such a way as to resemble the data of Postman et al. Thus, we may have discovered one of the reasons for the confusion that accompanied the experimental investigation of the recovery phenomenon. In this particular design, absolute recovery might be overlooked if the "wrong" points were selected (e.g., the intervals corresponding to 5 and 50).

For the more simple case of a two-list design, a similar factor may have been involved. This was examined by computing the predictions for the A–B, A–C MMFR recall paradigm. Figure 10 shows List 1 recall as a function of the retention interval. Consider first the two highest curves. The lower of these has been computed using the parameter values of Table 1, whereas the top curve was obtained with the interitem strength parameter set at 0.20. Detailed inspection of the sampling and recovery probabilities revealed the mechanism that generates the slightly nonmonotonic behavior of the middle curve. When the interitem strength is high, changes in the contextual strength have little effect on the recovery probability since this function will already be at the maximum value (1.0). However, when the interitem strength is relatively low, the recovery probability strongly depends on changes in the contextual strength. When the sampling and recovery probabilities that produced the nonmonotonic behavior were examined, it was observed that the main increase in the sampling probability occurred at a later point in time than the main decrease in the recovery probability. The latter starts immediately after the end of List 2 learning. This causes an initial decrease in the recall function followed by an increase. We have not been able to find any evidence for this prediction in the literature.

Moreover, it was found that the absolute recovery effect vanishes when the preexperimental associative strength (Z) is set at a somewhat higher value. The bottom curve of Figure 10 shows the results obtained with $Z = 0.01$. In this case, forgetting occurs at such a rapid rate (indirectly because of high preexperimental associations) that the spontaneous recovery process cannot compensate for it. This notion is in full agreement with Underwood (1948). He proposed that during the retention interval List 1 response strength is determined by two opposing processes: recovery and normal forgetting. Absolute recovery will only be observed when the normal forgetting effect is weaker than the spontaneous recovery effect. When this is taken

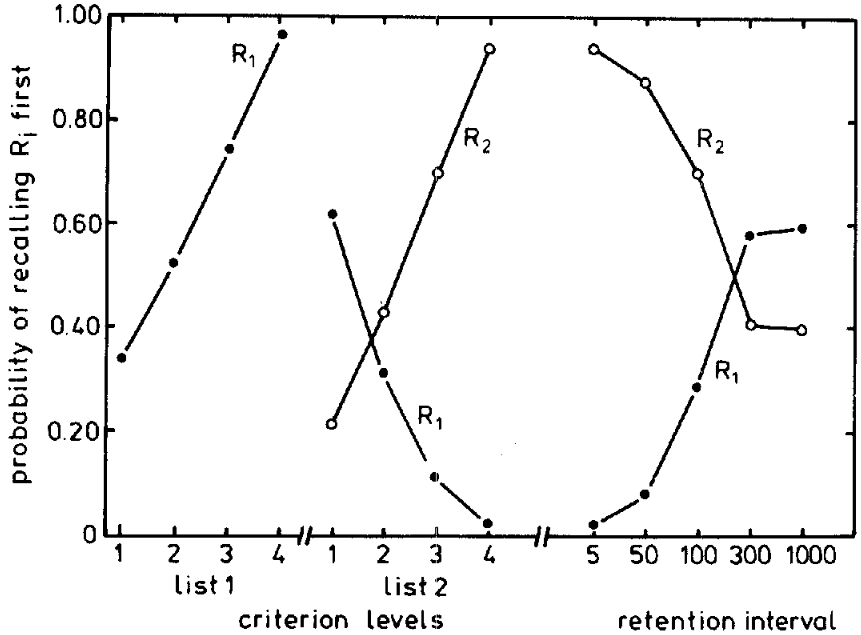

Figure 8. Analytically derived predictions for the Briggs (1954) study. (R_1 gives the List 1 modified free recall [MFR] response curves. R_2 symbolizes the second-list MFR response curves. Criterion level *i* corresponds to the MFR test given during list learning when the 24**i*% correct criterion was reached.)

into account, the evidence for spontaneous recovery is quite strong (see Brown, 1976, for a more extensive review).

The Independence Phenomenon

There has been much debate (e.g., Greeno, James, DaPolito, & Polson, 1978) about the so-called independence postulate

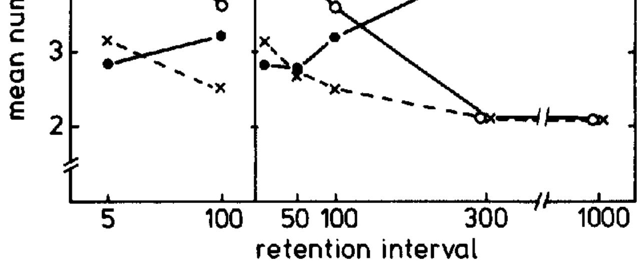

Figure 9. Predictions for Postman, Stark, and Fraser (1968) study. (Left panel: For two different retention intervals. Right panel: For five different retention intervals.)

proposed by McGeoch (1932). This postulate states that responses become associated to the same stimulus in an independent fashion. Intuitively, this is not compatible with the unlearning concept. It implies that the acquisition of R_2 does not affect the associative strength between the stimulus and the previously associated response R_1 (however, see Postman & Underwood, 1973).

Greeno et al. (1978) have presented impressive empirical evidence to support the independence assumption and have argued that this implies that the unlearning concept is contradicted. They argued that the unlearning assumption implies that the learning of R_2 should lead to the unlearning of R_1. The amount of unlearning should increase as the response strength of R_2 increases. Also, the higher the R_2 response strength, the higher its recall probability. Moreover, the more unlearning of R_1, the lower the probability of recalling this response. This leads to the conclusion that the recall probabilities should be dependent: $P(R_1 \mid R_2) < P(R_1)$. However, the majority of the data indicate independence. Indeed, the evidence is quite strong: The cumulative of chi-square values obtained in a number of experiments fits a chi-square distribution ($df = 1$) as would be expected if the assumption of independence holds (Greeno, James, & DaPolito, 1971).

Some potential artifacts concerning this result (i.e., subject–item selection effects) are discussed by Hintzman (1972). Moreover, Postman and Underwood (1973) have argued that the logic used by Greeno et al. (1971, 1978) is faulty and that the observed independence does not necessarily contradict the unlearning assumption. We will not join this debate but instead show relevant predictions made by our model that are in agree-

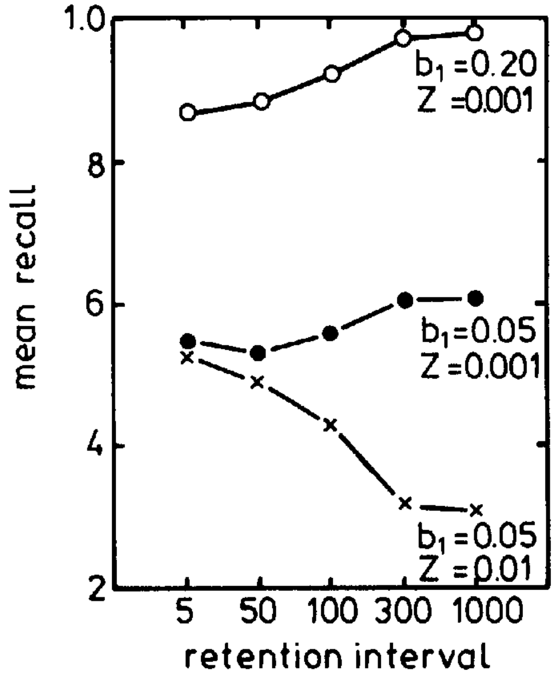

Figure 10. Spontaneous recovery for the A–B, A–C design. (The highest curve is generated using $b_1 = 0.20$, $z = 0.001$. The middle curve corresponds to $b_1 = 0.05$, $Z = 0.001$; and the lower curve is computed using $b_1 = 0.05$, $Z = 0.01$. All other parameters are set equal to their corresponding values given in Table 1.)

ment with the observations of Greeno et al. Using the algorithm presented in Figure 1 (without list discrimination), we generated predicted results which are classified here according to whether the first list was learned to a criterion of one perfect anticipation or was given a constant number of four study trials (the second list was always given four study trials), whether there was subject and/or item variability, and whether incrementing was used or not. Each predicted result is based on 100 simulated subjects. The lists consisted of eight pairs each. Thus, each ϕ coefficient is based on a total of 800 observations. Hence, an absolute value for ϕ of 0.069 would be significant at the 5% level.

According to our model, associating a second response to a stimulus does not affect the strength of the first response. Hence, the model predicts independence as long as the sampling counter L is reset to 0 if one of the two responses is recalled (assuming no incrementing takes place); that is, recalling one of the two responses does not have a negative effect on the probability of sampling the remaining response. This was confirmed by Monte Carlo simulation: Using learning to criterion on List 1 and assuming no subject–item variability, resulted in a nonsignificant correlation of 0.05.

Let us now turn to subject–item differences. These were created by sampling subject and item-difficulty values independently from a normal distribution. The sum of these two values determined the interitem associative strength for a particular subject–item combination. Thus, within the same subject, the "subject parameter" remains constant. The means of the two distributions were 0.1 and 0.05 for the subject and item distributions, respectively, if both were variable, and 0.2 for either if only one of them was varied. (These means are not exact since we resampled values if a value was selected that happened to be smaller than a certain criterion.) The variance of both distributions was 1.0. Thus, the variance of these distributions was given a relatively large value.

With learning to criterion on List 1, we observed a ϕ of −0.08 using both subject and item differences. Using only subject differences led to a ϕ of −0.07, and only item differences produced a ϕ of −0.14. Hence, both subject variability as well as item variability lead to negative correlations. With a fixed number of trials on List 1, the situation is somewhat different. In this case, the model predicts a similar result for item differences only: $\phi = -0.13$. However, subject variability now leads to a small positive correlation: $\phi = 0.04$. The magnitude of these effects will of course depend on the variance of the strength distributions.

The explanation for this pattern of results is fairly straightforward. Item differences always lead to a negative correlation since a strong List 1 association tends to inhibit the sampling of the List 2 image and vice versa. The effect of subject variability depends on whether List 1 is given a fixed number of trials or is learned to criterion. With a fixed number of trials, good subjects tend to recall both responses, leading to a positive correlation. With learning to criterion, the sign of the correlation changes from positive to negative. At first sight, this may seem strange, but it can be understood once it is realized that because of the learning to criterion, the List 1 strengths will not be very different, while the List 2 strengths will vary because of the subject variability. Hence, in this case high List 2 strengths are not accompanied by high List 1 strengths, thus eliminating the reason for a positive correlation. A negative correlation will be the result, since the probability of recall for List 1 will be inversely related to the strength of the List 2 images.

Furthermore, there exists yet another way for the model to generate dependence. This will be the case if we set the increment parameter at a sufficiently high value (note that there will be incrementing on all tests, hence also during the acquisition phase). Our simulation results show that the value of the increment parameter must be set at a quite high value compared with the value of the interitem strength parameter to obtain a significant effect. With learning to criterion and no subject–item variability, an increment of 0.05 leads to a ϕ of 0.01, while a high increment value (0.50) produces a negative ϕ of −0.12. The effect of incrementing is twofold: It introduces strength differences leading to a negative correlation as explained above, and second, if a response is recalled during the MMFR test, incrementing the strength of the retrieved image leads to a decrease in the probability of sampling the remaining image. Note that (a) only the interitem associative strengths are incremented and (b) with a relatively large number of trials on both lists, incrementing on the MMFR test will not have much effect since the interitem strengths are already at a high value.

This brings us to several conclusions. First, a model based on the independence of associations (i.e., without an "unlearning" assumption) may still predict small but significant correlations due to strength differences resulting from subject–item variability or to incrementing on test trials. However, reasonable values of these parameters lead only to small effects on the correlations. Second, as long as there are subject–item differences or incrementing processes, it will not be easy to prove the independence of associations (see also Hintzman, 1972). Finally, it is of some methodological interest that there appears to be an effect of the methods used: A design using learning to criterion will lead to different results compared with a design using a constant number of trials on List 1.

Osgood's Transfer and Retroaction Surface

Osgood (1949) proposed a descriptive model that predicts the degree of second list transfer following prior learning. According to this model, the amount and direction of transfer depend on the similarity between the stimuli and the responses of the two lists. It may be represented by a three dimensional surface in which the x axis represents the response similarity, the y axis the stimulus similarity, and the z axis the amount of transfer. Following the conventional terminology, the corners of the x–y plane represent the C–D, A–C, C–B, and A–B transfer designs.

The surface may be described as follows. Going from the A–B corner to the A–C corner, positive transfer gradually changes to negative transfer. Going from A–C to C–D, negative transfer is replaced by neutrality (the C–D design is defined as the zero transfer reference level). From the C–D corner to the C–B corner transfer remains zero, whereas positive transfer is induced if one moves from C–B to A–B.

It will now be shown that our model predicts the shape of this surface. Let us start with the induction of negative transfer as we move from A–B to A–C. We will first describe the representation of the A–B, A–B′ transfer design, where B′ denotes a response that is similar to the first list response. Positive transfer is accomplished in the following way. We will assume that if the List 2 response resembles the corresponding List 1 response, it will be incorporated in the same image. Thus, instead of the two images formed in the A–C design, one unitary image may be formed in the A–B′ design (as when the same list is presented twice). This leads to positive transfer because the contextual and interitem associative strengths will increase (as in the A–B transfer design) since the image is in fact given an additional number of study trials.

Thus, the decrease in response similarity is represented by assuming that the probability of forming a unitary image decreases as we move from A–B to A–C. Note that we treat the situation in an all-or-none manner: The List 2 response is either incorporated in the corresponding List 1 image or it is not. Figure 11 gives the predicted transfer results (the probability of recall on the first List 2 trial in comparison with the C–D design). Going from A–B to A–C, an initial positive amount of transfer is gradually replaced by negative transfer.

From A–C to C–D the stimulus similarity changes, which is represented by a gradually decreasing associative strength between the List 2 stimulus and the List 1 image. This results (for obvious reasons) in a decreasing amount of negative transfer (see Figure 11). Next we follow the x axis from C–D to C–B. As discussed in the section on retroactive inhibition, no changes

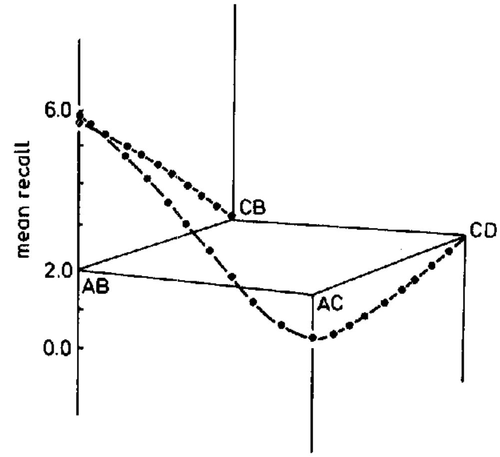

Figure 11. Application of the present model to Osgood's (1949) transfer surface. (The corners in the plane correspond to the four major transfer designs.)

are predicted because the List 2 stimuli remain different from the List 1 stimuli. Going from the C–B to the A–B corner, positive transfer is induced by a gradual increase in the probability of forming a unitary image. Hence, Osgood's results can be accommodated without any difficulty.

Forgetting Functions

As explained earlier, there are two basic factors in the SAM theory that account for forgetting: (a) an increase in the number of other, interfering, images that are associated to the retrieval cues; and (b) a decrease in the associative strengths of the cues to the image to be retrieved. In our model, the second of these two factors is explained by the contextual fluctuation process. Hence, in the absence of any interpolated learning, the model would still predict forgetting. Although such a condition of "pure" forgetting may be hard to realize experimentally, it is of some interest to investigate the pattern of results that would be predicted, in view of the recent controversy concerning the interpretation of the results obtained by Slamecka and McElree (1983; see Loftus, 1985a).

Although we shall not discuss it in detail, it might be noted that classical interference theory has some difficulty in providing a convincing explanation for this type of forgetting. Basically, it has been explained as being due to extraexperimental interference (see Postman, 1961). However, there is disappointingly little evidence to support this hypothesis.

Slamecka and McElree (1983) observed that as long as the recall probabilities are not too low, recall functions starting at different values (and hence based on different response strengths) decrease in a parallel fashion. This result seems to hold for a wide range of experimental manipulations affecting initial response strength. According to Slamecka and McElree, contemporary theories of memory (including SAM) have surprisingly little to say about this phenomenon. Our model, however, does predict this pattern of results. Forgetting functions

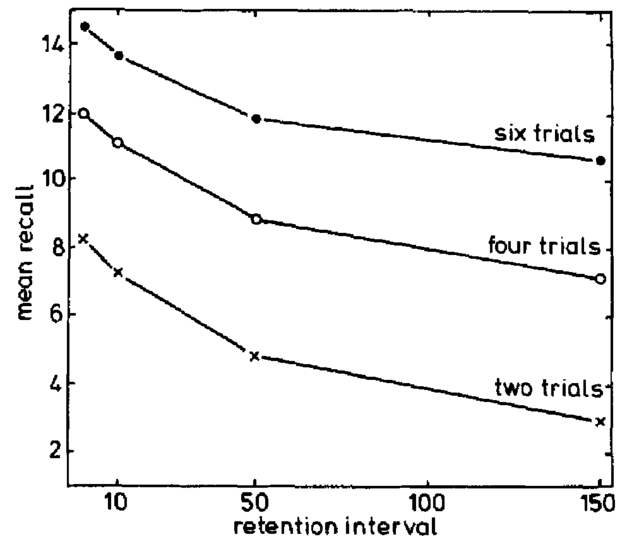

Figure 12. Predicted forgetting functions for lists differing in initial associative strength. (The differences in associative strengths were obtained by calculating the strengths for two, four, or six study trials.)

were derived for different associative strengths using Equation 4. The results are presented in Figure 12.

There has been a recent debate (Loftus, 1985a, 1985b; Slamecka, 1985) concerning the interpretation of these results. Slamecka and McElree (1983) described these results as indicating equal forgetting in all conditions. This would imply that forgetting is independent of associative strength. However, whether or not such a pattern of results is described as "equal forgetting" depends of course on the way "equal forgetting" is defined. Such statements are model dependent. That is, equality of forgetting should be defined in terms of the forgetting parameters defined in the model. In the present model, the rate of forgetting is controlled by the rate of the contextual fluctuation process. Since this is the same in all conditions, we may conclude that in terms of our model these results should indeed be interpreted as indicating equal forgetting. However, other models may lead to the conclusion of slower forgetting in the conditions starting at a higher value. In the end, this issue has to be decided in terms of the overall adequacy of the model.

A general model of this type that leads to the interpretation that higher learning produces slower forgetting, was presented by Loftus (1985a). A basic assumption of that model is that there is a one-to-one correspondence between the "state of the cognitive system" and memory performance. One of the reasons why the present SAM model is not compatible with Loftus's analysis is that it does not conform to this assumption. In the SAM model (and in a number of other models, such as Anderson's ACT* model), memory performance is a function of both the absolute and the relative strength of the memory trace of an item (see the section on PI). Different combinations of absolute and relative strengths may lead to equal recall performance. Hence, the assumption of a one-to-one relation is violated. In fact, there is some evidence that this assumption is indeed wrong. As discussed in a later section, there is no monotonic relation between different measures of memory performance. That is, response accuracy may be equal for two experimental conditions while the latencies are different. This would seem to be incompatible with the assumption made by Loftus (1985a).

Recognition

Next we will discuss the application of the model to some recognition tasks used in interference experiments. Recognition tasks became an issue after Postman et al. (1968) proposed the response set theory. It was argued that this theory could be tested with recognition tasks. We shall not go into this discussion here, but present only the basic data obtained using such recognition measures.

Recognition experiments have been performed using the four classic designs: C–D, C–B, A–C, and A–Br. Frequently, a four-choice method is used. Along with the stimulus, four responses are presented to the subject. One of the responses is the correct one, that is, the one that was associated to the stimulus during list learning. The following pattern of results has been obtained (Postman & Stark, 1969): The C–D, C–B, and A–C designs show about equally high recognition scores; only the A–Br design seems to suffer from interference effect. Thus, to the surprise of most investigators practically no deterioration in A–C recognition relative to the C–D performance could be detected. This observation severely undermined the unlearning hypothesis.

However, a number of comments concerning these results have to be made. First, A–C as well as C–B recognition seem to be consistently less compared with C–D recognition, in spite of the fact that the difference is usually not statistically significant. Second, the difference between the C–D design on the one hand and the C–B, A–C, and A–Br designs on the other hand is greater if the acquisition phase involves learning by recall instead of learning by recognition. Moreover, detailed examinations concerning these comparisons have revealed that A–Br recall mainly suffers from the effects of list discrimination (Anderson & Watts, 1971): If both the List 1 as well as the List 2 response are presented among the alternatives, the subject may recognize quite well which two responses belong to the presented stimulus, but may not correctly discriminate between them with respect to list membership. Thus, the model should account for three observations: (a) A–Br recognition deteriorates relative to the other designs if both responses are present in the set of alternatives, (b) there will be only small differences between these four designs when list discrimination problems are avoided, and (c) there is a consistent but small difference between the C–D design and the others.

To apply the model to recognition we first have to make an assumption about the use of response members as retrieval cues. In such experiments, the subject is presented with more than two cues: the usual context cue, the stimulus cue, and the response cues. We shall assume that the subject uses the response cues given in the set of alternatives one by one. This means that at any time only three associative strengths are involved:

$$S(I_i|C, S, R) = S(I_i, C)S(I_i, S)S(I_i, R).$$

Table 3
Predicted Results for Multiple-Choice Recognition

Design	Interfering response: Not excluded	Excluded
A–B, C–D	6.80	6.80
A–B, C–B	6.52	6.52
A–B, A–C	6.47	6.47
A–B, A–Br	5.66	6.29

It is assumed that the strength of the response cue to a particular image is the same as the strength of the corresponding stimulus to that image. For example, in the A–B, C–B design, the strength of the B response to the A–B image is equal to the strength of the corresponding stimulus A to that image. Similarly, its strength to the C–B image is equal to the strength of the stimulus C to that image.

We assume that there are two ways in which the subject may identify the correct response. The first is that the correct image is found (i.e., sampled and recovered) using the correct pair as cues. The second way is that the correct image is found using an incorrect S–R pair as cues. When this happens, it is assumed that as a check on the correctness of the response recovered from the retrieved image, the subject searches the set of alternatives, finds the matching response, and thus recognizes it. Finally, because we are interested only in the relative scores, no guessing assumptions have been made.

It is intuitively clear that such a model is in agreement with the fact that recognition is usually better than recall: The reason is that three cues are used instead of two. This focuses the search more efficiently on the correct image. The predictions generated by this model are presented in Table 3, the middle column, for the case that the interfering response in the A–Br design may be included in the set of alternatives. The values in Table 3 correspond to the number of correct recognitions (out of seven). The right-hand column gives the predicted score when A–Br is free from this negative factor. It should be noted that the usual method of making up the set of alternatives is used: Three randomly chosen responses from the same list are selected as distractors. Thus, in case of the A–Br design, the competing response will not always be among the alternatives.

From the middle column it can be concluded that the model correctly predicts that A–Br recognition is deteriorated. The right-hand column shows that the recognition of A–Br pairs improves as soon as the competitor is eliminated from the alternatives. In agreement with the data, there is a small difference between the C–D design and the other designs. In fact, the model predicts C–D > C–B = A–C > A–Br. The reason is that recognition performance is a function of the number of interfering relations. This prediction is supported by the observation that recognition of A–C lists is significantly less when compared with the C–D condition in a PI design (Postman et al., 1974).

Relation Between Response Accuracy and Response Latency

The traditional studies of interference have nearly always relied on accuracy measures as a dependent variable. The use of latency measures has apparently not been considered very useful. Presumably, it has been assumed that these two types of measures would show parallel effects. That is, a one-to-one relation was assumed between response accuracy and response latency. Anderson (1981) has shown, however, that this is not the case. He found a difference in latencies even when the interference and control conditions were equalized in terms of percent correct. Moreover, he demonstrated that this result was predicted by a model based on the ACT theory. In this section, we show that this relation is also predicted by the present SAM model. To show this, we shall fit our model to the data of Anderson's recall experiment (Anderson, 1981, Experiment 1). One additional reason for this exercise is that this experiment is one of the few that report the data in sufficient detail to make a quantitative comparison meaningful.

In this experiment, subjects were given eight anticipation trials on the first list (A–B) followed by eight anticipation trials on the second list (C–D or A–C). This was again followed by four relearning trials on List 1 and another four relearning trials on List 2. Since the set of responses consisted of the first 10 digits, it is perhaps better to describe these designs as C–B and A–Br. However, following Anderson (1981), we ignore this in the application of the model and treat the two conditions as regular C–D and A–C conditions. Hence, the effects of guessing will not be taken into account. The results of this experiment are shown in Figure 13. A closer look at these data reveals positive transfer on the C–D anticipation trials (Anderson, 1981), probably a learning-to-learn effect.

It will be assumed that such an effect influences the amount of associative information stored in LTS per second of (elaborative) rehearsal. One of the simplest ways to achieve this is by using a negative exponential formula for the interitem associative strength parameter b:

$$b(i) = b(1 - e^{-i\delta}),$$

where i denotes the *total* number of list presentations and δ is a scale parameter. The total strength accumulated for a specific item, say a List 1 item, after the nth study trial is given by

$$S_n(I_j, S) = t \sum_{i=1}^{n} b(i),$$

where the summation is taken only over the List 1 presentations.

Next, an equation has to be derived for response latency. We shall assume that the latency is linearly related to the number of sampling attempts. The expected number of samples given recall within a maximum of $m(=L_{\max})$ attempts, $E(L \mid R_m)$, is given by

$$E(L \mid R_m) = \frac{1 - (mp_s + 1)(1 - p_s)^m}{p_s[1 - (1 - p_s)^m]},$$

where p_s represents the probability of sampling in one draw. Furthermore, we have to transform $E(L \mid R_m)$ into real time. This will be accomplished by using a linear transformation and adding a constant denoted by v, representing encoding and decision processes:

$$RT = v + wE(L \mid R_m).$$

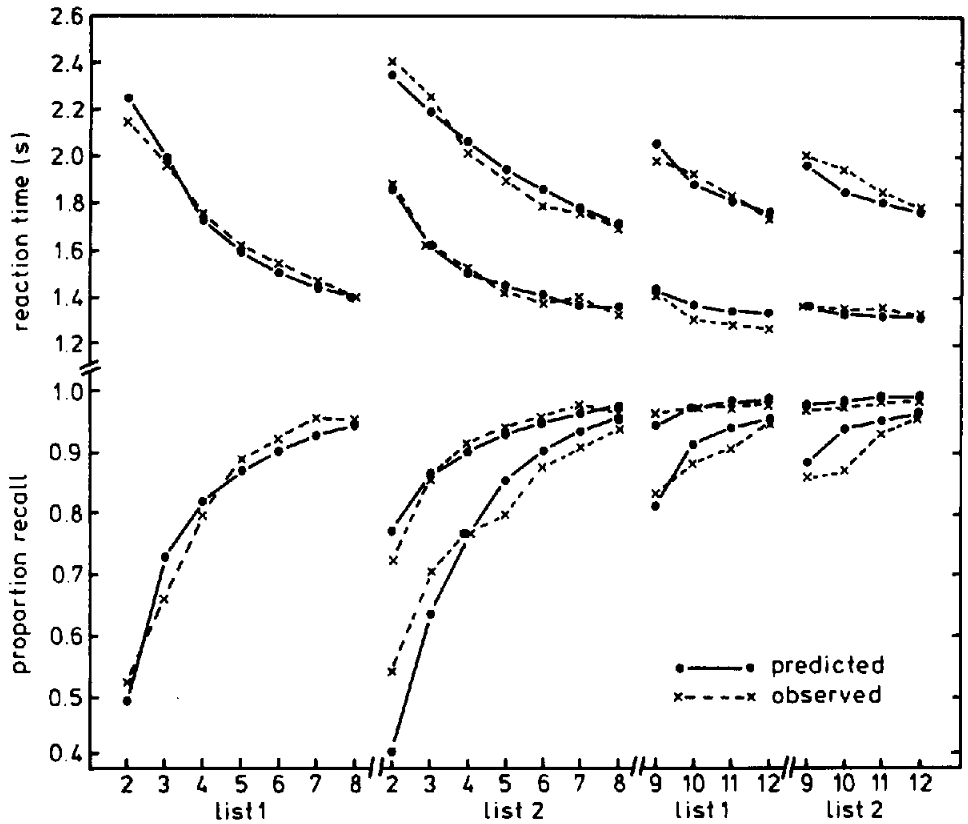

Figure 13. Predicted and observed results for the Anderson (1981) experiment. (The control condition is given by the reaction time curves that are lower and the percent recall curves that are higher.)

Since all recall tests were paced, we will assume that retrieval stops as soon as L_{max} has been reached.

The chi-square loss function used in fitting the model to the data was the same as used by Anderson (1981). Parameter estimation was performed using the STEPIT minimization routine (Chandler, 1965). The following parameter values were obtained: $a = 1.47$, $b = 0.07$, $d = 0.0$, $\alpha = 4.54$, $\beta = 0.011$, $\gamma = 0.012$, $L_{max} = 4.90$, $\delta = 0.31$, $v = 0.61$, and $w = 0.65$. The resulting chi-square value is 121.95. Figure 13 gives the predicted and observed data. Inspection of Figure 13 shows that the overall fit is satisfactory, although the chi-square value is a bit high (although not exactly, it should be approximately χ^2 distributed with a *df* of 77). We cannot compare our loss value with the one obtained by Anderson, because he fitted two data sets simultaneously. Therefore, we fitted his model to the same subset of the data considered above. For his model we obtained a loss value of 64.16, which is about half the value of our model.

However, the difference in loss value may not be so important, since our model predicts the data fairly well according to visual inspection. This conclusion is based on our experience that if a good qualitative fit is obtained, a good quantitative fit, that is, a significant decrease in the loss value, may also be obtained by using a number of auxiliary ad hoc assumptions. For example, Anderson made the assumption that there is a speed-up in the intercept time, the time for encoding and decision processes. We did not use such an assumption (v is a constant). If in the Anderson model we also fix the speed-up parameter b to zero, a loss value of 110.86 is obtained. Thus, a considerable increase in the loss value results if one of the auxiliary assumptions is deleted.

Such results strengthen our belief that a further decrease in the loss value may be obtained provided enough time and effort is invested to tailor the model to the specific details of the experiment. However, such an exercise would not add much to our knowledge, given that we have already shown that the general pattern of the results is quite well predicted by the model in its present form. We have therefore chosen not make such assumptions and to use the model as it has been presented in previous applications.

We now turn to the most interesting aspect of Anderson's (1981) data, the relation between the response latencies and response accuracies for the interference and control conditions. Anderson showed that there is a difference in latency between the two conditions even when they are equated in percent recall by giving extra study trials to the interference conditions. Anderson argued that this relation can be explained by models that are based on relative as well as absolute associative strengths. Since our model is of this general type, it is easy to show that it also predicts this result.

As described earlier, in the SAM model the probability of recall (PC) is a product of the probability of sampling, denoted by a function f, and a function describing the probability of recovery, g. f is a function of the relative strength, whereas g is a function of the absolute strength. Let the sampling probability be mapped into a response latency by a function h. That is, we assume that the response latency (RT) is only determined by the duration of the sampling process (plus an additive constant reflecting encoding and response processes). Hence, h is assumed to be a function of the relative strength. Then,

$$PC = f(R)g(R)$$

and

$$RT = h(R).$$

We assume that if two sampling probabilities are equal, the corresponding latencies will also be equal. Thus, f and h are monotonic functions of the relative strength and of each other:

$$f(R_1) > f(R_2) \Leftrightarrow h(R_1) < h(R_2).$$

Now, if we have a probability correct for the C–D condition and an equal probability correct for the interference condition then $f(R_{cd})g(R_{cd}) = f(R_{ac})g(R_{ac})$.

Note that the interference condition needs more trials to establish this equality because it suffers from negative transfer. This means that the absolute strength will be higher in the interference condition. Hence, $g(R_{cd})$ will be smaller than $g(R_{ac})$. To maintain equality, it must be the case that $f(R_{cd}) > f(R_{ac})$. From this we may conclude that

$$h(R_{cd}) < h(R_{ac}).$$

Thus, the latency in the interference condition will be larger than in the control condition, even when the conditions are equalized in terms of percent recall.

Discussion

We have presented a model for interference and forgetting that is based on the SAM theory for retrieval from long-term memory. The major new aspect of the present model is the incorporation of a process of contextual fluctuation. We have shown that a few relatively simple assumptions concerning the effects of contextual changes lead to a model that is capable of handling the major classical phenomena from the interference literature in a straightforward manner. It is perhaps worth emphasizing that we do not believe that this feat can only be accomplished by such a model based on the SAM theory. Instead, we are quite certain that other recent memory theories (e.g., Anderson's ACT* theory) will also be capable of handling these results.

What is important, however, is that it has been shown that these phenomena can be explained within a single framework. This is quite contrary to the impression one would get from a cursory review of the classical interference literature. In our opinion, the problem with the classical interference theories is that far-reaching conclusions were drawn on the basis of dubious interpretations of certain empirical observations. For example, the introduction of the unlearning concept was motivated by the observation that interference could be observed even when MMFR testing was used. Such a conclusion, however, is based on a number of tacit assumptions concerning the retrieval process. These may have seemed to be logical in the past but are not shared by contemporary theories of memory. As discussed previously by Baddeley (1976), the assumption that MMFR testing eliminates response competition is based on a model that assumes that retrieval involves a sampling-without-replacement process: Given enough time, subjects will recall all available information. Hence, when there is no need to discriminate between the responses with respect to list membership (as in MMFR testing), all response competition should be eliminated. Such a test should therefore constitute a pure measure of response availability.

Most contemporary theories of memory, on the other hand, do not make such an assumption. Instead, it is quite customary to assume some sort of sampling-with-replacement process: Recall of one of the responses does not increase the probability of recalling the remaining response. On the contrary, it is not uncommon to assume a negative effect due to output interference (as in the general SAM theory). However, if MMFR testing does not eliminate response competition (broadly defined), then it can be easily explained why proactive interference is observed on a MMFR test, a finding that has been particularly troublesome for the classical interference theories. Moreover, the finding that interference effects are quite weak when a recognition test is used is no longer puzzling. Hence, there is no need to introduce such assumptions as "response-set suppression" to explain the (near) absence of interference on recognition tests.

It is interesting to note that the present model for interference may be viewed as an updated mathematical version of McGeoch's response competition theory. As discussed in the introductory section, McGeoch assumed that responses were acquired independently and that the source of interference was located in the retrieval process. Moreover, McGeoch attached some importance to context changes as a source of forgetting. This is all quite consistent with the present formulation. The major difference between McGeoch's theory and our model is that in our model response competition is given a somewhat different interpretation. In his formulation, the competition seems to be mainly between available responses, whereas in the present model the competition is rooted in the retrieval process itself. We do not interpret this as a fundamental difference. Rather, it reflects the increased sophistication of modern memory theories. We believe that our formulation does not run counter to the basic spirit of McGeoch's theory, namely, that interference is not an encoding phenomenon.

Comparison with Other Models

In our presentation of the predictions of the SAM model we have (with one or two exceptions) not discussed the predictions made by other recent models for human memory. Although an extensive comparison of the present model to these other models is outside the scope of the present article, it is appropriate to discuss briefly the two major theoretical approaches that have similar aims and scope as the SAM theory, namely Anderson's ACT* theory (Anderson, 1983a, 1983b) and the holographic model advocated by Murdock (1982) and Eich (1982). We have chosen these two models for comparison because both (as well as the SAM theory) are general theories of memory, are not limited to a single paradigm, have been formalized in a quantitative manner, and have been explicitly applied to the traditional memory paradigms.

Anderson's ACT theory has undergone a number of modifications since its initial presentation (Anderson, 1976). The latest version, called ACT*, was shown by Anderson (1983a) to be able to generate a number of predictions for interference phenomena. It assumes an all-or-none trace formation mechanism and gradual strengthening of existing traces. In the case of paired associates, the model assumes, as does the SAM model, that the trace consists of stimulus, response, and context information. Once a trace has been established, its associated strength is increased by one unit on each subsequent study trial. These strengths determine the probability and speed of retrieval. Anderson assumes a spreading activation model for retrieval: When a stimulus is presented, it and the list context constitute sources of activation. The response will be retrieved if a trace connecting stimulus, response, and context has been formed and can be retrieved within a specific cut-off time. The amount of activation converging on a trace (that determines the probability and speed of retrieval) is a function of the relative strength of the target trace compared with the other traces that are connected to the same stimulus and context. The ACT* theory assumes that trace strengths are subject to decay. It is assumed that the strength is a power function of time.

There are a number of interesting similarities between the ACT* model and the present SAM model. First, an important aspect in both models is that performance is assumed to be a function of both relative and absolute strength. Both models assume that probability of recall is a function of relative as well as absolute strength, whereas reaction time is a function of relative strength only. In SAM, "absolute strength" affects the probability of recovery. In ACT*, a similar role is played by the probability that the trace has been formed, which is also a function of the absolute number of presentations. Given that a trace has enough "absolute strength," the probability of recall is determined by its "relative strength" (in comparison with the competitors). Second, both models assume that interference has no effect on encoding, only on retrieval. Thus, there is no unlearning assumption; that is, learning of List 2 does not lead to a differential effect on the decay of List 1 strengths. Given this similarity, it is reasonable to assume that the ACT* model will be able to generate most, if not all, of the qualitative predictions for interference phenomena that we have considered in this article.

However, there are certain differences between the two models that would seem to make a quantitative comparison quite informative. For example, the ACT* model assumes that trace strengths are subject to decay. In the SAM model, a similar role is played by the assumption that the contextual overlap is a decreasing function of the retention interval. Another, potentially important difference is that ACT* assumes that the effects of stimulus and context activation summate. In SAM, on the other hand, item and context strengths are combined in a multiplicative fashion. Hence, a high item strength cannot compensate for a zero (or near zero) context strength. A quantitative comparison of these two models might be quite interesting. In particular, it would be of interest to see how well the new ACT* model handles the Anderson (1981) data on interference, since the 1981 ACT model differs in several important respects from the 1983 version.

Eich (1982) proposed a model for associative recall that is based on the holographic metaphor. This model, called CHARM (Composite Holographic Associative Recall Model), is based on the convolution/correlation model proposed by Murdock (1979; see also Murdock, 1982). It assumes that items may be represented as patterns of features. An association between two such items is represented by the mathematical operation of convolution. The result of all these associations is stored in a single composite memory trace. Retrieval occurs by correlating the test stimulus with this composite memory trace. The resulting feature pattern is identified by being matched to every item in a lexicon (representing semantic memory).

Eich (1982) described a number of qualitative predictions for interference phenomena. The model predicts C–D interference compared to a control group. As in SAM, this prediction is based on a list-length effect. Everything else being equal, the level of recall is inversely related to the number of unrelated associations that have been stored in the composite memory trace. This is due to the increase in the level of noise in the memory trace. The model also predicts the Barnes and Underwood (1959) MMFR results in the A–B, A–D interference paradigm. This prediction is due to the fact that the recall probability (equivalent to the probability that the target item produces the best or second best match) is correlated with the number of times (study trials) the association is added to the memory trace. Finally, it is able to account for Osgood's (1949) transfer surface because the representation of similarity is an integral part of the model.

As appealing as the model may be, there remain certain disadvantages. First, it should be noted that all "fits" were qualitative and it remains to be seen whether the model can predict the correct magnitudes of the effects. Second, there are some conceptual problems with the prediction of interference in C–D paradigms (and list-length effects in general). It must be the case that the memory trace already incorporates thousands of associations when the subject enters the experiment. Adding 20 or so associations will not increase the amount of noise to a significant degree. The only way in which this explanation might be saved is to invoke a factor that focuses retrieval on the list of associations learned in the experimental session. In SAM, this focusing of the search is accomplished by the assumption that context is one of the retrieval cues. It is not evident how such a factor should be incorporated in the CHARM model.

In conclusion, it seems fair to say that the present application of the SAM theory represents an important new step toward the explanation of interference and forgetting. It is hoped that this work will arouse renewed interest in these old topics. We believe that these phenomena are too important to be neglected by any theory that aspires to be a truly general theory of memory.

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Appendix

In this Appendix, we will describe the relevant difference equations that are required to generate predictions for a two-list design. The following notation is used:

$A_v(i, j, t)$: expected number of contextual elements of type v that are active t s after the jth trial on List 2 and following i trials on List 1.

$K_v(i, j)$: expected total number of elements of type v (active plus inactive) after i trials on List 1 and j trials on List 2.

Thus, A_1 gives the expected number of active x_1 elements, A_2 the expected number of active x_2 elements, and similarly for A_o and A_y. We shall assume a constant intertrial interval of t_1 s. Hence, $A_v(i, j, t_1)$ gives the expected number of active elements of type v on the test phase that precedes the $(j + 1)$th study trial on List 2 (or, if $j = 0$, the $(i + 1)$th study trial on List 1). For $i = j = 0$, we have $A_1(0, 0, t_1) = K_1(0, 0) = 0$. The following equation gives the results after the ith trial on List 1.

$$A_1(i, 0, 0) = A_1(i - 1, 0, t_1) + [n - A_1(i - 1, 0, t_1)]w,$$

$$K_1(i, 0) = K_1(i - 1, 0) + [n - A_1(i - 1, 0, t_1)]w,$$

where w gives the probability of encoding an element on a given study trial.

At the end of the intertrial interval of t_1 s, A_1 will have a different value because of the contextual fluctuation process (see Equation 1):

$$A_1(i, 0, t_1) = A_1(i, 0, 0)e^{-(\gamma+\beta)t_1} + K_1(i, 0)h(t_1),$$

where

$$h(t) = \frac{\gamma}{\gamma + \beta}[1 - e^{-(\gamma+\beta)t}].$$

Assume that a total of I List 1 study trials are given, followed by an interlist interval of t_2 s. At the end of this interval we have

$$A_1(I, 0, t_2) = A_1(I, 0, 0)e^{-(\gamma+\beta)t_2} + K_1(I, 0)h(t_2),$$

$$A_2(I, 0, t_2) = A_o(I, 0, t_2) = 0,$$

$$A_y(I, 0, t_2) = n - A_1(I, 0, t_2),$$

$$K_2(I, 0) = K_o(I, 0) = 0.$$

The next set of difference equations describes the state of affairs after the jth trial on List 2.

$$A_1(I, j, 0) = (1 - w)A_1(I, j - 1, t_1),$$

$$K_1(I, j) = K_1(I, j - 1) - wA_1(I, j - 1, t_1),$$

$$A_2(I, j, 0) = A_2(I, j - 1, t_1) + wA_y(I, j - 1, t_1),$$

$$K_2(I, j) = K_2(I, j - 1) + wA_y(I, j - 1, t_1),$$

$$A_o(I, j, 0) = A_o(I, j - 1, t_1) + wA_1(I, j - 1, t_1),$$

$$K_o(I, j) = K_o(I, j - 1) + wA_1(I, j - 1, t_1),$$

$$A_y(I, j, 0) = (1 - w)A_y(I, j - 1, t_1).$$

At the end of the intertrial interval of t_1 s, the A_v will be equal to

$$A_v(I, j, t_1) = A_v(I, j, 0)e^{-(\gamma+\beta)t_1} + K_v(I, j)h(t_1).$$

Finally, assume that the final testing follows t_3 s after the last List 2 trial. The number of active elements of type v on this test is given by

$$A_v(I, J, t_3) = A_v(I, J, 0)e^{-(\gamma+\beta)t_3} + K_v(I, J)h(t_3),$$

where J equals the total number of List 2 study trials.

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